

Closed-loop Performance Diagnosis Using Prediction Error Identification

Ali Mesbah, Xavier Bombois, Jobert H.A. Ludlage and Paul M.J. Van den Hof

Abstract—This paper presents a methodology to detect the origin of closed-loop performance degradation of model-based control systems. The approach exploits the statistical hypothesis testing framework. The decision rule consists of examining if an identified model of the true system lies in a set containing all models that fulfill the closed-loop performance requirements. This allows us to determine whether performance degradation arises from changes in system dynamics or from variations in disturbance characteristics. The probability of making an erroneous decision is estimated a posteriori using the known distribution of the identified model with respect to the unknown true system.

I. INTRODUCTION

The life time performance of model-based control systems such as model predictive controllers and real-time dynamic optimizers is often limited. This primarily arises from the quality of their underlying models that affect the closed-loop performance. Models are almost always prone to plant-model mismatch. In addition, various changes typically occur in system dynamics over time that may increase the mismatch and consequently invalidate the models identified at the commissioning stage of these control systems.

Performance monitoring and diagnosis comprises a crucial step in maintenance of model-based control systems. In the event of performance degradation, diagnostic tools should allow us to verify if the unsatisfactory closed-loop operation results from plant-model mismatch. Hypothesis testing can be utilized to assess whether an observed deviation from nominal performance is due to a system change and/or variations in disturbance characteristics. Hypothesis testing is a classical statistical methodology to make a decision between contradictory hypotheses by comparing their likelihood of occurrence [10]. The foundations of research on the use of hypothesis testing for performance diagnostics have primarily been laid by Basseville and her coworkers, who proposed a systematic approach for on-line fault detection and isolation; see [2] and the references therein. Huang and Tamayo [9] extended the so-called asymptotic local approach presented in [3] by deriving a new detection statistic for on-line model validation of model predictive control systems.

The research leading to these results has received funding from the European Union's Seventh Framework Programme (FP7/2007-2013) under grant agreement 257059 (www.fp7-autoprofit.eu).

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Gustafsson and Graebe [8] were first to highlight the need to detect control relevant system changes in closed-loop operation and to distinguish them from variations in disturbances. They applied hypothesis testing to examine whether an observed performance degradation results from a system change that has deteriorated the closed-loop stability margins. A new closed-loop stability criterion was defined in order to use a standard CUSUM change detector. Owing to the deterministic nature of disturbances, explicit expressions were derived for the probability of mistaking a disturbance for a system change. Recently, Badwe et al. [1] proposed a different methodology for detection and isolation of plant-model mismatch based on the analysis of partial correlations between the model residuals and system inputs. The approach is applicable to MIMO systems. It can however be used when it is decided that the root-cause of poor closed-loop performance is significant plant-model mismatch.

This work presents a novel approach to address the problem of closed-loop performance diagnosis. Similar to [8], the statistical hypothesis testing framework is applied to detect whether performance degradation originates from control relevant system changes or from variations in disturbance characteristics. We exploit prediction error identification to define a decision rule based on which the hypotheses are distinguished. In contrast to [8], the presented approach is capable of dealing with stochastic disturbances.

In the proposed performance diagnosis methodology, we identify a model $G(z, \hat{\theta}_N)$ of the true system $G_0(z)$ using input-output data collected from the existing closed-loop system. The decision rule consists of examining whether the identified model lies in the set \mathcal{D}_{adm} containing all models $G(z)$ that result in a satisfactory closed-loop performance. In case that the identified model does not lie in \mathcal{D}_{adm} , we decide that the observed performance degradation is due to a change in the system dynamics. On the contrary, the deviation from nominal performance arises from variations in disturbance characteristics when $G(z, \hat{\theta}_N) \in \mathcal{D}_{adm}$. The decision rule may however lead to erroneous decisions since $G(z, \hat{\theta}_N)$ is only an estimate of $G_0(z)$. Thus, different procedures are proposed to estimate a posteriori the probability of making a wrong decision. For this purpose, we utilize the known distribution of the identified model with respect to the unknown true system.

II. THE PERFORMANCE DIAGNOSIS METHODOLOGY

The key objective of the performance diagnosis methodology presented in this paper is to detect whether an observed

closed-loop performance deterioration is due to control relevant system changes or due to variations in disturbance characteristics. This is not necessarily the same as on-line model validation and detection of any changes in the system dynamics. We therefore aim to assess the closed-loop performance instead of directly evaluating the model quality.

