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Experiment Design for Closed-loop Performance Diagnosis ⋆

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Abstract: This paper uses prediction error identification to distinguish control-relevant system changes in closed-loop operation from variations in disturbance characteristics. The approach consists of a hypothesis test to verify whether an identified model of the true system lies in a set containing all models that exhibit adequate closed-loop performance. To increase the detection probability, i.e. the probability of choosing the correct hypothesis, experiment design is performed to devise an excitation signal for closed-loop identification of the system dynamics. For a given identification cost, this allows us to maximize the probability that an identified model of the system lies in the performance-related region of interest in accordance with the hypothesis test and, therefore, decrease the probability of opting for an erroneous hypothesis.

1. INTRODUCTION

Performance assessment of controllers is of paramount importance in practical control applications. Various changes often occur in system dynamics over time, e.g. changes in equipment or operating conditions. Such variations typically lead to deterioration of the existing controller which originally performed satisfactorily at the commissioning stage. Performance monitoring and diagnosis is therefore a crucial step in maintenance of control systems to restore the closed-loop performance of existing controllers.

The problem of performance monitoring from closed-loop data has received considerable attention in the literature. The research on performance monitoring is primarily based on the performance benchmark of a minimum variance controller [Harris, 1989]; see, e.g., [Qin, 1998] for an extensive survey. On the other hand, very few researchers have explored the performance diagnostics aspects in maintenance of control systems. Performance diagnostic tools should allow us to assess whether an observed deviation from the nominal performance is due to a system change and/or variations in disturbance characteristics.

The foundations of research on performance diagnosis have been laid by Basseville and her coworkers who proposed a systematic approach for on-line fault detection and isolation; see [Basseville, 1998] and the references therein. The so-called local approach transforms the detection problems related to a parameterized stochastic process into the universal problem of monitoring the mean of a Gaussian vector. Therefore, the local approach is particularly suited to detect any changes in the system under study. In performance assessment of most control applications, we however aim to verify whether an observed closed-loop performance drop is due to control-relevant system changes. This is not necessarily the same as on-line model validation and detection of any changes in the system dynamics.

Recently, Mesbah et al. [2011] have proposed a novel methodology for closed-loop performance diagnosis using prediction error identification. They exploit hypothesis testing to distinguish control-relevant system changes in closed-loop operation from variations in disturbance characteristics. The hypothesis testing framework is a classical statistical methodology to make decisions between contradictory hypotheses by comparing their probability of occurrence [Kay, 1998]. The proposed decision rule consists of verifying whether an identified model $G(z, \hat{\theta}_N)$ of the true system $G_0(z)$ lies in a set $\mathcal{D}_{adm}$ containing all models that exhibit satisfactory closed-loop performance. The closed-loop performance at commissioning is deemed to be satisfactory when the system disturbances are adequately rejected, i.e. system inputs and outputs have a sufficiently small variance. Clearly when the identified model lies in $\mathcal{D}_{adm}$, it can be decided that the observed performance degradation arises from variations in the disturbance characteristics. On the contrary, the deviation from nominal performance is due to control-relevant system changes in case that $G(z, \hat{\theta}_N) \notin \mathcal{D}_{adm}$.

A similar approach as in [Mesbah et al., 2011] has been utilized by [Tyler and Morari, 1996] for the problem of performance monitoring using routine closed-loop operating data. They defined a complicated hypothesis test in terms of constraints on the impulse response coefficients of the closed-loop transfer function. Based on the closed-loop performance criterion, the performance assessment problem was formulated as a generalized ratio test that involved identifying the true system. Gustafsson and Graebe [1998] have also applied the hypothesis testing framework

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to examine whether an observed performance drop results from a system change that has deteriorated the closed-loop stability margins. They designed a standard CUSUM change detector on the basis of a closed-loop stability criterion. The specific definition of the closed-loop performance measure however restricts the applicability of the methodology.

This work aims to explore the use of experiment design as a means to increase the detection probability in the closed-loop performance diagnosis methodology presented in [Mesbah et al., 2011]. The decision rule of the diagnosis methodology may lead to erroneous decisions as it relies on an identified model $G(z, \hat{\theta}_N)$ of the true system. We exploit experiment design [Gevers, 2005] to devise the excitation signal used for identifying the system dynamics. For a prespecified closed-loop identification cost, the experiment design problem is intended to maximize the probability that an identified model of the true system lies in the performance-related region of interest, i.e. either inside or outside $\mathcal{D}_{adm}$ depending on the cause of the performance drop. A proper design of the excitation signal will therefore allow us to increase the probability of opting for the correct decision and, consequently, increase the detection probability.

