# Closed-loop Identification

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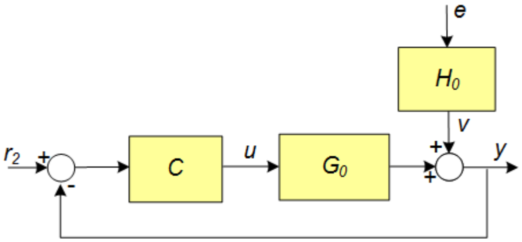
# Introduction



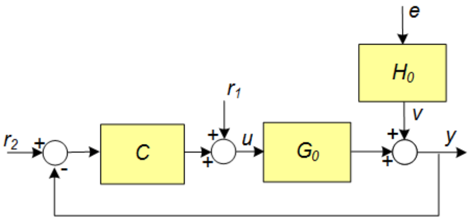

- Usual operation of many plants
- Particularly when processes are unstable
- Sometimes controller intrinsically present (biomedical, economic systems)
- Linearizing effect of controller

$r_2$ is reference or setpoint signal

Principle difference with the open-loop situation: $u$ and $v$ correlated.

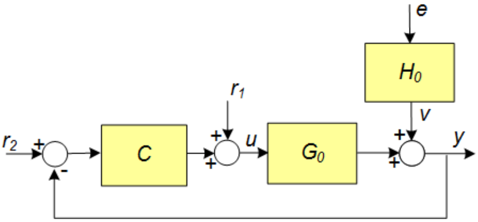
## The closed-loop problem

System set-up:

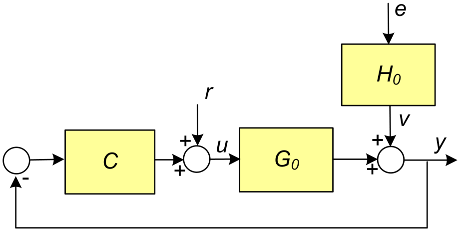



### Available data:

- $(u(t), y(t)),\ \ t = 1, \cdots N$ are measured
- Additionally $r_1(t)$ and/or $r_2(t),\ \ t = 1, \cdots N$ might be present and/or measured
- Knowledge of $C(q)$ might be available / used
- $r$ and $v$ are assumed to be uncorrelated
- $u$ and $v$ are correlated




For ease of notation:
introduce $r(t) := r_1(t) + C(q)r_2(t)$.



## System's equations:

$$\begin{aligned} u &= r - Cy \\ y &= G_0 u + v \end{aligned}$$

leading to:

$$\begin{aligned} y &= \frac{G_0}{1 + CG_0} r + \frac{1}{1 + CG_0} v \\ u &= \frac{1}{1 + CG_0} r - \frac{C}{1 + CG_0} v \end{aligned}$$

Using the sensitivity function: $S_0 := \dfrac{1}{1 + CG_0}$

the system relations become:

$$\begin{aligned} y &= G_0 S_0 \ r + S_0 \ v \\ u &= S_0 \ r - CS_0 \ v \end{aligned}$$

# Example ("is there a problem?") – Spectral analysis

Suppose that we make a nonparametric spectral analysis/ETFE estimate on the basis of $u$ and $y$ only

$$\hat{G}(e^{i\omega}) = \frac{\Phi_{yu}(\omega)}{\Phi_u(\omega)};$$

$$\begin{aligned} y(t) &= S_0\left[G_0 r(t) + v(t)\right] \\ u(t) &= S_0\left[r(t) - Cv(t)\right] \end{aligned}$$

Then

$$\begin{aligned} \Phi_u(\omega) &= |S_0|^2[\Phi_r(\omega) + |C|^2\Phi_v(\omega)] \\ \Phi_{yu}(\omega) &= |S_0|^2[G_0\Phi_r(\omega) - C^*\Phi_v(\omega)] \end{aligned}$$

and so:

$$\hat{G} = \frac{\Phi_{yu}(\omega)}{\Phi_u(\omega)} = \frac{G_0\Phi_r(\omega) - C^*\Phi_v(\omega)}{\Phi_r(\omega) + |C|^2\Phi_v(\omega)}$$

$$\hat{G}(e^{i\omega}) = \frac{G_0\Phi_r(\omega) - C^*\Phi_v(\omega)}{\Phi_r(\omega) + |C|^2\Phi_v(\omega)}$$

