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Single module identification – local direct method

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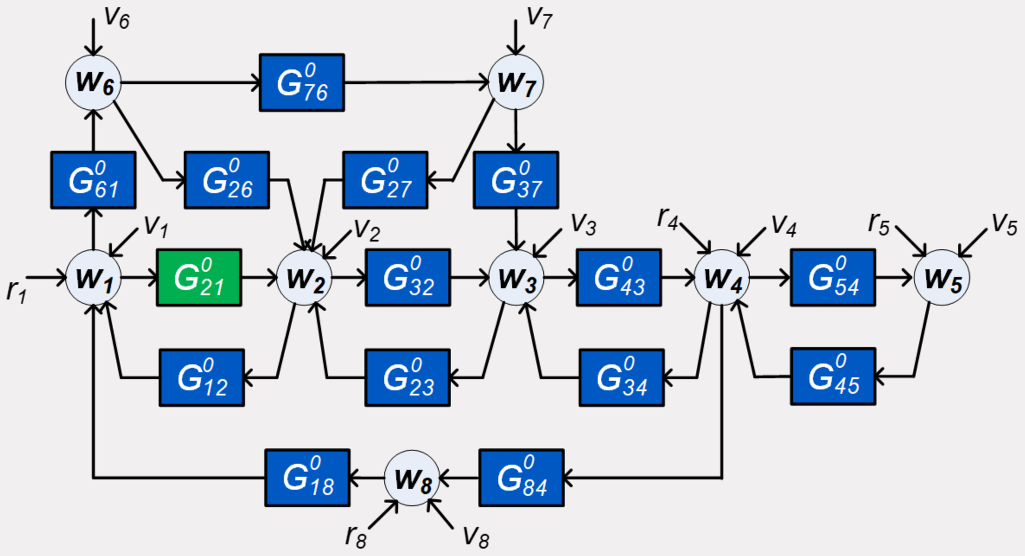
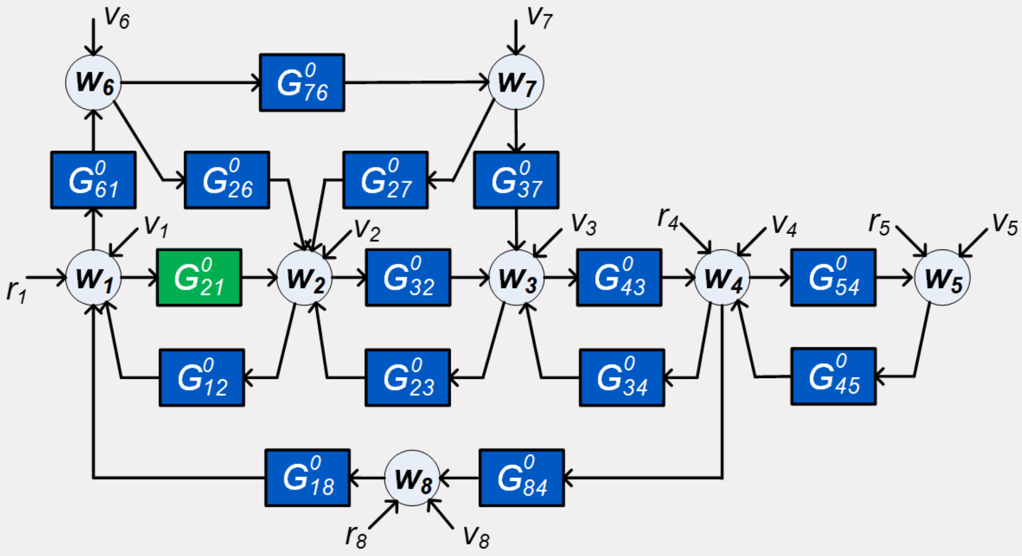
Single module identification



For a network with
known topology:

- Identify G^0_{21} on the basis of measured signals
- Which signals to measure? Preference for local measurements
- When is there enough excitation / data informativity?

Single module identification



Different types of methods:

Indirect methods [1,2,3]

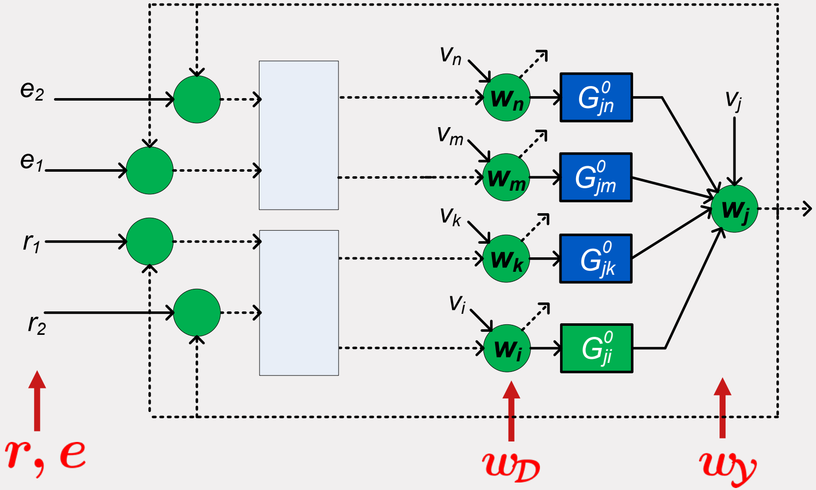
- Rely on mappings $\boldsymbol{r} \rightarrow \boldsymbol{w}$ and on sufficient excitation signals $\boldsymbol{r}$

Direct methods [1,2,4]

- Rely on mappings $\boldsymbol{w} \rightarrow \boldsymbol{w}$ and use excitation from both $\boldsymbol{r}$ and $\boldsymbol{v}$ signals

[1] PVdH et al., Automatica, 2013.
[2] A.G. Dankers et al., IEEE-TAC, 2016.
[3] M. Gevers et al., SYSID 2018.
[4] K.R. Ramaswamy et al., IEEE-TAC, 2021.

Local direct method



$$\varepsilon(t,\theta) = \bar{H}(q,\theta)^{-1}[w_\mathcal{Y}(t) - \bar{G}(q,\theta)w_\mathcal{D}(t)]$$

- Estimate transfer $w_\mathcal{D} \rightarrow w_\mathcal{Y}$ and model the disturbance process on the output.
- consistent estimate and ML properties

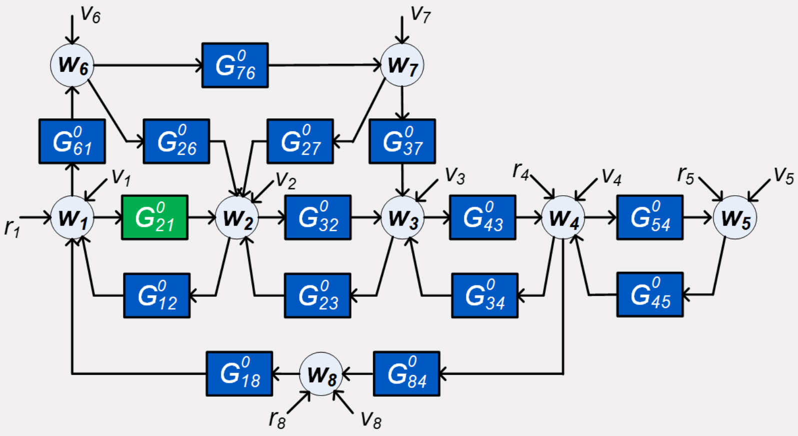
Additional problem:

If:
- v signals are correlated, i.e. $\Phi_v(\omega)$ non-diagonal, or
- some in-neighbors of $w_\mathcal{Y}$ are not included in $w_\mathcal{D}$

then confounding variables can occur, destroying the consistency results

[1] Dankers et al., IEEE-TAC, 2016; Dankers et al., IFAC 2017

Single module identification



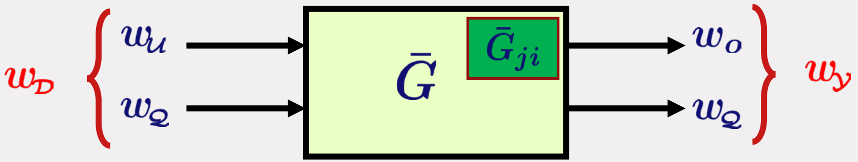
Local direct method:

(consistency and minimum variance properties)

Select a subnetwork:

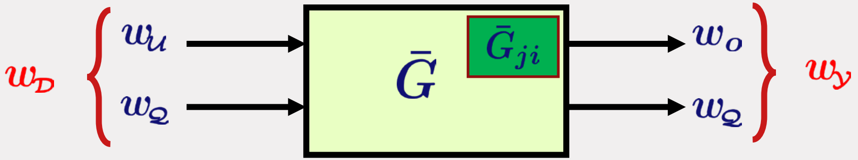
- Predicted outputs: $w_{\mathcal{Y}}$
- Predictor inputs: $w_{\mathcal{D}}$

such that prediction error minimization leads to an accurate estimate of G^0_{21}



Note: same node signals can appear in input and output

Single module identification

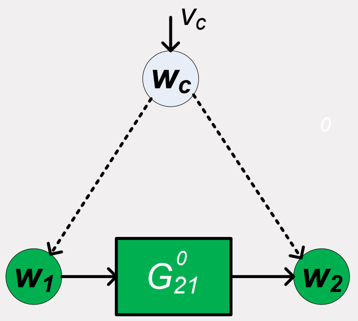


Conditions for arriving at an accurate (consistent) model estimate:

1. Module invariance: $\bar{G}_{ji} = G^0_{ji}$ when removing discarded nodes (immersion)
2. Handling of confounding variables
3. Data-informativity
4. *Technical condition on presence of delays (avoiding algebraic loops)*

Confounding variables

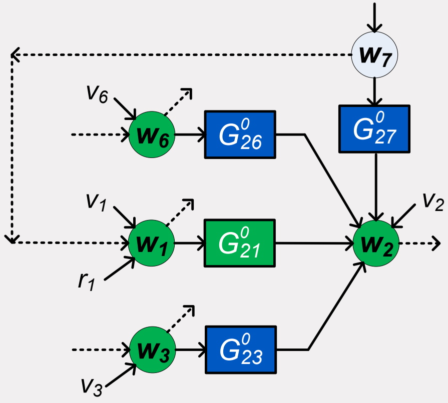
Single module identification - confounding variables



Confounding variables [1][2]**:**

Unmeasured signal that has (unmeasured) paths to both the input and output of an estimation problem.