2. PERFORMANCE DIAGNOSIS

The performance diagnosis methodology is intended to detect whether an observed closed-loop performance degradation, i.e. an increase in the variance of system inputs and outputs, originates from control-relevant system changes or from variations in disturbance characteristics. We restrict our attention to stable linear time-invariant single input single output systems. The true system is represented as follows:

$$y(t) = \underbrace{G(z,\theta_0)}_{G_0(z)}u(t) + \underbrace{H(z,\theta_0)e(t)}_{v(t)}, \tag{1}$$

where $\theta_0 \in \mathbf{R}^k$ is an unknown parameter vector; $e(t)$ is a white noise signal with variance σ_e^2; $G(z,\theta_0)$ and $H(z,\theta_0)$ are stable discrete-time transfer functions. Furthermore, $H(z,\theta_0)$ is assumed to be monic and minimum-phase.

We analyze the performance of the closed-loop system shown in Fig. 1; $r(t)$ represents an excitation signal used for identification. In this work, the performance of a stable closed-loop system made up of a system $G(z,\theta)$ and an existing controller $C(z)$ is expressed as:

$$J(G,C,W_l,W_r) = \sup_{\omega} \bar{J}(\omega,G,C,W_l,W_r) \tag{2}$$

with

$$\bar{J}(\omega,G,C,W_l,W_r) = \bar{\sigma}\Big(W_l(e^{j\omega})F\big(G(e^{j\omega}),C(e^{j\omega})\big)W_r(e^{j\omega})\Big)$$
$$F(G,C) \triangleq \begin{pmatrix} \dfrac{GC}{1+GC} & \dfrac{G}{1+GC} \\ \dfrac{C}{1+GC} & \dfrac{1}{1+GC} \end{pmatrix}, \tag{3}$$

where $\bar{\sigma}(A)$ denotes the largest singular value of A; $W_l(z)$ and $W_r(z)$ are chosen diagonal performance weighting filters. In fact, the performance measure of Eq. (2) quantifies the ability of the loop $[C\ G]$ in coping with some system disturbances. The performance level is satisfactory when $J(G,C,W_l,W_r) \leq 1$. Note that the performance filters can be selected such that the performance measure is stated as a weighted function of $\frac{C}{1+GC}$ or $\frac{1}{1+GC}$. This allows us to relate the disturbance $v(t)$ to the system input $u(t)$ and system output $y(t)$, respectively.

The hypothesis test of the performance diagnosis methodology is based on the following definitions.

Definition 1: Given the existing controller $C(z)$ in the closed-loop system of Fig. 1, the region $\mathcal{D}_{adm}$ is the set of all transfer functions $G(z)$ that are stabilized by $C(z)$ and achieve the nominal performance $J(G,C,W_l,W_r) \leq 1$.

Definition 2: The set $\mathcal{V}_J$ contains the power spectrum $\Phi_v(\omega)$ of all disturbances $v(t)$ which are sufficiently rejected by all loops $[C\ G]$ that satisfy $J(G,C,W_l,W_r) \leq 1$.

It follows from the aforementioned Definitions that the performance of a closed-loop system $[C\ G]$ cannot be described merely based on the performance measure $J(G,C,W_l,W_r)$ as the performance is also dependent on the disturbance spectrum $\Phi_v(\omega)$. Therefore, the closed-loop performance of the system is deemed to be satisfactory when $G(z,\theta) \in \mathcal{D}_{adm}$ and $\Phi_v(\omega) \in \mathcal{V}_J$.

At the commissioning stage, the controller $C(z)$ has been constructed such that it stabilizes $G_0(z)$ and achieves the nominal performance level, i.e. $J(G,C,W_l,W_r) \leq 1$ with $G = G_0(z)$, while the system disturbances are adequately rejected. This implies that the loop $[C\ G_0]$ at commissioning ensures $G_0(z) \in \mathcal{D}_{adm}$ and $\Phi_v(\omega) \in \mathcal{V}_J$.

Nonetheless, various changes typically occur in the system $G_0(z)$ and/or the disturbance spectrum $\Phi_\nu(\omega)$ that may lead to increased variance of input and output signals. In the event of such a closed-loop performance degradation, one of the following scenarios holds:

(1) the system $G_0(z)$ remains in $\mathcal{D}_{adm}$, suggesting that the disturbance spectrum $\Phi_v(\omega)$ no longer lies in $\mathcal{V}_J$;
(2) the system $G_0(z)$ moves outside $\mathcal{D}_{adm}$.

Hence, the hypothesis test of the detection problem under study is stated as:

$$\begin{aligned} \mathcal{H}_0 &: G_0(z) \in \mathcal{D}_{adm} \\ \mathcal{H}_1 &: G_0(z) \notin \mathcal{D}_{adm}. \end{aligned} \tag{4}$$

Further details of the performance diagnosis methodology can be found in [Mesbah et al., 2011].

