Asymmetric risk measures for optimizing economic performance of oil reservoirs☆

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Abstract

Model-based economic optimization of the water-flooding process in oil reservoirs suffers from high levels of uncertainty both in geological reservoir parameters and in economic parameters such as (future) oil price. In order to improve robustness of the solutions, a scenario-based approach is used, where the geological uncertainty is characterized by an ensemble of model realizations and the economic uncertainty is defined by an ensemble of varying oil prices. Different robust approaches have been introduced in the literature, such as mean and mean-variance optimization (MVO). Due to the symmetric nature of variance, MVO equally penalizes both the worst and the best cases. In this paper we apply concepts from the theory of risk, as a systematic approach to handling uncertainty, allowing an asymmetric treatment of uncertainty, and targeting at improving the worst-case economic performance without heavily compromising the best-cases. Besides the earlier mentioned

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approaches, we investigate worst-case robust optimization and Conditional Value-at-Risk (CVaR), as well as a deviation measure, semi-variance, to evaluate their performance in the considered water-flooding process. Both geological and economic uncertainty are considered. Simulations are performed on a literature benchmark problem (the 'standard egg model').

Keywords: Water-flooding optimization, Theory of risk, Uncertainty, Risk measures, Robust optimization, Economic optimization

1. Introduction

Water-flooding involves the injection of water in an oil reservoir to increase oil production. Dynamic optimization of the water-flooding process has shown significant scope for improvement of the economic life-cycle performance of oil fields compared to more conventional strategies, see e.g., [1–6]. In these studies a financial measure, i.e., Net Present Value (NPV), is maximized. One of the key challenges in this model-based economic optimization is the high level of uncertainty arising from the limited knowledge of model parameters and from strongly varying economic conditions. As a result, the potential advantages of this optimization are not fully realized and the risk of loosing the expected economic objective is very high.

Among various approaches to optimization under uncertainty, the theory of risk provides a systematic approach to handling uncertainty. It helps in modeling (or defining) risk of uncertainty, measuring it, and also provides tools to minimize or manage the effect of it, see e.g., [7, 8]. Risk is a broad concept being applied in different social sciences and humanities, e.g., ethics, psychology, medicine, economics etc. The typical definition of risk as used in the oil industry is 'probability times consequence', i.e., the probability of occurrence of an undesirable event times some quantitative measure of the outcome of that event. In the early 50's, Markowitz [9] has introduced a 'risk-return' portfolio selection approach, where the risk is characterized as the variance of the returns. In [7], an axiomatic approach to define risk measures has been introduced. In [10] and [11], the relevant properties of risk and deviation measures have been presented.

In the petroleum engineering literature, various robust approaches to handle uncertainty have been discussed from various perspectives. In [12], a so-called robust optimization approach has been introduced, which maximizes an average NPV over an ensemble of geological model realizations. In [13], a symmetric mean-variance optimization (MVO) approach has been implemented honoring geological uncertainty. MVO quantifies the risk as the variance of the NPV distribution and with a maximization of the average NPV also minimizes the risk (variance of NPV distribution). In [14], these approaches have been extended to consider economic uncertainty characterized by varying oil price scenarios. Similar MVO approaches, e.g., for a well-placement problem have been described in [15–17]. One of the drawbacks of the risk-averse mean-variance optimization is the symmetric nature of the objective which equally penalizes both the best and the worst-case values. The decision maker, in a maximization problem, is mainly concerned with the objective function values below the average value, i.e., the lower tail of the objective function distribution, thus the use of downside or asymmetric risk-averse approaches in water-flooding optimization becomes highly relevant. A multi-objective optimization that maximizes the average of the objective function and the worst-case value with only geological uncertainty has been implemented in [18]. In [19], different symmetric and asymmetric risk measures have been reviewed and their suitability for water-flooding optimization has been studied. The authors proposed the use of conditional value-at-risk (CVaR) ([20]) and the worst-case approach ([21]) as appropriate measures for risk minimization. In [22], a set of time-explicit (TE) methods has been introduced which offers a reduction of geological and economic risks using concepts of multi-objective optimization and risk mitigation techniques. In [23], CVaR is used for constraint handling with daily production optimization by adjusting the gas lift rates and wellhead pressure under capacity constraints. Model-predictive control (MPC) has been used as a reference tracking controller in [24], which balances short-term and long-term objectives based on a time-varying oil price. In [25], an augmented Lagrangian method has been used with Stochastic-Simplex-Approximate-Gradient (StoSAG) algorithm which maximizes the expected NPV and minimizes the associated risk. In these studies, a detailed analysis of asymmetric risk and deviation measures for improving the downside NPV values (worst-cases) for water-flooding optimization and their affect on the best-cases and average NPV values has not been addressed. Furthermore, these studies focus on geological uncertainty and an explicit inclusion of economic uncertainty is generally not considered.

