

Robust optimization of water-flooding in oil reservoirs using risk management tools

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Abstract: The theory of risk provides a systematic approach to handling uncertainty with well-defined risk and deviation measures. As the model-based economic optimization of the water-flooding process in oil reservoirs suffers from high levels of uncertainty, the concepts from the theory of risk are highly relevant. In this paper, the main focus is to offer an asymmetric risk management, i.e., to maximize the lower tail (worst cases) of the economic objective function distribution without heavily compromising the upper tail (best cases). Worst-case robust optimization and Conditional Value-at-Risk (CVaR) risk measures are considered with geological uncertainty to improve the worst case(s). Furthermore, a deviation measure, semi-variance, is also used with both geological and economic uncertainty to maximize the lower tail. The geological uncertainty is characterized by an ensemble of geological model realizations and the economic uncertainty is defined by an ensemble of varying oil price scenarios.

Keywords: Water-flooding optimization, Theory of risk, Handling uncertainty, Risk measures

1. INTRODUCTION

Risk is a broad concept covering different social and human sciences, e.g., ethics, psychology, medicine, economics etc. As a general definition, risk is an unexpected result or the probability of a failure. Theory of risk helps in modeling (or defining) risk, measuring it, and also provides tools to minimize or manage it, see e.g., Artzner et al. (1999), Krokmal et al. (2011). From a financial viewpoint, risk can be defined as the unpredicted variability or a potential loss of the expected economic objective. Markowitz (1952) in the early 50's has proposed a 'risk-return' portfolio selection approach, where the risk is characterized as the variance of the individual assets.

In the oil reservoir water-flooding optimization, a financial objective, e.g., Net Present Value (NPV) is maximized, see e.g., Brouwer and Jansen (2004), Foss (2012) and Van den Hof et al. (2012). Due to the limited knowledge of the reservoir model parameters and the varying economic conditions, this model-based economic optimization suffers from high levels of uncertainty. As risk management plays an important role in decision making under uncertainty (Rockafellar (2007)), water-flooding optimization becomes a natural candidate to use concepts from the theory of risk. In the petroleum engineering literature, decision making under uncertainty has been discussed from various perspectives. In Van Essen et al. (2009), a so-called robust optimization approach has been introduced, which maximizes an average NPV over an ensemble of geological model realizations. In Capolei et al. (2015b), a symmetric mean-variance optimization approach has been implemented honoring geological uncertainty. In Siraj et al. (2015a), these approaches have been extended to consider

the economic uncertainty characterized by varying oil price scenarios. Similar strategies have been described in Yeten et al. (2003), Bailey et al. (2005) and Yasari et al. (2013). One of the main limitations of the mean-variance optimization approach is the symmetric nature of the variance which also penalizes the best cases, while generally, in a maximization problem, the decision maker is mainly concerned with the lower tail of the objective function distribution. In Xin and Albert (2015), a multi-objective optimization has been implemented that maximize the average of the objective function and the worst case with respect to the geological uncertainty. As an early work of using the theory of risk in water-flooding optimization, different risk measures with their pros and cons have been reviewed in Capolei et al. (2015a) and their suitability for the production optimization is studied. In Siraj et al. (2015b), asymmetric risk measures have been studied and implemented with economic uncertainty.

The main contribution of this work is to address the question of how the well-defined risk and deviation measures in the theory of risk can be beneficial in providing an asymmetric risk management of the objective function, i.e., NPV distribution? Both geological and economic uncertainties are considered. The asymmetric risk measures such as the worst-case max-min approach (Bertsimas et al. (2011)) and the Conditional Value-at-Risk (CVaR) (Rockafellar and Uryasev (2000)) are implemented with geological uncertainty characterized by an ensemble of reservoir models. The worst-case approach, that maximizes the worst-case in a given uncertainty set, and the CVaR, defined as the average of some percentage of the worst-case scenarios, allow for an asymmetric shaping of the objective

function distribution. The asymmetric deviation measure, semi-variance, originally proposed in Markowitz (1952), provides a measure for the return being below the expected return. It is also considered and implemented with both geological and economic uncertainties. The economic uncertainty is characterized by an ensemble of varying oil prices.