Remark 1: In certain practical control applications, the cause of an observed performance drop can be readily detected on the basis of expert knowledge on the system. In such cases, the choice of the true hypothesis is straightforward.

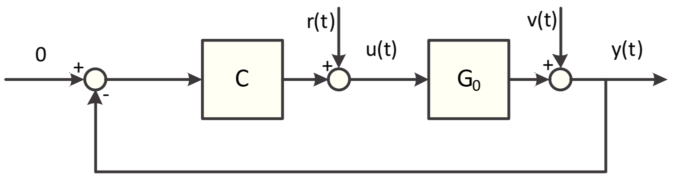


Fig. 1. The closed-loop system $[C\ G_0]$.

3. DECISION RULE AND EXPERIMENT DESIGN

3.1 Decision Rule

To apply the hypothesis test given in Eq. (4), we should identify the unknown true system $G_0(z)$ in closed-loop operation with the existing controller $C(z)$. In Fig. 1, the signal $r(t)$ is zero in normal operation but can be used to excite the system for a closed-loop identification experiment. By applying an excitation signal $r(t)$ for $(t = 0, \dots, N-1)$ to the closed-loop system and measuring signals $\{u(t), y(t)\,|\, t = 0, \dots, N-1\}$, a model $\{G(z, \hat{\theta}_N), H(z, \hat{\theta}_N)\}$ of the true system can be identified using prediction error identification. We shall assume throughout this paper that a full order model structure can be constructed such that θ_0 is the only value of the parameter vector for which $\{G(z,\theta), H(z,\theta)\}$ represents the true system. The identified parameter vector is defined as:

$$\hat{\theta}_N = \arg\min_{\theta} \frac{1}{N} \sum_{t=0}^{N-1} \epsilon^2(t,\theta), \tag{5}$$

where $\epsilon(t,\theta) = H(z,\theta)^{-1}(y(t) - G(z,\theta)u(t))$. Note that $\epsilon(t,\theta)$ depends on the excitation signal $r(t)$ via the measured signals $\{u(t), y(t)\}$, i.e.

$$y(t) = S_0 v(t) + \underbrace{G_0 S_0 r(t)}_{y_r(t)} \tag{6}$$

$$u(t) = -CS_0 v(t) + \underbrace{S_0 r(t)}_{u_r(t)} \tag{7}$$

with S_0 being the sensitivity function. Assuming that the closed-loop identification experiment is sufficiently informative, the identified parameter vector $\hat{\theta}_N$ is asymptotically normally distributed around the true parameter vector θ_0, i.e. $\hat{\theta}_N \sim \mathcal{N}(\theta_0, P_\theta)$ with

$$P_\theta = \frac{\sigma_e^2}{N}\left(\mathbb{E}\Big[\big(\frac{\partial \epsilon(t,\theta)}{\partial \theta}|_{\theta_0}\big)\big(\frac{\partial \epsilon(t,\theta)}{\partial \theta}|_{\theta_0}\big)^T\Big]\right)^{-1} \tag{8}$$

being a strictly positive definite matrix that can be estimated from $\hat{\theta}_N$ and the measured signals $\{u(t), y(t)\}$ [Ljung, 1999].

Once a model $G(z, \hat{\theta}_N)$ of the true system is identified, we can utilize the hypothesis test of Eq. (4) to detect the cause of the closed-loop performance degradation. The decision rule that allows us to decide between $\mathcal{H}_0$ and $\mathcal{H}_1$ is stated as:

$$\begin{aligned} G(z,\hat{\theta}_N) \in \mathcal{D}_{adm} &\Rightarrow \text{choose} \quad \mathcal{H}_0 \\ G(z,\hat{\theta}_N) \notin \mathcal{D}_{adm} &\Rightarrow \text{choose} \quad \mathcal{H}_1. \end{aligned} \tag{9}$$

The above decision rule may however lead to erroneous decisions since $G(z, \hat{\theta}_N)$ is an estimate of the true system $G_0(z)$. For instance, there is a risk that we choose $\mathcal{H}_0$, whereas the identified model $G(z, \hat{\theta}_N) \in \mathcal{D}_{adm}$ has been generated by a true system outside $\mathcal{D}_{adm}$. Clearly this is an erroneous decision as the deviation from nominal performance originates from changes in the system dynamics, while the decision rule attributes the performance drop to variations in $\Phi_\nu(\omega)$. On the contrary, the alternative hypothesis $\mathcal{H}_1$ may be opted for erroneously when $G(z, \hat{\theta}_N) \notin \mathcal{D}_{adm}$ has been generated by $G_0(z) \in \mathcal{D}_{adm}$. This is in effect a wrong decision as the performance degradation does not result from changes in the system dynamics.

