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Value of information in parameter identification and optimization of hydrocarbon reservoirs

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Abstract: This paper describes a recently introduced methodology to perform value of information (VOI) analysis within a closed-loop reservoir management (CLRM) framework, and adds a first step to improve the computational efficiency of the procedure. CLRM is a combination of model-based optimization and model-parameter identification applied to large-scale models of subsurface hydrocarbon reservoirs. The approach is illustrated with a simple two-dimensional model of an oil reservoir produced with water injection. The results are compared with previous work on other measures of information valuation. We show that our method is a more complete, although also more computationally intensive, approach to VOI analysis in a CLRM framework. We recommend it to be used as the reference for the development of more practical and less computationally demanding tools for VOI assessment in real fields.

Keywords: value of information, value of clairvoyance, decision making, uncertainty, closed-loop control, model-based optimization, EnKF, parameter estimation, reservoir management, well production data.

1. INTRODUCTION

Over the past decades, numerical techniques for model-based optimization and 'history matching' (i.e. parameter identification) of subsurface hydrocarbon reservoirs have developed rapidly, while it also has become possible to obtain increasingly detailed reservoir information by deploying different types of well-based sensors and field-wide sensing methods. Many of these technologies come at significant costs, and an assessment of the associated value of information (VOI) becomes therefore increasingly important. In particular assessing the value of future measurements during the field development planning (FDP) phase of an oil field requires techniques to quantify the VOI under geological uncertainty. An additional complexity arises when it is attempted to quantify the VOI for closed-loop reservoir management (CLRM), i.e., under the assumption that frequent life-cycle optimization will be performed using frequently updated reservoir models. Recently we introduced a new methodology to assess the VOI in a such a CLRM context (Barros et al, 2014). Here we repeat the description, and, in addition, propose a modification to improve the computational efficiency of the procedure.

In the Background section we introduce the most relevant concepts and review some previous work on information measures. Next, in the Methodology section, we present the proposed workflow for VOI analysis and thereafter, in the Examples section, we illustrate it with some case studies in which the results of the VOI calculations are analyzed. Finally, in the Discussion and conclusion section, we address the computational aspects of applying this workflow to real field cases, and we suggest a direction for further research.

2. BACKGROUND

2.1 Closed-loop reservoir management

CLRM is a combination of frequent life-cycle production optimization and parameter identification (also known as 'data assimilation' or 'computer-assisted history matching'). Life-cycle optimization aims at maximizing a financial measure, typically net present value (NPV), over the producing life of the reservoir by optimizing the production strategy. This may involve well location optimization, or, in a more restricted setting, optimization of well rates and pressures for a given configuration of wells, on the basis of one or more numerical reservoir models. History matching involves modifying the parameters of one or more reservoir models, or the underlying geological models, with the aim to improve their predictive capacity, using measured data from a potentially wide variety of sources such as production data or time-lapse seismics. For further information on CLRM see, e.g., Jansen et al. (2005, 2008, 2009), Naevdal et al. (2006), Sarma et al. (2008); Chen et al. (2009) and Wang et al. (2009).

2.2 Robust optimization

An efficient model-based optimization algorithm is one of the required elements for CLRM. Because of the inherent uncertainty in the geological characterization of the subsurface, a non-deterministic approach is necessary. Robust life-cycle optimization uses one or more ensembles of geological realizations (reservoir models) to account for uncertainties and to determine the production strategy that maximizes a given objective function over the ensemble; see, e.g., Yeten et al. (2003) or Van Essen et al (2009). The objective function J_{NPV} is defined as

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$$J_{NPV} = \mu_{NPV} - \lambda \sigma_{NPV} , \quad (1)$$

where μ_{NPV} and σ_{NPV} are the ensemble mean (expected value) and the ensemble standard deviation of the objective function values J_i of the individual realizations:

$$\mu_{NPV} = \frac{1}{N} \sum_{i=1}^{N} J_i , \quad (2)$$

$$\sigma_{NPV} = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} \left(J_i - \mu_{NPV} \right)^2} . \quad (3)$$