In this work, we analyze the performance of the closed-loop system depicted in Fig. 1. Our attention is restricted to a stable linear time-invariant single input single output system. The true system is represented as:

$$y(t) = \overbrace{G_0(z)}^{G_0(z)} u(t) + \overbrace{H(z, \theta_0)}^{v(t)} e(t), \quad (1)$$

where $\theta_0 \in \mathbf{R}^k$ is an unknown parameter vector; $e(t)$ is a white noise with variance σ_e^2 ; $G(z, \theta_0)$ and $H(z, \theta_0)$ are stable discrete-time transfer functions. Note that $H(z, \theta_0)$ is assumed to be monic and minimum-phase. In Fig. 1, $r(t)$ represents an excitation signal used for identification.

The performance of a closed-loop system can be expressed in various ways. We adopt the following performance measure for a stable closed-loop system made up of a system $G(z, \theta)$ and an existing controller $C(z)$:

$$J(G, C, W_l, W_r) = \sup_{\omega} \bar{J}(\omega, G, C, W_l, W_r) \quad (2)$$

with

$$\begin{aligned} \bar{J}(\omega, G, C, W_l, W_r) &= \bar{\sigma}(W_l(e^{j\omega})F(G(e^{j\omega}), C(e^{j\omega}))W_r(e^{j\omega})) \\ F(G, C) &\triangleq \begin{pmatrix} \frac{GC}{1+GC} & \frac{G}{1+GC} \\ \frac{1}{1+GC} & \frac{1}{1+GC} \end{pmatrix}, \end{aligned} \quad (3)$$

where $\bar{\sigma}(A)$ denotes the largest singular value of A ; $W_l(z)$ and $W_r(z)$ are chosen diagonal performance weighting filters. It is evident that $J(G, C, W_l, W_r) \leq 1$ ensures that the four entries of $W_l(z)F(G, C)W_r(z)$ have an H_∞ -norm smaller than or equal to one. Note that Eq. (3) gives the most general form of the adopted performance measure. The performance filters can be selected such that the performance measure is expressed as a weighted function of $\frac{C}{1+GC}$ or $\frac{1}{1+GC}$. The latter transfer functions relate the disturbance $v(t)$ to the system input $u(t)$ and system output $y(t)$, respectively.

The weightings W_l and W_r are chosen at the commissioning stage such that any loop $[C \ G]$ achieving $J(G, C, W_l, W_r) \leq 1$ is able to reject adequately the disturbance $v(t)$. This is to ensure that signals $u(t)$ and $y(t)$ have a sufficiently small variance in the presence of the disturbance $v(t)$ to fulfill some pre-specified requirements.

The controller $C(z)$ in the closed-loop system of Fig. 1 has been constructed based on the knowledge of system dynamics at the commissioning stage. The controller $C(z)$ stabilizes $G_0(z)$ and ensures the nominal performance level:

$$J(G, C, W_l, W_r) \leq 1 \quad (4)$$

with $G = G_0(z)$. The loop $[C \ G_0]$ at commissioning thus exhibits a satisfactory performance in terms of coping with the disturbance $v(t)$.

To present our performance diagnosis methodology, we first introduce the sets \mathcal{D}_{adm} and \mathcal{V}_J .

Definition 1: Given the existing controller $C(z)$, the region \mathcal{D}_{adm} is the set of all transfer functions $G(z)$ that are stabilized by $C(z)$ and achieve the nominal performance $J(G, C, W_l, W_r) \leq 1$.

Definition 2: The set \mathcal{V}_J contains the power spectrum $\Phi_v(\omega)$ of all disturbances $v(t)$ which are sufficiently rejected by all loops $[C \ G]$ satisfying $J(G, C, W_l, W_r) \leq 1$. The disturbance $v(t)$ is deemed to be sufficiently rejected by a loop if the corresponding input and output signals have a reasonably small variance in accordance with the pre-specified requirements.

At the commissioning stage, the controller $C(z)$ ensures that $G_0 \in \mathcal{D}_{adm}$ and $\Phi_v(\omega) \in \mathcal{V}_J$. In the course of operation, situations may arise that the system dynamics $G_0(z)$ and/or the disturbance spectrum $\Phi_v(\omega)$ change. In the event of an observed performance drop, i.e. an increase in the variance of input and output signals, one of the following two scenarios holds:

- 1) the system dynamics $G_0(z)$ remains in \mathcal{D}_{adm} which implies that the disturbance spectrum $\Phi_v(\omega)$ no longer lies in \mathcal{V}_J ;
- 2) the system dynamics $G_0(z)$ moves outside \mathcal{D}_{adm} .