3.2 Input Design

In this work, we aim to maximize the probability of making a correct decision according to the decision rule of Eq. (9) for a predetermined closed-loop identification cost. This is done by designing the power spectrum $\Phi_r(\omega)$ of the excitation signal $r(t)$ to maximize the probability that an identified model $G(z, \hat{\theta}_N)$ of the true system $G_0(z)$ lies in the region of interest, i.e. either inside or outside $\mathcal{D}_{adm}$, and, consequently, maximize the detection probability of the decision rule. We design the power spectrum $\Phi_r(\omega)$ for a fixed experiment duration N and an admissible identification cost β to keep $u_r(t)$ and $y_r(t)$ sufficiently small. The cost of the closed-loop identification experiment is expressed as:

$$\mathcal{J}_r = \beta_y\Big(\frac{1}{2\pi}\int_{-\pi}^{\pi} \Phi_{y_r}(\omega) d\omega\Big) + \beta_u\Big(\frac{1}{2\pi}\int_{-\pi}^{\pi} \Phi_{u_r}(\omega) d\omega\Big), \tag{10}$$

where $\Phi_{y_r}(\omega)$ and $\Phi_{u_r}(\omega)$ are the power spectra of the disturbance signals $y_r(t)$ and $u_r(t)$, respectively; see Eqs. (6)-(7). In Eq. (10), β_y and β_u are some arbitrarily chosen scalars. The user specified admissible experimentation cost allows us to limit the perturbations caused by the excitation signal $r(t)$ during an identification experiment.

We restrict our attention to excitation signals whose power spectrum $\Phi_r(\omega)$ can be defined as:

$$\Phi_r(\omega) = R_r(0) + 2\sum_{i=1}^{m} R_r(i) cos(i\omega) \geq 0 \quad \forall \omega, \tag{11}$$

where m is a positive integer. The parameters $R_r(i)$ with $(i = 0, \dots, m)$ can be regarded as the auto-correlation sequence of a signal which has been generated by a white noise signal passing through an FIR filter of length $m+1$. The spectrum given in Eq. (11) allows us to ensure that the closed-loop identification experiment is sufficiently informative and, therefore, the assumption $\hat{\theta}_N \sim \mathcal{N}(\theta_0, P_\theta)$ holds.

In the sequel, the problem of experiment design is addressed for each of the hypotheses stated in the decision rule. Note that the choice of the experiment design problem to be solved depends on the unknown system dynamics $G_0(z)$.

Situation 1: When $G_0(z)$ lies in $\mathcal{D}_{adm}$, the power spectrum $\Phi_r(\omega)$ of the excitation signal is designed such that it maximizes the probability that $G(z, \hat{\theta}_N) \in \mathcal{D}_{adm}$. Under the null hypothesis $\mathcal{H}_0$, the experiment design problem is therefore formulated as:

$$\begin{aligned} &\max_{R_r(i)} \quad Pr\Big(G(z,\hat{\theta}_N) \in \mathcal{D}_{adm}\Big) \\ &\text{s.t.:} \quad \mathcal{J}_r < \beta \end{aligned} \tag{12}$$

with $R_r(i)$ for $(i = 0, \dots, m)$ being the coefficients of the power spectrum $\Phi_r(\omega)$ defined in Eq. (11).

Situation 2: If $G_0(z)$ does not lie in $\mathcal{D}_{adm}$, the experiment design problem aims to maximize the probability that an identified model $G(z, \hat{\theta}_N)$ of the true system will lie outside the set $\mathcal{D}_{adm}$. Hence, in the case of the alternative hypothesis $\mathcal{H}_1$ the experiment design problem is expressed as:

$$\begin{aligned} \max_{R_r(i)} \quad & Pr\Big(G(z,\hat{\theta}_N) \notin \mathcal{D}_{adm}\Big) \\ \text{s.t.:} \quad & \mathcal{J}_r < \beta. \end{aligned} \tag{13}$$

3.3 The Choice of the Initial Model

As in every experiment design problem, design of the power spectrum $\Phi_r(\omega)$ requires the knowledge of the true system $G_0(z)$ to be identified. The fact that $G_0(z)$ is not known in practice poses an additional difficulty in the experiment design problems under study. This is due to the fact that the choice of the experiment design problem, i.e. Situation 1 or 2, depends on the selected hypothesis in the decision rule of Eq. (9) and, consequently, on $G_0(z)$. Hence, this is a 'chicken and egg' problem where the knowledge of the unknown $G_0(z)$ determines the cause of the performance drop and, thereof, how we can maximize the detection probability by a proper design of $\Phi_r(\omega)$.