The symbol λ in equation (1) is a risk attitude parameter to represent risk-averse or risk-prone decision strategies with positive or negative values respectively. The objective function J_i, for a single realization i is defined as

$$J_i = \int_{t=0}^{T} \frac{q_o(t,\mathbf{m}_i)\, r_o - q_{wp}(t,\mathbf{m}_i)\, r_{wp} - q_{wi}(t,\mathbf{m}_i)\, r_{wi}}{(1+b)^{t/\tau}} dt , \quad (4)$$

where $\mathbf{m}_i$ is a realization of the vector of uncertain model parameters (e.g. grid block permeabilities or fault multipliers), t is time, T is the producing life of the reservoir, q_o is the oil production rate, q_{wp} is the water production rate, q_{wi} is the water injection rate, r_o is the price of oil produced, r_{wp} is the cost of water produced, r_{wi} is the cost of water injected, b is the discount factor expressed as a fraction per year, and τ is the reference time for discounting (typically one year). The outcome of the optimization procedure is a vector $\mathbf{u}$ containing the settings of the control variables over the producing life of the reservoir. Note that, although the optimization is based on N models, only a single strategy $\mathbf{u}$ is obtained. Typical elements of $\mathbf{u}$ are monthly or quarterly settings of well head pressures, water injection rates, valve openings etc.

2.3 Data assimilation

Efficient data assimilation algorithms are also an essential element of CLRM. Many methods for reservoir-focused data assimilation have been developed over the past years, and we refer to Oliver et al. (2008), Evensen (2009), Aanonsen et al. (2009) and Oliver and Chen (2011) for overviews. An essential component of data assimilation is accounting for uncertainties, and it is generally accepted that this is best done in a Bayesian framework:

$$p(\mathbf{m} \mid \mathbf{d}) = \frac{p(\mathbf{d} \mid \mathbf{m})\, p(\mathbf{m})}{p(\mathbf{d})} , \quad (5)$$

where p indicates the probability density, and $\mathbf{d}$ is a vector of measured data (e.g. oil and water flow rates or saturation estimates from time-lapse seismic). In equation (5) the terms $p(\mathbf{m})$ and $p(\mathbf{m}|\mathbf{d})$ represent the prior and posterior probabilities of the model parameters $\mathbf{m}$, which are, in our setting, represented by initial and updated ensembles respectively. The underlying assumption in data assimilation is that a reduced uncertainty in the model parameters leads to and improved predictive capacity of the models, which, in turn, leads to improved decisions. In our CLRM setting, decisions take the form of control vectors $\mathbf{u}$, aimed at maximizing the objective function J.

2.4 Information valuation

Previous work on information valuation in reservoir engineering focused on analyzing how additional information impacts the model predictions. One way of valuing information is proposed by Krymskaya et al. (2010). They use the concept of observation impact, which was first introduced in atmospheric modelling. Starting from a Bayesian framework, they derive an observation sensitivity matrix, which contains self and cross-sensitivities (diagonal and off-diagonal elements of the matrix, respectively). The self-sensitivities, which quantify how much the observation of measured data impacts the prediction of these same data by a history-matched model, provide a measure of the information content in the data.

Another approach is taken by Le et al. (2014) who address the usefulness of information in terms of the reduction in uncertainty of a variable of interest (e.g. NPV). They introduce a method to estimate, in a computationally feasible way, how much the assimilation of an observation contributes to reducing the spread in the predictions of the variable of interest, expressed as the difference between P10 and P90 percentiles, i.e. between the 10% and 90% cumulative probability density levels.

Both approaches are based on data assimilation to obtain a posterior ensemble which forms the basis to compute various measures of information valuation. In this case, the measurements are obtained in the form of synthetic data generated by a synthetic truth. This preempts our proposed method of information valuation in which we will use an ensemble of models in the FDP stage, of which each realization will be selected as a synthetic truth in a consecutive set of twin experiments.

2.5 VOI and decision making

The two studies that we referred to above (Krymskaya et al., 2010 and Le et al., 2014) only measure the effect of additional information on model predictions and do not explicitly take into account how the additional information is used to make better decisions. In these studies it is simply assumed that history-matched models automatically lead to better decisions. However, there seems to be a need for a more complete framework to assess the VOI, including decision making, in the context of reservoir management. VOI analysis originates from the field of decision theory. It is an abstract concept, which makes it into a powerful tool with many potential applications, although implementation can be complicated.