Limit cases:

$\Phi_v(\omega) = 0$ no noise $\Rightarrow$ $\hat{G} = G_0$

$\Phi_r(\omega) = 0$ no excitation $\Rightarrow$ $\hat{G} = -1/C$

There are two dynamical relationships between $u$ and $y$ (forward and backward)

A linear combination of the two is estimated dependent on the signal to noise ratio

Note that the model structure used, does not have intrinsic causality (as predictor models do)

Two principle approaches to identification in closed-loop:

- Direct identification (based on $u$ and $y$ only)
  Which part of the present PE theory can still be used?
- Indirect identification (based on $u$, $y$ and ($r$ and/or $C$))

Which results are we going to focus on:

(a) Consistency of $(\hat{G}, \hat{H})$ (situation $\mathcal{S} \in \mathcal{M}$)

(b) Consistency of $\hat{G}$ (situation $G_0 \in \mathcal{G}$)

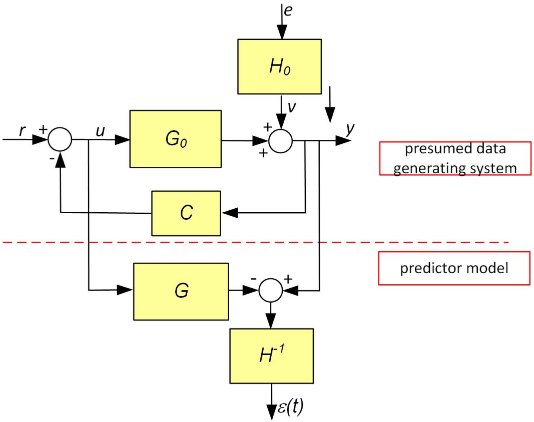
(c) Asymptotic variance

(d) Additional topics: model approximations, validation

# Direct identification method

## Leading principle:

Use measured $y, u$ and identify a "standard" PE model, irrespective of the presence of $C$



Available results from p.e. identification theory:

- **Convergence result**

$$\hat{\theta}_N \xrightarrow[N\to\infty]{w.p.1} \theta^* = \arg\min_{\theta} \bar{V}(\theta) = \arg\min_{\theta} \bar{\mathbb{E}}\varepsilon^2(t,\theta)$$

- **Minimum of asymptotic cost function**

  If $C(q)G_0(q)$ and $C(q)G(q,\theta)$ are strictly proper $\forall\theta$

  then: $\bar{V}(\theta) \geq \sigma_e^2$, with equality for $\hat{\theta}$ if

$$\begin{aligned} G(q,\hat{\theta}) &= G_0(q) \\ H(q,\hat{\theta}) &= H_0(q) \end{aligned}$$

- **Consistency / uniqueness**

  If additionally $\mathcal{S} \in \mathcal{M}$ and $Z^\infty$ is informative with respect to $\mathcal{M}$ then

$$\begin{aligned} G(q,\theta^*) &= G_0(q) \\ H(q,\theta^*) &= H_0(q) \end{aligned}$$

## Justification of consistency / uniqueness:

Combining:

$$\begin{aligned} y(t) &= G_0 u(t) + H_0 e(t) \\ u(t) &= r(t) - Cy(t) \\ \varepsilon(t,\theta) &= H(\theta)^{-1}[y(t) - G(\theta)u(t)] \end{aligned}$$

delivers:

$$\varepsilon(t,\theta) = \underbrace{\frac{(G_0 - G(\theta))}{H(\theta)(1 + CG_0)}}_{T_{\varepsilon r}(q,\theta)} r(t) + \underbrace{\frac{H_0(1 + CG(\theta))}{H(\theta)(1 + CG_0)}}_{T_{\varepsilon e}(q,\theta)} e(t)$$

If $CG_0$ and $CG(\theta)$ are strictly proper (i.e. the products contain a delay), then

$T_{\varepsilon e}(q,\theta)$ is *monic*

This requires that there is no algebraic loop in the system: $\lim_{z\to\infty} C(z)G_0(z) = 0$.