The paper is organized as follows: In the next section, uncertainty in model-based economic optimization of the water-flooding process is explained. In Section 3, the worst-case optimization approach is presented with a simulation example. CVaR optimization with a simulation example under geological uncertainty is presented in Section 4. Section 5 discusses the Semi-variance approach in detail with a simulation example with both geological and economic uncertainty. Finally the conclusions of the presented results are given in Section 6.

2. UNCERTAINTY IN WATER-FLOODING OPTIMIZATION

Water-flooding involves the injection of water in an oil reservoir to increase oil production. NPV, as an objective for the dynamic optimization of the water-flooding process, can be mathematically represented in the usual fashion as:

$$J = \sum_{k=1}^K \left[\frac{r_o \cdot q_{o,k} - r_w \cdot q_{w,k} - r_{inj} \cdot q_{inj,k}}{(1+b)^{\frac{t_k}{\tau_t}}} \cdot \Delta t_k \right] \quad (1)$$

where r_o , r_w and r_{inj} are the oil price, the water production cost and the water injection cost in $\frac{\$}{m^3}$ respectively. K represents the production life-cycle i.e., the total number of time steps k and Δt_k the time interval of time step k in days. The term b is the discount rate for a certain reference time τ_t . The terms $q_{o,k}$, $q_{w,k}$ and $q_{inj,k}$ represent the total flow rate of produced oil, produced water and injected water at time step k in $\frac{m^3}{day}$.

The limited information contents in seismic, well logs and production data about the true reservoir parameters result in highly uncertain reservoir models. Similarly, the NPV objective function contains economic variables such as interest rate, oil price, etc., which fluctuate with time and can not be precisely predicted. The first step in handling uncertainty is the modeling (quantification) of the uncertainty space Θ . A general practice of quantifying uncertainty in the water-flooding optimization is by considering an ensemble of uncertain parameters, see e.g., Van Essen et al. (2009), Capolei et al. (2015b). It is equivalent to discretizing the uncertainty space, i.e., $\Theta_{N_{geo}} := \{\theta_1, \theta_2, \dots, \theta_{N_{geo}}\}$, where θ_i is a realization of uncertain parameter in an ensemble of N_{geo} members.

Water-flooding optimization is a highly complex large-scale non-linear optimization problem. In this work, a gradient-based optimization approach is used where the gradients are obtained by solving a system of adjoint equations, see e.g., Jansen (2011). An optimization solver KNITRO (Byrd et al. (2006)) is then used with an interior point method to iteratively converge to a (possibly local) optimum.

In the next sections, various risk/deviation measures, i.e., worst-case, CVaR and semi-variance are discussed in details with simulation examples.

3. WORST-CASE ROBUST OPTIMIZATION

Worst-case robust optimization (WCO) assumes that the uncertainty is known only within certain bounds, i.e., uncertainty set Θ , and the solution is robust for any realization of the uncertainty in the given set. Hence it focuses only on the worst-case in Θ and solves a max-min (or min-max) problem. The worst-case or a max-min optimization objective can be written as:

$$\max_{\mathbf{u}} \min_{\theta_i} J_i(\mathbf{u}, \theta_i) \quad (2)$$

where \mathbf{u} is control input and $\theta_i \in \Theta_{N_{geo}}$ is the uncertain parameter. It can easily be seen that the above optimization problem is non-differentiable, so a common approach to reformulate the above *max-min* problem is by adding a slack variable z with additional constraints as follows: (Ben-Tal et al. (2009))

$$\begin{aligned} \max_{\mathbf{u}, z} \quad & z \\ \text{s.t.} \quad & z \leq J_i(\mathbf{u}, \theta_i) \quad \forall i. \end{aligned} \quad (3)$$

Therefore, for a total number of ensemble members N_{geo} , there will be N_{geo} additional constraints. As the worst-case optimization only focuses on the lowest value of the NPV distribution, it provides an asymmetric risk management. The main limitation of the worst-case approach is that it provides a very conservative solution. In the next subsection, the simulation example for the worst-case approach (3) implemented with geological uncertainty is presented.