An early reference to VOI originates from Howard (1966) who considered a bidding problem and was one of the first to formalize the idea that information could be economically valued within a context of decision under uncertainties. Since then, several applications have appeared in many different fields, including the petroleum industry. Bratvold et al. (2009) produce an extensive literature review on VOI in the oil industry and also identify several potential misconceptions and misunderstandings in the use of VOI analysis. Through examples with a petroleum-oriented perspective they show how a VOI analysis should be carried out rigorously. They affirm that “VOI attributes no value to ‘uncertainty reduction’ or ‘increased confidence’” and that “value is added by enabling the decision maker to better 'tune' his/her choice to the underlying uncertainty". Thus, their main message is that "one cannot value information outside of a particular decision context", and they continue "The fundamental question for any information-gathering process is then whether the likely improvement in decision making is worth the cost of obtaining the information." (All citations from Bratvold et al., 2009). Finding an answer to this question is the ultimate goal that drives the work described in this paper.

3. METHODOLOGY

In our setting, decisions in CLRM take the form of optimizing the production strategy **u** under uncertainty which involves repeated robust optimization of a large number of variables: the vector **u** typically has tens to hundreds of elements and needs to be updated when new information becomes available. As noted by Bratvold et al. (2009), in many cases the reported work on VOI in the petroleum industry is related to other types of decisions and uncertainties. Most of the examples are about whether to drill or not to drill a well in a certain location (Bhattacharjya et al., 2010), or about whether a fault is sealing or not. These problems contain limited numbers of decision alternatives and uncertainty scenarios. The tools used to solve them involve decision trees and influence diagrams, which are feasible when dealing with binary or simple discrete scenarios. The CLRM problem seems to contain too many variables to be approached in the same way. However, the question to be answered is essentially the same and so should be the conceptual approach.

Reducing uncertainty in a model prediction has no value by itself, and therefore one cannot assign a value to information without modelling the decisions that are made based on the model forecasts. VOI is decision-dependent. We therefore propose to combine data assimilation and decision making (in the form of optimization) to create a more complete workflow to value information. By doing that, we intend to not only quantify how information changes knowledge (through data assimilation), but also how it influences the results of decision making (through optimization).

In the proposed workflow, the analysis is performed in the design phase – when no real data are yet available. Note that classical CLRM is performed during the operation of the field whereas we are considering here an a-priori evaluation of the value of CLRM (i.e. in the design phase). The workflow starts with a prior ensemble of realizations which characterizes the initial uncertainty associated with the model parameters. From this ensemble, one realization is selected to be the synthetic truth and the remaining realizations form the prior ensemble for a robust optimization procedure to maximize the economic value of the ensemble. The resulting strategy is applied to the synthetic truth, and synthetic data from the analyzed measurements are generated by running a reservoir simulation over the specified control time interval (typically one or more months). With these, data assimilation is performed and a posterior ensemble obtained. As a next step robust optimization is carried out on this posterior to find an new optimal production strategy (from the time the data became available to the end of the reservoir life-cycle), and the procedure is repeated while gradually progressing over the producing life of the reservoir in time steps equal to the specified control time interval. The exercise of matching data generated by a synthetic 'truth' model, a common practice in the data assimilation community, is in this way extended to include the effects of the model updates on the reservoir management actions.

The strategies obtained for the prior and the posterior ensembles are then tested on the synthetic truth and their economic outcomes (NPV values $J_{NPV,prior}$ and $J_{NPV,post}$) are evaluated. The difference between these outcomes is a measure of the VOI incorporated through the CLRM procedure for this particular choice of the synthetic truth.

The choice of one of the realizations to be the synthetic truth in the procedure is completely random. In fact, because the analysis is conducted during the FDP phase, any of the models in the initial ensemble could be selected to be the 'truth'. Note that this also implies that the VOI is a random variable. One of the underlying assumptions of our proposed workflow is that the truth is captured by the initial ensemble. Hence, the methodology only allows to quantify the VOI under uncertainty in the form of known unknowns. Obviously, specifying uncertainty in the form of unknown unknowns is impossible, which therefore is a fundamental shortcoming in any VOI analysis.