Therefore, the detection problem under study in this paper is to decide which one of the following hypotheses holds when a performance drop is observed:

$$\begin{aligned} \mathcal{H}_0 &: G_0(z) \in \mathcal{D}_{adm} \\ \mathcal{H}_1 &: G_0(z) \notin \mathcal{D}_{adm}. \end{aligned} \quad (5)$$

In a nutshell, the hypothesis test can be restated as:

$$\begin{aligned} \mathcal{H}_0 &: \text{performance drop does not result from changes in } G_0(z) \\ \mathcal{H}_1 &: \text{performance drop results from changes in } G_0(z). \end{aligned} \quad (6)$$

Remark: Under the null hypothesis \mathcal{H}_0 , the system dynamics $G_0(z)$ is not necessarily the same as that at commissioning, but the eventual changes in $G_0(z)$ do not lead to a degraded performance level, i.e. $J(G_0, C, W_l, W_r)$ remains smaller than or equal to 1. On the contrary, $J(G_0, C, W_l, W_r) > 1$ under the alternative hypothesis. When \mathcal{H}_1 is true, the disturbance spectrum may not be identical to that at the commissioning stage as $\Phi_v(\omega)$ might have also moved outside \mathcal{V}_J . In this case, we could consider a subsequent hypothesis test distinguishing between $\Phi_v(\omega) \in \mathcal{V}_J$ and $\Phi_v(\omega) \notin \mathcal{V}_J$. This hypothesis test can be performed in a similar way as discussed in this paper.

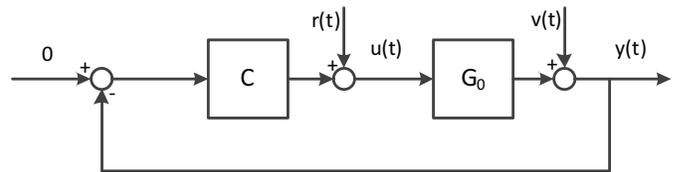


Fig. 1. The closed-loop system $[C \ G_0]$.

To be able to discriminate between the two hypotheses in Eq. (5), an identification experiment is performed in closed-loop with the existing controller to identify a model of the unknown true system $G_0(z)$. The identification experiment consists of exciting the system with a sequence $r(t)$ ($t = 0 \cdots N-1$), as depicted in Fig. 1, and generating for instance the data set $Z^N = \{u(t), y(t) | t = 0 \cdots N-1\}$. It is assumed that we can construct a full order model structure $\mathcal{M} = \{G(z, \theta), H(z, \theta)\}$ such that θ_0 is the only value of the parameter vector for which $\{G(z, \theta), H(z, \theta)\}$ represents the true system. The identified parameter vector $\hat{\theta}_N$ can then be defined as:

$$\hat{\theta}_N = \arg \min_{\theta} \frac{1}{N} \sum_{t=0}^{N-1} \epsilon^2(t, \theta), \quad (7)$$

where $\epsilon(t, \theta) = H(z, \theta)^{-1}(y(t) - G(z, \theta)u(t))$. Provided that N is sufficiently large, $\hat{\theta}_N$ is asymptotically normally distributed around the true parameter vector θ_0 . This suggests that $\hat{\theta}_N \sim \mathcal{N}(\theta_0, P_{\theta})$ with P_{θ} being a strictly positive definite matrix:

$$P_{\theta} = \frac{\sigma_e^2}{N} (\mathbb{E}[(\frac{\partial \epsilon(t, \theta)}{\partial \theta}|_{\theta_0})(\frac{\partial \epsilon(t, \theta)}{\partial \theta}|_{\theta_0})^T])^{-1}, \quad (8)$$

which can be estimated from $\hat{\theta}_N$ and Z^N [11].

We use the identified model $G(z, \hat{\theta}_N)$ to determine the most likely hypothesis. The decision rule that allows us to decide between \mathcal{H}_0 and \mathcal{H}_1 is formulated as:

$$\begin{aligned} G(z, \hat{\theta}_N) \in \mathcal{D}_{adm} &\Rightarrow \text{choose } \mathcal{H}_0 \\ G(z, \hat{\theta}_N) \notin \mathcal{D}_{adm} &\Rightarrow \text{choose } \mathcal{H}_1. \end{aligned} \quad (9)$$

A decision rule is typically characterized by probabilities that quantify the probability of making wrong and right choices. Fig. 2 illustrates the two possibilities of making an erroneous decision in the decision rule stated in Eq. (9). As can be seen, there is a risk that we opt for \mathcal{H}_0 while the true system $G_0(z)$ does not lie in \mathcal{D}_{adm} . Clearly this is a wrong decision as the closed-loop performance degradation arises from changes in the system dynamics, whereas we have attributed the cause of deviation from the nominal performance to variations in $\Phi_v(\omega)$. On the other hand, \mathcal{H}_1 might be chosen erroneously when in effect \mathcal{D}_{adm} includes the true system $G_0(z)$ as shown in Fig. 2(b). This is also a wrong decision since the system dynamics are not responsible for the performance deterioration.