$$\varepsilon(t,\theta) = \underbrace{\frac{(G_0 - G(\theta))}{H(\theta)(1+CG_0)}}_{T_{\varepsilon r}(q,\theta)} r(t) + \underbrace{\frac{H_0(1+CG(\theta))}{H(\theta)(1+CG_0)}}_{T_{\varepsilon e}(q,\theta)\ \text{monic}} e(t)$$

Minimum of $\bar{E}\varepsilon(t,\theta)^2$ is achieved for

- $T_{\varepsilon r}(q,\theta^*) = 0$
- $T_{\varepsilon e}(q,\theta^*) = 1$

provided that $r$ is persistently exciting, of a sufficiently high order.

$$\{T_{\varepsilon r}(q,\theta^*)=0\} \Rightarrow \left\{ \frac{(G_0 - G(\theta^*))}{H(\theta^*)(1+CG_0)} = 0 \right\} \quad \Rightarrow \quad \{G(q,\theta^*) = G_0(q)\}$$

This, together with

$$\{T_{\varepsilon e}(q,\theta^*)=1\} \Rightarrow \left\{ \frac{H_0(1+CG(\theta^*))}{H(\theta^*)(1+CG_0)} = 1 \right\}$$

implies that $H(q,\theta^*) = H_0(q)$.

## Consistency result for direct method

If $\mathcal{S} \in \mathcal{M}$, $r$ is sufficiently exciting, and there are no algebraic loops in closed-loop system and parametrized models, then

$$G(q,\theta^*) = G_0(q); \qquad H(q,\theta^*) = H_0(q)$$

i.e. $G(q,\hat{\theta}_N)$ and $H(q,\hat{\theta}_N)$ are consistent estimates.

- For the situation $\mathcal{S} \in \mathcal{M}$ the existing consistency result is fully valid for the closed-loop case
- Two remaining questions:
  - What happens in the situation $G_0 \in \mathcal{G}$?
  - Relaxation of the excitation condition on $r$ towards a condition for informative data with repect to $\mathcal{M}$.

## The situation $G_0 \in \mathcal{G}$

$$\varepsilon(t,\theta) = \underbrace{\frac{(G_0 - G(\theta))}{H(\theta)(1 + CG_0)}}_{T_{\varepsilon r}(q,\theta)} r(t) + \underbrace{\frac{H_0(1 + CG(\theta))}{H(\theta)(1 + CG_0)}}_{T_{\varepsilon e}(q,\theta)\ \text{monic}} e(t)$$

- If $H(\theta) \neq H_0$, then bringing $T_{\varepsilon e}(q,\theta)$ "close to monic" needs to be comprised with making $T_{\varepsilon r}(q,\theta)$ "close to 0"; $\bar{V}(\theta) > \sigma_e^2$.
- This is due to the presence of $G(\theta)$ in $T_{\varepsilon e}(q,\theta)$.
- Conclusion that $G(q,\theta^*) = G_0(q)$ cannot be drawn anymore

**Direct method - situation $G_0 \in \mathcal{G}$**

In the closed-loop situation there is no consistency result for the situation $G_0 \in \mathcal{G}$ anymore.

## Relaxation of the persistence of excitation condition on $r$

Rather than $r$ being persistently exciting, it is sufficient to require that the data set is informative with respect to $\mathcal{M}$ (see earlier lecture).

This is covered by the condition that

$$\Phi_z(\omega) > 0 \quad \text{for a sufficient number of frequencies}$$

where

$$\Phi_z(\omega) = \begin{bmatrix} \Phi_u(\omega) & \Phi_{uy}(\omega) \\ \Phi_{yu}(\omega) & \Phi_y(\omega) \end{bmatrix}$$

A data set is informative with respect to the set of all LTI models, if

$$\Phi_z(\omega) > 0 \quad \forall\, \omega$$

# Data informativity (background)

Consider a data sequence: $Z^\infty := \{u(1), y(1), u(2), y(2), \cdots\}$

and a model set $\mathcal{M}$ determined by: $\hat{y}(t|t-1;\theta) = W(q,\theta)\begin{bmatrix} u(t) \\ y(t) \end{bmatrix}$

