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Risk management in oil reservoir water-flooding under economic uncertainty

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Abstract— **Model-based economic optimization of the water-flooding process in oil reservoirs suffers from high levels of uncertainty. The achievable economic objective is highly uncertain due to the varying economic conditions and the limited knowledge of the reservoir model parameters. For improving robustness, different approaches, e.g., mean or mean-variance optimization have been proposed. One of the drawbacks of the mean-variance approach is the symmetric nature of the variance and hence the reduction of the best cases. In this work, we focus only on the lower tail, i.e., the worst-case(s) and aims to maximize the lower tail of the economic objective function without heavily compromising the best cases. Concepts from robust optimization (max-min approach) and the theory of risk (a risk averse mean-CVaR approach) are considered to offer an asymmetric shaping of the objective function distribution with respect to the given uncertainty. A scenario-based approach is used, where an ensemble of oil price scenarios characterizes the economic uncertainty.**

I. INTRODUCTION

Water-flooding involves the injection of water in an oil reservoir to increase oil production. Dynamic optimization of the water-flooding process has shown significant scope for improvement of the economic life-cycle performance of oil fields compared to a more conventional reactive strategy, see e.g., [1], [2] and [3]. In these studies a financial measure, i.e., Net Present Value (NPV), is maximized. One of the key challenges in this model-based economic optimization is the high level of uncertainty arising from the varying economic conditions and the limited knowledge of the model parameters. Economic variables such as oil price, interest rate, etc., that are involved in defining the NPV, fluctuate with time and can not be precisely predicted. As a result, the potential advantages of model-based economic optimization are not fully realized and the risk of loosing the expected economic objective is very high.

Various approaches to handle uncertainty in an optimization framework can be broadly divided into two categories: Stochastic Optimization (SO), see e.g., [4], which assumes a probabilistic description of the uncertainty and Robust optimization (RO), see e.g., [5], [6], which is a deterministic set-based approach and mainly concerned with the worst-case value of the parameters in the uncertainty set $\mathcal{U}$. Risk measures are an important tool in decision making under uncertainty. Markowitz [7] in the early 50's proposed a 'risk-return' portfolio selection approach, where the risk is characterized as the variance of the individual assets. One of the drawbacks of the mean-variance optimization is the symmetric nature of the variance which also penalizes the best-cases, while generally, in a maximization problem, the decision maker is mainly concerned with the lower tail of the objective function distribution. Robust optimization approaches provide an asymmetric way of shaping an objective function distribution by maximizing the worst-case in the given uncertainty set. Conditional Value-at-Risk (CVaR), introduced in [8] and defined as the average of some percentage of worst-case scenarios, also allows the asymmetric shaping of the objective function distribution.

In the petroleum engineering literature, a so-called robust optimization approach has been introduced in [9] which uses a scenario-based averaging scheme to maximize an average objective function with respect to an ensemble of geological model realizations. In [10], a symmetric mean-variance optimization approach has been implemented honoring geological uncertainty. In [11], these approaches have been extended to consider the economic uncertainty characterized by the varying oil price scenarios. Similar strategies have been described in [12], [13] and [14]. In [15] multi-objective optimization approaches have been implemented that maximize the average of the objective function and the worst-case with respect to the geological uncertainty. In [16] different risk averse objectives in oil reservoir water-flooding have been studied.

In this paper, we aim to explore the question of an asymmetric shaping of the objective function, i.e., the NPV distribution under economic uncertainty. The focus is to maximize the lower tail (worst cases) without largely compromising the upper tail (best cases). For this purpose, the worst-case (max-min) robust optimization scheme is considered. The economic uncertainty is characterized by an ensemble of varying oil price scenarios. It is shown that, when considering the uncertainty in the oil prices with a specific set of scenarios, the max-min problem is simplified to a single optimization with the worst-case realization of the oil price. Later on, a mean-worst case multi-objective optimization is implemented which gives decision makers a tool to maximize the worst-case for a given value of the mean or vice versa. Furthermore, a risk averse mean-CVaR multi-objective optimization approach is formulated that aims to maximize the average value of the economic objective function while minimizing the CVaR risk. These worst-case and mean-CVaR approaches are then compared with a symmetric mean-variance scheme.