The main contribution of this work is to address the question: how can well-defined risk and deviation measures in the theory of risk be beneficial in reducing the effect of uncertainty in the achieved NPV and in particular in improving the worst-case values without heavily penalizing the best-case values? We explicitly consider both geological as well as economic uncertainty. Asymmetric or downside risk and deviation measures such as the worst-case max-min approach ([26]), CVaR ([20]) and semi-variance optimization ([9]) are considered and their suitability for the water-flooding optimization in terms of improving the worst-case values are compared and validated on a standard benchmark ensemble of reservoir models (the standard egg model, see [27]). The worst-case approach, that maximizes the worst-case in a given uncertainty set, and the CVaR, defined as the average of some percentage of the worst-case scenarios, allow for an asymmetric shaping of the objective function distribution. The asymmetric deviation measure, semi-variance, originally proposed in [9], provides a measure for the return being below the expected return. Geological and economic uncertainties are characterized by an ensemble of reservoir models and oil price scenarios respectively. To analyse and evaluate the particular effect of each form of uncertainty on the obtained NPV, both forms of uncertainties are not considered simultaneously. The joint geological-economic uncertainty reduction is computationally expensive and considered as a future research direction. This paper is based on the preliminary results of [28, 29] but developed and formulated here in a systematic and unified framework.

The paper is organized as follows: Section 2 introduces water-flooding optimization and uncertainty quantification. Section 3 discusses worst-case optimization. In section 4, CVaR optimization with simulation results is presented. Semi-variance optimization is discussed in Section 5, followed by conclusions in Section 6.

2. Handling uncertainty using risk and deviation measures

Nominal water-flooding optimization without uncertainty is a deterministic optimization problem given as:

$$\max_{\mathbf{u}} J(\mathbf{u}, \boldsymbol{\theta}), \tag{1}$$

where $\mathbf{u}$ is the decision variable, J is the economic objective, e.g., NPV and $\boldsymbol{\theta}$ represents model and/or economic parameters. NPV can be mathematically represented as follows:

$$J = \sum_{k=1}^{K} \left[\frac{r_o \cdot q_{o,k} - r_w \cdot q_{w,k} - r_{inj} \cdot q_{inj,k}}{(1+b)^{\frac{t_k}{\tau_t}}} \cdot \Delta t_k \right], \tag{2}$$

where r_o, r_w and r_{inj} are the oil price, the water production cost and the water injection cost in $[\$/m^3]$ respectively. K represents the production life-cycle, i.e., the total number of time steps k, and Δt_k the time interval of time step k in $[days]$. The term b is the discount rate (expressed as a fraction) for a certain reference time τ_t. The terms $q_{o,k}, q_{w,k}$ and $q_{inj,k}$ represent the flow rates of produced oil, produced water and injected water at time step k in $[m^3/day]$.

