Control Systems
Lecture 7

Feedforward Control and
Fundamental Performance Limitations of Control

- Should we solve all control problems by feedback?
- Fundamental performance limitations
  What limits the achievable performance?
  Why not make a control system infinitely fast
  (bandwidth ∞, closed-loop response to a step is a step)?

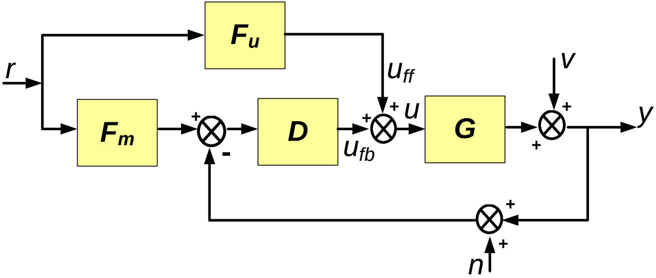
Feedforward Control

Rather than acting on observed differences between realized and desired behaviour:

feedforward control

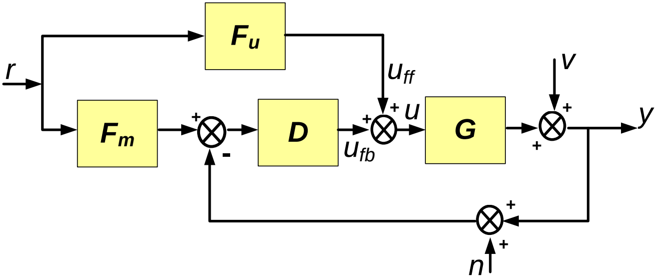
steers the system in the (presumed) right direction

It allows to *separate* the design for reference tracking from the design for disturbance rejection



$F_m :=$ the desired response to the command reference r
$F_u :=$ the feedforward controller

By choosing F_u to satisfy $F_u G = F_m$, the feedforward controller can directly provide the correct input signal to the plant G.

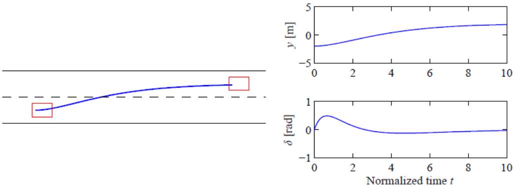


Transfer from $r \rightarrow y$ becomes:

$$F_m \frac{DG}{1+DG} + F_u \frac{G}{1+DG} = F_m + \frac{F_u G - F_m}{1+DG}$$

i.e. by choosing $F_u = F_m G^{-1}$ the desired closed-loop response is realized, and feedback can handle the deviations/uncertainties that appear in the second term.

Example: vehicle steering (AM, chapter 11)



(a) Overhead view (b) Position and steering

Figure 11.4: Feedforward control for vehicle steering. The plot on the left shows the trajectory generated by the controller for changing lanes. The plots on the right show the lateral deviation y (top) and the steering angle δ (bottom) for a smooth lane change control using feedforward (based on the linearized model).

Transfer from steering angle δ to lateral deviation y:
$G(s) = (\gamma s + 1)/s^2$
Desired response: $F_m(s) = a^2/(s+a)^2$ with $a > 0$ a measure for the aggressiveness of the steering.

$$F_u = \frac{F_m}{G} = \frac{a^2 s^2}{(\gamma s + 1)(s+a)^2} \quad \text{stable as long as } \gamma > 0.$$

Note that the "desired response"

$$F_m(s) = \frac{a^2}{(s+a)^2}$$

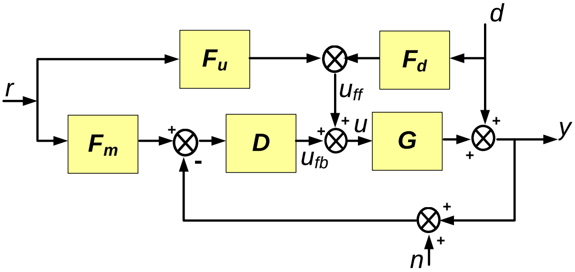
corresponds to the second order system

$$F_m(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

with $\omega_n = a$ and $\zeta = 1$.