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The paper is organized as follows: In the next section, the model-based economic optimization of water-flooding is explained. This is followed by the quantification of economic uncertainty in Section III. In Section IV, different optimization schemes under uncertainty are discussed in details with simulation examples. Specifically, Sub-sections IV-A and IV-B discuss the mean and the mean-variance optimization schemes with simulation examples. In Sub-section IV-C a worst-case robust optimization approach is presented in detail with simulation examples while Sub-section IV-D discusses the mean-CVaR approach in detail. Finally the conclusions of the presented results are given in Section V.

II. MODEL-BASED ECONOMIC OPTIMIZATION

The economic objective, in the form of Net Present Value (NPV), can be represented as follows:

$$J = \sum_{k=1}^{K} \left[\frac{r_o \cdot q_{o,k} - r_w \cdot q_{w,k} - r_{inj} \cdot q_{inj,k}}{(1+b)^{\frac{t_k}{\tau_t}}} \cdot \Delta t_k \right] \quad (1)$$

where r_o, r_w and r_{inj} are the oil price, the water production cost and the water injection cost in $\left[\$/m^3\right]$ respectively. K represents the production life-cycle, i.e., the total number of time steps k and Δt_k the time interval of time step k in $[days]$. The term b is the discount rate for a certain reference time τ_t. The terms $q_{o,k}, q_{w,k}$ and $q_{inj,k}$ represent the flow rate of produced oil, produced water and injected water at time step k in $\left[m^3/day\right]$.

Water-flooding optimization is a highly complex large-scale non-linear optimization problem. In this work, a gradient-based optimization approach is used where the gradients are obtained by solving a system of adjoint equations, see e.g., [17]. An optimization solver KNITRO [18] is then used with an interior point method to iteratively converge to a (possibly local) optimum.

In the next section, a scenario-based approach to quantify economic uncertainty is discussed. Later on, different optimization frameworks are presented for improving robustness.

III. QUANTIFYING ECONOMIC UNCERTAINTY

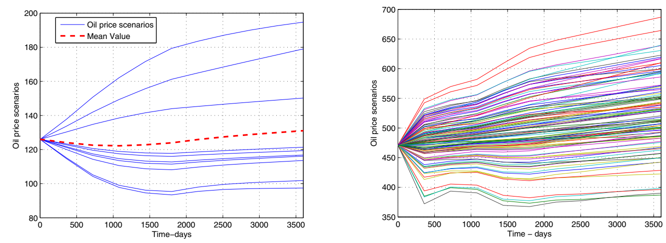
Oil reservoirs typically have a long life cycle from 10 to 100 years. The economic variables that govern the NPV, especially the oil price r_o, vary drastically over time and can not be precisely predicted. These unknown variations of future oil prices are the key source of economic uncertainty. Therefore in this work, only varying oil prices are used to characterize economic uncertainty. They have a time-varying dynamic nature and their negative effect on the control strategy increases with the time horizon. Due to the complexity and the nature of the water-flooding optimization problem, a common approach in this optimization is to consider a finite number of scenarios $\eta_i, i = 1, \cdots, N_{eco}$ from an uncertainty set $\mathcal{U}$. These samples represent finite discrete points in the uncertainty space.

There are various ways to predict the future values of the changing oil prices. For this work a simplified AutoRegressive-Moving-Average (ARMA) model is used to generate oil price time-series. The ARMA model is shown below:

$$r_{o_k} = a_0 + \sum_{i=1}^{6} a_i r_{o_{k-i}} \quad (2)$$

where a_i are the randomly selected coefficients.

Two different oil price scenarios with the same base oil price of 471 $\left[\$/m^3\right]$ and an ensemble size, N_{eco}, of 10 and 100 respectively are generated as shown in Fig. 1.



(a) $N_{eco} = 10$ **with mean value** **(b)** $N_{eco} = 100$

Fig. 1: Oil price scenarios

IV. HANDLING ECONOMIC UNCERTAINTY

The following strategies are considered to handle economic uncertainty and to offer asymmetric risk management.