In the presence of uncertainty, the parameter vector belongs to uncertainty space (Θ) and it may be represented by a random variable with some probability distribution. Consequently, the objective J becomes a random variable. A risk measure is defined as functional $\mathcal{R} : J \rightarrow \mathbb{R}$ and it can be quantified as a surrogate for the overall cost. Risk management aims at minimizing the risk measure given as follows ([11], [19]):

$$\min_{\mathbf{u}} \mathcal{R}(J(\mathbf{u}, \boldsymbol{\theta} \in \Theta)). \tag{3}$$

The pioneering work of [7, 30] and [31] has provided axiomatic properties to ensure that the functional $\mathcal{R}$ is a good quantifier of the risk and has framed the concept of coherency of the risk measures. Coherency implies that if the nominal optimization problem, e.g., eq. (1), is convex, minimization of the coherent risk measure, as in eq. (3), is also convex. The optimization of NPV in water-flooding is a highly complex large-scale non-convex optimization problem and therefore, irrespective of coherency of the risk measures, the overall problem stays non-convex. One of the first steps in handling uncertainty is the modeling (quantification) of the uncertainty space. A general practice of quantifying uncertainty in water-flooding optimization involves considering an ensemble of uncertain parameters [12, 13]. This is equivalent to descretizing the uncertainty space, i.e., $\Theta_N := \{\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \cdots, \boldsymbol{\theta}_N\}$, where $\boldsymbol{\theta}_i$ is a realization of the uncertain parameter in an ensemble of N members.

In [12], a so-called robust optimization approach has been introduced where an average NPV over an ensemble of geological model realizations has been maximized, which is given as follows:

$$J_{MO} = \frac{1}{N_{geo}} \sum_{i=1}^{N_{geo}} J_i, \tag{4}$$

where N_{geo} is the number of model realizations. We reserve the name robust optimization for worst-case optimization and refer to the work of [12] as mean optimization (MO). It can easily be seen that the averaging approach includes uncertainty in the optimization framework but does not minimize the effect of it on the achieved NPV. MO is also referred to as a risk-neutral strategy. The Markowitz risk-return portfolio selection approach involves a quantitative characterization of risk in terms of the variance of the returns distribution [9]. With the reduction of variance of the objective function (NPV) distribution, the MVO offers some robustness and will reduce the uncertainty in the achieved NPV. The MVO approach can be written as:

$$J_{MVO} = J_{MO} - \gamma J_{VAR}, \tag{5}$$

where J_{VAR} represents the variance of the NPV distribution and $\gamma > 0$ is a weighting parameter. As a symmetric risk measure, MVO also penalizes the upper tail of the NPV distribution, i.e., reduces the best-case values. In the next sections, the focus will be on the asymmetric shaping of the NPV distribution. Among various risk/deviation measures, worst-case, CVaR and the semi-variance are considered.

3. Worst-case robust optimization

Worst-case (robust) optimization (WCO) is a deterministic approach and assumes that the uncertain parameter is known only within certain bounds, i.e., it is assumed to belong to an uncertainty set Θ. It optimizes the worst possible case of the considered problem and solves a max-min problem. A typical unconstrained worst-case robust optimization problem can be stated as:

$$\max_{\mathbf{u}} \min_{\boldsymbol{\theta}} J(\mathbf{u}, \boldsymbol{\theta}), \tag{6}$$

where $\boldsymbol{\theta} \in \Theta$ is the vector of uncertain parameters. The worst-case approach obtains an optimal solution $\mathbf{u}^*$ by maximizing the objective J for the worst-case value of the uncertain parameter $\boldsymbol{\theta}$. The above max-min problem can not be differentiated, and therefore it can not be directly optimized using a gradient-based scheme. For a scenario-based approach with an ensemble of model realizations, a common approach is to reformulate the above max-min problem in an epigraph form by adding a slack variable z with additional constraints given as follows [21]:

$$\begin{aligned} \max_{\mathbf{u},z} \quad & z, \\ \text{s.t.} \quad & z \leq J_i(\mathbf{u}, \boldsymbol{\theta}_i) \quad \forall i, \quad i = 1, 2, \cdots, N_{geo}. \end{aligned} \tag{7}$$

Therefore, for a total number of ensemble members N_{geo}, there will be N_{geo} additional constraints, which also increases the computational complexity. As the worst-case optimization only focuses on the lowest value of the NPV distribution, it does not penalize the best cases and provides an asymmetric shaping.