In networks they can appear in two different ways:



Direct:

- If disturbances on inputs and outputs are correlated.

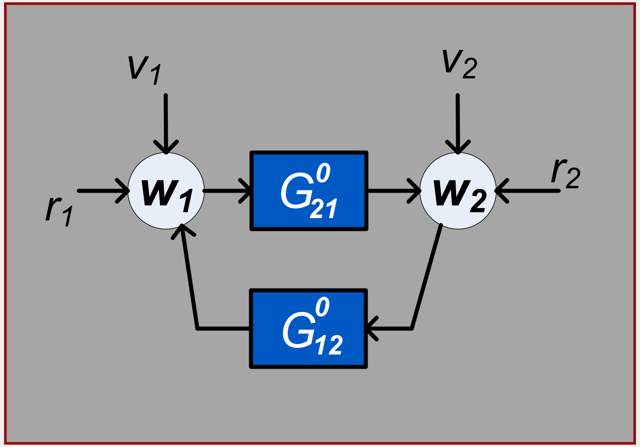
Indirect:

- If non-measured in-neighbors of an output affect signals in the inputs.

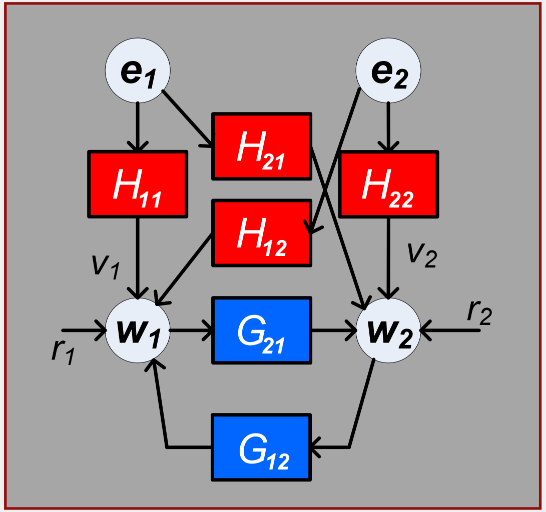
[1] J. Pearl, *Stat. Surveys, 3,* 96-146, 2009
[2] A.G. Dankers et al., *Proc. IFAC World Congress*, 2017.

Confounding variables

- Direct confounding variable:



v_1, v_2
correlated

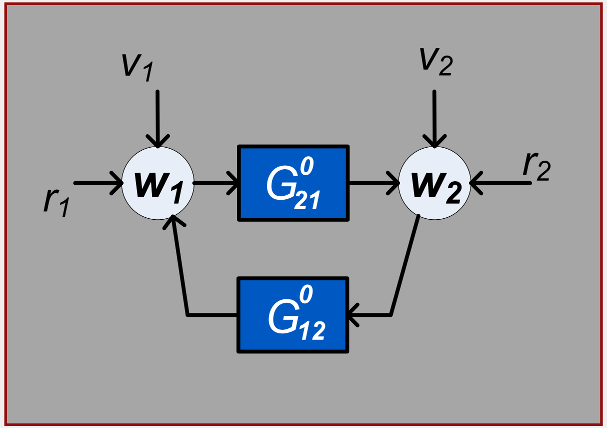


Typically not treated in direct methods of closed-loop identification

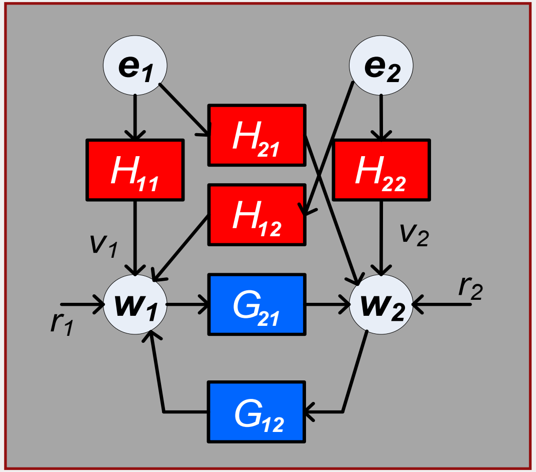
→ loss of consistency

Confounding variables

- Direct confounding variable:



v_1, v_2
correlated



Multivariate noise model: $\begin{bmatrix} w_2 \\ w_1 \end{bmatrix} = \begin{bmatrix} G_{21} \\ 0 \end{bmatrix} w_1 + \begin{bmatrix} H_{22} & H_{21} \\ \tilde{H}_{12} & \tilde{H}_{11} \end{bmatrix} \begin{bmatrix} e_2 \\ e_1 \end{bmatrix}$

Predict both w_1 and w_2 ⟶ **Adding predicted outputs** [1],[2]

Becomes a multi output local identification problem.

[1] P.M.J. Van den Hof et al. , CDC 2019.
[2] K.R. Ramaswamy et al., IEEE-TAC, 2021.

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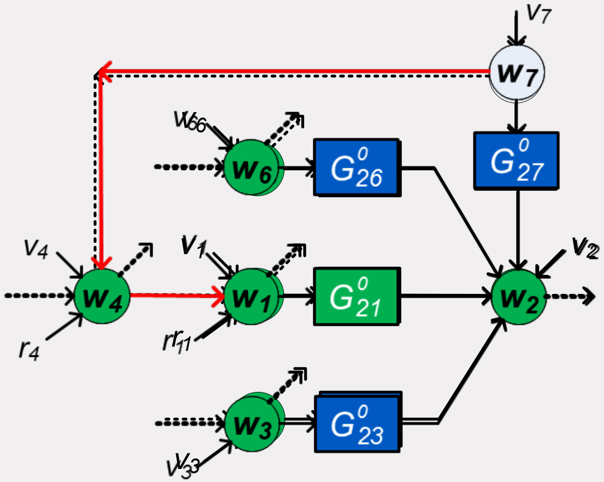
Confounding variables

- Indirect confounding variable:



Non-measurable w_7 is a confounding variable

Two possible solutions:

1. Include w_4 ➡ add predictor input

$$w_{\mathcal{D}} = \{w_1, w_3, w_4, w_6\} \quad w_{\mathcal{Y}} = \{w_2\}$$

2. Predict w_1 too ➡ add predictor output

$$w_{\mathcal{D}} = \{w_1, w_3, w_6\} \quad w_{\mathcal{Y}} = \{w_1, w_2\}$$

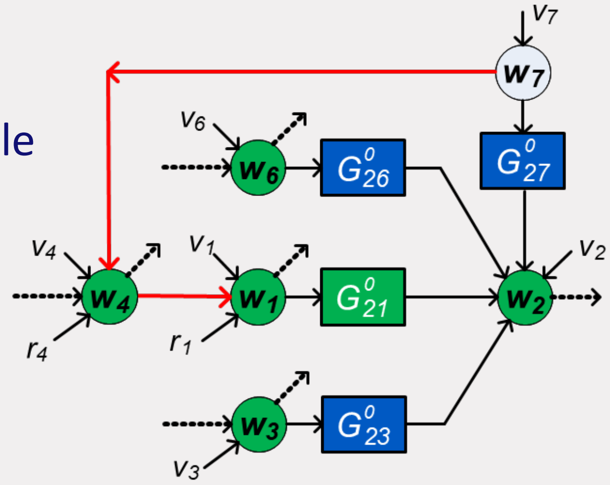
- There are degrees of freedom in choosing the predictor model

Handling confounding variables in local module identification

``Blocking'' confounding variables by adding predictor inputs

By adding w_4 as predictor input, new confounding variable for $w_4 \rightarrow w_2$.
Does this help?

Yes. Since we do not need an accurate model of G_{24}

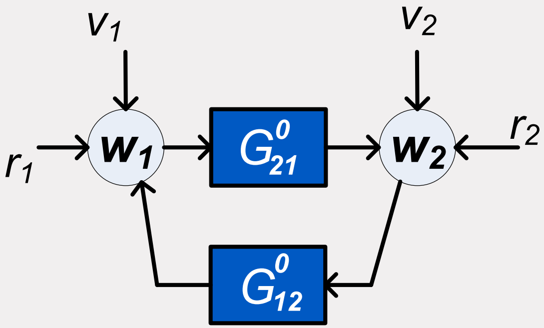


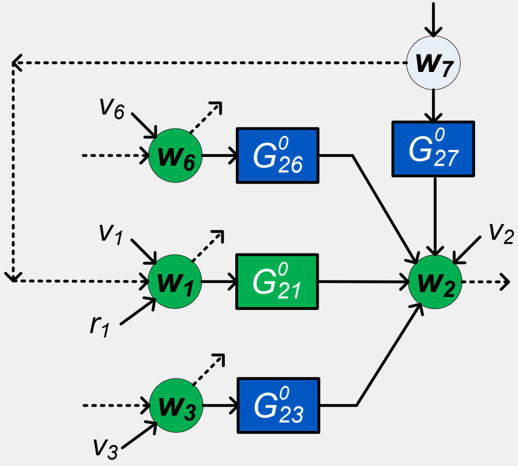
Handling confounding variables in local module identification

Confounding variables and closed-loop mechanisms

In closed-loop case
(when predicting only w_2):

- Correlation between w_1 and v_2 is no problem, as long as it passes through w_2.
- Correlation between v_1 and v_2 is a problem.