There are various ways to alleviate the 'chicken and egg' problem. The most common procedure is to perform closed-loop identification with a white noise excitation signal to arrive at an initial model estimate using prediction error identification, i.e. Eq. (5). The initial model estimate will subsequently be exploited to conduct the experiment design according to either Situation 1 or 2 in order to maximize the detection probability of the decision rule. Alternatively, the Bayesian approach to system identification [Eykhoff, 1974] can be applied to utilize prior knowledge of the system in combination with the routine closed-loop data to identify an initial model of the true system.

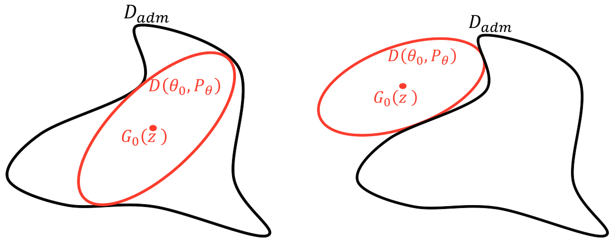
We propose another approach to circumvent the 'chicken and egg' problem. In the two scenario approach, we consider two hypothetical scenarios:

(1) the performance drop results only from the changes in disturbance characteristics, while the true system dynamics remain identical to that of the commissioning, i.e. $G_0(z) = G(z, \theta_{com})$;
(2) the performance drop results only from the changes in the true system dynamics, while the disturbance characteristics remain identical to that of the commissioning, i.e. $H_0(z) = H(z, \theta_{com})$ and σ_e^2 is intact.

It is self-evident that the above scenarios correspond to the null hypothesis $\mathcal{H}_0$ and the alternative hypothesis $\mathcal{H}_1$, respectively. Under scenario 1 the model used to construct the controller at commissioning can be utilized to describe the true system dynamics, whereas under scenario 2 an estimate of the true system dynamics $G_0(z)$ can be obtained from routine closed-loop data with a reasonable accuracy. In the two scenario approach, these models are exploited as an initial estimate of $G_0(z)$ to perform experiment design according to Situations 1 and 2. The latter experiment design problems in fact correspond to two extreme cases, namely when the performance drop is only due to changes in disturbance characteristics and when it only arises from changes in system dynamics.

3.4 Approximation of the Experiment Design Problems

Numerical solution of the above discussed experiment design problems is cumbersome. Suppose that we can



(a) $\mathcal{H}_0$ (b) $\mathcal{H}_1$

Fig. 2. Approximation of the experiment design problems by using the confidence set $\mathcal{D}(\theta_0, P_\theta, \mathcal{X})$.

construct a confidence set $\mathcal{D}(\theta_0, P_\theta, \mathcal{X})$ centered around $G_0(z)$ on the basis of $\hat{\theta}_N \sim \mathcal{N}(\theta_0, P_\theta)$. The problem of maximizing the probability that $G(z,\hat{\theta}_N)$ lies in a certain region is then approximated by maximizing the probability associated with the size of the confidence set $\mathcal{D}(\theta_0, P_\theta, \mathcal{X})$ within the region of interest, i.e. the region inside $\mathcal{D}_{adm}$ in $\mathcal{H}_0$ or the region outside $\mathcal{D}_{adm}$ in $\mathcal{H}_1$. This is illustrated in Fig. 2. The set $\mathcal{D}(\theta_0, P_\theta, \mathcal{X})$ contains any identified model $G(z,\hat{\theta}_N)$ of the true system $G_0(z)$ at a prespecified probability level α [Gevers, 2005]:

$$\mathcal{D}(\theta_0, P_\theta, \mathcal{X}) = \Big\{ G(z,\hat{\theta}_N) \mid \hat{\theta}_N \in U, \tag{14}$$
$$U = \{\hat{\theta}_N \mid (\hat{\theta}_N - \theta_0)^T P_\theta^{-1} (\hat{\theta}_N - \theta_0) < \mathcal{X}\} \Big\}$$

where $\mathcal{X}$ is a real constant such that

$$Pr\Big(\chi^2(k) < \mathcal{X}\Big) = \alpha \tag{15}$$

with $\chi^2(k)$ being a chi-square distribution with k degrees of freedom.