The WCO formulation as in eq. (7) may result in an undesired reduction of the average objective function value. Therefore, in order to give the decision maker a preference to choose a worst-case improvement for a given level of average NPV, a mean-worst case optimization (MWCO) problem can be formulated as follows:

$$J_{MWCO} = J_{MO} + \lambda J_{WCO}, \tag{8}$$

where J_{WCO} is the worst-case objective and $\lambda > 0$ is a weighting parameter which balances the average and the worst-case objectives.

For the case of economic uncertainty, as the uncertain parameter (varying oil prices, $r_{o_k}, k = 1, 2, \cdots K$) is affecting NPV linearly, the worst-case formulation in eq. (7) is simply equivalent to a nominal optimization with the worst oil price scenario in the ensemble, provided that it can be identified in the ensemble, i.e., $r^i_{o_k} \neq r^j_{o_k}, k = 1, 2, \cdots K, i \neq j$, and therefore, it simplifies the constrained WCO approach in eq. (7) to a single optimization problem.

In order to investigate whether the worst-case optimization leads to attractive results for the water-flooding process, simulation examples with geological and economic uncertainties are considered in the next sub-sections. The concepts of mean-CVaR and mean-semi variance optimization are discussed afterwards.

3.1. Simulation example under geological uncertainty

In all the simulation examples under geological uncertainty, the only source of uncertainty is the unknown model parameters. The economic parameters are considered as fixed.

3.1.1. Simulation tools

All the simulation experiments in this work are performed using MATLAB Reservoir Simulation Toolbox (MRST) [32]. In this work, a gradient-based optimization approach is used where the gradients are obtained by solving a system of adjoint equations, see, e.g., [33]. An optimization solver KNITRO [34] is then used with an interior point method to iteratively converge to a (possibly local) optimum.

3.1.2. Reservoir models

We use an ensemble of $N_{geo} = 100$ geological realizations of the standard egg model, see [27]. Each model is a three-dimensional realization of a channelized reservoir produced under water flooding conditions with eight water injectors and four producers based on the original egg model proposed in [12]. The true permeability field is considered to be the unknown parameter and the number of 100 realizations is assumed to be large enough to be a good representation of this parametric uncertainty space. The life cycle of each reservoir model is 3600 days. The absolute-permeability field of the first realization in the set is shown in Fig. 1. Fig. 2 shows the permeability fields

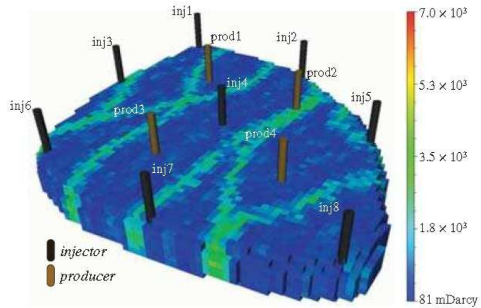


Figure 1: Permeability field of realization 1 **of a set of** 100 **realizations ([12],[27]). The permeability is expressed in mDarcy** $= 9.87 \times 10^{-16} m^2$**.**

of six randomly chosen realizations of the standard egg model in an ensemble of 100 realizations. Each realization in the set is considered as equiprobable.