So a directly reflects the rise-time of the step-response.

Disturbance correction with feedforward



If disturbances d can be measured, then we do not have to rely on *feedback* to attenuate them.

Transfer from $d \to y$ becomes: $\dfrac{1 + F_d G}{1 + DG}$

By choosing $F_d = -G^{-1}$ the disturbance term d is completely rejected in the output y.

Feedforward Control

The effect of measured disturbances as well as known signals to track is best treated by feedforward control, leaving feedback control for handling the effects of all unmeasured/disturbance signals.

Limitations for implementation

The optimal feedforward controller typically contains a plant inverse.
Implementation is limited to stable and causal controllers, and therefore approximations have to be made if the plant has RHP zeros, is strictly proper or has time-delays.

Fundamental limitations of feedback control

Performance specifications:

- Stability
- Tracking reference signals (bandwidth)
- Disturbance rejection
- Insensitivity to modelling errors (robustness)
- Insensitivity to sensor noise

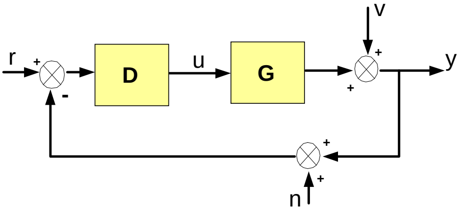
but also

- Limited control effort u (actuator power)

Limiting factors in performance

1. Sensor noise (how accurate can you measure?)
2. Maximum actuator power (control effort)
3. Disturbances
4. Non-minimum phase zeros
5. Time delays in the system (how fast can you actuate?)
6. Model uncertainty (modelling errors)
7. Unstable poles
8. Limited bandwidth of sensor

1. Sensor noise (how accurate can you measure?)



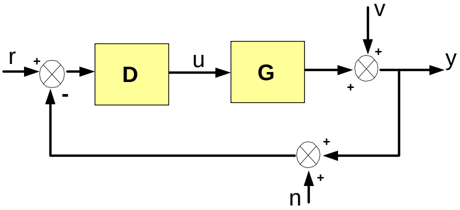
Sensor noise: $n \rightarrow y$: $\quad -T = \dfrac{-DG}{1+DG}$

$|T(j\omega)|$ should be small in the ω-region where n has substantial power (generally the high frequency area).

> Sensor noise puts a limitation on the achievable closed-loop bandwidth

The sensor system should be accurate (low noise level) in the frequency range where the controller is active (bandwidth).

2. Maximum actuator power u (control effort)

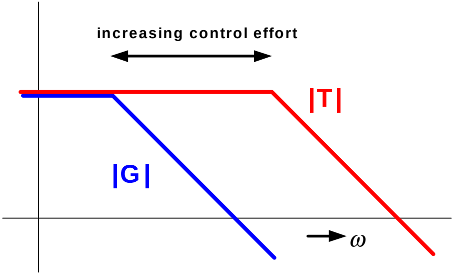


Power of u is induced by transfer from r, n, v to u, determined by

$$\frac{D}{1+DG} = \frac{T}{G}$$

Relation between closed-loop transfer and open-loop plant transfer drives the required control effort.

$$\frac{D}{1+DG} = \frac{T}{G}$$



Increase of closed-loop bandwidth requires high-frequency amplification of input u

Limited actuator power puts a limitation on the achievable closed-loop bandwidth

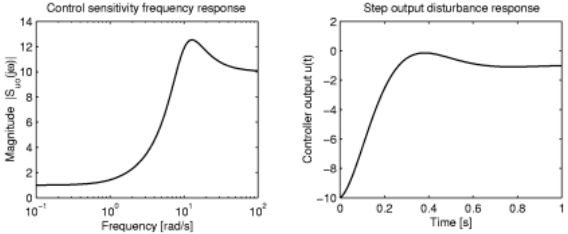
It takes a huge control effort to turn a slow system into a fast one by feedback control

Example (Goodwin, Graebe & Salgado, Ex 8.1)

$$\text{Open-loop plant: } G(s) = \frac{10}{(s+10)(s+1)}; \qquad \omega_n^2 = 10$$

$$\text{Closed-loop: } T(s) = \frac{100}{s^2+12s+100}; \qquad \omega_n^2 = 100$$

$\rightarrow$ bandwidth shifts from ≈ 1 to ≈ 10 rad/sec.