Because any of the N models in the initial ensemble could be the truth, the procedure has to be repeated N times, consecutively letting each one of the initial models act as the synthetic truth. This allows us to quantify the expected VOI over the entire ensemble:

$$\bar{J}_{NPV} = \frac{1}{N}\sum_{i=1}^{N}\left(J^{i}_{NPV,post} - J^{i}_{NPV,prior}\right). \tag{6}$$

We note that in the literature on VOI, most of the times the term VOI is used to refer to the expected VOI. The flowchart in Fig. A.1 (Appendix A) shows the complete procedure including the aforementioned repetition.

The workflow can be adapted to compute the expected value of clairvoyance (VOC), which gives a feeling for the technical limit (i.e. the maximum possible expected VOI) that could be obtained from measurements. In this case, data assimilation does not form part of the loop. Instead, perfect information is assumed to become available through a revelation of the truth at a certain moment in time. Such a clairvoyance implies the availability of completely informative data without observation errors, and the expected VOC therefore forms a theoretical upper bound to the expected VOI. Moreover, because this modified workflow does not require data assimilation, and, after the truth has been revealed, only requires optimization of a single (true) model, it is computationally significantly less demanding.

4. EXAMPLE

As a next step, we applied the proposed VOI workflow to a simple reservoir simulation model representing a two-dimensional (2D) inverted five-spot water flooding configuration; see Fig. 1. In a 21 × 21 grid (700 × 700 m), with heterogeneous permeability and porosity fields, the model simulates the displacement of oil to the producers in the corners by the water injected in the center. Table 1 lists the values of the physical parameters of the reservoir model. We used $N = 50$ realizations of the porosity and permeability fields, conditioned to hard data in the wells, to model the geological uncertainties. The simulations were used to determine the set of well controls (bottom hole pressures) that maximizes the NPV. The economic parameters considered in this example are also indicated in Table 1. The optimization was run for a 1500-day time horizon with well controls updated every 150 days, i.e. $M = 10$, and, with five wells, u has 50 elements. We applied bound constraints to the optimization variables (200 bar $\leq p_{prod} \leq$ 300 bar and 300 bar $\leq p_{inj} \leq$ 500 bar). The whole exercise was performed in the open-source reservoir simulator MRST (Lie et al., 2012), by modifying the adjoint-based optimization module to allow for robust optimization and combining it with the EnKF module to create a CLRM environment for VOI analysis. The average NPV for the ensemble is \$ 65.1 million when using baseline control (fixed bottom hole pressures: 400 bar in the injector and 250 bar in the producers) and \$ 70.2 million when using robust optimization over the prior (i.e. without additional information). The workflow was repeated for different observation times, t_{data} = {150, 300, … , 1350} days. For this 2D model we assessed the VOI of the production data (total flow rates and water-cuts) with absolute measurement errors (ε_{flux} = 5 m³/day and ε_{wct} = 0.1). The VOI and the VOC were computed for each of the nine observation times.

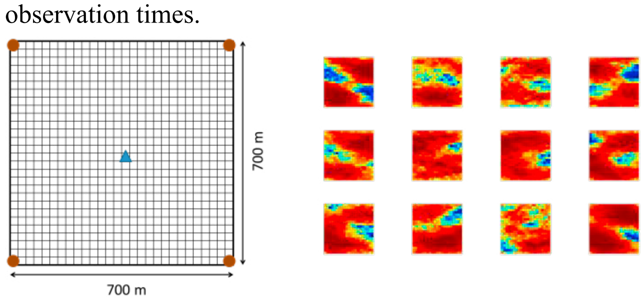

Fig. 1. 2D five-spot model (left); 12 randomly chosen realizations of the uncertain permeability field (right).