3.1 Simulation example under geological uncertainty

Simulation tools: All the simulation experiments in this work are performed using MATLAB Reservoir Simulation Toolbox (MRST) (Lie et al. (2012)) while KNITRO (Byrd et al. (2006)) is used for subsequent optimization.

Reservoir models: We use an ensemble of $N_{geo} = 100$ geological realizations of the Standard egg model, see Jansen et al. (2014). Each model is a three-dimensional realization of a channelized reservoir produced under water flooding conditions with eight water injectors and four producers based on the original Egg model proposed in Van Essen et al. (2009). The true permeability field is considered to be the only unknown parameter and the number of 100 realizations is assumed to be large enough to be a good representation of this parametric uncertainty space. The life-cycle of each reservoir model is 3600 days. The absolute-permeability field of the first realization in the set is shown in Fig. 1. Fig. 2 shows the

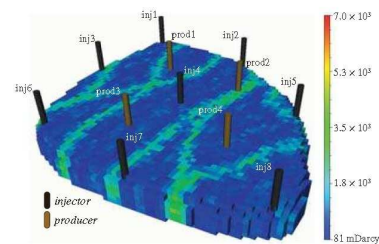


Fig. 1. Permeability field of realization 01 of a set of 100 realizations
permeability fields of six randomly chosen realizations of

the standard egg model in an ensemble of 100 realizations. Each realization in the set is considered as equiprobable.

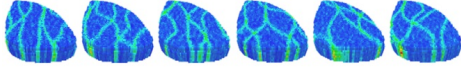


Fig. 2. Permeability fields of 6 randomly chosen realizations

Economic data for NPV: In this example, all economic parameters are considered as fixed. An un-discounted NPV, i.e., with discount factor $b = 0$ is used. Other economic parameters, e.g., oil price r_o , water injection r_{inj} and production cost r_w are chosen as $126 \frac{\$}{m^3}$, $6 \frac{\$}{m^3}$ and $19 \frac{\$}{m^3}$ respectively.

Control input: The control input \mathbf{u} involves injection flow rate trajectories for each of the eight injection wells. The minimum and the maximum rate for each injection well are set as $0.2 \frac{m^3}{day}$ and $79.5 \frac{m^3}{day}$ respectively. The production wells operate at a constant bottom-hole pressure of $395bar$. The control input \mathbf{u} is reparameterized in control time intervals with input parameter vector φ . For each of the eight injection wells, the control input \mathbf{u} is reparameterized into ten time periods of t_φ of 360 days during which the injection rate is held constant at value φ_i . Thus the input parameter vector φ consists of $N_u = 8 \times 10 = 80$ elements.

Control strategies: A mean optimization (MO) has been proposed in Van Essen et al. (2009), where an average NPV is maximized with an ensemble of geological model realization given as follows:

$$J_{MO} = \frac{1}{N_{geo}} \sum_{i=1}^{N_{geo}} J_i(\mathbf{u}, \theta_i) \quad (4)$$

This mean optimization (MO) approach is a so-called risk-neutral approach. It includes the uncertainty in the optimization framework and shows its effect by a distribution of NPV. But it does not aim at minimizing the negative effects, i.e., it does not reduce/shape the obtained NPV distribution. It also provides an upper bound on the maximal achievable average NPV for a given uncertainty ensemble. MO is compared with the worst-case optimization.

3.2 Results

The obtained MO and the WCO optimal strategies are applied to the ensemble of 100 reservoir model realizations, resulting in 100 different NPVs. The corresponding PDFs are obtained by approximating a non-parametric Kernel Density Estimation (KDE) with MATLAB routine 'ksdensity' on these NPV data values as shown in Fig. 3. MO shows the highest achieved average NPV with a longer lower tail. It does not attempt to reduce/shape the NPV distribution. With WCO, as expected, the worst-case value (lower bound) is increased with a decrease in the average value. The increase in worst case also effect the achievable best case in this case. The results are summarized in Table 1.