## Data informativity

Data sequence $Z^\infty$ is *informative with respect to* $\mathcal{M}$ if for any two models $(G_1, H_1)$, $(G_2, H_2)$ with predictor filters $W_1$, $W_2$, it holds that

$$\bar{\mathbb{E}}\left\{\left|[W_1(q) - W_2(q)]\begin{bmatrix} u(t) \\ y(t) \end{bmatrix}\right|^2\right\} = 0 \implies \left\{ \begin{array}{c} W_1(e^{i\omega}) = W_2(e^{i\omega}) \\ \text{for almost all } \omega \end{array} \right.$$

The expression on the left hand side can be rewritten (by Parsseval's relation) to:

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} \tilde{W}(e^{i\omega})\Phi_z(\omega)\tilde{W}^T(e^{-i\omega})d\omega = 0$$

with $\tilde{W} = W_1 - W_2$ and

$$\Phi_z(\omega) = \begin{bmatrix} \Phi_u(\omega) & \Phi_{uy}(\omega) \\ \Phi_{yu}(\omega) & \Phi_y(\omega) \end{bmatrix}.$$

The implication in the data informativity definiton is guaranteed if $\Phi_z(\omega) > 0$ for almost all $\omega$.

This holds for *any* model set, with models of *any* order.
For models of restricted order, the condition for data informativity can be relaxed.

## Relaxation of the persistence of excitation condition on $r$

The spectrum condition cannot be applied is $r$ is absent

Note that $\begin{bmatrix} u \\ y \end{bmatrix} = \frac{1}{1+CG_0} \begin{bmatrix} 1 & C \\ G_0 & 1 \end{bmatrix} \begin{bmatrix} r \\ v \end{bmatrix}.$

If $r \equiv 0$ then excitation has to come from $\begin{bmatrix} C \\ 1 \end{bmatrix} v$, i.e. $\Phi_z \sim \begin{bmatrix} |C|^2 & C \\ C^* & 1 \end{bmatrix}$,

$\Phi_z$ is always rank deficient

Besides the spectrum condition there are other conditions to guarantee data-informativity:

- A sufficiently complex, nonlinear or time-varying controller[1]

[1] T. Söderström and P. Stoica, *System Identification*, Prentice-Hall, 1989; and M. Gevers et al., IEEE Trans. Autom. Control, *54*, 2828-2840.

Data informativity can be achieved by

- A persistently exciting input signal $u$ in the open-loop situation;
- In the closed-loop situation:
  - Presence of a persistently exciting $r$, or
  - A controller of sufficiently high order, or
  - A time-varying or nonlinear controller

Question:
Can we use (frequency domain) identification methods that rely on a periodic input signal $u$?

## Unstable plant $G_0$

For all presented results is required:

- Predictor
$$\varepsilon(t,\theta) = H(\theta)^{-1}G(\theta)u(t) + (1 - H(\theta)^{-1})y(t)$$
is (uniformly) stable.

For unstable $G_0$ this can be satisfied if system can be modelled in an ARX or ARMAX structure.
Then unstable dynamics in $G(q,\theta)$ is cancelled out in $H(q,\theta)^{-1}G(q,\theta)$:

$$\varepsilon(t,\theta) = C(\theta)^{-1}B(\theta)u(t) + (1 - \frac{A(\theta)}{C(\theta)}y(t)$$

and predictor filters remain stable for $A(z,\theta)$ having unstable roots.

## Summary direct identification method

- Consistent estimates in the situation $\mathcal{S} \in \mathcal{M}$, under excitation conditions
- Excitation conditions can be realized by either presence of an exciting $r$-signal, or by excitation with the noise through a sufficiently complex controller.
- No consistency when only $G_0 \in \mathcal{G}$
- No "free" excitation of input $u$; periodic excitation of $u$ is not feasible
- Unstable plants can be modelled only with particular model structures (ARX,ARMAX)
- In situation of consistency, maximum likelihood results remain valid (Cramer Rao lower bound)
- But noise models need to be accurately estimated!
- Results remain valid for nonlinear and/or time-varying controllers

# Indirect identification methods

Main step with respect to direct methods:

- Additional use of measured $r$, and
- Possibly use knowledge of $C$
- Utilizing the linearity of the closed loop system (linear controller)

Several indirect methods are all closely related.