To assess the quality of our decision a posteriori, we first use the confidence region that is constructed based on $\hat{\theta}_N \sim \mathcal{N}(\theta_0, P_{\theta})$. The confidence region $\mathcal{D}(\hat{\theta}_N, P_{\theta})$ is a set of systems $G(z, \theta)$ centered around the identified model $G(z, \hat{\theta}_N)$. The set contains the unknown true system $G_0(z)$ at a pre-specified probability level α [7]:

$$\begin{aligned} \mathcal{D}(\hat{\theta}_N, P_{\theta}) &= \{G(z, \theta) | \theta \in U, \\ U &= \{\theta | (\theta - \hat{\theta}_N)^T P_{\theta}^{-1} (\theta - \hat{\theta}_N) < \mathcal{X}\}. \end{aligned} \quad (10)$$

In Eq. (10), \mathcal{X} is a real constant such that

$$Pr(\chi^2(k) < \mathcal{X}) = \alpha, \quad (11)$$

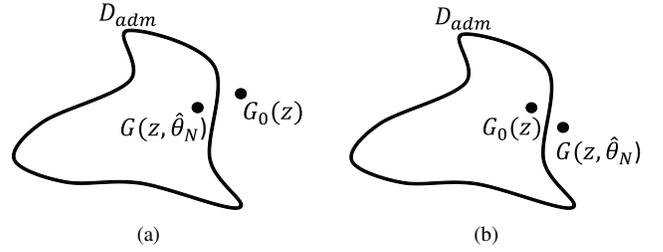


Fig. 2. The possibilities of making an erroneous decision in the decision rule stated in Eq. (9).

where $\chi^2(k)$ is a chi-square distribution with k degrees of freedom. In fact, $\mathcal{D}(\hat{\theta}_N, P_{\theta})$ comprises a set of transfer functions that are parameterized by the real vector θ belonging to an uncertainty ellipsoid. In case we can verify that for instance not only $G(z, \hat{\theta}_N)$ lies in \mathcal{D}_{adm} , but also the whole set $\mathcal{D}(\hat{\theta}_N, P_{\theta})$ is within \mathcal{D}_{adm} , the confidence in our choice of \mathcal{H}_0 will at least be equal to the probability level α . This test can be performed by computing the so-called worst and best case performance achieved over all closed-loop systems made up of the existing controller $C(z)$ and the systems lying in $\mathcal{D}(\hat{\theta}_N, P_{\theta})$.

Definition 3: Consider an uncertainty set $\mathcal{D}(\hat{\theta}_N, P_{\theta})$ of the parameterized transfer functions $G(z, \theta)$. For the existing controller $C(z)$, the worst and best case performance achieved over all systems in $\mathcal{D}(\hat{\theta}_N, P_{\theta})$ are defined as:

$$J_{WC}(G, C, W_l, W_r) = \sup_{\omega} \bar{J}_{WC}(\omega, G, C, W_l, W_r) \quad (12)$$

and

$$J_{BC}(G, C, W_l, W_r) = \sup_{\omega} \bar{J}_{BC}(\omega, G, C, W_l, W_r), \quad (13)$$

respectively, where

$$\bar{J}_{WC}(\omega, G, C, W_l, W_r) = \max_{G(z, \theta) \in \mathcal{D}} \bar{J}_{(\omega, G, C, W_l, W_r)} \quad (14)$$

$$\bar{J}_{BC}(\omega, G, C, W_l, W_r) = \min_{G(z, \theta) \in \mathcal{D}} \bar{J}_{(\omega, G, C, W_l, W_r)}. \quad (15)$$

A procedure to compute the worst and best case performance is described in Section III.