Algorithm for dealing with confounding variables

For estimating target module G_{ji}

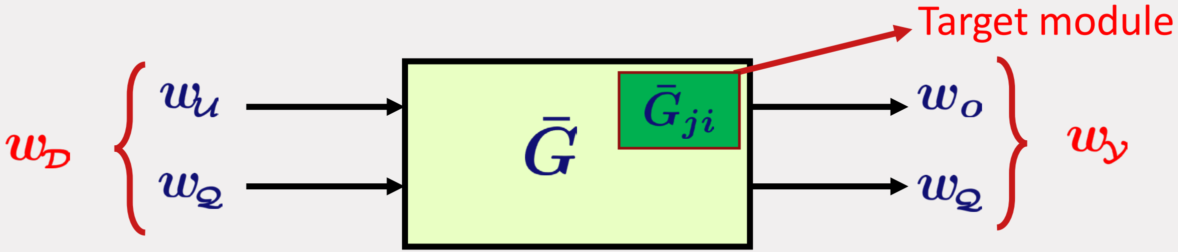
1. Select input w_i and output w_j
2. Add inputs to satisfy the parallel path and loop condition
3. Check on direct confounding variables → add output and return to step 2
4. Check on indirect confounding variables
   a) Add output and return to step 2, **OR**
   b) Add input

Algorithm always reaches a convergence point where conditions are satisfied.

The choice options lead to different end-results for signals to be included
→ **different predictor models**
that all can reach consistency of $\hat{G}_{ji}$

Local direct method

General setup:



Different algorithms for satisfying the 2 conditions (module invariance and conf. var.):

- Full input case: include all in-neighbors of $w_{\mathcal{Y}}$ [1]
- Minimum input: maximize number of outputs[2]
- User selection case (inputs first) : dedicated choice based on measurable nodes[2]
- User selection case (outputs first) : dedicated choice based on measurable nodes[3]

[1] A.G. Dankers et al., TAC 2016.
[2] K.R. Ramaswamy et al., TAC 2021.
[3] S. Shi et al., IFAC 2023.

Local direct method – Explanation of algorithms

All nodes are measurable:

Full input case: For every (added) output, include all in-neighbors as inputs

Minimum input case: Every (added) input is copied to the output in case of a confounding variable

Preselected set of measured nodes (satisfying PPL test):

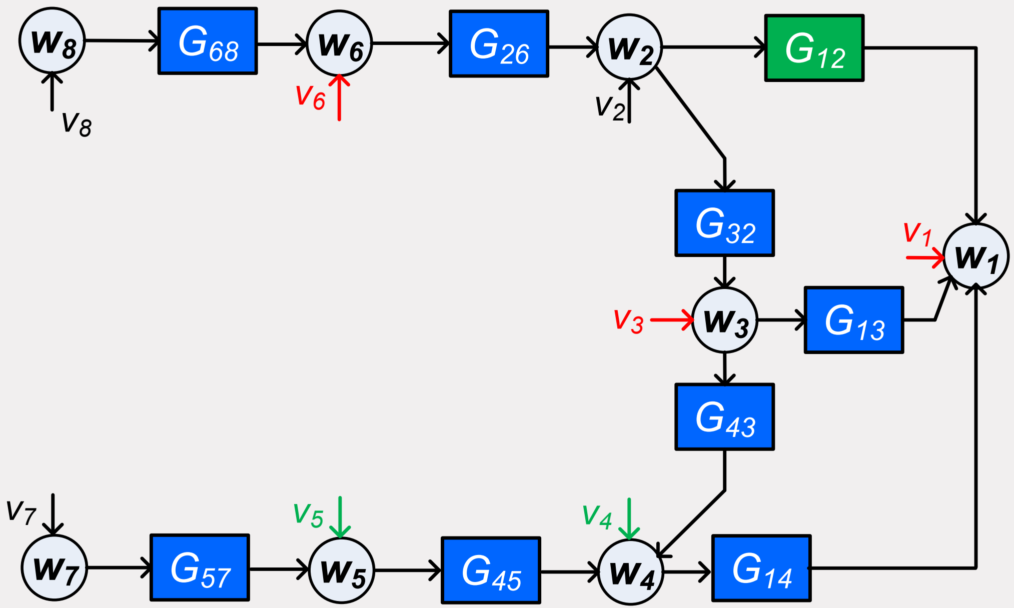
User selection (inputs first): For every (added) output, include all in-neighbors *in the immersed network* as inputs

User selection (outputs first): All signals that have a (sequence of linked) confounding variable(s) to the target output are included in the output. All in-neighbors *in the immersed network* are included as inputs

[1] A.G. Dankers et al., TAC 2016.
[2] K.R. Ramaswamy et al., TAC 2021.
[3] S. Shi et al., IFAC 2023.

Different strategies – direct method

- Full input case
- User selection case (inputs first)
- Minimum input case



Network with v_1 correlated with v_3 and v_6.
v_4 correlated with v_5.

Full input case

We include all in-neighbors of the predicted outputs as predictor inputs

Maximum use of information in signals

$$w_{\mathcal{D}} = \{2, 3, 4\} \quad w_{\mathcal{Y}} = \{1\}$$

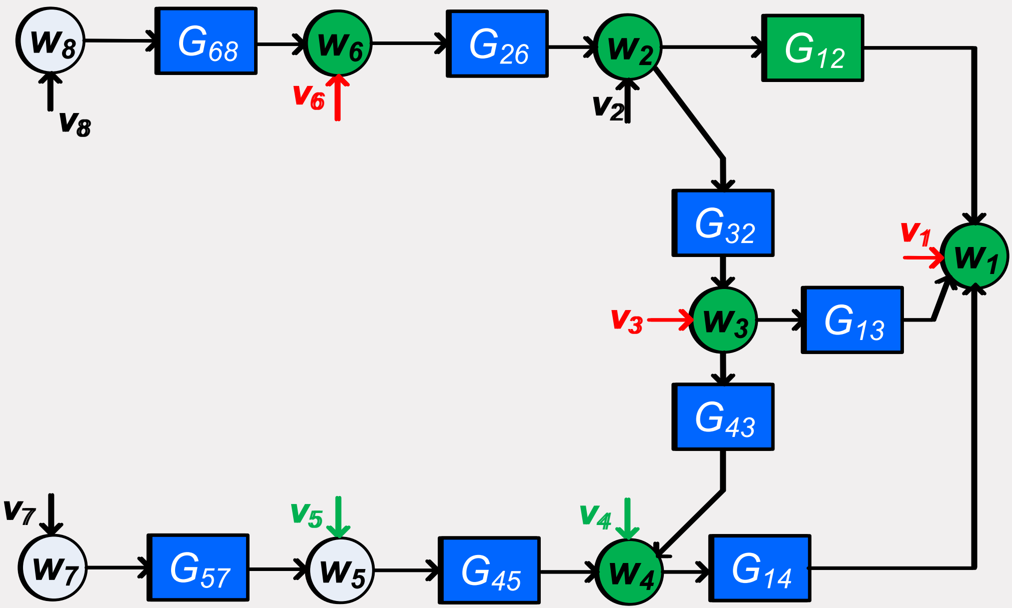
Handling direct confounding variable:

$$w_{\mathcal{D}} = \{2, 3, 4\} \quad w_{\mathcal{Y}} = \{1, 3\}$$

Handling indirect confounding variable:

$$w_{\mathcal{D}} = \{2, 3, 4, 6\} \quad w_{\mathcal{Y}} = \{1, 3\}$$

Direct identification $w_{\mathcal{D}} \rightarrow w_{\mathcal{Y}}$

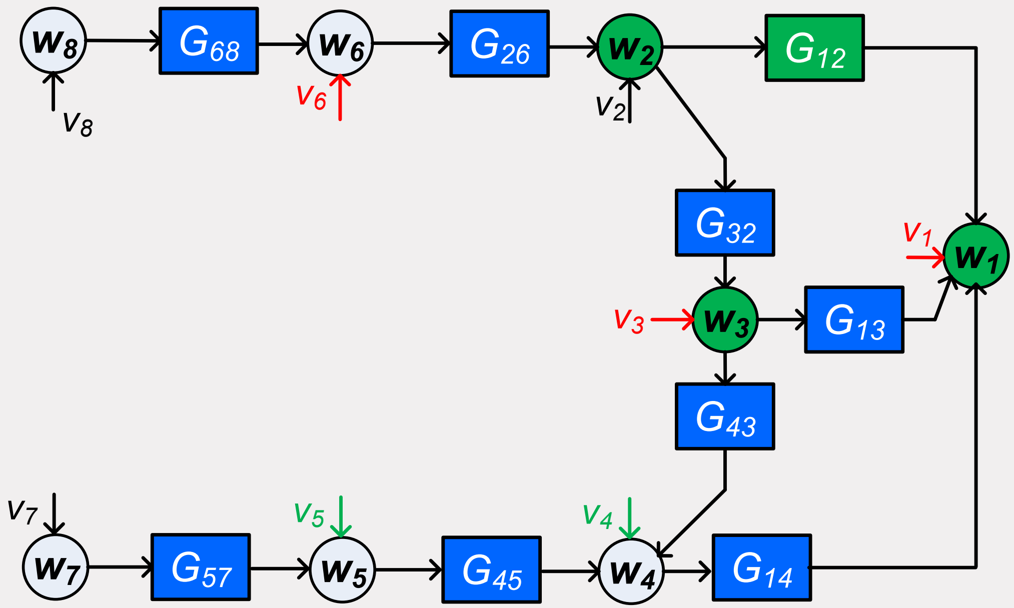


Minimum input case

- Select signals to satisfy the parallel path and loop condition
- Handle all confounding variables by including signals in output