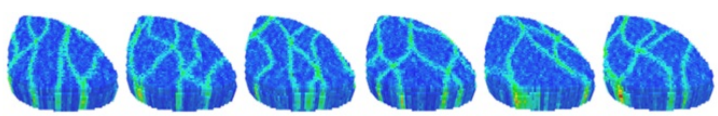


Figure 2: Permeability fields of 6 **randomly chosen realizations ([12],[27])**

3.1.3. Economic data for NPV

In this example, all economic parameters are considered as fixed. An undiscounted NPV, i.e., with discount factor $b = 0$, is used. The other economic parameters, i.e., oil price r_o, water injection r_{inj} and production cost r_w, are chosen as $126 \frac{\$}{m^3}$, $6 \frac{\$}{m^3}$ and $19 \frac{\$}{m^3}$ respectively.

3.1.4. Control input

The control input $\mathbf{u}$ involves injection flow rate trajectories for each of the eight injection wells. The minimum and the maximum rate for each injection well are set as $0.2\frac{m^3}{day}$ and $79.5\frac{m^3}{day}$ respectively. The production wells operate at a constant bottom-hole pressure of $395bar$. The control input $\mathbf{u}$ is reparameterized in control time intervals with input parameter vector $\boldsymbol{\varphi}$. For each of the eight injection wells, the control input $\mathbf{u}$ is reparameterized into ten time periods of t_φ of 360 days during which the injection rate is held constant at value φ_i. Thus the input parameter vector $\boldsymbol{\varphi}$ consists of $N_u = 8 \times 10 = 80$ elements.

Results

The results for the WCO approach are compared to MO and MVO with the same control and economic parameters. MVO, as given in eq. (5), is performed for different values of γ, i.e., $\gamma \in [1 \times 10^{-6}, 3 \times 10^{-6}, 7 \times 10^{-6}]$. These MVO results have also been presented in [14]. The optimal strategies obtained from solving WCO, MO and MVO for different values of γ are applied to the ensemble of 100 reservoir model realizations, resulting in 100 different NPVs for each strategy. The corresponding approximate PDFs are obtained by applying a non-parametric Kernel Density Estimation (KDE) with MATLAB routine '*ksdensity*' on these NPV values, with results as shown in Fig. 3. In this case, MO results in the highest achieved average NPV with a longer lower tail than the other two methods. It does not attempt to shape the NPV distribution, and therefore MO is also known as a risk-neutral approach. MVO is a symmetric measure and aims at minimizing the variance of NPV distribution irrespective of the worst-case and/or the

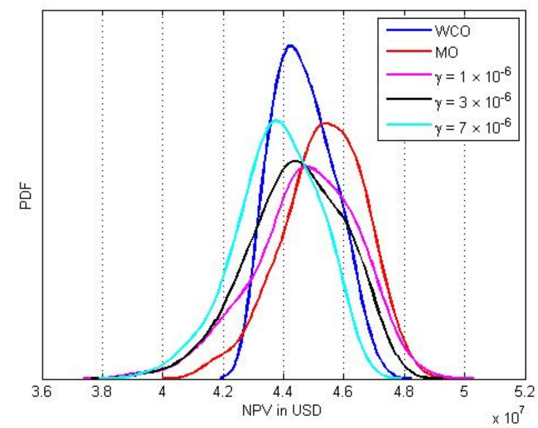


Figure 3: NPV distribution by applying optimal inputs from MO, MVO (with different γ) and WCO to each ensemble member under geological uncertainty

best-case performance. In this case, for all solutions of MVO, the reduction of variance results in lower best-case and worst-case NPV values compared to MO. Hence MVO does not offer an attractive solution. With WCO, as expected, the worst-case performance is improved, which is achieved at the cost of reducing the mean performance and also with an effect on the achievable best-case values. Interestingly, in this particular example, the WCO solution also results in a minimum variance which is a co-incidence and a result that can not be generalized. The numerical results are summarized in Table 1.