We use the set $\mathcal{D}(\theta_0, P_\theta, \mathcal{X})$ to recast the experiment design problems of Eqs (12) and (13) as:

$$\begin{aligned} \max_{\mathcal{X}, R_r(i)} \quad & \mathcal{X} \\ \text{s.t.:} \quad & \mathcal{D}(\theta_0, P_\theta, \mathcal{X}) \subseteq \mathcal{D}_{adm} \\ & \mathcal{J}_r < \beta \end{aligned} \tag{16}$$

and

$$\begin{aligned} \max_{\mathcal{X}, R_r(i)} \quad & \mathcal{X} \\ \text{s.t.:} \quad & \mathcal{D}(\theta_0, P_\theta, \mathcal{X}) \subseteq \mathcal{C}_{\mathcal{D}_{adm}} \\ & \mathcal{J}_r < \beta, \end{aligned} \tag{17}$$

respectively, where $\mathcal{C}_{\mathcal{D}_{adm}}$ is the set containing all systems that do not belong to $\mathcal{D}_{adm}$. Note that the covariance matrix P_θ is dependent on the power spectrum of the excitation signal $r(t)$ via Eqs. (6)-(8). Therefore, the largest set $\mathcal{D}(\theta_0, P_\theta, \mathcal{X})$ within the performance-related region of interest can be obtained by designing the power spectrum $\Phi_r(\omega)$ to dictate the orientation of the ellipsoidal region $\mathcal{D}(\theta_0, P_\theta, \mathcal{X})$ via P_θ and, subsequently, by maximizing the real constant $\mathcal{X}$ which governs the size of the ellipsoidal region. Clearly, the detection probability will be increased by maximizing the ellipsoidal set $\mathcal{D}(\theta_0, P_\theta, \mathcal{X})$ in the region inside $\mathcal{D}_{adm}$ and in the region outside $\mathcal{D}_{adm}$ when the hypothesis $\mathcal{H}_0$ and $\mathcal{H}_1$ are chosen, respectively; see Fig. 2.

The performance constraint $\mathcal{D}(\theta_0, P_\theta, \mathcal{X}) \subseteq \mathcal{D}_{adm}$ in Eq. (16) requires that $\bar{J}(\omega, G, C, W_l, W_r) \leq 1 \; \forall \; G(z,\hat{\theta}_N) \in \mathcal{D}(\theta_0, P_\theta, \mathcal{X})$ at all frequencies. It has been demonstrated in

[Bombois et al., 2006] that such performance requirement can be stated as an LMI expression, linear in P_θ^{-1} and $\mathcal{X}$, at one particular frequency ω. In addition, $\mathcal{J}_r$ is shown to be an LMI expression. The experiment design problem of Eq. (16) can therefore be recast as an LMI optimization problem in which the performance constraint has to be computed at all frequencies. Note that in practice we approximate the frequency domain by a finite frequency grid in order to evaluate the LMI constraints at each frequency ω.

$\mathcal{D}(\theta_0, P_\theta, \mathcal{X}) \subseteq \mathcal{C}_{\mathcal{D}_{adm}}$ implies that there exists certain frequencies ω^* where $\bar{J}(\omega^*, G, C, W_l, W_r) > 1 \; \exists \; G(z, \hat{\theta}_N) \in \mathcal{D}(\theta_0, P_\theta, \mathcal{X})$. As in the previous case, the latter performance constraint can be written as an LMI expression at each frequency [Mesbah et al., 2011]. To solve the experiment design problem of Eq. (17), we first perform the following LMI optimization at each frequency ω^* for which $\bar{J}(\omega^*, G_0, C, W_l, W_r) > 1$:

$$\mathcal{X}^*_{opt}(\omega^*) = \arg \max_{\mathcal{X}, R_r(i)} \quad \mathcal{X} \tag{18}$$

$$\text{s.t.:} \quad \bar{J}(\omega^*, G, C, W_l, W_r) > 1 \quad \forall \, G(z,\hat{\theta}_N) \in \mathcal{D}(\theta_0, P_\theta, \mathcal{X})$$

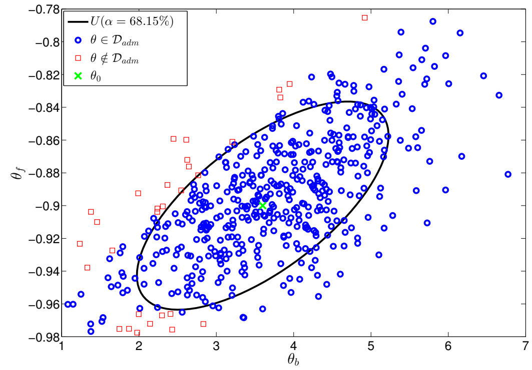
$$\mathcal{J}_r < \beta.$$

The optimum spectrum of the excitation signal will then be the $\Phi_r(\omega)$ that results in the maximum $\mathcal{X}^*_{opt}(\omega^*)$ over all frequencies ω^*.