$$w_{\mathcal{D}} = \{2, 3\} \quad w_{\mathcal{Y}} = \{1, 2, 3\}$$

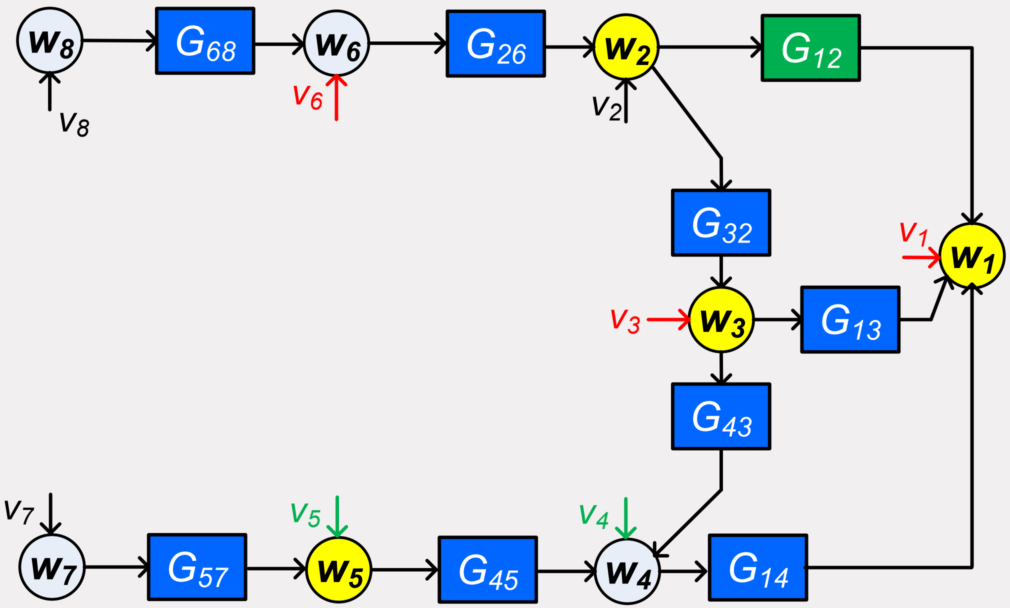
Direct identification $w_{\mathcal{D}} \rightarrow w_{\mathcal{Y}}$



User selection case

- The user does not have access to all node signals
- Four node signals can be measured
- Parallel path and loop condition is satisfied
- Start with:

$$\mathcal{w}_{\mathcal{D}} = \{2, 3\} \quad \mathcal{w}_{\mathcal{Y}} = \{1\}$$



User selection case (inputs first)

$$w_{\mathcal{D}} = \{2,3\} \quad w_{\mathcal{Y}} = \{1\}$$

Adding input from immersed network:

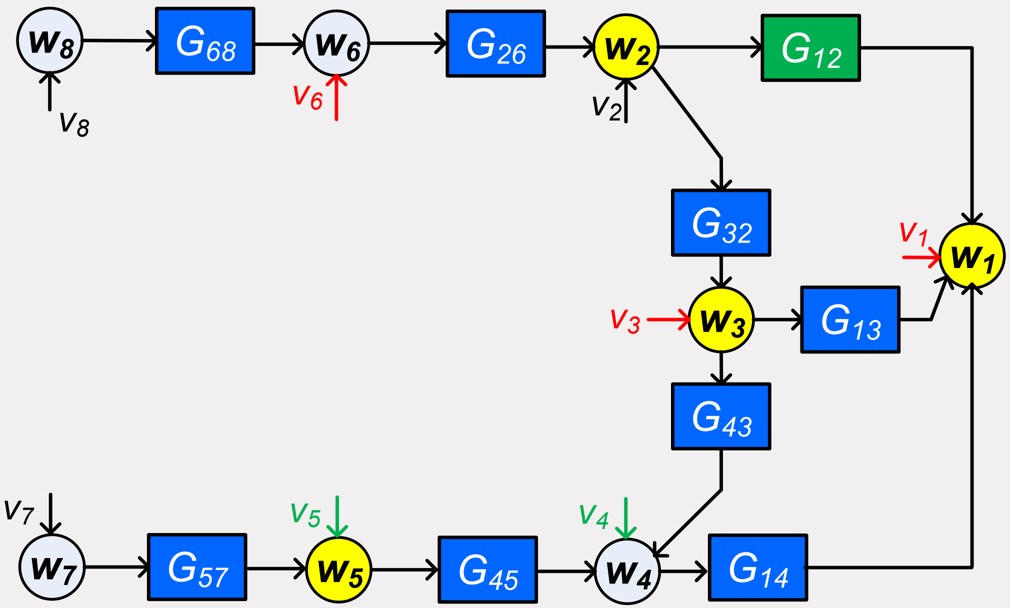
$$w_{\mathcal{D}} = \{2,3,5\} \quad w_{\mathcal{Y}} = \{1\}$$

Handling direct confounding variable:

$$w_{\mathcal{D}} = \{2,3,5\} \quad w_{\mathcal{Y}} = \{1,3\}$$

Indirect confounding variables: v_6:

$$w_{\mathcal{D}} = \{2,3,5\} \quad w_{\mathcal{Y}} = \{1,2,3\}$$



Confounding variable on w_5 induced by (v_4, v_5) is OK as it can be moved to set $\mathcal{B}$

Direct identification $w_{\mathcal{D}} \rightarrow w_{\mathcal{Y}}$

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User selection case (outputs first)

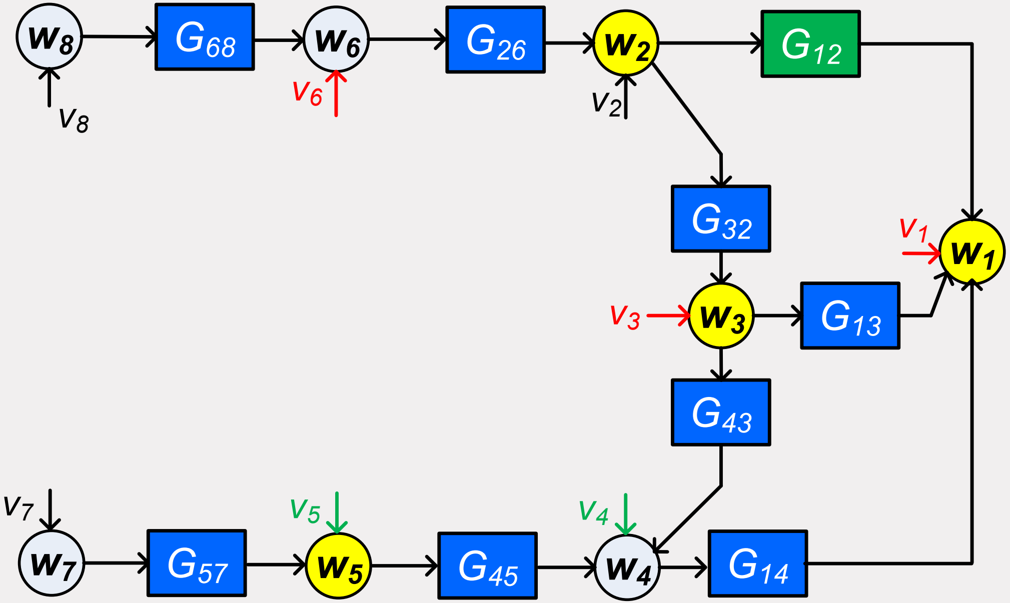
Select as outputs those node signals with confounding variables with the target output node:

$$w_{\mathcal{Y}} = \{1, 2, 3, 5\}$$

Add appropriate inputs to each of these output nodes:

$$w_{\mathcal{D}} = \{2, 3, 5\} \quad w_{\mathcal{Y}} = \{1, 2, 3, 5\}$$

Direct identification $w_{\mathcal{D}} \rightarrow w_{\mathcal{Y}}$



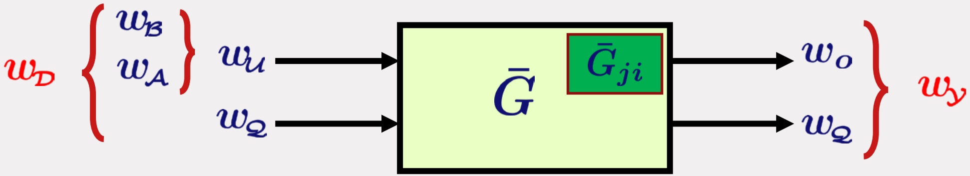
Different strategies for same network and target module

Same network with different identification setups that lead to **consistent estimate of the target module** with **Maximum likelihood properties** based on the strategy used.

Full input case	Minimum input case	User selection case (inputs)	User selection case (outputs)
$\begin{bmatrix} w_2 \\ w_3 \\ w_4 \\ w_6 \end{bmatrix} \rightarrow \begin{bmatrix} w_1 \\ w_3 \end{bmatrix}$	$\begin{bmatrix} w_2 \\ w_3 \end{bmatrix} \rightarrow \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$	$\begin{bmatrix} w_2 \\ w_3 \\ w_5 \end{bmatrix} \rightarrow \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$	$\begin{bmatrix} w_2 \\ w_3 \\ w_5 \end{bmatrix} \rightarrow \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_5 \end{bmatrix}$

Data informativity conditions might be different (see later)

Structural conditions for consistency of target G_{ji}



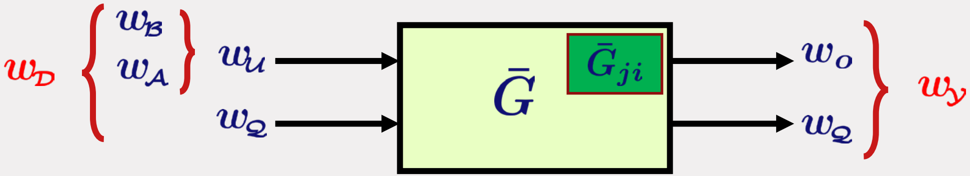
1) PPL condition

2) Confounding variable conditions:

- $i \in \mathcal{Q} \cup \mathcal{A}$; $j \in \mathcal{Y}$
- No confouding variables between $w_{\mathcal{A}}$ and $w_{\mathcal{Y}}$
- No confouding variables between $w_{\mathcal{A}}$ and $w_{\mathcal{B}}$
- No unmeasured paths from $\{i, j\}$ to $w_{\mathcal{B}}$

These conditons can always be satisfied by appropriate choices of $w_{\mathcal{A}}, w_{\mathcal{B}}, w_{\mathcal{Q}}$ and influence the selection of the predictor model

Structural conditions for consistency of target G_{ji}



1) PPL condition

2) Confounding variable conditions:

- $i \in \mathcal{Q} \cup \mathcal{A};\ j \in \mathcal{Y}$
- No confouding variables between $w_{\mathcal{A}}$ and $w_{\mathcal{Y}}$
- No confouding variables between $w_{\mathcal{A}}$ and $w_{\mathcal{B}}$
- No unmeasured paths from $\{i, j\}$ to $w_{\mathcal{B}}$

Confounding variables between $w_{\mathcal{B}}$ and $w_{\mathcal{Y}}$ do not hurt.