Since $G_0(z) \in \mathcal{D}(\hat{\theta}_N, P_{\theta})$, the worst and best case performance represent a lower bound and an upper bound for the closed-loop performance achieved with the true system, respectively. These bounds allow us to assess the probability of making a wrong decision by examining whether the worst and best case performance satisfy the nominal performance level, i.e. $J(G, C, W_l, W_r) \leq 1$. Hence, we define the following likelihood measure on the basis of the decision rule given in Eq. (9):

$$\begin{aligned} \text{If } \mathcal{H}_0 \text{ is chosen, verify } J_{WC} &\leq 1 \\ \text{If } \mathcal{H}_1 \text{ is chosen, verify } J_{BC} &> 1. \end{aligned} \quad (16)$$

In case that the above measure holds for the chosen hypothesis, the probability of making a correct decision is at least α , i.e. the probability level at which $\mathcal{D}(\hat{\theta}_N, P_{\theta})$ contains

$G_0(z)$. Eq (16) indicates that when the worst case performance meets the nominal performance level, i.e. Eq (4), the likelihood that variations in $\Phi_v(\omega)$ cause the observed performance degradation is at the probability level α . This results from the fact that the uncertainty region $\mathcal{D}(\hat{\theta}_N, P_\theta)$ lies entirely in \mathcal{D}_{adm} and therefore the probability that \mathcal{D}_{adm} contains $G_0(z)$ is at least α . On the contrary, if $J_{BC} > 1$, $\mathcal{D}(\hat{\theta}_N, P_\theta)$ is fully outside \mathcal{D}_{adm} . The latter suggests that the deviation from nominal performance is due to changes in the true system, i.e. $G_0(z) \notin \mathcal{D}_{adm}$, for a probability of at least α .

Nonetheless, the likelihood measure stated in Eq. (16) is a conservative tool to determine the chances of making a wrong decision. Situations may occur that the uncertainty region $\mathcal{D}(\hat{\theta}_N, P_\theta)$ lies neither entirely in \mathcal{D}_{adm} nor entirely outside \mathcal{D}_{adm} . For instance in Fig. 3, \mathcal{D}_{adm} fully contains $\mathcal{D}(\hat{\theta}_N, P_\theta)$ only for small probability levels, whereas the probability that $G(z, \hat{\theta}_N)$ has been generated by a true system $G_0(z)$ inside \mathcal{D}_{adm} can be much larger. Thus, the likelihood measure becomes misleading particularly when $G(z, \hat{\theta}_N)$ lies close to the boundary of \mathcal{D}_{adm} . We therefore employ the following procedure to estimate the actual probability of making a wrong decision directly based on the distribution of the identified parameter vector, i.e. $\hat{\theta}_N \sim \mathcal{N}(\theta_0, P_\theta)$. This procedure no longer relies on the uncertainty region $\mathcal{D}(\hat{\theta}_N, P_\theta)$.

The probability of making an erroneous decision when opting for \mathcal{H}_0 and \mathcal{H}_1 can be expressed as:

$$Pr_{\mathcal{H}_0}\{G(z, \hat{\theta}_N) \text{ generated by } G_0(z) \notin \mathcal{D}_{adm}\} \quad (17)$$

and

$$Pr_{\mathcal{H}_1}\{G(z, \hat{\theta}_N) \text{ generated by } G_0(z) \in \mathcal{D}_{adm}\}, \quad (18)$$

respectively. $Pr_{\mathcal{H}_0}$ is the probability that $G(z, \hat{\theta}_N)$ has been generated by a true system, which does not exhibit satisfactory closed-loop performance. It therefore represents the likelihood of making a wrong decision when $G(z, \hat{\theta}_N) \in \mathcal{D}_{adm}$ as shown in Fig. 2(a). On the contrary, Eq. (18) corresponds to the probability of making an erroneous decision when \mathcal{H}_1 is chosen; see Fig. 2(b). $Pr_{\mathcal{H}_1}$ denotes the probability that $G(z, \hat{\theta}_N)$ has been generated by a true system lying in \mathcal{D}_{adm} . Hence, $Pr_{\mathcal{H}_1}$ represents the probability of attributing the performance degradation to changes in the system dynamics while in effect $G_0(z) \in \mathcal{D}_{adm}$.

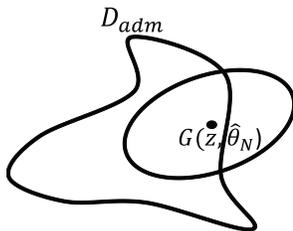


Fig. 3. The situation in which the measure stated in Eq. (16) will become conservative to assess the probability of making an erroneous decision.

