Combining Experiments for Linear Dynamic Network Identification in the Presence of Nonlinearities

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Abstract. In many practical applications it might be desirable to excite only point at a time in an interconnection of multiple dynamic subsystems (e.g. large-scale system). Therefore multiple experiments need to be combined to successfully identify one or more subsystems in the network of subsystems. This papers illustrates how the identification of a linear subsystem of a dynamical network containing one or more nonlinear subsystems can result in biased estimates when multiple experiments are combined using the Best Linear Approximation (BLA) based approach.

1. Introduction

Large scale mechanical systems consisting of many components, the electrical grid, biological systems or industrial plant can be interpreted as the interconnection of multiple subsystems, i.e. a dynamic network setting.

The identification of linear dynamical networks has received quite some attention over the last years focusing on e.g. network structure detection [1, 2, 3], identification of one or more subsystems in the network [3, 4, 5, 6], input selection [7], and multiple noise frameworks [8, 6]. However, the identification of systems operating in a nonlinear dynamic network has received consideringly less attention [9, 10, 11, 12].

In many practical applications it might be desirable to excite only one node at a time in a dynamic network, e.g. for safety reasons, limited actuation capabilities, or due to a geographical spread of the different nodes. Therefore multiple experiments need to be combined to successfully identify of one or more subsystems in the network. This papers illustrates how the identification of a linear subsystem of a dynamical network containing one or more nonlinear subsystems can result in biased estimates when multiple experiments are combined using the Best Linear Approximation (BLA) based approach presented in [12].

A short introduction to nonlinear dynamic networks is given in Section 2. The BLA framework is discussed next in Section 3. Section 4 discusses how multiple experiments can be combined to estimate the subsystems in a dynamic network. A simulation example illustrates that effect of the presence of one or more nonlinearities in the dynamic network on the obtained estimates in Section 5.

2. Dynamic Networks

The dynamic networks considered here follow the same definitions and visualization as in [4, 13]. A dynamic network (see Figure 1) consists of a total of L nodes, representing internal variables of the network, which are interconnected with other nodes by (nonlinear) dynamic systems. A node signal, denoted $w_i(t)$, is obtained as the sum of the outputs of the incoming (nonlinear)

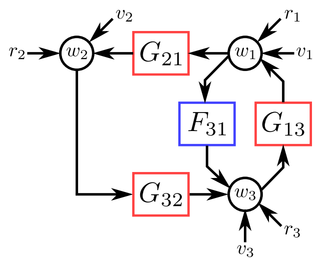

Figure 1. An example of a nonlinear dynamic network with 3 nodes. The node signal $w_i(t)$ is obtained as the sum of the outputs y_{ij} of the linear G_{ij} and nonlinear F_{ij} subsystems connecting to it, the noise signal $v_i(t)$ and the known reference signal $r_i(t)$.

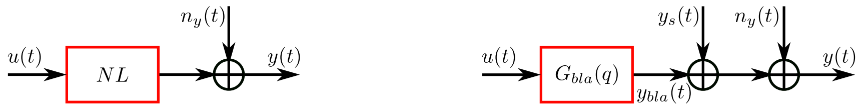

Figure 2. A nonlinear dynamical system with noise $n_y(t)$ at the output only (left) and its Best Linear Approximation (right). The BLA represents a nonlinear system as a linear approximation G_{bla}, best in least squares sense, and a stochastic nonlinearity source y_s.

dynamic subsystems ($y_{ij}(t)$ denotes the output of the subsystem connecting node j to node i), an external reference signal $r_i(t)$, and a noise signal $v_i(t)$: $w_i(t) = \sum_{j=1,\, j\neq i}^{L} y_{ij}(t) + r_i(t) + v_i(t)$. Only the node signals $w_i(t)$ and the reference signals $r_i(t)$ are known.

The node noise signal $v_i(t)$ is assumed to be zero-mean and to have a finite variance $\sigma_{v_i}^2$. Note that only noise at the network nodes are considered. No measurement noise is present in the networked system.

3. Best Linear Approximation

The BLA model of a nonlinear system is a linear time-invariant (LTI) approximation of the behavior of that system, best in least squares sense (Figure 2). For the open-loop, single-input single-output case, the BLA is defined as [14, 15, 16, 17]: $G_{bla}(q) = \underset{G(q)}{\arg\min}\, E_{u,n_y}\left\{|\tilde{y}(t) - G(q)\tilde{u}(t)|^2\right\}$, where, $\tilde{u}(t) = u(t) - E_u\{u(t)\}$, $\tilde{y}(t) = y(t) - E_{u,n_y}\{y(t)\}$, and $E_{u,n_y}\{.\}$ denotes the expected value operator taken w.r.t. the random variations due to the input $u(t)$ and the output noise $n_y(t)$ and $G(q)$ belongs to the set of all possible LTI systems. The extension of the BLA framework to the dynamical network setting is presented in [12].