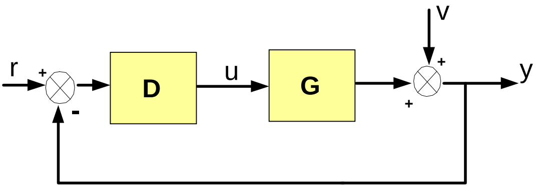


Right plot shows the controller output response to a unit step output disturbance. (Agressive response with amplitude 10)

3. Disturbances

For disturbances on plant output:

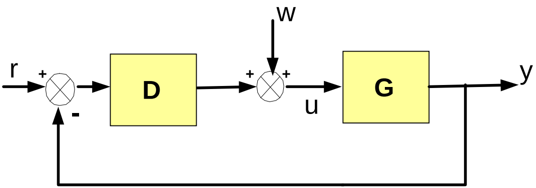
$$\text{Transfer } v \to y : \quad \frac{1}{1 + DG}$$



S has to be small (and T has to be ≈ 1) in the frequency region where v has substantial power

For disturbances on plant input:

$$\text{Transfer } w \to y: \quad \frac{G}{1 + DG}$$



S has to be small in frequency area where w has power; disturbances beyond the bandwidth of G are suppressed by G also.

If G contains an integrator, D has to have high gain at low frequencies

Disturbances put a limitation on the minimum required closed-loop bandwidth (but location of disturbances remains important)

4. Non-minimum-phase zeros

G has a zero in the right half plane (RHP) $\Longrightarrow$ G^{-1} is unstable

- Limit on controller gain, as for $|DG| \gg 1$:

$$\text{transfer } r \to u: \quad \frac{D}{1+DG} \approx G^{-1} \quad \Longrightarrow \text{ instability}$$

- Same mechanism is visible in root-locus: for high gain, closed-loop poles move to plant zero's $\Longrightarrow$ instability

Result: upper bound on closed-loop bandwidth

Effect can also be understood by the phase lag introduced by a (real-valued) RHP zero.
If $G(s)$ has a RHP zero in $z > 0$, then we can write:

$$G(s) = G_{mp}(s)\frac{z-s}{z+s}$$

such that $|G(j\omega)| = |G_{mp}(j\omega)|$ for all ω.
The phase-effect of the RHP zero is then quantified by

$$\angle\frac{z-j\omega}{z+j\omega}$$

i.e. a phase that starts with 0 and decreases towards -180^o.
At $\omega = z$ there is an additional phase lag of 90^o.

It will generally be hard to create sufficient phase margin for $\omega_c > z$.

Non-minimum-phase zeros

Non-minimum-phase zeros in the plant, lead to an upper bound on the closed-loop bandwidth;
rule-of-thumb for real-valued zeros: $\omega_{bw} \leq \textit{smallest RHP zero}$

5. Time delays

Impulse response of a time-delay:

$$g(t) = \delta(t - T_d)$$

(an impulse at time $t = T_d$)

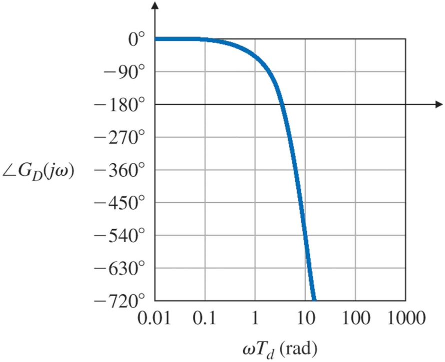
After Fourier transform:

$$\begin{aligned} G(j\omega) &= \int_0^\infty \delta(t - T_d) e^{-j\omega t} dt \\ &= e^{-j\omega T_d} \end{aligned}$$