Table 1. Parameter values for 2D five-spot model

Rock-fluid parameters		Initial conditions	
$\rho_o = 800$	kg/m³	$p_0 = 300$	bar
$\rho_w = 1{,}000$	kg/m³	$S_{oi} = 0.8$	[–]
$\mu_o = 0.5$	cP	$S_{wi} = 0.2$	[–]
$\mu_w = 1$	cP		
$n_o = 2$	[–]	**Economic parameters**	
$S_{or} = 0.2$	[–]	$r_o = 80$	\$/bbl
$k_{ro,or} = 0.9$	[–]	$r_{wp} = 5$	\$/bbl
$n_w = 2$	[–]	$r_{wi} = 5$	\$/bbl
$S_{wc} = 0.2$	[–]	$b = 0.15$	[–]
$k_{rw,wc} = 0.6$	[–]		

Figs. 2 and 3 depict the results of the analysis for production data. Dashed lines correspond to expected values and solid lines to percentiles quantifying the uncertainty of the information measures. The markers correspond to the observation times at which the analysis was carried out. In Fig. 2 we note that clairvoyance loses value with observation time, following a stepwise behavior. In addition, by observing the percentiles, we realize that, in this case, the VOC has a non-symmetric probability distribution. The high values of P10 indicate that, for some realizations of the truth, knowing the truth can be considerably more valuable than indicated by the expected VOC; however, the P50 values, which are always below those of the expected VOC, indicate what is more likely to occur. The same holds for the VOI, as can be observed in Fig. 3. Here, Px is defined as the probability that x % of the outcomes exceeds this value. The observation that provides the best VOI is the one at t_{data} = 150 days, followed by a second modest peak at t_{data} = 450 days. The non-monotonous decrease of the VOI may be caused by the nature of the optimization and parameter identification procedures (which search for a local optimum). However, some of it may also be caused by different observations having different VOI (e.g. water measurements before and after water breakthrough). Note that in our example the earliest observation seems to be the most valuable one, but that this may be case-specific.

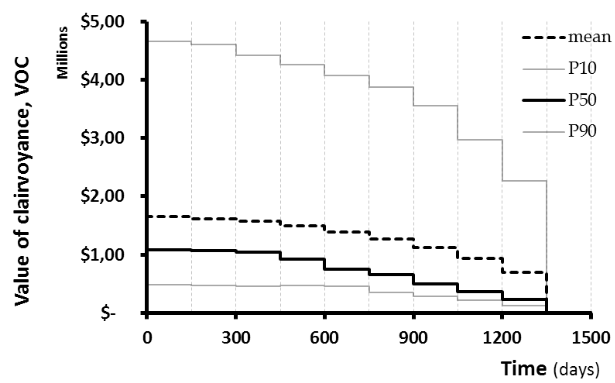

Fig. 2. Value of clairvoyance (VOC) as a function of clairvoyance time in the 2D model.

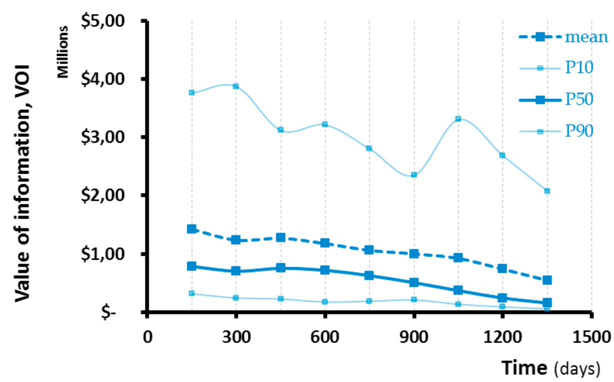

Fig. 3. Value of information (VOI) as a function of observation time for production data in the 2D model.

Fig. 4 depicts the expected values of VOI (blue dots) and VOC (black line). The plot confirms that clairvoyance can be considered the technical limit for any information gathering strategy and that the expected VOC forms an upper-bound to the expected VOI. We also note that the expected VOI comes closer to the expected VOC with time. Indeed, as water breakthrough is observed in more producers, the production data of this five-spot pattern become more effective in revealing the main features of the true permeability and porosity fields.