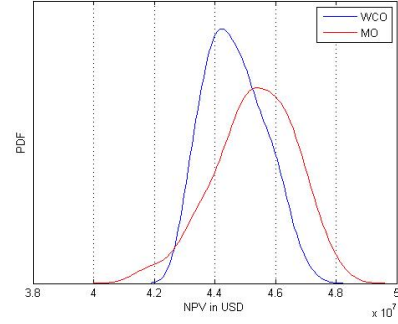


Fig. 3. NPV distribution comparison of MO and WCO

Table 1. % change of the worst-case and the average values with WCO

	MO	WCO	% change
Average in million USD	45.3	44.6	1.54% decrease
Worst-case in million USD	41.6	43.1	3.60% increase

Another way to use the WCO problem is to optimize a weighted objective of worst-case and the MO as follows:

$$J_{MWCO} = J_{MO} + \lambda J_{WCO} \quad (5)$$

where J_{WCO} represents the WCO optimization objective, as in (2) and $\lambda \in \mathbb{R}$ is a weighting parameter. The formulation in (5), gives a decision maker a preference to obtain an increase in the worst case for a given value of the mean or vice versa. This approach is not implemented in this work. The WCO can easily be extended with the economic uncertainty, see e.g., Siraj et al. (2015b).

4. CONDITIONAL VALUE-AT-RISK (CVAR) OPTIMIZATION

Conditional Value-at-Risk (CVaR), introduced in Rockafellar and Uryasev (2002), is a popular tool for managing risk in finance. CVaR indicates the average of the β -tail of the worst cases of a distribution. It addresses the overly conservative solution of the worst case optimization by considering a class of worst cases and offers an asymmetric shaping of the objective function distribution. It is similar to the Value-at-Risk (VaR) or chance constrained optimization (Schwarm and Nikolaou (1999)), which is a percentile of a loss/gain distribution.

For a random variable Ψ with cumulative distribution function $F_\Psi(z) = P\{\Psi \leq z\}$, the VaR (α_β) and CVaR (ϕ_β) of Ψ with confidence level $\beta \in]0, 1[$ are given as:

$$\alpha_\beta(\Psi) = \min\{z | F_\Psi(z) \leq \beta\},$$

$$\phi_\beta(\Psi) = \mathbb{E}[\Psi | \Psi \leq \alpha_\beta].$$

For a function $f(u, \theta)$ that represents a loss distribution, where $u \in U \subseteq \mathbb{R}^m$ is the decision vector and $\theta \in \mathbb{R}^n$ is a random vector representing uncertainties, Rockafellar and Uryasev (2002) have introduced a simpler auxiliary function F_β on $U \times \mathbb{R}$ defined as follows:

$$F_\beta(u, \alpha) = \alpha + \frac{1}{1 - \beta} \int_{\theta \in \mathbb{R}^n} [f(u, \theta) - \alpha]^+ p(\theta) d\theta \quad (6)$$

where $[t]^+ := \max\{t, 0\}$. Rockafellar and Uryasev (2002) have shown that the β -CVaR of the loss associated with any $u \in U$ can be determined as follows:

$$\phi_\beta(u) = \min_{\alpha \in \mathbb{R}} F_\beta(u, \alpha). \quad (7)$$

Furthermore, minimizing the β -CVaR of the loss associated with u is equivalent to minimizing $F_\beta(u, \alpha)$ over all $(u, \alpha) \in U \times \mathbb{R}$, in the sense that

$$\min_{u \in U} \phi_\beta(u) = \min_{(u, \alpha) \in U \times \mathbb{R}} F_\beta(u, \alpha). \quad (8)$$