As it is shown, WCO provides an attractive option to improve the worst-case NPV value compared to MVO under geological uncertainty. However, uncertain model parameters have a large range of variability, as is also indicated by the long tail of the NPV distribution in Fig. 3. Therefore, in this case, WCO may provide a conservative solution to improve the worst-case

Table 1: % change of the worst case and the average NPV values with WCO in comparison with MO under geological uncertainty

	MO	WCO	% change
Average NPV in million USD	45.3	44.6	1.54% decrease
Worst-case NPV in million USD	41.6	43.1	3.60% increase

value. The conservatism of WCO is connected to the modeling of uncertainty, i.e., if the considered ensemble contains an 'outlier', the WCO is concerned only with the outlier and the solution is conservative. Furthermore, as it is observed that the mean value also decreases with improving worst-case value, an alternative way to maintain a given level of mean value is to use the formulation as given in eq. (8). For the case of geological uncertainty, as considered in this section, the mean-worst case approach J_{MWCO}, as in eq. (8), is not considered because it requires multiple WCO solutions. This is computationally too expensive because of the large number of constraints required to cope with the slack variables; see eq. (7).

3.2. Simulation examples under economic uncertainty

Oil reservoirs typically have a producing life cycle between 10 and 100 years. The economic variables that govern the NPV, especially the oil price r_o, vary drastically over time and can not be precisely predicted. These unknown variations of oil prices are the key source of *economic* uncertainty. Therefore in this work, only varying oil prices are used to characterize economic uncertainty. A finite number of oil price scenarios $\eta_i, i = 1, \cdots, N_{eco}$ from an uncertainty set $\mathcal{U}$ is used. There are various ways to predict the future values of the changing oil prices. Different models, e.g., Prospective Outlook on Long-term Energy Systems (POLES) used by European Union and the French government, National Energy Modeling System (NEMS) of United States energy markets created at the U.S. Department of Energy, Energy Information Administration (EIA) etc., are used for energy prices prediction, for details see [35–38]. However, for this example a simplified Auto-Regressive-Moving-Average model (ARMA) model, see [39], is used to generate oil price time-series. This simplified approach can still represent the effect of varying oil prices. The ARMA model is defined as:

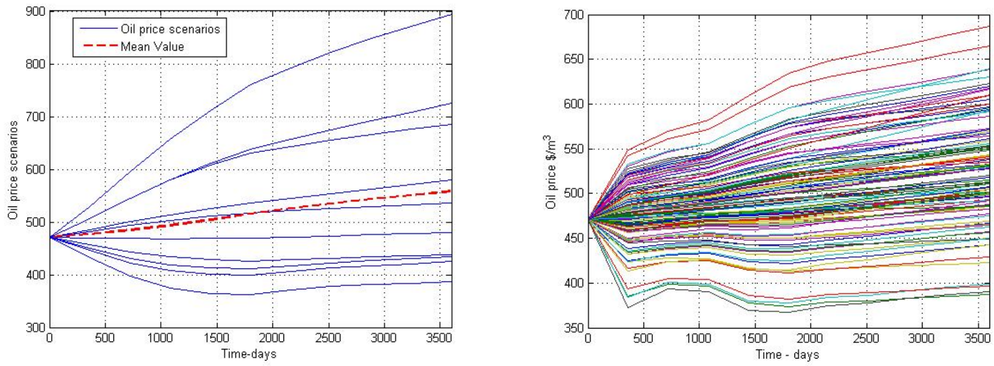
$$r_{o_k} = a_0 + \sum_{i=1}^{6} a_i r_{o_{k-i}} + \sum_{i=1}^{6} b_i e_{k-i}, \qquad (9)$$

where e_k is a white-noise sequence and a_i, b_i are coefficients selected in an ad-hoc way.

Two different oil price scenarios with the same base oil price of 471 [$\$/m^3$] and ensemble sizes, N_{eco}, of 10 and 100 respectively are generated as shown in Fig. 4. A single realization, i.e., realization number 1, of the standard egg model as shown in Fig. 1 is used as a fixed model. In this example, an ensemble size, N_{eco}, of 10 as shown in Fig. 4(a) is considered. The remaining economic parameters and the control inputs are the same as in the previous example.