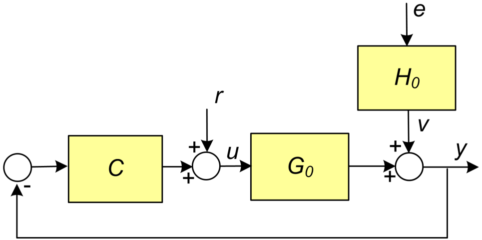
- Explanation of the principle
- Consistency results
- Variation of approaches and algorithms

# 1. Indirect method (coprime factor approach)

System equations:

$$\begin{aligned} y &= G_0 S_0 \; r + S_0 \; v \\ u &= S_0 \; r - C S_0 \; v \end{aligned}$$

with $S_0 = \dfrac{1}{1 + CG_0}$




The transfers $r \rightarrow \begin{bmatrix} y \\ u \end{bmatrix}$ can be estimated with open-loop methods.

Predictor model: $\varepsilon(t, \theta) = H_{ind}(q, \eta)^{-1} \left[ \begin{bmatrix} y \\ u \end{bmatrix} - \begin{bmatrix} G_{yr}(q, \theta) \\ G_{ur}(q, \theta) \end{bmatrix} r(t) \right]$

Two separate SISO problems when $H_{ind}(q, \eta)$ chosen diagonal.

## Indirect method (coprime factor approach)

If $G_{yr}$ and $G_{ur}$ are estimated consistently, it follows from

$$\begin{aligned} G_{yr}(q,\theta_0) &= G_0 S_0 \\ G_{ur}(q,\theta_0) &= S_0 \end{aligned}$$

that

$$\hat{G} := \frac{\hat{G}_{yr}(q,\hat{\theta}_N)}{\hat{G}_{ur}(q,\hat{\theta}_N)}$$

is a consistent estimate of $G_0$.

## Consistency result

The prediction error analysis results (earlier lecture) lead to the following statement:

**Consistency for indirect method in closed-loop**

If model structures $G_{yr}(q, \theta)$ and $G_{ur}(q, \theta)$ are chosen such that they contain the underlying true system dynamics, and $H_{ind}(q, \eta)$ is parametrized independently from $\theta$, then

$\hat{G}$ is a consistent estimate of $G_0$

provided that $r$ is persistently exciting of a sufficiently high order.

Note: We do not need to estimate a noise model to obtain a consistent estimate of $G_0$.

# Algorithms for indirect methods

There are different variations of indirect methods,

- Focussing on controlling the model set and model order of the final model $\hat{G}$:
  - Projection methods (two-stage method / instrumental variable (IV) method)
- Focussing on constraining the models to be stabilized by $C$
  - Dual-Youla parametrization

# Projection methods (two-stage / IV)

## Approach

- Decompose the input signal $u$ into

$$u(t) = u^r(t) + u^e(t)$$

  i.e. the components of $u$ that result from $r$ and $e$ respectively.
  Since $r$ and $e$ are uncorrelated this implies that both $u$-components are uncorrelated too.

- Then

$$y(t) = G_0 u^r(t) + \underbrace{G_0 u^e(t) + H_0 e(t)}_{\text{disturbance terms}}.$$

- Identify $G_0$ (and possibly a noise model), based on input $u^r$ and output $y$. This is basically an open-loop problem.

## 1. Projection methods (two-stage / IV)

$$y(t) = G_0 u^r(t) + \underbrace{G_0 u^e(t) + H_0 e(t)}_{\text{disturbance terms}}.$$

- Typically $G_0$ is identified with a parametric model
- The model order of $\hat{G}$ can directly be prespecified
- By "shifting" part of signal $u$ to "noise", the SNR will decrease $\rightarrow$ higher variance

### How to construct (an estimate of) $u^r$?

- Identify the transfer function $G_{ur}$ on the basis of $r$ and $u$ (open-loop problem)
- Simulate:

$$\hat{u}^r(t) = \hat{G}_{ur}(q) r(t)$$

- Use $\hat{u}^r(t)$ as input signal in the identification of $G_0$.