As \mathcal{D}_{adm} cannot be explicitly described, the probability of making a wrong decision is approximated by the use of randomized algorithms; see, e.g., [12]. We utilize the procedure presented in [6] to estimate $Pr_{\mathcal{H}_0}$ and $Pr_{\mathcal{H}_1}$. According to the known distribution $\Delta\theta = \hat{\theta}_N - \theta_0 \sim \mathcal{N}(0, P_\theta)$, we generate n random realizations $\Delta\theta^{(i)}$ ($i = 1 \dots n$) of $\Delta\theta$ and subsequently construct the parameter vectors $\theta^{(i)} = \hat{\theta}_N + \Delta\theta^{(i)}$ ($i = 1 \dots n$). The latter parameter vectors in fact include possible values for the unknown true parameter vector θ_0 that parameterizes the true system $G_0(z)$. This allows us to estimate the probabilities $Pr_{\mathcal{H}_0}$ and $Pr_{\mathcal{H}_1}$ as:

$$\begin{aligned} \hat{Pr}_{\mathcal{H}_0} &= \frac{\text{Number of realizations when } G(z, \theta^{(i)}) \notin \mathcal{D}_{adm}}{n} \\ \hat{Pr}_{\mathcal{H}_1} &= \frac{\text{Number of realizations when } G(z, \theta^{(i)}) \in \mathcal{D}_{adm}}{n}. \end{aligned} \quad (19)$$

It is proven in [12] that if we choose

$$n > \frac{1}{2\epsilon^2} \ln \frac{2}{\delta}, \quad (20)$$

the probability that $|\hat{Pr}_{\mathcal{H}_j} - Pr_{\mathcal{H}_j}| > \epsilon$ ($j = 0, 1$) will be smaller than δ .

III. CLOSED-LOOP PERFORMANCE ANALYSIS

It is demonstrated that closed-loop performance analysis can be used as a tool to assess the quality of our decision. In this section, we present a procedure to compute the best case performance achieved over all closed-loop systems made up of the existing controller and systems lying in an uncertainty set $\mathcal{D}(\hat{\theta}_N, P_\theta)$. The performance of a closed-loop system $[C \ G]$ is defined as the largest singular value of a weighted version of the matrix containing the four closed-loop transfer functions, i.e. Eq. (2). This implies that our description of the worst and best case performance given in Definition 3 is general since the most commonly used performance measures can be derived by an appropriate choice of the weightings.

Bombois et al. [4] presented a procedure to compute the worst case performance. They showed that the worst case performance achieved over all systems in such an uncertainty set is the solution of a convex optimization problem involving LMI constraints. In the sequel, it is demonstrated that the best case performance $\bar{J}_{BC}(\omega, G, C, W_l, W_r)$ can be computed exactly at each frequency by translating Eq. (15) into an LMI-based optimization problem. We exploit the fact that all model structures can be rewritten as:

$$G(z, \theta) = \frac{Z_N(z)\theta}{1 + Z_D(z)\theta}, \quad (21)$$

where Z_N and Z_D are row vectors containing delays and zeros. It is evident that the numerator and denominator of the transfer function $G(z, \theta)$ are linearly dependent on the uncertain parameter vector. The following proposition summarizes the procedure to compute $\bar{J}_{BC}(\omega, G, C, W_l, W_r)$.

Proposition 4: Consider an uncertainty set $\mathcal{D}(\hat{\theta}_N, P_\theta)$ and a controller $C(z) = X(z)/Y(z)$. The best case closed-loop

performance achieved at a frequency ω over all systems in $\mathcal{D}(\hat{\theta}_N, P_\theta)$ is $\sqrt{\gamma_{opt}}$, where γ_{opt} is the solution of the following convex optimization problem involving LMI constraints evaluated at the frequency ω :

$$\begin{aligned} & \max_{\gamma, \tau} \gamma \\ & \text{s.t.: } \tau \geq 0 \\ & \begin{pmatrix} \text{Re}(a_{11}) & \text{Re}(a_{12}) \\ \text{Re}(a_{12}^*) & \text{Re}(a_{22}) \end{pmatrix} - \tau \begin{pmatrix} R & -R\hat{\theta} \\ (-R\hat{\theta})^T & \hat{\theta}^T R \hat{\theta} - 1 \end{pmatrix} < 0, \end{aligned} \quad (22)$$

where

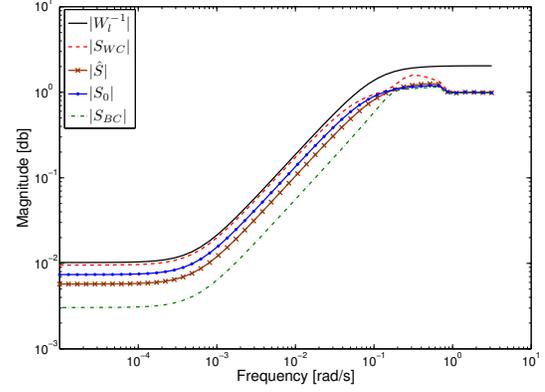
$$\begin{aligned} a_{11} &= \gamma(QZ_1^*Z_1) - (Z_N^*W_{l1}^*W_{l1}Z_N - Z_D^*W_{l2}^*W_{l2}Z_D) \\ a_{12} &= \gamma(QZ_1^*Y) - W_{l2}^*W_{l2}Z_D^* \\ a_{22} &= \gamma(QY^*Y) - W_{l2}^*W_{l2} \\ Z_1 &= XZ_N + YZ_D \\ Q &= \frac{1}{X^*W_{r1}^*W_{r1}X + Y^*W_{r2}^*W_{r2}Y} \\ R &= \frac{P_\theta^{-1}}{\mathcal{X}}. \end{aligned} \quad (23)$$