In the water-flooding optimization, the mean-CVaR approach can be written by considering the objective as:

$$J_{MCVaR} = J_{MO} - \omega J_{\phi_\beta} \quad (9)$$

where J_{ϕ_β} represents the CVaR objective and $\omega \in \mathbb{R}$ is the weighting parameter. As the sampling of the uncertainty space generates a collection of scenarios $\theta_1, \dots, \theta_{N_{geo}}$, the integral in the CVaR optimization formula Eq. (6) can be approximated by a sum. For the NPV distribution, the CVaR (i.e., the negative of the CVaR value) is then given by:

$$J_{\phi_\beta}(\mathbf{u}, \alpha) = -\alpha - \frac{1}{N_{geo}(1-\beta)} \sum_{i=1}^{N_{geo}} \min\{J_i(\mathbf{u}, \theta_i) - \alpha, 0\}$$

The J_{ϕ_β} is still non-differentiable. A common approach, like in the max-min problem considered before, is to reformulate the problem using slack variables and additional constraints as follows:

$$J_{\phi_\beta}(\mathbf{u}, \alpha) = \left\{ -\alpha - \frac{1}{N_{geo}(1-\beta)} \sum_{i=1}^{N_{geo}} t_i, \right. \\ \left. \text{s.t. } \begin{cases} t_i \leq J_i(\mathbf{u}, \theta_i) - \alpha \\ t_i \leq 0 \end{cases} \quad \forall i. \right.$$

Therefore, the optimization problem Eq. (9) (excluding the system dynamics, initial conditions and input bound constraints) can be re-written as:

$$\max_{\mathbf{u}, \alpha, t} \left\{ \frac{1}{N_{geo}} \sum_{i=1}^{N_{geo}} J_i(\mathbf{u}, \theta_i) + \omega \alpha + \omega \frac{1}{N_{geo}(1-\beta)} \sum_{i=1}^{N_{geo}} t_i, \right. \\ \left. \text{s.t. } \begin{cases} t_i \leq J_i(\mathbf{u}, \theta_i) - \alpha \\ t_i \leq 0 \end{cases} \quad \forall i. \right.$$

4.1 Simulation example under geological uncertainty

The geological model uncertainty ensemble, economic parameters and the control inputs are the same as used in the previous simulation example. The CVaR problem is optimized for different values of $\omega \in \{0.5, 2, 2.5\}$. As there are no well-defined rules on how to choose ω , these values are randomly chosen. The confidence interval β is 80%. Hence the CVaR equals the average of the last 20% of the worst-case values. In the case of an ensemble size of 100, it is the average of the worst 20 NPV values from the ensemble. The obtained CVaR optimal strategies with different ω are applied to the ensemble of 100 reservoir model realizations, resulting in 100 different NPVs. The corresponding PDFs are obtained by approximating KDE with MATLAB routine *'ksdensity'* on these NPV data values as shown in Fig. 4. It can be observed in the figure that the NPV distribution resulting from MO has a long lower tail. All CVaR strategies provide an improvement in the worst-cases but this improvement is achieved at an expense of compromising the best-cases. The results highly depend upon the chosen uncertainty ensemble and on the weighting parameter ω . In this case, $\omega = 0.5$ provides better results in terms of improving the worst-cases with

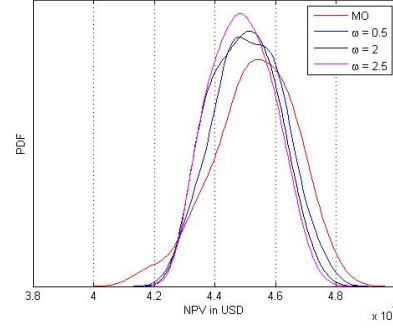


Fig. 4. NPV distribution comparison of MO and CVaR for different ω

a minimum decrease of best-cases. The The average value is also decreased with increasing ω .

These optimization problems are highly non-convex and computationally very demanding. A server computer having 20 physical cores is used with the MATLAB parallel computing toolbox to reduce the time of computation. Due to the non-convexity of the optimization problem, many local optima are attained with different values of ω . But as it is computationally extremely demanding optimization problem, we have not solved the problem for local minima.