A. Mean optimization (MO)

In this approach, the expected value of the objective function, i.e., NPV is maximized. It can be written as follows:

$$J_{MO} = \sum_{i=1}^{N_{eco}} p_i J_i \quad (3)$$

where p_i is the probability vector associated with N_{eco} scenarios. Generally, the probability distribution of the underlying uncertainty is poorly known. In [9], an averaging approach is proposed, where a uniform distribution of the geological model realizations, representing geological uncertainty, is assumed. Therefore, the problem is simplified to an arithmetic average as follows:

$$J_{MO} = \frac{1}{N_{eco}} \sum_{i=1}^{N_{eco}} J_i \quad (4)$$

It can easily be seen that averaging approach includes uncertainty in the optimization framework but does not improve robustness to the solution. Mean optimization is also categorized as a risk neutral approach. This seriously limits the performance of this approach to handle uncertainty. One important point to consider here is that due to the linearity of the oil price in the NPV objective in addition to the certainty of the geological model, the average of the individual objective function values from each realization is equal to a single objective function value with the average value of all oil price realizations as shown below:

$$\frac{1}{N_{eco}} \sum_{i=1}^{N_{eco}} [J(\mathbf{u}, \eta_i)] = J(\mathbf{u}, \frac{1}{N_{eco}} \sum_{i=1}^{N_{eco}} [\eta_i]). \quad (5)$$

B. Mean-variance optimization (MVO)

The Markowitz risk-return portfolio selection approach involves a quantitative characterization of risk in terms of the variance of the returns distribution [7]. It results in an efficient frontier, i.e., a set of optimal portfolios that offers the highest expected return for a defined level of risk or the lowest risk for a given level of expected return. With the reduction of variance of the objective function (NPV) distribution, the MVO offers some robustness and will reduce the sensitivity of the optimal solution to the uncertainties. The MVO approach can be written as:

$$J_{MVO} = J_{MO} - \gamma J_{VAR} \tag{6}$$

where J_{VAR} represents the variance of the NPV distribution and γ is a weighting parameter. MVO stems from stochastic optimization theory and requires a-priori knowledge of the uncertainty distribution. Furthermore, as a symmetric risk measure, it also penalizes the upper tail of the NPV distribution, i.e., reduces the best cases.

The simulation example using MVO is presented and compared with MO in the next subsection.

1) Simulation example: All simulation experiments are performed using MRST, see [19], which is a MATLAB-based reservoir simulator. The following reservoir model, economic data and the control input are used in all subsequent simulation examples.

Reservoir model and economic data: A single model realization of the Standard Egg model, [20], is used. The standard egg model is a three-dimensional realization of a channelized reservoir produced under water flooding conditions with eight water injectors and four producers based on the original Egg model proposed in [9]. The life-cycle of this reservoir model is $3600[days]$. The absolute-permeability field and well locations of the model realization are shown in Fig. 2.

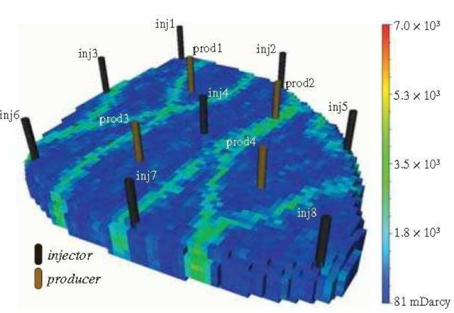


Fig. 2: Permeability field and well locations of the model realization

An un-discounted NPV, i.e., with discount factor $b = 0$, is used. Other economic parameters, e.g., the water injection cost r_{inj} and the production cost r_w are kept fixed at $23 \left[\$/m^3\right]$ and $72 \left[\$/m^3\right]$ respectively. Both oil price scenarios as shown in Fig. 1 are considered in this example. All scenarios are given equal probabilities, i.e., a uniform distribution of the uncertainty scenarios is assumed.