Analysis – from network to predictor model

Theory for single module direct method (MIMO)

Separate the node variables of the network into

$$w = \begin{bmatrix} w_{\mathcal{Q}} \\ w_{o} \\ w_{\mathcal{U}} \\ w_{\mathcal{Z}} \end{bmatrix} = \begin{bmatrix} \text{nodes that appear in input and output} \\ \text{output of target module, if not present in } w_{\mathcal{Q}} \\ \text{nodes that appear only in the input} \\ \text{unmeasured nodes} \end{bmatrix}$$

and write the network equations:

$$\begin{bmatrix} w_{\mathcal{Q}} \\ w_{o} \\ w_{\mathcal{U}} \\ w_{\mathcal{Z}} \end{bmatrix} = \begin{bmatrix} G_{\mathcal{Q}\mathcal{Q}} & G_{\mathcal{Q}o} & G_{\mathcal{Q}\mathcal{U}} & G_{\mathcal{Q}\mathcal{Z}} \\ G_{o\mathcal{Q}} & G_{oo} & G_{o\mathcal{U}} & G_{o\mathcal{Z}} \\ G_{\mathcal{U}\mathcal{Q}} & G_{\mathcal{U}o} & G_{\mathcal{U}\mathcal{U}} & G_{\mathcal{U}\mathcal{Z}} \\ G_{\mathcal{Z}\mathcal{Q}} & G_{\mathcal{Z}o} & G_{\mathcal{Z}\mathcal{U}} & G_{\mathcal{Z}\mathcal{Z}} \end{bmatrix} \begin{bmatrix} w_{\mathcal{Q}} \\ w_{o} \\ w_{\mathcal{U}} \\ w_{\mathcal{Z}} \end{bmatrix} + R(q)r + \begin{bmatrix} H_{\mathcal{Q}\mathcal{Q}} & H_{\mathcal{Q}o} & H_{\mathcal{Q}\mathcal{U}} & H_{\mathcal{Q}\mathcal{Z}} \\ H_{o\mathcal{Q}} & H_{oo} & H_{o\mathcal{U}} & H_{o\mathcal{Z}} \\ H_{\mathcal{U}\mathcal{Q}} & H_{\mathcal{U}o} & H_{\mathcal{U}\mathcal{U}} & H_{\mathcal{U}\mathcal{Z}} \\ H_{\mathcal{Z}\mathcal{Q}} & H_{\mathcal{Z}o} & H_{\mathcal{Z}\mathcal{U}} & H_{\mathcal{Z}\mathcal{Z}} \end{bmatrix} \begin{bmatrix} e_{\mathcal{Q}} \\ e_{o} \\ e_{\mathcal{U}} \\ e_{\mathcal{Z}} \end{bmatrix}$$

Then remove node variables $w_{\mathcal{Z}}$ from the equations through immersion

Theory for single module direct method (MIMO)

Upon immersing node variables $w_{\mathcal{Z}}$ there exists a system transform into the equivalent network representation

$$\begin{matrix} w_{\mathcal{Y}} \left\{ \begin{matrix} \\ \\ \end{matrix} \right. \\ \\ \end{matrix} \underbrace{\begin{bmatrix} w_{\mathcal{Q}} \\ w_{\mathcal{O}} \\ \hline w_{\mathcal{U}} \end{bmatrix}}_{w_m} = \underbrace{\left[\begin{array}{c|c} \bar{G} & 0 \\ \hline \bar{G}_{\mathcal{UD}} & \bar{G}_{\mathcal{UO}} \end{array}\right]}_{\bar{G}_m} \begin{bmatrix} w_{\mathcal{Q}} \\ w_{\mathcal{U}} \\ \hline w_{\mathcal{O}} \end{bmatrix} + \underbrace{\left[\begin{array}{c|c} \bar{H} & 0 \\ \hline 0 & \bar{H}_{\mathcal{U}} \end{array}\right]}_{\bar{H}_m} \underbrace{\begin{bmatrix} \xi_{\mathcal{Q}} \\ \xi_{\mathcal{O}} \\ \hline \xi_{\mathcal{U}} \end{bmatrix}}_{\xi_m}$$

with ξ_m a white noise process, $\bar{H}_m$ square, monic, stable and stably invertible.

I.e. the number of noise sources is reduced to match $dim(w_m)$.

Under the conditions on the confounding variables, the disturbances on $w_{\mathcal{Y}}$ and $w_{\mathcal{U}}$ can be decoupled → $\bar{H}_m$ becomes block-diagonal.

Theory for single module direct method (MIMO)

$$\color{red}{w_{\mathcal{Y}}\left\{\begin{matrix} \\ \\ \end{matrix}\right.}\underbrace{\left[\begin{array}{c} w_{\mathcal{Q}} \\ w_{\mathcal{O}} \\ \hline w_{\mathcal{U}} \end{array}\right]}_{w_m} = \underbrace{\left[\begin{array}{c|c} \bar{G} & 0 \\ \hline \bar{G}_{\mathcal{U}\mathcal{D}} & \bar{G}_{\mathcal{U}\mathcal{O}} \end{array}\right]}_{\bar{G}_m} \left[\begin{array}{c} w_{\mathcal{Q}} \\ w_{\mathcal{U}} \\ \hline w_{\mathcal{O}} \end{array}\right] + \underbrace{\left[\begin{array}{c|c} \bar{H} & 0 \\ \hline 0 & \bar{H}_{\mathcal{U}\mathcal{U}} \end{array}\right]}_{\bar{H}_m} \underbrace{\left[\begin{array}{c} \xi_{\mathcal{Q}} \\ \xi_{\mathcal{O}} \\ \hline \xi_{\mathcal{U}} \end{array}\right]}_{\xi_m}$$

Upper part of the equation leads to:

$$\underbrace{\begin{bmatrix} w_{\mathcal{Q}} \\ w_{\mathcal{O}} \end{bmatrix}}_{w_{\mathcal{Y}}} = \underbrace{\begin{bmatrix} \bar{G}_{\mathcal{Q}\mathcal{Q}} & \bar{G}_{\mathcal{Q}\mathcal{U}} \\ \bar{G}_{\mathcal{O}\mathcal{Q}} & \bar{G}_{\mathcal{O}\mathcal{U}} \end{bmatrix}}_{\bar{G}} \underbrace{\begin{bmatrix} w_{\mathcal{Q}} \\ w_{\mathcal{U}} \end{bmatrix}}_{w_{\mathcal{D}}} + \underbrace{\begin{bmatrix} \bar{H}_{\mathcal{Q}\mathcal{Q}} & \bar{H}_{\mathcal{Q}\mathcal{O}} \\ \bar{H}_{\mathcal{O}\mathcal{Q}} & \bar{H}_{\mathcal{O}\mathcal{O}} \end{bmatrix}}_{\bar{H}} \underbrace{\begin{bmatrix} \xi_{\mathcal{Q}} \\ \xi_{\mathcal{O}} \end{bmatrix}}_{\xi_{\mathcal{Y}}}$$

to be used for identification

TU/e

Completing the predictor model with excitation signals

Local direct method

Incorporating the role of external signals:

Original (full) network model: $w(t) = G(q)w(t) + \underbrace{u(t)}_{R(q)r(t)} + H(q)e(t);$

Predictor model (subset of nodes):

$$w_{\mathcal{Y}}(t) = \bar{G}(q)w_{\mathcal{D}(t)} + \bar{J}(q)u_{\mathcal{K}}(t) + \bar{S}u_{\mathcal{P}}(t) + \bar{H}(q)\xi_{\mathcal{Y}}(t)$$

Effect of u on $w_{\mathcal{Y}}$ can appear in three different ways:

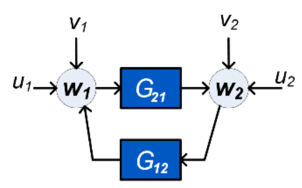
1. Incorporated in input $w_{\mathcal{D}}$
2. With a dynamic term $\bar{J}(q)$
3. With a constant unit-term in $\bar{S}$ (binary matrix)

[1] K.R. Ramaswamy et al., IEEE-TAC, 2021.