5. Time delays

$$G(j\omega) = e^{-j\omega T_d}$$

$$|G(j\omega)| = 1 \qquad \forall\omega$$
$$\angle(G(j\omega)) = -\omega T_d$$



ωT_d	$\angle G$
$\pi/2$	-90^o
π	-180^o
$3\pi/2$	-270^o

Because of the phase-lag, it will be very hard to stabilize a time delay system with cross-over $\omega_c T_d \geq \pi$ (because of phase lag of 180^o)

Time delays

A time delay of T_d leads to an upper bound on the closed-loop bandwidth;
rule-of-thumb: $\omega_{bw} \leq 3/T_d$

6. High-frequency model uncertainty (modelling errors)

In the frequency range where $G(j\omega)$ is uncertain, take care that it does not lead to instability, i.e. $|DG| \ll 1$.

> High-frequency model uncertainty limits the achievable closed-loop bandwidth, in order to guarantee robust stability

Actually for higher frequencies ($\omega > \omega_c$):

$$|DG_0(j\omega)| < W_2^{-1}(\omega)$$

with W_2 an upper bound on the (multiplicative) uncertainty bound on G_0 (see Lecture 6)

7. Unstable poles

G has a pole in the RHP $\Longrightarrow$ sufficient control gain is required to move the (closed-loop) pole to the LHP

For small controller gains, i.e. $|DG| \ll 1$,

$$\text{transfer } w \to y : \quad \frac{G}{1+DG} \approx G \quad \Longrightarrow \text{ instability}$$

A minimum controller gain is required, leading to a lower bound on the required bandwidth.

> Unstable poles in the plant, lead to a lower bound on the closed-loop bandwidth;
> rule-of-thumb: $\omega_{bw} \geq \text{Re}\{\textit{any unstable pole}\}$

Cancelling unstable poles?

Can't we simply compensate an RHP pole in G by an RHP zero in D, so that DG does not have any RHP poles?

Answer:

- Exact cancellation is hardly possible in practice
- Unstable pole is cancelled in S and T but NOT in

$$\frac{G}{1+DG}$$

$\Rightarrow$ unstable transfer from (load) disturbance on plant input to output

Similar phenomenon when cancelling RHP zero in G by a RHP pole in D: $\rightarrow$ unstable $D/(1+DG)$.

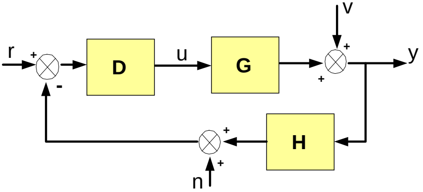
7. Unstable poles

With reference to the Bode sensitivity integral (Lecture 6):

$$\int_0^\infty \ln(|S(j\omega)|)d\omega = \pi \sum_{i=1}^{n_p} \text{Re}\{p_i\}$$

- Sensitivity integral is positive, and so disturbance attenuation "looses the battle" from disturbance amplification in the waterbed effect

8. Limited bandwidth of sensor



Design controller on the basis of GH, and consider

$$S = \frac{1}{1+DGH} \qquad T = \frac{DGH}{1+DGH}$$

The "actual" closed-loop is: $\dfrac{DG}{1+DGH}$

Approach allowed if bandwidth of H exceeds the bandwidth of T

> Bandwidth of output sensor limits the closed-loop bandwidth

QUIZ: Fundamental limitations on bandwidth

For which plants $G_i(s)$ can a controller be designed with a closed-loop bandwidth equal to around 10 Hz.

1. $G_1(s) = 1/(s-200)$
2. $G_2(s) = \dfrac{s-20}{(s+200)(s+20)}$
3. $G_3(s) = \dfrac{s+20}{(s+200)(s-2)}$

This is only possible for

A. $G_2(s)$, **B.** $G_3(s)$, or **C.** $G_1(s)$ and $G_3(s)$.

Summary

- Feedforward control is more effective for designing a response to known signals (tracking and measured disturbances)
  "Feedback is always too late", so reserved for handling uncertain/unpredicted effects
- Feedback control can contribute a lot to realizing a high-performance system, but there are clear "physical" limitations
- Actuator effort, quality of sensors/actuators, and system properties (time delays, RHP zeros and poles) limit the achievable performance of the closed-loop system
- Poorly designed systems can not be given any high-performance through (feedback) control.