Control input: The control input $\mathbf{u}$ reflects injection flow rate trajectories for each of the eight injection wells. The minimum and the maximum flow rate constraints are $0.2 \left[m^3/day\right]$ and $79.5 \left[m^3/day\right]$ respectively. The production wells operate at a constant bottom-hole pressure of $395[bar]$. The control input $\mathbf{u}$ is reparameterized into an input parameter vector φ with ten time periods of $t_\varphi = 360[days]$, during which the injection rate is held constant at value φ_i. Thus the input parameter vector φ consists of $8 \times 10 = 80$ elements. The initial input guess for optimization routine is considered as $78 \left[m^3/day\right]$ for all input parameters.

Results: The results for the ensemble size $N_{eco} = 100$ (Fig. 1(b)) are shown in this subsection with both MO and MVO. Different values of γ, i.e., $\gamma \in \{1 \times 10^{-7}, 1 \times 10^{-8}, 0.5 \times 10^{-8}\}$ in the MVO objective (6) are used.

The obtained MO and the MVO optimal strategies are applied to the reservoir model realization with each oil price scenario, resulting in 100 different NPVs. The corresponding PDFs are obtained by approximating a non-parametric Kernel Density Estimation (KDE) with MATLAB routine '*ksdensity*' on these NPV data values as shown in Fig. 3. In case of a perfect normal NPV distribution, the MVO offers a symmetric reduction but in the case of a non-normal distribution, as in Fig. 3, it is observed that the best cases are highly penalized in order to reduce the variance.

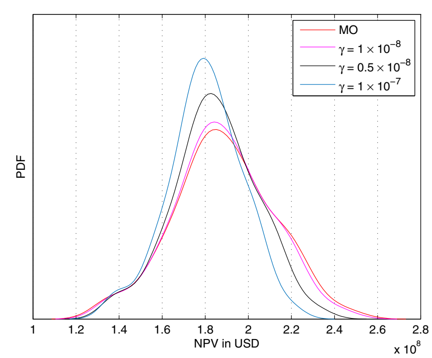


Fig. 3: Comparison between MVO and MO

In the next subsections, asymmetric risk measures are discussed with simulations examples.

C. Worst-case optimization (WCO)

Worst-case robust optimization assumes that the uncertainty is known only within certain bounds, which is called the uncertainty set $\mathcal{U}$, and the robust solution is optimal for any realization of the uncertainty in the given set. Hence it focuses only on the worst-case in $\mathcal{U}$ and solves a max-min (or min-max) problem. The worst-case or a max-min optimization objective can be written as:

$$\max_{\mathbf{u}} \min_{\eta_i} J_i(\mathbf{u}, \eta_i) \tag{7}$$

where η_i is the uncertainty ensemble. It can easily be seen that the above optimization problem is non-differentiable, so a common approach to reformulate the above *max-min* problem is by adding a slack variable z with additional constraints as follows [21]:

$$\max_{\mathbf{u},z} z \quad \text{s.t.} \quad z \leq J_i(\mathbf{u}, \eta_i) \quad \forall i \tag{8}$$

Therefore, with the total number of ensemble members N_{eco}, there will be N_{eco} additional constraints. As the worst-case optimization only focuses on the lowest value of the NPV distribution, it offers an asymmetrical shaping of the NPV distribution.

1) Simulation examples: The reservoir model, the economic data and the control input are the same as used in Subsection IV-B.1. The oil price ensemble with 10 members as shown in Fig. 1(a) is used in this example.

Results: The MO and the WCO strategies are applied to the model with 10 oil price realizations resulting in 10 different NPVs. The corresponding histogram is shown in Fig. 4. It can be seen that the worst-case value of the NPV distribution is improved at the cost of a decrease in the mean value.