Proof. The proof is similar to that of the worst case performance given in [4]. ■

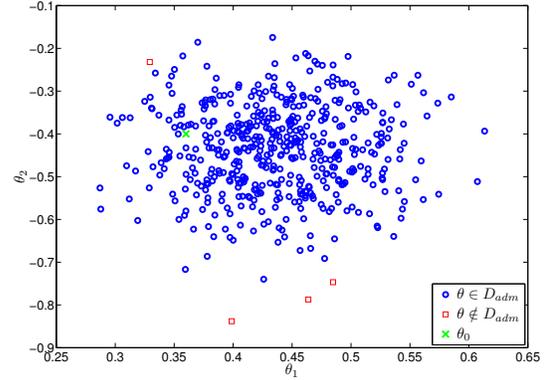
IV. NUMERICAL ILLUSTRATIONS

The performance diagnosis methodology is applied to a simulation case study. Two scenarios are considered to demonstrate the adequacy of the proposed methodology under different circumstances. We take the following Box-Jenkins system as the true system: $y(t) = B_0(z)/F_0(z)u(t) + C_0(z)/D_0(z)e(t)$ with $B_0(z) = 0.36z^{-1}$, $F_0(z) = 1 - 0.4z^{-1}$, $C_0(z) = 1 + 0.6z^{-1}$, $D_0(z) = 1 + 0.1z^{-1}$, and $e(t)$ being a realization of a white noise process with variance $\sigma_e^2 = 1.0$. The control performance measure of interest is related to the sensitivity function. Hence, the filters in Eq. (3) are chosen as $W_l(z) = \text{diag}(0, W(z))$ and $W_r(z) = \text{diag}(0, 1)$ with $W(z) = (0.52 - 0.46z^{-1})/(1 - 0.99z^{-1})$. The true system is in closed-loop operation with a controller which has been devised based on the 4-block H_∞ control design method. Note that the nominal performance level is initially satisfied. The variance of the system output is originally 1.44.

Scenario 1: We alter the disturbance characteristics of the true system by changing the parameters of the noise transfer function, i.e. $C(z) = 1 + 0.75z^{-1}$ and $D(z) = 1 - 0.8z^{-1}$. This results in a drastic change in the variance of the system output, i.e. 6.74. The performance diagnosis methodology is applied to verify whether it can detect the original cause of the observed deviation from nominal performance. We first identify a model of the true system by applying a white noise excitation signal with $\sigma_r^2 = 0.1$ and measuring 500 samples of the signals $\{u(t), y(t)\}$ under the closed-loop operation. The signal to noise ratio is approximately equal to that of the nominal operation. This implies that the excitation signal barely disturbs the system output and therefore the identification cost is reasonably low.



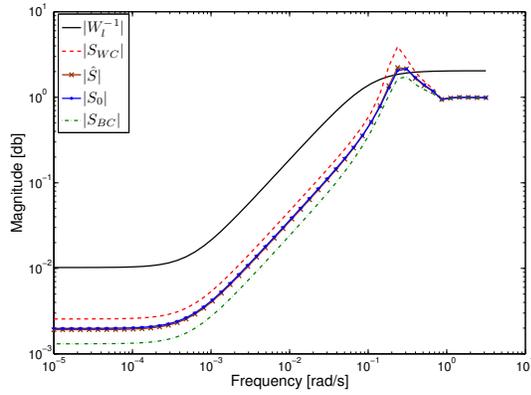
(a) Closed-loop performance: Modulus of the sensitivity function



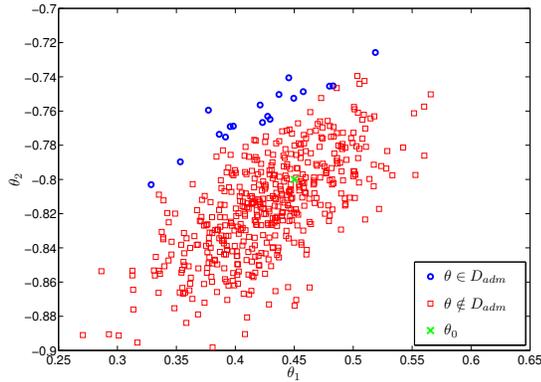
(b) Depiction of the probability of making an erroneous decision

Fig. 4. Scenario 1: $G(z, \hat{\theta}_N) \in \mathcal{D}_{adm}$.