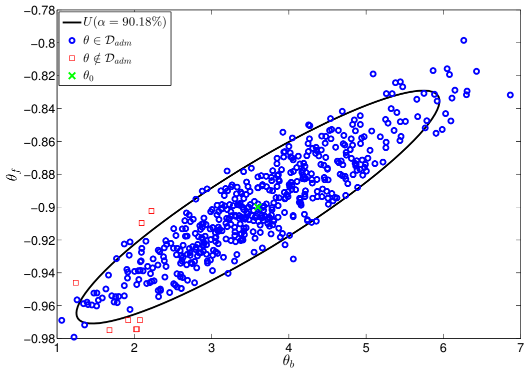
4. NUMERICAL ILLUSTRATIONS

The performance diagnosis methodology is applied to a simulation case study. We consider the following Box-Jenkins true system: $y(t) = G_0(z)u(t) + H_0(z)e(t)$ where $G_0(z) = \theta_b z^{-1}/(1 + \theta_f z^{-1})$ and $H_0(z) = 1 + \theta_c z^{-1}$; $\theta_0 = (\theta_b \; \theta_c \; \theta_f)^T$ and $e(t)$ are the true parameter vector and a realization of a white noise process with variance $\sigma_{e,0}$, respectively. The control performance measure of interest is related to the sensitivity function. The performance weighting filters of Eq. (3) are therefore chosen as $W_l(z) = diag(0, W(z))$ and $W_r(z) = diag(0, 1)$ with $W(z) = (0.52 - 0.46z^{-1})/(1 - 0.99z^{-1})$. The true system $G_0(z)$ is in closed-loop operation with a H_∞-controller. At commissioning, the controller has been constructed based on a Box-Jenkins model with $\theta_{com} = (3.6 \quad -0.9 \quad -0.9)^T$ and $\sigma_{e,com} = 1.0$. Note that the nominal performance level is initially satisfied. The variance of the system output $y(t)$ is originally 0.73. In the sequel, we consider two scenarios to investigate the extent to which the detection probability can be increased by designing the power spectrum $\Phi_r(\omega)$. To illustrate the benefits of experiment design regardless of the 'chicken and egg' problem, it is assumed that the true system $G_0(z)$ is known. We therefore know a priori which experiment design problem to solve. In both scenarios, the experiment duration N and the admissible closed-loop identification cost β are 500 and 0.15, respectively.

Scenario 1: We alter the disturbance characteristics by varying the noise transfer function $H_0(z)$ and the variance of the white noise signal $e(t)$ with respect to the commissioning stage, i.e. $\theta_0 = (3.6 \quad -0.05 \quad -0.9)^T$ and $\sigma_{e,0} = 5.0$. This leads to a drastic change in the variance of the system output, i.e. 7.26.



(a) $\Phi_r(\omega)$ is designed for $m = 0$, i.e. $r(t)$ is a white noise signal

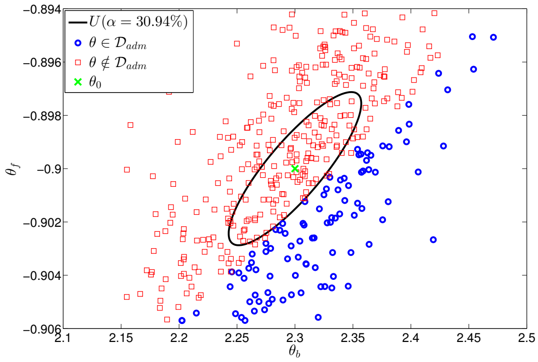


(b) $\Phi_r(\omega)$ is designed for $m = 35$

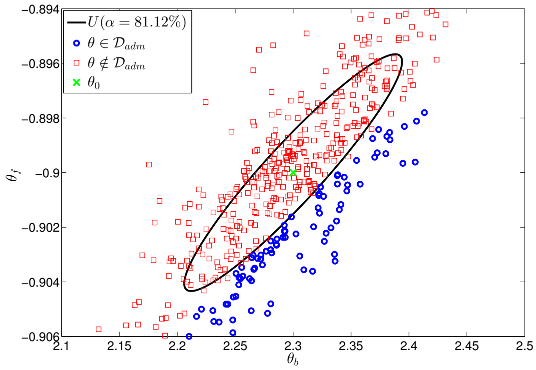
Fig. 3. Scenario 1: $G(z, \hat{\theta}_N) \in \mathcal{D}_{adm}$.