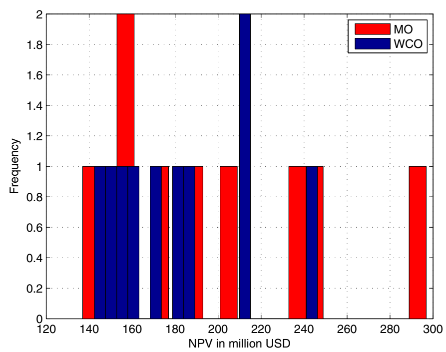


Fig. 4: Comparison of WCO and MO

Table I summarizes the percentage increase and decrease of the worst-case NPV and the average NPV respectively. For the sake of comparison of the MVO and the WCO approaches, non-parametric KDE is used to approximate the density function as shown in Fig. 5. It can be observed that the WCO improves the worst-case without heavily compromising the best-case compared to the MVO approach. An important point to note is that, due to the scalar nature of the economic uncertainty with the linearity in the objective function, the worst-case formulation in Eq. (8) is simply equivalent to a single optimization with the worst oil price scenario in the ensemble. Especially, in the case of oil price scenarios as shown in Fig. 1(a), it is easy to identify the worst-case oil price scenario. Fig. 6 shows a comparison of the histogram of NPV values resulting from the worst-case formulation as in Eq. (8) and a single optimization with the worst oil price scenario.

TABLE I: Results comparison of WCO with MO

	MO	WCO	% change
Mean in million USD	194	182	6.18% decrease
Worst-case in million USD	136	142	4.41% increase

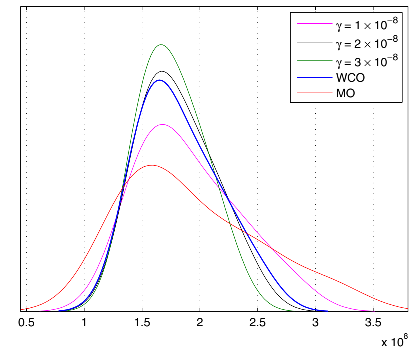


Fig. 5: Comparison of MO, MVO (with different γ) and WCO optimization

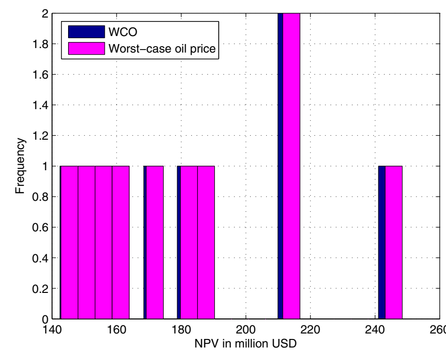


Fig. 6: Comparison of the WCO and the optimization with the worst oil price realization

In order to give the decision maker a preference to chose between a worst-case improvement for a given level of average NPV, a mean-worst case optimization problem is formulated as follows:

$$J_{MWCO} = J_{MO} + \lambda J_{WCO} \tag{9}$$

where λ is a weighting parameter. Fig. 7 shows the variation of the mean and the worst-case values with respect to the changing λ, also known as an efficient frontier. As the λ increases the worst-case is improved at the cost of reducing the mean value. MO has the highest mean with the lowest worst-case value. At $\lambda = 5$, a different result is observed probably because the optimization has gotten stuck in a lower local optimum. The results of the efficient frontier are also

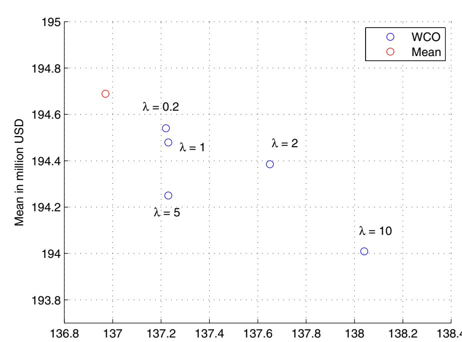


Fig. 7: Efficient frontier (Mean Vs. worst-case values with changing λ)

summarized in Table II.