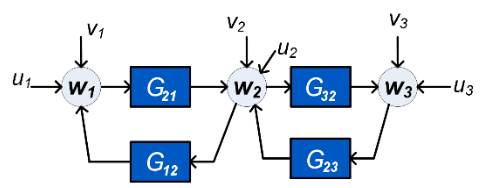
Local direct method

Examples for different roles of u:

$$w_{\mathcal{Y}}(t) = \bar{G}(q)w_{\mathcal{D}(t)} + \bar{J}(q)u_{\mathcal{K}}(t) + \bar{S}u_{\mathcal{P}}(t) + \bar{H}(q)\xi_{\mathcal{Y}}(t)$$



$$w_2 = G_{21}(q)w_1 + u_2 + v_2 \qquad u_{\mathcal{P}} = u_2 \qquad u_{\mathcal{K}} = \emptyset$$



$$w_2 = \frac{1}{1 + G_{32}G_{23}}\left[G_{21}w_1 + u_2 + v_2 + G_{23}(u_3 + v_3)\right]$$

$$u_{\mathcal{P}} = \emptyset \qquad u_{\mathcal{K}} = \{u_2, u_3\}$$

Dynamic term $\bar{J}(q)$ can be left unmodelled → higher level of ``disturbances''

Alternative: estimate the term with measured input $u_{\mathcal{K}}(t)$

TU/e

Local direct method – predictor model

Based on:

$$w_{\mathcal{Y}}(t) = \bar{G}(q)w_{\mathcal{D}(t)} + \bar{J}(q)u_{\mathcal{K}}(t) + \bar{S}u_{\mathcal{P}}(t) + \bar{H}(q)\xi_{\mathcal{Y}}(t)$$

we construct the (parametrized) prediction error:

$$\varepsilon(t,\theta) = \bar{H}(q,\theta)^{-1}[w_{\mathcal{Y}}(t) - \bar{G}(q,\theta)w_{\mathcal{D}}(t) - \bar{J}(q,\theta)u_{\mathcal{K}}(t) - \bar{S}u_{\mathcal{P}}(t)]$$

Quadratic identification criterion: $\hat{\theta}_N := \arg\min_{\theta} \frac{1}{N}\sum_{t=0}^{N-1} \varepsilon(t,\theta)^T Q \varepsilon(t,\theta)$ $\qquad Q > 0$

Characteristic of 'direct' method: no postprocessing of estimate required.

The target module G_{ji} is directly estimated as one of the modules in $\bar{G}(q)$.

Local direct method

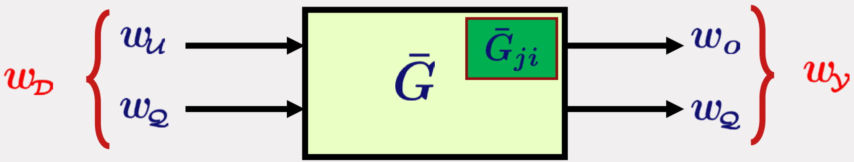
Determining the different roles of excitation signals:

Given $\mathcal{Y}$ and $\mathcal{D}$, the sets $\mathcal{P}$ and $\mathcal{K}$ are determined through graphical conditions[1,2,3]:

- For $\ell \in \mathcal{Q}$, $u_\ell \in u_\mathcal{P}$ if all loops around w_ℓ pass through a node in $w_\mathcal{D}$
- $u_o \in u_\mathcal{P}$ if all loops around w_o pass through a node in $w_\mathcal{D}$ and all paths from w_o to $w_\mathcal{Q}$ pass through a node in $w_\mathcal{U}$.
- $u_\mathcal{Y} \in u_\mathcal{K}$ if $u_\mathcal{Y} \notin u_\mathcal{P}$
- For $\ell \notin \{\mathcal{Y} \cup \mathcal{D}\}$, $u_\ell \in u_\mathcal{K}$ if w_ℓ has a direct or unmeasured path to $w_\mathcal{Y}$

[1] Simple case where set $\mathcal{B} = \emptyset$
[2] Ramaswamy, PhD thesis 2022;
[3] VdH et al, IFAC 2023.

Consistency result



$G_{ji}(q,\hat{\theta}_N)$ is a consistent estimate of G^0_{ji}, if

- $\mathcal{S} \in \mathcal{M}$
- Structural conditions on the predictor model are satisfied
  - Parallel path and loop (PPL) condition
  - Confounding variable conditions
- Data set is informative with respect to $\mathcal{M}$
- A technical condition on presence of delays is satisfied

According to PEM/ML theory, the estimator can achieve the CRLB

[1] K.R. Ramaswamy et al., IEEE-TAC, 2021.
[2] VdH et al., CDC-2020.

TU/e

Data-informativity

Data informativity (classical definition)

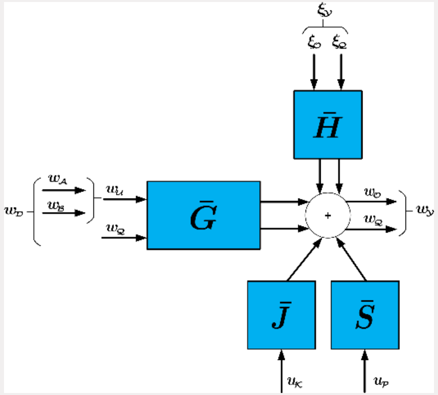
$$w_{\mathcal{Y}}(t) = \bar{G}(q,\theta)w_{\mathcal{D}}(t) + \bar{H}(q,\theta)\xi_{\mathcal{Y}}(t) + \bar{J}(q,\theta)u_{\mathcal{K}}(t) + \bar{S}u_{\mathcal{P}}(t)$$

Predictor model: $\hat{w}_{\mathcal{Y}}(t,\theta) = W(q,\theta)z(t)$

for a model set $\mathcal{M} := (\bar{G}(q,\theta), \bar{H}(q,\theta), \bar{J}(q,\theta))_{\theta\in\Theta}$ with $z(t) := \begin{bmatrix} w_{\mathcal{D}}(t) \\ \xi_{\mathcal{Y}}(t) \\ u_{\mathcal{K}}(t) \end{bmatrix}$

Then a quasi-stationary data sequence $\{z(t)\}_{t=0,\ldots}$ is informative with respect to $\mathcal{M}$ if for any two models in $\mathcal{M}$:

$$\boxed{\bar{\mathbb{E}}[(W_1(q) - W_2(q))z(t)]^2 = 0 \implies W_1(e^{i\omega}) \equiv W_2(e^{i\omega})}$$



A sufficient condition for this is that z is persistently exciting:

$$\boxed{\Phi_z(\omega) > 0 \text{ for almost all } \omega}$$

[1] L. Ljung, Englewood Cliffs, NJ: Prentice-Hall, 1999

Single module identification – data-informativity

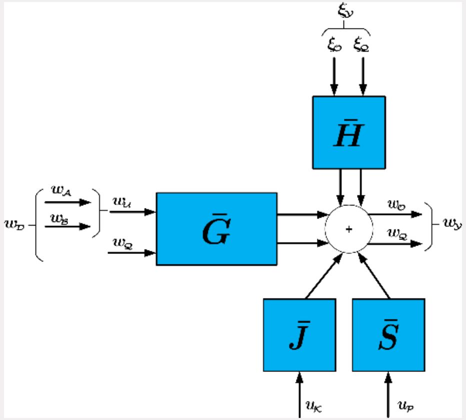
Predictor model equation:

$$w_{\mathcal{Y}}(t) = \bar{G}(q,\theta)w_{\mathcal{D}}(t) + \bar{H}(q,\theta)\xi_{\mathcal{Y}}(t) + \bar{J}(q,\theta)u_{\mathcal{K}}(t) + \bar{S}u_{\mathcal{P}}(t)$$

Typical data-informativity condition:

κ persistently exciting $\quad \boxed{\Phi_{\kappa}(\omega) > 0 \text{ for almost all } \omega}$

$$\kappa(t) := \begin{bmatrix} w_{\mathcal{D}}(t) \\ \xi_{\mathcal{Y}}(t) \\ u_{\mathcal{K}}(t) \end{bmatrix}$$ inputs of the predictor model



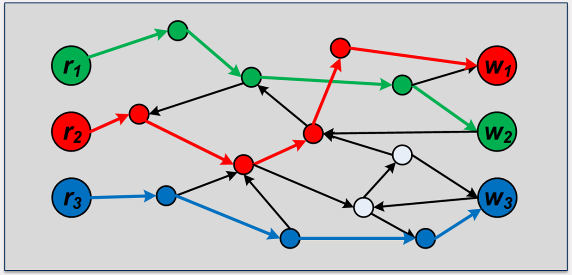
Rank-based condition can generically be satisfied based on a graph-based condition

[1] L. Ljung, 1989.
[2] K.R. Ramaswamy et al., IEEE-TAC, 2021; VdH and Ramaswamy, CDC 2020.
[3] X.Bombois et al., Automatica, 2023.

Data informativity (path-based condition)

A signal $w(t) = F(q)r(t)$ with r persistently exciting, is persistently exciting iff F has **full row rank**.

This condition can be verified in a generic sense, by considering the **generic rank** of F [1],[2]



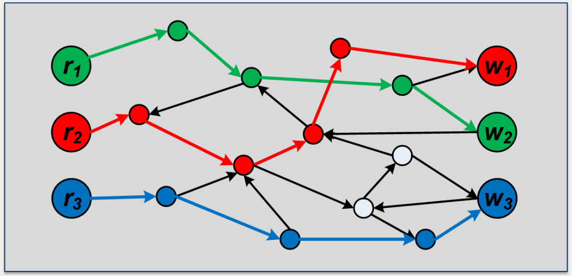
$b_{\mathcal{R}\to\mathcal{W}} = 3$

linking to the maximum number of **vertex disjoint paths** between inputs and outputs

F has generic full row rank if the number of vertex disjoint paths $b_{\mathcal{R}\to\mathcal{W}}$ satisfies $b_{\mathcal{R}\to\mathcal{W}} = dim(w)$

[1] Van der Woude, 1991
[2] Hendrickx, Gevers & Bazanella, CDC 2017, TAC 2019.