4. Combining Multiple Experiments

The indirect networked identification approach in the frequency domain, using multiple experiments M, is given in practice as follows. First, the FRF from the reference to the node is obtained: $\hat{\mathbf{S}}_{\mathbf{bla}}(j\omega) = \left[\mathbf{R}^H(j\omega)\mathbf{R}(j\omega)\right]^{-1}\mathbf{R}^H(j\omega)\mathbf{W}(j\omega)$, where $.^H$ denotes the Hermitian operator, and $\mathbf{R}(j\omega)$, $\mathbf{W}(j\omega)$ are given by (for a 3-node network):

$$\mathbf{R}(j\omega) = \begin{bmatrix} R_1^{[1]}(j\omega) & 0 & 0 \\ \vdots & \vdots & \vdots \\ R_1^{[M_1]}(j\omega) & 0 & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & R_3^{[M_2+1]}(j\omega) \\ \vdots & \vdots & \vdots \\ 0 & 0 & R_3^{[M]}(j\omega) \end{bmatrix}, \quad \mathbf{W}(j\omega) = \begin{bmatrix} W_1^{[1]}(j\omega) & W_2^{[1]}(j\omega) & W_3^{[1]}(j\omega) \\ W_1^{[2]}(j\omega) & W_2^{[2]}(j\omega) & W_3^{[2]}(j\omega) \\ \vdots & \vdots & \vdots \\ W_1^{[M]}(j\omega) & W_2^{[M]}(j\omega) & W_3^{[M]}(j\omega) \end{bmatrix}. \tag{1}$$

Note that in the considered setting, only one reference signal is active simultaneously in each experiment, resulting in a sparse matrix $\mathbf{R}(j\omega)$. $\mathbf{W}(j\omega)$, unlike $\mathbf{R}(j\omega)$ is typically not sparse, since, depending on the interconnections that are present in the network, each reference can evoke a response at each node in the network. In the purely linear case one could also combine the experiments by using the superposition principle.

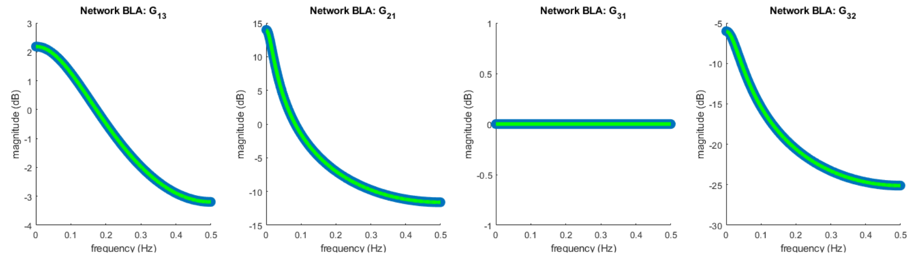

Figure 3. Case 1: linear dynamic network. Unbiased estimates are obtained: the true linear subsystems (green) and the estimates (blue) coincide.

A noiseless estimate of the node signals is obtained: $\hat{W}_j^{[i]}(j\omega) = \sum_{k=1}^{n_w} \hat{S}_{bla,j,r_k}(q) R_k^{[i]}(j\omega)$. The networked BLA is now obtained in practice as: $\hat{\mathbf{G}}_{\mathbf{bla,i}}(j\omega) = \left[\mathbf{K}_i^H(j\omega)\mathbf{K}_i(j\omega)\right]^{-1} \mathbf{K}_i^H(j\omega)\mathbf{W}_i(j\omega)$, where $\mathbf{K}_i(j\omega)$, $\mathbf{W}_i(j\omega)$, $\hat{\mathbf{G}}_{\mathbf{bla,i}}(j\omega)$ are given by (the $j\omega$ notation is dropped for compactness):

$$\mathbf{K}_i(j\omega) = \begin{bmatrix} \hat{W}_1^{[1]} & \dots & \hat{W}_{i-1}^{[1]} & \hat{W}_{i+1}^{[1]} & \dots & \hat{W}_{n_w}^{[1]} \\ \hat{W}_1^{[2]} & \dots & \hat{W}_{i-1}^{[2]} & \hat{W}_{i+1}^{[2]} & \dots & \hat{W}_{n_w}^{[2]} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \hat{W}_1^{[M]} & \dots & \hat{W}_{i-1}^{[M]} & \hat{W}_{i+1}^{[M]} & \dots & \hat{W}_{n_w}^{[M]} \end{bmatrix}, \mathbf{W}_i(j\omega) = \begin{bmatrix} \hat{W}_i^{[1]} \\ \hat{W}_i^{[2]} \\ \vdots \\ \hat{W}_i^{[M]} \end{bmatrix}. \quad \hat{\mathbf{G}}_{bla,i}(j\omega) = \begin{bmatrix} \hat{G}_{bla,i,1} \\ \dots \\ \hat{G}_{bla,i,i-1} \\ \hat{G}_{bla,i,i+1} \\ \dots \\ \hat{G}_{bla,i,n_w} \end{bmatrix}. \quad (2)$$