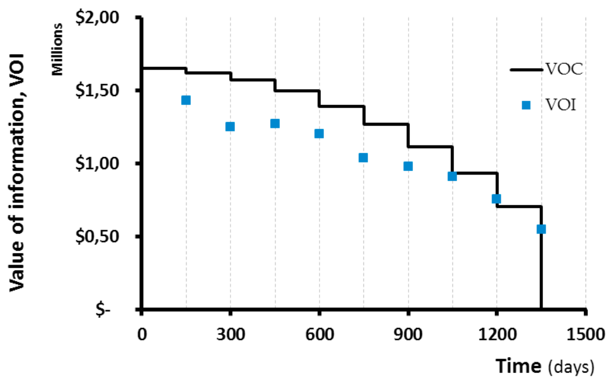

Fig. 4. Results for the 2D model; the expected VOI is upper-bounded by expected VOC.

Using the proposed workflow as the reference for VOI assessment, for this case, we recommend the production data to be collected at t_{data} = 150 days and we estimate this additional information to be worth $ 1.4 million.

Another important finding after running all the simulations is that the optimal production strategies obtained for the different prior ensembles are almost equal. Fig. 5 depicts the optimal well controls, in the form of bottom-hole pressures (BHP), for one of the producers in the 2D model example. This occurs because, for every repetition of the VOI procedure (Fig. A.1), the prior ensembles differ only by one realization; and, when we are dealing with considerably large ensemble sizes (N = 50 in the example), replacing only one realization tends to have a minor impact on the ensemble. Consequently, the outcome of the robust optimization over these different prior ensembles is almost the same. These results suggest that there is an opportunity to reduce the number of simulations required in the proposed workflow. For instance, in our example, we could reduce the number of prior robust optimizations from 50 to 1, which represents a significant improvement regarding the computational costs associated with the VOI assessment procedure: approximately 420,000 simulations for the original formulation and 215,000 for the modified formulation to compute the VOI for one observation time; and approximately 2,100,000 simulations for the original formulation and 1,895,000 simulations for the modified formulation to compute all the VOI values depicted in Fig. 4 (9 observation times).

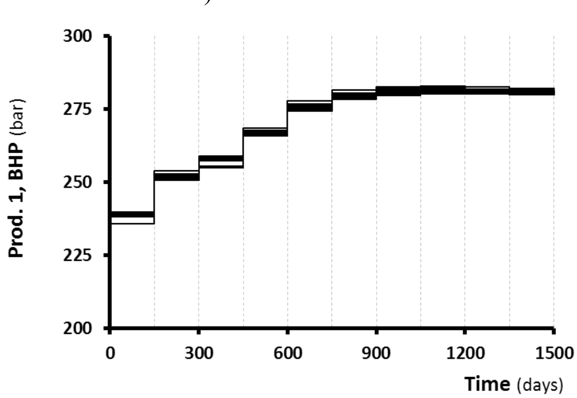

Fig. 5. Optimal well controls (BHP) at producer 1 for the 50 different prior ensembles in the 2D model example.

5. DISCUSSION AND CONCLUSION

We proposed a new workflow for VOI assessment in CLRM. The method uses elements available in the CLRM framework, such as history matching and robust optimization. First, we identified the opportunity to combine these elements with concepts of information value theory to create a VOI analysis instrument. We then designed a generic procedure that can, in theory, be simply implemented in a variety of applications, including our optimal reservoir management problem. Next, the workflow was illustrated with an example. Because we take into account that the production strategy is updated periodically after new information has been assimilated in the models, we believe that our proposed method is more complete than previous work to estimate the VOI in a reservoir engineering context.

The main drawback of our proposed VOI workflow is its computational costs; it involves the repeated application of robust optimization and data assimilation, which requires a very large number of reservoir simulations. Depending on the types of optimization and data assimilation methods used (e.g. adjoint-based, ensemble-based, or gradient-free) there may be large differences in the computational requirements, but even in case of using the most efficient (i.e. adjoint-based) algorithms, the computational load of the workflow will be huge. Hence, if the method is to be applied to real-field cases, some serious improvements regarding the number of simulations required are necessary. In this paper, we showed a first step in this direction by suggesting a way to reduce the number of robust optimizations necessary. However, more has to be done. One potential method could be to use clustering techniques to select a few representative realizations rather than a full ensemble. Furthermore, reduced-order modelling or response surface techniques to generate surrogate models could facilitate the application of our workflow to larger reservoir models by reducing the number of full reservoir simulations. Despite its computational cost, we conclude that our approach constitutes a rigorous VOI assessment for CLRM. For this reason, we recommend that it be used as the reference for the development of more practical and less computationally demanding tools to be applied in real-field cases.