Data informativity (path-based condition)



For persistence of excitation of $\boldsymbol{\kappa}$ this implies:

$\boldsymbol{\kappa}$ persistently exciting holds **generically** if there are $dim(\kappa)$**vertex disjoint paths** between external signals $\{u, e\}$and $\kappa = \begin{bmatrix} w_{\mathcal{D}} \\ \xi_{\mathcal{Y}} \\ u_{\mathcal{K}} \end{bmatrix}$

and since $\{\xi_{\mathcal{Y}}, u_{\mathcal{K}}\}$ are external signals too, this is equivalent to:

$dim(w_{\mathcal{D}})$ vertex disjoint paths between $\{u, e\}\backslash\{\xi_{\mathcal{Y}}, u_{\mathcal{K}}\}$ and $w_{\mathcal{D}}$

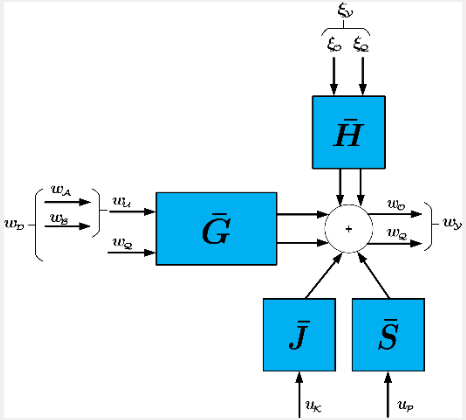
Data informativity (path-based condition)

$dim(w_{\mathcal{D}})$ vertex disjoint paths between $\{u, e\} \backslash \{\xi_{\mathcal{Y}}, u_{\mathcal{K}}\}$ and $w_{\mathcal{D}}$

Excitation is provided by all external signals $\{u, e\}$ except for

- $\xi_{\mathcal{Y}}$: the white noise signals e that have a path to an output node or to a node that has a confounding variable with an output node
- $u_{\mathcal{K}}$: the excitation signals u that affect an output through unknown dynamics

Data informativity (path-based condition)



Specific result for networks with **full rank disturbances**:

Every node signal in $w_{\mathcal{Q}}$ requires an excitation in $u_{\mathcal{P}}$ having a 1-transfer to $w_{\mathcal{Y}}$

$$w_{\mathcal{Y}}(t) = \bar{G}(q,\theta)w_{\mathcal{D}}(t) + \bar{H}(q,\theta)\xi_{\mathcal{Y}}(t) + \bar{J}(q,\theta)u_{\mathcal{K}}(t) + \bar{S}u_{\mathcal{P}}(t)$$

- For every node in $w_{\mathcal{Q}}$ we need a *u*-excitation
- More expensive experiments with growing # outputs
- A node $w_{\mathcal{Q}}$ whose excitation appears in $u_{\mathcal{K}}$ can never be sufficiently excited

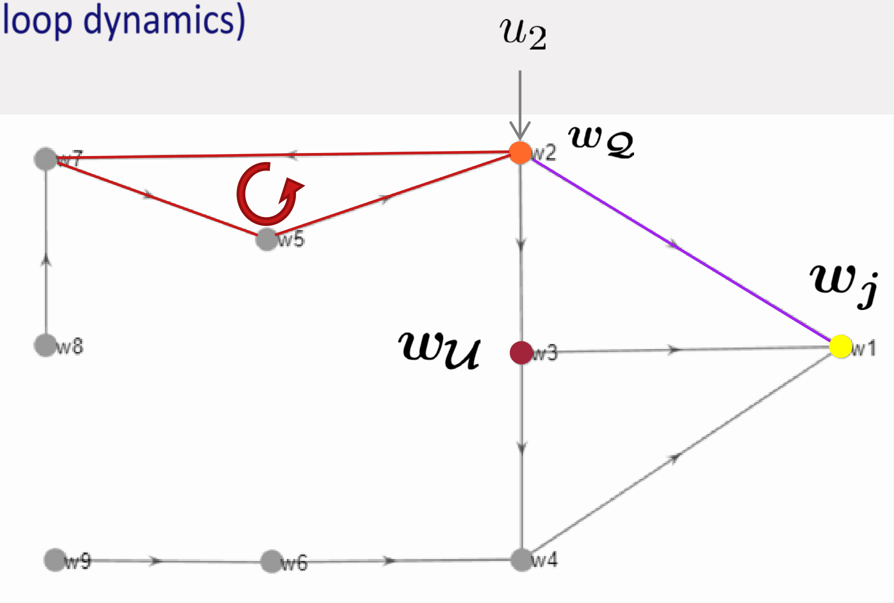
[1] VdH et al., IFAC 2023.

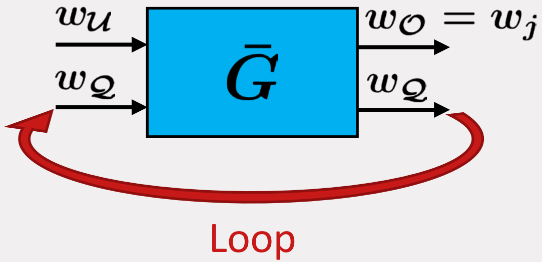
Data-informativity - Example

Target module: G_{12}

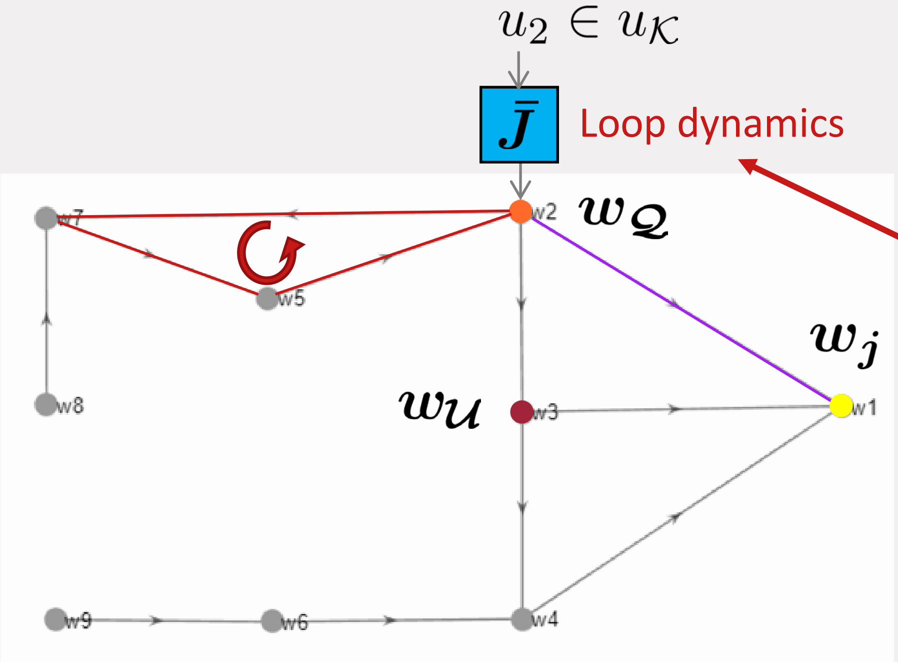
Because of confounding variable between w_2 and w_1, predictor model is $(w_2, w_3) \rightarrow (w_1, w_2)$

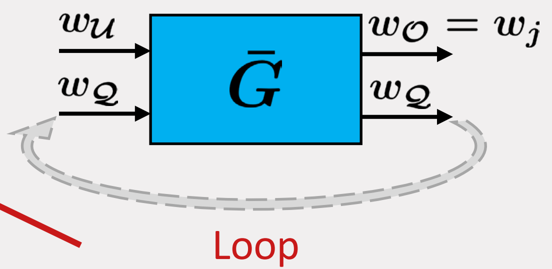
An excitation u_2 on w_2 will affect w_2 through an unknown dynamic transfer funtion (loop dynamics)





Data-informativity - Example



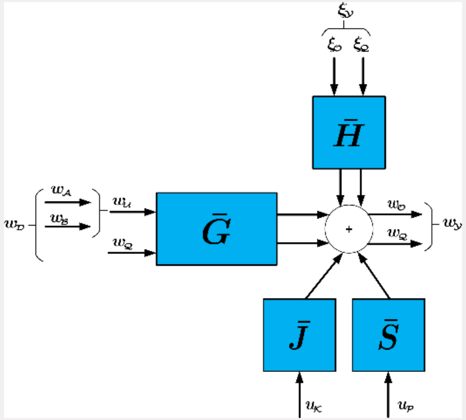


$w_{\mathcal{Q}}$ can never be sufficiently excited!

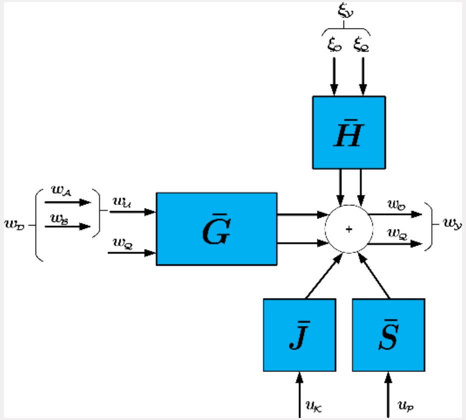
Data informativity (path-based condition)

Observations:

- Since $w_{\mathcal{Q}}$ is an output, unmeasured disturbances on $w_{\mathcal{Q}}$ are modelled through a noise model. Their white noise sources are not available anymore more for excitation of $\bar{G}$.
- Data-informativity cannot always be guaranteed by providing a sufficient number of external excitation signals.
- (Additional) structural conditions on the predictor model need to be satisfied



Data informativity (path-based condition)



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Additional condition for a node $w_{\mathcal{Q}}$ to be effectively ``excitable'':

Every loop around a node in $w_{\mathcal{Q}}$ should be blocked by a node in $w_{\mathcal{D}}$.