TABLE II: Results for the $J_{MO} + \lambda J_{WCO}$ approach

λ	Mean in million USD	Worst-case in million USD	% decrease of mean w.r.t MO	% increase of worst-case w.r.t MO
MO	194.68	136.97	-	-
0.2	194.54	137.22	0.07%	0.18%
1	194.47	137.23	0.10%	0.19%
2	194.38	137.65	0.15%	0.49%
5	194.25	137.23	0.22%	0.18%
10	194.00	138.04	0.34%	0.78%

D. Conditional Value-at-Risk (CVaR) optimization

Conditional Value-at-Risk (CVaR), introduced in [22], is a popular tool for managing risk in finance. CVaR indicates the average of the $\beta-$tail of the worst cases of a distribution. It addresses the overly conservative solution of the worst case optimization by considering a class of worst cases. The CVaR risk measure is the negative of the CVaR value. It is similar to the value-at-risk (VaR) or chance constrained optimization [23], which is a percentile of a loss/gain distribution.

For a random variable X with cumulative distribution function $F_X(z) = P\{X \leq z\}$, the VaR (α_β) and CVaR (ϕ_β) of X with confidence level $\beta \in]0,1[$ are given as:

$$\alpha_\beta(X) = \min\{z | F_X(z) \geq \beta\},$$
$$\phi_\beta(X) = \mathbb{E}[X | X \geq \alpha_\beta].$$

For a function $f(u,y)$ that represents a loss distribution, where $u \in U \subseteq \mathbb{R}^m$ is the decision vector and $y \in \mathbb{R}^n$ is a random vector representing uncertainties, [22] introduces a simpler auxiliary function F_β on $U \times \mathbb{R}$ defined as follows:

$$F_\beta(u,\alpha) = \alpha + \frac{1}{1-\beta} \int_{y \in \mathbb{R}^n} [f(u,y) - \alpha]^+ p(y) dy \quad (10)$$

where $[t]^+ := \max\{t, 0\}$. [22] shows that the $\beta-$CVaR of the loss associated with any $u \in U$ can be determined as follows:

$$\phi_\beta(u) = \min_{\alpha \in \mathbb{R}} F_\beta(u,\alpha). \quad (11)$$

Furthermore, minimizing the $\beta-$CVaR of the loss associated with u is equivalent to minimizing $F_\beta(u,\alpha)$ over all $(u,\alpha) \in U \times \mathbb{R}$, in the sense that

$$\min_{u \in U} \phi_\beta(u) = \min_{(u,\alpha) \in U \times \mathbb{R}} F_\beta(u,\alpha). \quad (12)$$

In the water-flooding optimization, the mean-CVaR approach can be written as:

$$J_{MCVaR} = J_{MO} - \omega J_{\phi_\beta} \quad (13)$$

where ω is the weighting parameter.

As the sampling of the uncertainty space generates a collection of scenarios $\eta_i, \cdots, \eta_{N_{eco}}$, the integral in the CVaR optimization formula Eq. (10) can be approximated by a sum. For the NPV distribution, the CVaR (i.e., the negative of the CVaR value) is then given by:

$$J_{\phi_\beta}(\mathbf{u},\alpha) = -\alpha - \frac{1}{N_{eco}(1-\beta)} \sum_{i=1}^{N_{eco}} \min\{J_i(\mathbf{u},\eta_i) - \alpha, 0\}$$

The definition of J_{ϕ_β} is still non-differentiable. A common approach, like in the max-min problem considered before, is to reformulate the problem using slack variables and additional constraints as follows:

$$\begin{aligned} J_{\phi_\beta}(\mathbf{u},\alpha) &= \{-\alpha - \frac{1}{N_{eco}(1-\beta)} \sum_{i=1}^{N_{eco}} t_i\}, \\ \text{s.t.} &\begin{cases} t_i \leq J_i(\mathbf{u},\eta_i) - \alpha \\ t_i \leq 0 \end{cases} \quad \forall i. \end{aligned} \quad (14)$$

Therefore, the optimization problem Eq. (13) (excluding the system dynamics, initial conditions and input bounds constraints) can be re-written as:

$$\begin{aligned} &\max_{u,\alpha,t} \{\frac{1}{N_{eco}} \sum_{i=1}^{N_{eco}} J_i + \gamma\alpha + \gamma \frac{1}{N_{eco}(1-\beta)} \sum_{i=1}^{N_{eco}} t_i\}, \\ &\text{s.t.} \begin{cases} t_i \leq J_i(\mathbf{u},\eta_i) - \alpha \\ t_i \leq 0 \end{cases} \quad \forall i. \end{aligned} \quad (15)$$

1) Simulation example: In order to better approximate Eq. (10), a larger ensemble of 100 scenarios of oil prices, as shown in Fig. 1(b), is chosen. The confidence interval β is 80%. Hence the CVaR equals the average of the last 20% of the worst-case values. In the case of an ensemble size of 100, it is the average of the worst 20 NPV values from the ensemble.