6. ACKNOWLEDGEMENTS

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The EnKF module for MRST was developed by Olwijn Leeuwenburgh (TNO) and can be obtained from http://www.isapp2.com/data-sharepoint/enkf-module-for-mrst.

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Appendix A. WORKFLOW FOR VOI ASSESSMENT

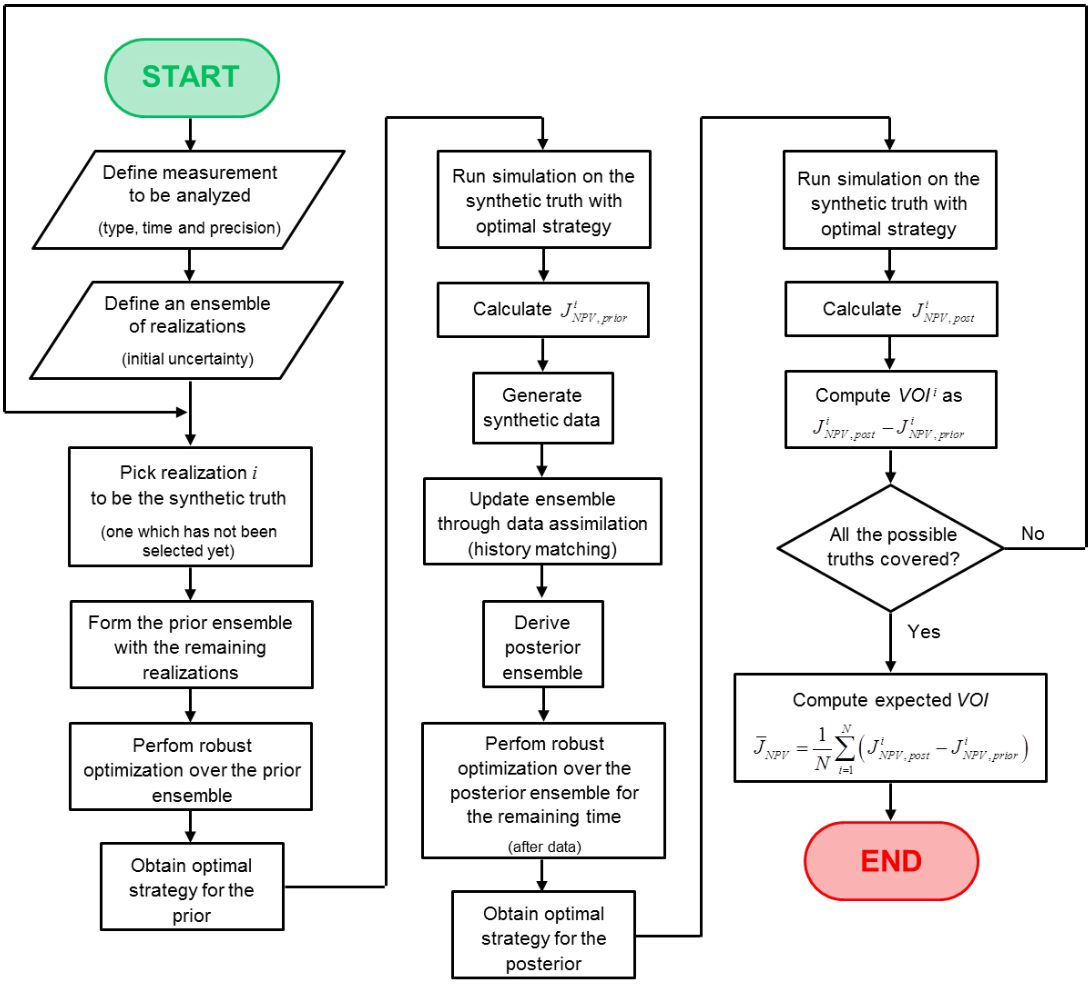

Fig. A.1. Proposed workflow to compute the expected VOI.