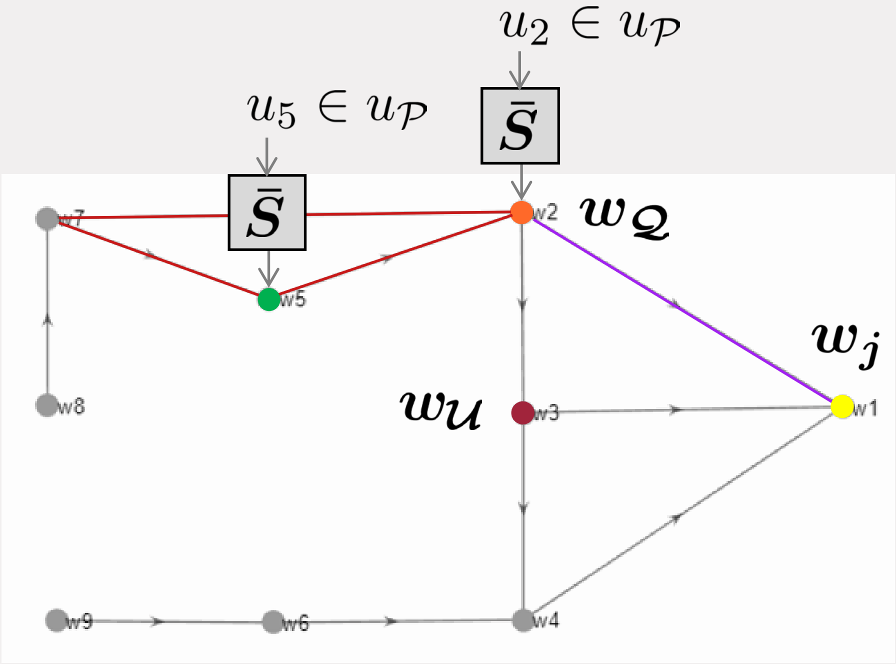
This additional graph-based condition needs to be integrated in the predictor model algorithms

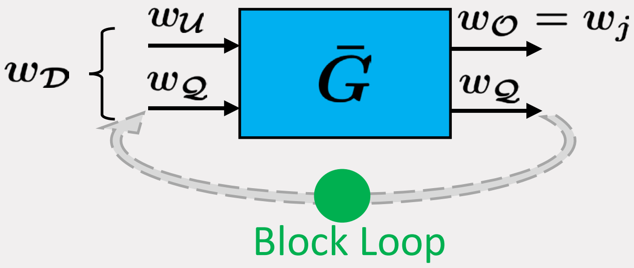
[1] VdH et al., IFAC 2023

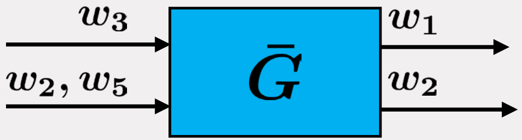
Data-informativity - Example

Every loop around a node in $w_{\mathcal{Q}}$ should be blocked by a node in $w_{\mathcal{D}}$

$\longrightarrow$ add w_5 to $w_{\mathcal{D}}$







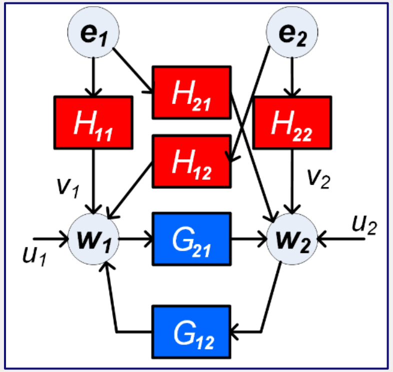
w_2, w_3 and w_5 need excitation, and u_2 and u_5 can be used for that

2-node example

Target: identify G_{21} with direct method

Predictor model: $\underbrace{\{w_1\}}_{w_{\mathcal{D}}} \rightarrow \underbrace{\{w_1, w_2\}}_{w_{\mathcal{Y}}}$

$w_{\mathcal{Q}} = \{w_1\} \quad w_{\mathcal{U}} = \emptyset$



Step 1:

Both u_1 and u_2 contribute to $w_{\mathcal{K}}$ ⟶ data informativity condition is not satisfied

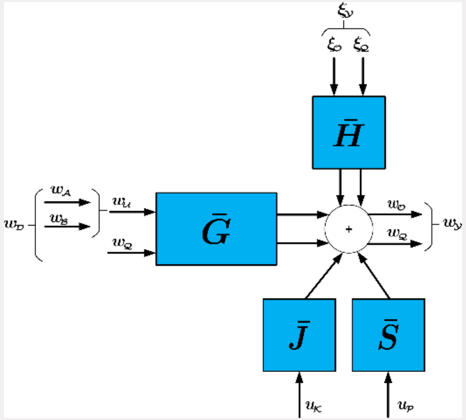
Step 2: Change predictor model to: $\underbrace{\{w_1, w_2\}}_{w_{\mathcal{D}}} \rightarrow \underbrace{\{w_1, w_2\}}_{w_{\mathcal{Y}}}$

Both u_1 and u_2 contribute to $w_{\mathcal{P}}$ ⟶ data informativity condition is satisfied

Both u_1 and u_2 need to be present, while an indirect method requires only u_1!

Data informativity - summary

$\boxed{\Phi_{\kappa}(\omega) > 0 \text{ for almost all } \omega}$ $\quad \kappa(t) := \begin{bmatrix} w_{\mathcal{D}}(t) \\ \xi_{\mathcal{Y}}(t) \\ u_{\mathcal{K}}(t) \end{bmatrix}$

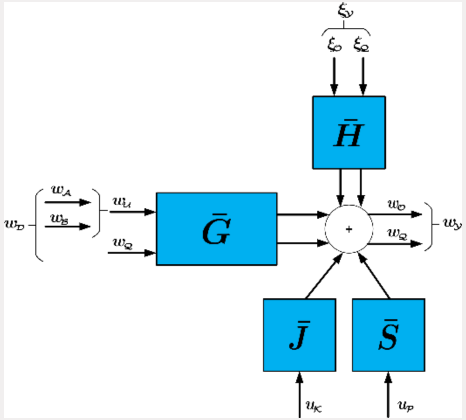


$dim(w_{\mathcal{D}})$ vertex disjoint paths between $\{u, e\}\backslash\{\xi_{\mathcal{Y}}, u_{\mathcal{K}}\}$ and $w_{\mathcal{D}}$

- Disturbances $\xi_{\mathcal{Y}}$ can not be used for exciting $w_{\mathcal{D}}$
  They are used for exciting the noise model
- For every signal in $w_{\mathcal{Q}}$ we need an u-excitation
- More “expensive” experiments with growing # outputs
- Additional structural condition:

Every loop around a node in $w_{\mathcal{Q}}$ should be blocked by a node in $w_{\mathcal{D}}$.

Data informativity - extension



So far, data-informativity conditions have been based on consistent estimation of the full predictor model, i.e. all entries in $\bar{G}$

It appears that the conditions can be further relaxed when focusing on the target module only!

I.e., rather than requiring:

$$\boxed{\bar{\mathbb{E}}[(W^{(1)}(q)-W^{(2)}(q))z(t)]^2 = 0 \implies W^{(1)}(e^{i\omega}) \equiv W^{(2)}(e^{i\omega})}$$

we can require:

$$\boxed{\bar{\mathbb{E}}[(W^{(1)}(q)-W^{(2)}(q))z(t)]^2 = 0 \implies G^{(1)}_{ji}(e^{i\omega}) \equiv G^{(2)}_{ji}(e^{i\omega})}$$

These single module DI conditions are also implemented in the app/toolbox.

[1] VdH et al., in preparation, 2024

Algebraic loop condition

Algebraic loop condition

Well known in a standard closed-loop problem with the direct method:
$G(q)C(q)$ should be strictly proper, in the system and in the parametrized model.

In the local direct method for networks this becomes:

The following paths should have at least a delay:

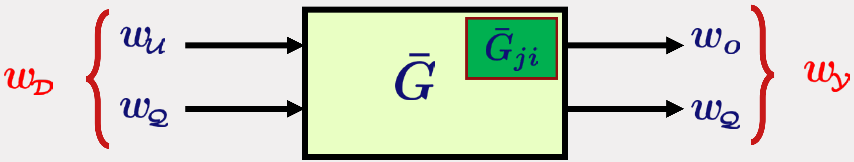
1) All paths/loops from $w_{\mathcal{Y}\cup\mathcal{B}}$ to $w_{\mathcal{Y}}$ in the original network and in the parametrized model;

2) For every $w_k \in w_{\mathcal{A}}$, all paths from $w_{\mathcal{Y}\cup\mathcal{B}}$ to w_k in the original network, or all paths from w_k to $w_{\mathcal{Y}}$ in the parametrized model.

[1] Ramaswamy et al., TAC 2021

Summary

Consistency result



$G_{ji}(q,\hat{\theta}_N)$ is a consistent estimate of G_{ji}^0, if

- $\mathcal{S} \in \mathcal{M}$
- Structural conditions on the predictor model are satisfied
  - Parallel path and loop (PPL) condition
  - Confounding variable conditions
- Data set is informative with respect to $\mathcal{M}$
- A technical condition on presence of delays is satisfied

According to PEM/ML theory, the estimator can achieve the CRLB

[1] K.R. Ramaswamy et al., IEEE-TAC, 2021.

[2] VdH et al., CDC-2020.

Summary local direct method for single module ID

- Flexible algorithm for selecting measured signals in a predictor model
- that leads to consistent (and minimum variance) module estimates
- Verifiable conditions on the network topology (assumed a priori known)
- Path-based conditions also for (generic) data informativity
- For the actual identification algorithm: preferably regularized techniques
- Extensions:
  - effective use of u-signals can further relax the conditions for signal selection [1]
  - include topology estimation as a first step [2]

[1] Ramaswamy, VdH, Dankers, CDC 2019.
[2] Rajagopal et al., CDC 2021.

Identifiability and data informativity

- For a **particular** identification method:
  Consistency conditions include aspects of data-informativity
  and underlying conditions of identifiability (implicitly)

- Current consistency conditions can be split in (a) identifiability conditions and (b) data informativity conditions

- Network identifiability is identification method - independent
  Reflects choice of predictor model:

  – presence and location of excitation and disturbance signals

  – parametrized model set (fixed modules and disturbance correlations)