Furthermore, the approximation of the CVaR risk measure in (6) provides a numerically stable estimate of CVaR with a higher number of uncertainty samples (Rockafellar and Uryasev (2002)). In water-flooding optimization, increasing the number of geological realizations will give a better CVaR approximation but at the cost of increasing the computational complexity. This approach can also be easily extended to include economic uncertainty, see e.g., Siraj et al. (2015b).

5. SEMI-VARIANCE OPTIMIZATION

Standard semi-deviation or semi-variance has been originally proposed by Markowitz (1952). It measures the downside risk of an objective function distribution. According to the axiomatic approach of deviation measures (Artzner et al. (1999)), it is not a coherent or averse measure but it still provides an asymmetric treatment of the NPV distribution. For the random variable Ψ , the semi variance can be defined as:

$$\text{Var}_+(\Psi) = \mathbb{E}[\max\{\Psi - \mathbb{E}\Psi, 0\}]^2, \quad (10)$$

$$\text{Var}_-(\Psi) = \mathbb{E}[\max\{\mathbb{E}\Psi - \Psi, 0\}]^2, \quad (11)$$

where Var_+ defines the spread of the values of Ψ greater than the mean $\mathbb{E}\Psi$, and Var_- characterizes the spread of the lower tail. As with the NPV gain distribution, the worst-case loses, i.e., the lower tail of the NPV distribution, is the main concern. Therefore, Var_- is minimized in a weighted mean-semi-variance (MSV) optimization given as follows:

$$J_{MSV} = J_{MO} - \gamma J_{\text{Var}_-} \quad (12)$$

where J_{MO} is the average objective as in (4), J_{Var_-} represents the semi variance objective and $\gamma \in \mathbb{R}$ is the weighting parameter of the semi-variance term. Due to the different units of semi variance and mean, γ plays a role of a scaling parameter as well.

Consider the case of geological uncertainty with an ensemble of N_{geo} model realizations. The sample semi-variance can be given as:

$$J_{Var-} = \frac{1}{(N_{geo} - 1)} \sum_{i=1}^{N_{geo}} \max\{J_{MO} - J_i, 0\}^2. \quad (13)$$

Therefore, Eq. (12) becomes:

$$J_{MSV} = J_{MO} - \gamma \frac{1}{(N_{geo} - 1)} \sum_{i=1}^{N_{geo}} \max\{J_{MO} - J_i, 0\}^2.$$

The above J_{MSV} is non-differential due to the 'max' operator. A common approach, as seen in the previous worst-case and CVaR cases, is to replace the 'max' operator with additional constraints. The Mean-semi variance optimization problem can then be written as follows:

$$J_{MSV} = J_{MO} - \gamma \frac{1}{(N_{geo} - 1)} \sum_{i=1}^{N_{geo}} t_i^2, \quad (14)$$

$$s.t. \quad \begin{cases} t_i \geq J_{MO} - J_i \\ t_i \geq 0 \end{cases} \quad \forall i.$$

5.1 Simulation example under geological uncertainty

The geological model ensemble, economic parameters and the control inputs are the same as used in the previous simulation examples. The Mean-semi variance problem is optimized for different values of $\gamma \in \{2 \times 10^{-8}, 3 \times 10^{-8}, 5 \times 10^{-8}\}$. These values are randomly chosen. The corresponding PDFs are obtained by approximating KDE with MATLAB routine 'ksdensity' and are displayed in Fig. 5. In this case, the NPV distributions are shifted to

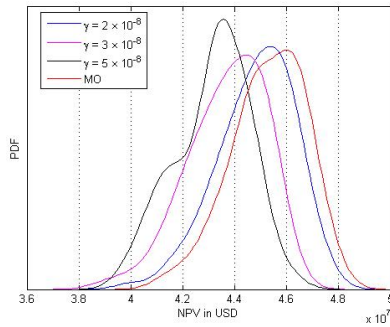


Fig. 5. NPV distribution comparison of MO and Mean-semi variance for different γ

the left to minimize the semi variance. Though the semi variance (the lower-tail spread) is reduced, both the worst-cases and the mean are also reduced. Hence this does not provide an attractive solution with the given geological uncertainty.