# Discussion - indirect method

- The consistency results for $G_0 \in \mathcal{G}$ carry over to the closed-loop case
- Effective use is made of external signal $r$ which needs to be measured and persistently exciting of a sufficiently high order
- Because of the postprocessing
$$\hat{G}(q,\hat{\theta}_N) := \frac{\hat{G}_{yr}(q,\hat{\theta}_N)}{\hat{G}_{ur}(q,\hat{\theta}_N)}$$
it is hard to prespecify the model order of $\hat{G}(q,\hat{\theta}_N)$ (result of taking quotient of two estimates)
- The method can also be applied on the basis of non-parametric frequency response estimates of $G_{yr}$ and $G_{ur}$
- Any desired excitation signal can be used for $r$ (e.g. periodic)

# Consistency

## Direct method

Consistency of $(G, H)$ can be obtained in the situation $\mathcal{S} \in \mathcal{M}$.

- Condition on excitaton of $\begin{bmatrix} u \\ y \end{bmatrix}$

## Indirect methods

Consistency of $G$ can be obtained in the situation $G_0 \in \mathcal{G}$.

- Condition on excitation of $r$

# Asymptotic variance

## Direct method

- The variance results of the open-loop situation remain valid, provided that we have consistency ($\mathcal{S} \in \mathcal{M}$).
- This includes the Maximum Likelihood properties of the estimates (minimum variance asymptotically)
- Both $r$ and $e$ contribute to variance reduction of $\hat{G}$, unless $n \to \infty$ (then all $e$-excitation is required to estimate $H$)
- Asymptotic-in-order-of-$G$-and-$H$ result for $n, N \to \infty$,

$$var\,\hat{G}(e^{i\omega}) \sim \frac{n}{N}\frac{\Phi_v(\omega)}{\Phi_{u^r}(\omega)}$$

# Asymptotic variance

## Indirect methods

- Typically the reference signal $r$ is used as input for identification;
- Typical variance result (asymptotic in model order $n$ and in $N$):

$$var\,\hat{G}(e^{i\omega}) \sim \frac{n}{N}\frac{\Phi_v(\omega)}{\Phi_{u^r}(\omega)}$$

  valid in the situation $G_0 \in \mathcal{G}$.
- Only the reference-part of the input signal contributes to variance reduction.
- For finite model orders:
  neglecting $u^e$ as input signal contributes to a worse SN-ratio.

# Asymptotic variance

## Reasoning behind asymptotic variance result

Asymptotic $(n, N \to \infty)$ result is:

$$E\begin{pmatrix} \hat{G}(e^{i\omega}) - G_0(e^{i\omega}) \\ \hat{H}(e^{i\omega}) - H_0(e^{i\omega}) \end{pmatrix}\begin{pmatrix} \\ \end{pmatrix}^*$$

$$\sim \frac{n}{N}\Phi_v(\omega)\cdot\begin{bmatrix} \Phi_u(\omega) & \Phi_{eu}(\omega) \\ \Phi_{ue}(\omega) & \sigma_e^2 \end{bmatrix}^{-1}.$$

Using

$$\Phi_u = \Phi_u^r + \Phi_u^e$$

and direct use of the system's equations delivers:

$$var(\hat{G}(e^{i\omega})) \sim \frac{n}{N}\frac{\Phi_v(\omega)}{\Phi_u^r(\omega)}$$

# Bias expressions - Approximations

Bias in direct closed-loop identification

$$\hat{\theta}_N \to \theta^* = \arg\min_{\theta} \bar{V}(\theta); \qquad \bar{V}(\theta) = \bar{\mathbb{E}}\varepsilon_f^2(t,\theta)$$

By Parsseval, $\bar{V}(\theta) =$ (see slide 13)

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\left\{\left|\frac{G_0 - G(\theta)}{1+CG_0}\right|^2 \Phi_r + \left|\frac{1+CG(\theta)}{1+CG_0}\right|^2 \Phi_v\right\}\frac{|L|^2}{|H(\theta)|^2}d\omega$$

No explicit (tunable) approximation criterion for $G(\theta)$,
since $G(\theta)$ appears in both terms of the integrand

## Bias in indirect closed-loop identification

Bias expressions for "all" indirect alternatives

$$\theta^* = \arg\min_{\theta} \int_{-\pi}^{\pi} \left| \frac{G_0}{1 + CG_0} - \frac{G(\theta)}{1 + CG(\theta)} \right|^2 \frac{\Phi_r |L|^2}{|K|^2} d\omega$$