Results: Fig. 8 shows the change in the CVaR values and the corresponding change in the return (average NPV) with the increasing values of ω. MO has the highest return with the lowest CVaR value. With increasing value of ω, as the CVaR risk measure is the negative of CVaR value, risk is reduced at the cost of compromising return. At $\omega = 10$, a different result is observed because probably the optimization has become stuck in a lower local optimum. The results are summarized in Table III. MO and mean-CVaR optimal

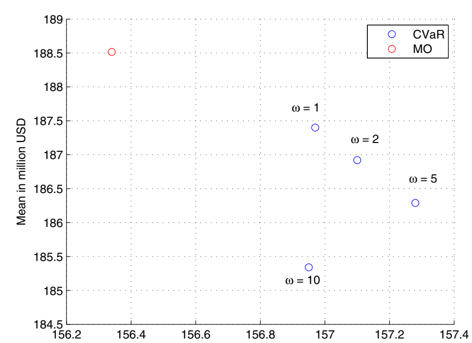


Fig. 8: Efficient frontier (Mean Vs. CVaR values with changing ω)

strategies are applied to the reservoir model realization with each oil price scenarios resulting in 100 different NPVs. As in the previous simulation examples, the corresponding PDFs are obtained by approximating a non-parametric KDE on these NPV data values as shown in Fig. 9. The CVaR risk measure, depending upon the confidence interval β, offers an asymmetric shaping of the NPV distribution by focusing

TABLE III: Results for the $J_{MO} - \omega J_{\phi_\beta}$ approach

ω	Mean in million USD	CVaR value in million USD	% decrease of mean w.r.t MO	% increase of CVaR value w.r.t MO
MO	188.51	156.34	-	-
1	187.40	156.97	0.58%	0.40%
2	186.92	157.10	0.84%	0.48%
5	186.29	157.28	1.17%	0.60%
10	185.34	156.95	1.68%	0.39%

only on the worst cases. In Fig. 9, it can be observed that in order to maximize the worst cases, the best cases are not highly penalized as compared to the MVO.

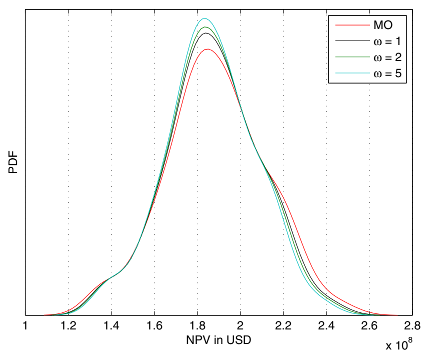


Fig. 9: Comparison of PDFs between MO and mean-CVaR

V. CONCLUSIONS

An asymmetric shaping of the NPV objective function distribution with respect to the economic uncertainty is performed using concepts from robust optimization and the theory of risk. A max-min robust optimization problem is implemented with an ensemble of varying oil price scenarios and the performance is demonstrated with a simulation example. Later a mean-worst-case multi-objective optimization is formulated that gives a decision maker a choice of selecting a worst-case value for the given value of the mean. In order to address the conservatism of the worst-case approach, a mean-CVaR approach is considered that maximizes some percentage of the worst cases. An efficient frontier, i.e., the value of CVaR at a given value of mean is constructed. Both mean-CVaR and worst-case are compared with the mean-variance approach. It is shown that the mean-variance approach has a higher tendency of largely compromising best cases in order to minimize the variance compared to the worst-case and the mean-CVaR approaches.

VI. ACKNOWLEDGEMENT

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