5. Simulation Example

The structure of the simulated system is visualized in Figure 1. The linear subsystems G_{21}, G_{32} and G_{13} are first order systems of the form:

$$\begin{array}{rcl} x_{ij}(t+1) & = & A_{ij}x_{ij}(t) + B_{ij}w_j(t) \\ w_i(t) & = & C_{ij}x_{ij}(t), \end{array} \qquad \begin{array}{lll} A_{21} = 0.9, & B_{21} = 1.0, & C_{21} = 0.5 \\ A_{32} = 0.8, & B_{32} = 0.1, & C_{32} = 1.0 \\ A_{13} = 0.3, & B_{13} = 1.0, & C_{13} = -0.9 \end{array} \quad (3)$$

The nonlinear subsystem F_{31} is given by $w_3(t) = \tanh(w_1(t-1))$, in the linear simulation case discussed below F_{31} is replaced by a unit delay: $w_3(t) = w_1(t-1)$.

The system is excited by three reference signals $r_1(t)$, $r_2(t)$ and $r_3(t)$. These signals are all three random phase multisine signals [17] exciting all the frequencies $]0, f_s/2[$ with a flat amplitude spectrum. The random phases are uniformly distributed between 0 and 2π. Only one reference signal is active simultaneously. $M = 20$ realizations of the multisines are applied to the system for each reference signal, each realization contains $P = 2$ steady state periods of $N = 4096$ points per period. The reference signals have each a standard deviation of 0.5. No noise is present in the presented simulation example.

Case 1: the proposed framework is tested on a linear dynamic network. As can be expected from [13, 17], an unbiased estimate is obtained, see Figure 3.

Case 2: the BLA framework is applied on a nonlinear dynamic network. Figure 4 indicates the clear presence of a bias on the estimate of the linear network module G_{32}. The estimate of the network module G_{31} represents the linear approximation of the nonlinear system. The observed bias can be explained due to the different setpoints of the nonlinearity for the different experiments: each reference signal excites a different range of the nonlinear module of the network. Combining these measurements can lead to a bias on the estimates of the linear subsystems connecting to the same node as the nonlinear subsystem, in this case node 3.

Case 3: the nonlinear dynamic network is excited by reference signals with different power: while r_1, r_3 have a std $= 0.5$, r_2 has a std of 4.5 in this setup. The different reference amplitudes used ensure that the nonlinearity is in a similar setpoint for all experiments, as can be observed in Figure 5, this reduces the bias on the estimate of module G_{32} significantly.

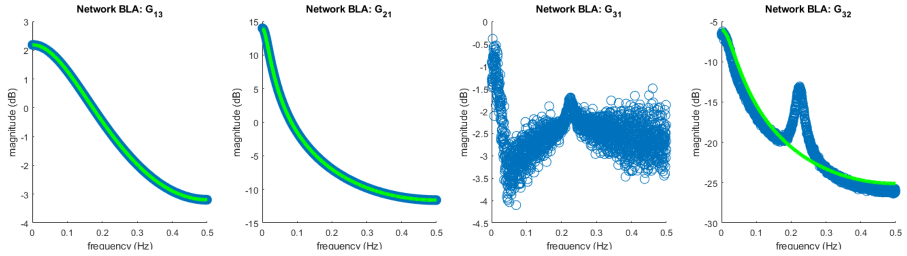

Figure 4. Case 2: nonlinear dynamic network. A bias is observed between the true linear subsystems (green) and the estimates (blue) for the G_{32} estimate.

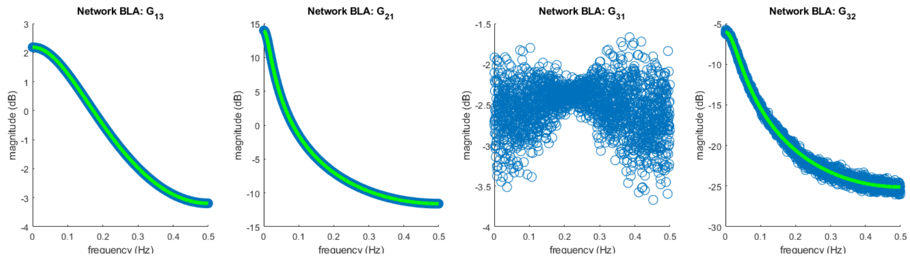

Figure 5. Case 3: nonlinear dynamic network, varying reference amplitudes. The true linear subsystems (green) and the estimates (blue) coincide.

6. Conclusion

While combining experiments for the estimation of dynamic modules in linear dynamic networks still leads to consistent estimates using the BLA-framework, a bias can be introduced on the BLA-framework estimates in case one or more of the network modules behaves nonlinear. This nonlinearity is caused by combining measurements where the nonlinearity operates in a different setpoint, leading to different approximations of this nonlinear behavior.

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