It appears that the identified model lies in \mathcal{D}_{adm} . We therefore choose \mathcal{H}_0 in the decision rule stated in Eq. (9), attributing the performance degradation to the changes in $\Phi_v(\omega)$. To assess the quality of our decision a posteriori, we analyze the worst case performance according to the likelihood measure given in Eq. (16). Fig. 4(a) shows the modulus of sensitivity functions. In fact, $|S|$ allows us to examine the closed-loop performance at different frequencies. As can be seen, the identified as well as the true modulus of the sensitivity function is bounded by the worst and best case performance levels. It is evident that $J_{WC} < 1$ since the worst case modulus of the sensitivity function, i.e. $|S_{WC}|$, over all systems in $\mathcal{D}(\hat{\theta}_N, P_\theta)$ obtained for the pre-specified probability level 95% is smaller than $|W_l^{-1}|$. This suggests that the probability that we have opted for the correct hypothesis is at least 95%, i.e. the probability level at which $G_0(z) \in \mathcal{D}(\hat{\theta}_N, P_\theta)$. We also estimate the actual probability of making an erroneous decision $\hat{P}_{r\mathcal{H}_0}$. Fig. 4(b) depicts 500 realizations of the parameter vector θ computed around $\hat{\theta}_N$ according to the known normal distribution. This figure indicates that over 99% of the generated parameter vectors lead to a system $G(z, \theta)$ that lies in \mathcal{D}_{adm} . Therefore, the chances that the identified model $G(z, \hat{\theta}_N)$ has been generated by a true system outside \mathcal{D}_{adm} are in fact less



(a) Closed-loop performance: Modulus of the sensitivity function



(b) Depiction of the probability of making an erroneous decision

Fig. 5. Scenario 2: $G(z, \hat{\theta}_N) \notin \mathcal{D}_{adm}$.

than 1%, more precisely $\hat{Pr}_{\mathcal{H}_0} = 0.8\%$.

Scenario 2: We induce changes in the system dynamics by adopting $G_0(z) = 0.36z^{-1}/(1 - 0.8z^{-1})$ as the true system while the disturbance characteristics remain intact. The changes in the system dynamics lead to a slight variation in the variance of the output, i.e. 1.70. We exploit the performance diagnosis methodology to examine if the alteration to the original system can be detected. Like in Scenario 1, the identification exercise is performed using a white noise excitation signal with $\sigma_r^2 = 0.1$ to collect 500 samples of the signals $\{u(t), y(t)\}$. The signal to noise ratio in this case is somewhat larger than the nominal operation, suggesting a higher identification cost.

The identified model of the true system lies outside \mathcal{D}_{adm} . This implies that \mathcal{H}_1 is the most likely hypothesis. Thus, we analyze the best case performance to assess the probability of making a wrong decision. The modulus of sensitivity functions is shown in Fig. 5(a). It is evident that the modulus of the best case sensitivity function, i.e. $|S_{BC}|$, over all systems in $\mathcal{D}(\hat{\theta}_N, P_\theta)$ when $\alpha = 95\%$ is smaller than $|W_l^{-1}|$. The use of the likelihood measure given in Eq. (16) can therefore be conservative since $J_{BC} < 1$. We estimate the actual probability $Pr_{\mathcal{H}_1}$ by 500 realizations of θ around the identified parameter vector $\hat{\theta}_N$ as depicted in Fig. 5(b). It

appears that about 94% of the generated parameter vectors result in a system that is outside \mathcal{D}_{adm} . This suggests that the probability that $G(z, \hat{\theta}_N)$ has been generated by a true system lying in \mathcal{D}_{adm} , i.e. $Pr_{\mathcal{H}_1}$, is approximately 6%.

V. CONCLUSIONS

We have presented a novel methodology to distinguish control relevant system changes in closed-loop operation from variations in disturbance characteristics. The approach consists of verifying whether an identified model of the true system lies in a set containing all models that exhibit satisfactory closed-loop performance. We have exploited the known distribution of the identified model to characterize a posteriori our confidence in the detected origin of performance degradation.

In future, this work will be extended for performance diagnosis of model predictive controllers. In addition, we will investigate the use of prior information to reduce the identification costs associated with this methodology.

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