Knowing that $G_0(z) \in \mathcal{D}_{adm}$, we design the excitation signal $r(t)$ according to the experiment design problem of Eq. (16) for the null hypothesis $\mathcal{H}_0$. Initially, the excitation signal is chosen to be a white noise signal whose variance is determined by solving Eq. (16), i.e. $m = 0$ in the power spectrum $\Phi_r(\omega)$ given in Eq. (11). Experiment design with the white noise signal reveals that the probability associated with the ellipsoidal confidence set $\mathcal{D}(\theta_0, P_\theta, \mathcal{X})$ centered around the true system $G_0(z)$ is 68.15%. The latter probability indicates that at least 68.15% of the models identified using the white noise excitation signal are within the set $\mathcal{D}_{adm}$ and, therefore, will lead to the choice of the null hypothesis $\mathcal{H}_0$ in the decision rule. This suggests that the detection probability is at least 68.15%. To verify the results of experiment design, we perform a Monte Carlo simulation to identify the true system dynamics 500 times by applying the designed white noise excitation signal and measuring the signals $\{u(t), y(t)\}$. Fig. 3(a) depicts the identified parameters $\theta = (\theta_b \; \theta_f)^T$ in the Monte Carlo simulation. The Monte Carlo simulation indicates that about 81% of the identified models $G(z, \hat{\theta}_N)$ lie in the set $\mathcal{D}_{adm}$.

We now design the spectrum $\Phi_r(\omega)$ of the excitation signal for $m = 35$, implying that the designed $r(t)$ is no longer a white noise signal. For the same admissible identification cost, i.e. $\beta = 0.15$, the probability associated with the confidence set $\mathcal{D}(\theta_0, P_\theta, \mathcal{X})$ is 90.18%, i.e. the least detection probability. By comparing the latter detection probability with that achieved using the white



(a) $\Phi_r(\omega)$ is designed for $m = 0$, i.e. $r(t)$ is a white noise signal



(b) $\Phi_r(\omega)$ is designed for $m = 35$

Fig. 4. Scenario 2: $G(z, \hat{\theta}_N) \notin \mathcal{D}_{adm}$.

noise excitation signal, i.e. 68.15%, we clearly observe that the probability of choosing the correct hypothesis can be significantly increased through a proper design of the excitation signal $r(t)$. The results of the Monte Carlo simulation are illustrated in Fig. 3(b). It follows that over 91% of the identified models $G(z, \hat{\theta}_N)$ lie in the set $\mathcal{D}_{adm}$. This is also larger than the probability 81% obtained with the white noise signal.

Scenario 2: We induce changes in the system dynamics with respect to commissioning by defining the parameter vector θ_0 as $(2.3 \quad -0.9 \quad -0.9)^T$. It is evident that the disturbance characteristics remain intact in this case. The change in system dynamics results in a slight variation in the variance of the output signal $y(t)$.

Like in Scenario 1, we assume that the true system $G_0(z)$ is known. It follows from the fact that $G_0(z) \notin \mathcal{D}_{adm}$ we should design the excitation signal $r(t)$ such that the probability that the identified model $G(z, \hat{\theta}_N)$ lies outside $\mathcal{D}_{adm}$ is maximized, i.e. Eq. (17). It turns out that the least detection probability achieved with the white noise signal is 30.94%, i.e. the probability associated with the elliposoidal set $\mathcal{D}(\theta_0, P_\theta, \mathcal{X})$. A Monte Carlo simulation is performed to identify the true system dynamics 500 times by means of the white noise excitation signal. The results of the Monte Carlo simulation are depicted in Fig. 4(a). This reveals that approximatly 70% of the identified models lie outside $\mathcal{D}_{adm}$, leading to the choice of the alternative hypothesis $\mathcal{H}_1$. On the other hand, by designing the excitation signal $r(t)$ with $m = 35$ we can increase the least detection probability to 81.12%. This is a drastic improvement with respect to the white noise case. Fig. 4(b) indicates that about 84% of the models identified with the designed excitation signal are outside $\mathcal{D}_{adm}$.

5. CONCLUSIONS

A methodology has been presented to address the problem of closed-loop performance degradation using prediction error identification. The approach exploits the statistical hypothesis testing framework to detect whether an observed performance drop results from changes in the system dynamics or is due to variations in the disturbance characteristics. This is performed by examining if an identified model of the true system lies in a set which contains all models leading to satisfactory closed-loop performance with the existing controller. To increase the detection probability, we design the excitation signal used for system identification according to the chosen hypothesis in order to maximize the probability that the identified model lies in the region of interest. The simulation studies show that the detection probability can be significantly increased by a proper design of the excitation signal for a prespecified closed-loop identification cost.

In future, we will investigate various approaches to address the 'chicken and egg' problem of determining an initial model estimate for experiment design. In addition, the work will be extended for closed-loop performance diagnosis of model predictive controllers.

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