5.2 Simulation example under economic uncertainty

Economic uncertainty is also considered for the semi-variance optimization. The results of economic uncertainty with worst-case and CVaR optimization have been given in Siraj et al. (2015b). The economic variables that govern the NPV, especially the oil price r_o , vary drastically over time and can not be precisely predicted. These unknown

variations of future oil prices are the key source of economic uncertainty. Therefore in this work, only varying oil prices are used to characterize economic uncertainty. A finite number of scenarios $\eta_i, i = 1, \dots, N_{eco}$ are considered to characterize the economic uncertainty. There are various ways to predict the future values of the changing oil prices. For this work a simplified Auto-Regressive-Moving-Average (ARMA) model is used to generate oil price time-series. The ARMA model is shown below:

$$r_{ok} = a_0 + \sum_{i=1}^6 a_i r_{ok-i} \quad (15)$$

where a_i are the randomly selected coefficients.

The oil price scenarios with a base oil price of $471 \frac{\$}{m^3}$ and an ensemble size, $N_{eco} = 100$ are generated as shown in Fig. 6. The remaining economic parameters and the

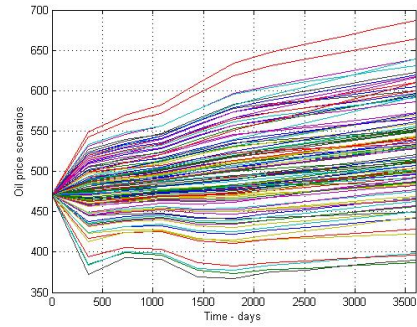


Fig. 6. Oil price scenarios with ensemble size $N_{eco} = 100$

control inputs are the same as in the previous examples. A single realization of the standard egg model as shown in Fig. 1 is used.

5.3 Results

The Mean-semi variance problem with economic uncertainty is optimized for different values of $\gamma \in \{5 \times 10^{-8}, 8 \times 10^{-8}, 1 \times 10^{-7}\}$. These values are randomly chosen. The obtained strategies are applied to the single reservoir model with 100 oil price scenarios, resulting in 100 NPV values. The corresponding PDFs are shown in Fig. 7. In this

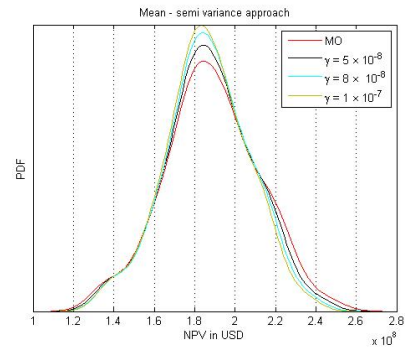


Fig. 7. NPV distribution comparison of MO and Mean-semivariance for different γ

case, the semi variance is reduced and it still improves the worst cases unlike the case in geological uncertainty. The improvement is achieved at the cost of compromising the mean and the best cases.

6. CONCLUSION

An asymmetric shaping of the NPV objective function distribution is performed using concepts from the theory of risk. The simulation results with geological uncertainty show that the asymmetric risk measures, i.e., the worst-case and the CVaR improve the worst case(s) without highly compromising the best cases. The results, especially for the CVaR case, depend largely on the selection of uncertainty ensemble and the ensemble size. The semi variance does not provide a very attractive solution for geological uncertainty as it reduces the worst cases. As all these risk measures involves constraint optimization with large-scale physical model ensemble, they are computationally very involved. If computation is not a problem, for example because of the availability of fast computing clusters, CVaR risk measure due to its better mathematical properties and with less conservative solution, is a preferred asymmetric risk measure.

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