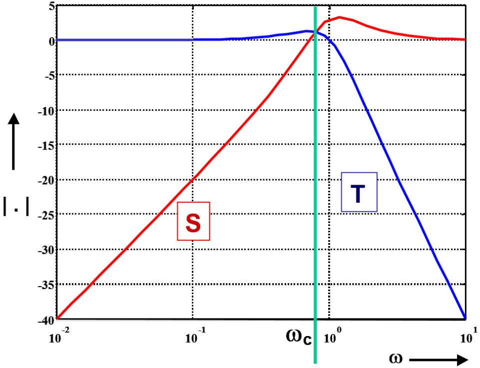
(with slight variations) and $K$ a fixed noise model.

Closed-loop properties of the plant ($G_0 S_0$) are best approximated.

Note

$$\frac{G_0}{1 + CG_0} - \frac{G(\theta)}{1 + CG(\theta)} = \frac{G_0 - G(\theta)}{(1 + CG_0)(1 + CG(\theta))}$$

"Additive error" is weighted with sensitivity of plant and model.



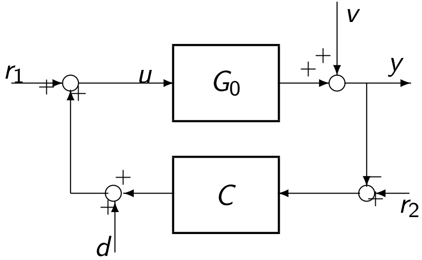

Typical curve for Bode magnitude plot of sensitivity function $S_0$ and related complementary sensitivity $G_0C/(1+CG_0)$.

Model errors are highly weighted around the cross-over frequency of the closed-loop.

## Main properties of the different methods

| | Direct | Indirect | | |
|---|---|---|---|---|
| | | Indirect (CF) | Projection | dual Youla |
| Consistency $(\hat{G}, \hat{H})$ | $+$ | $+$ | $+$ | $+$ |
| Consistency $\hat{G}$ | $-$ | $+$ | $+$ | $+$ |
| Tunable bias | $-$ | $+$ | $+$ | $+$ |
| Fixed model order | $+$ | $-$ | $+$ | $-$ |
| Variance | $+$ | $-$ | $-$ | $-$ |
| $C$ assumed known | no | no | no | yes |
| $C$ assumed linear | no | yes | yes | yes |
| $G(\hat{\theta}_N)$, $C$ stable | no | no | no | yes |

For noise disturbed controller output:




- If $C$ known: use $u + Cy$ as external signal
  $\rightarrow$ $d$ can effectively be used as external signal reducing the variance
- If $C$ unknown, $r$ measured: $d$ acts as additional disturbance

Knowledge of $C$ is more informative than knowledge of $r$

## Model validation in closed-loop

- For all indirect methods:
  validation with correlation tests as in open-loop
- For direct method: Careful with test on $R_{\varepsilon u}(\tau)$.

$$\varepsilon(t,\theta) = H(\theta)^{-1}[(G_0 - G(\theta))u(t) + H_0 e(t)]$$
$$R_{\varepsilon u}(\tau) = H(\theta)^{-1}[G_0 - G(\theta)]R_u(\tau) + H(\theta)^{-1}H_0 R_{eu}(\tau).$$

Can the second term influence the cross-correlation test?

$R_{eu}(\tau)$ will have a contribution for $\tau < 0$ only.
The second term can then have a contribution for $\tau > 0$ if the filter $H(\theta)^{-1}H_0$ has dynamics, i.e. when the noise model is incorrect.

> For the direct method the residual tests should not be interpreted independently (validation of $\hat{G}$ and $\hat{H}$ simultaneously).

## Summary - Closed-loop identification

- Parametric models can be consistently identified with a direct method
- but only through modelling $G$ and $H$ simultaneously ($\mathcal{S} \in \mathcal{M}$)
- Indirect methods can provide consistent estimates in the situation $G_0 \in \mathcal{G}$
- Direct methods lead to smaller variance (ML-properties)
- In closed-loop identification, the frequency area around the cross-over frequency of the closed-loop system, typically is most dominantly present in the plant data/models.
- Indirect methods rely on linearity of the closed-loop system, while direct methods can handel nonlinear/time-varying controllers