Results

The MO and the WCO strategies are applied to the single model and the objective function, i.e., NPV, is evaluated with 10 oil price realizations, resulting in 10 different NPVs. The corresponding histogram is shown in Fig. 5. It can be seen that the worst-case value of the NPV distribution is



(a) $N_{eco} = 10$ **with mean value as indicated by red dotted line**

(b) $N_{eco} = 100$

Figure 4: Oil price according to scenarios for uncertainty characterization

improved at the cost of a decrease in the mean value. Table 2 summarizes

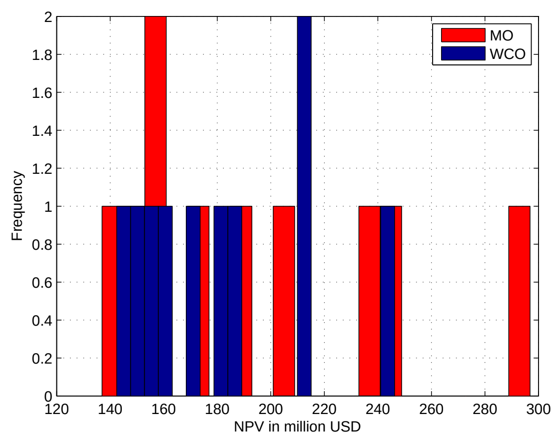


Figure 5: NPV histogram by applying optimal inputs from WCO and MO to the single model realization with each member of the oil price ensemble (economic uncertainty)

the percentage increase and decrease of the worst-case NPV and the average NPV respectively.

Table 2: % change of the worst-case and the average NPV values with WCO in comparison with MO under economic uncertainty

	MO	WCO	% change
Average NPV in million USD	194	182	6.18% decrease
Worst-case NPV in million USD	136	142	4.41% increase

The MVO strategy as given in eq. (5), with the considered model realization and oil price ensemble, is implemented for various values of weighting parameter, i.e., $\gamma \in \{1 \times 10^{-8}, 2 \times 10^{-8}, 3 \times 10^{-8}\}$. The results for the MVO and the WCO approaches are compared in Fig. 6. The corresponding approximate PDFs are obtained by applying a non-parametric KDE to these NPV values. The first observation is that, for this case of economic uncertainty, MVO does not reduce the worst-case values compared to the MO solution as was the case of for geological uncertainty. MVO is a symmetric measure, and hence the best-case values are also heavily penalized. For WCO, it can be observed that it improves the worst-case value without heavily compromising the best-case value compared to the MVO approach.

The mean-worst case approach, J_{MWCO}, as formulated in eq. (8), has now also been implemented. Fig. 7 shows the variation of the mean and the worst-case NPV values with respect to λ. As λ increases, the worst-case value is improved at the cost of reducing the mean value. MO has the highest mean with the lowest worst-case value. One of the challenges of using risk

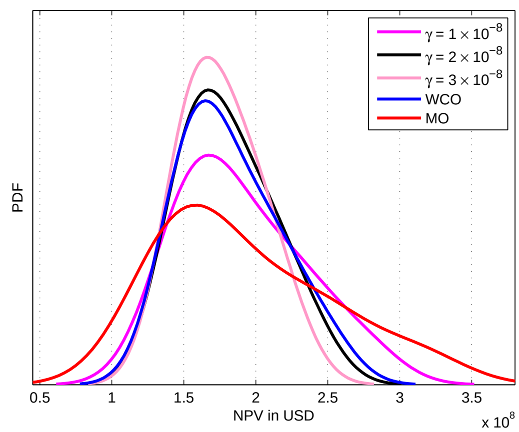


Figure 6: NPV distribution by applying optimal inputs from MO, MVO (with different γ) and WCO optimization to the single model realization with each member of the oil price ensemble (economic uncertainty)

measures in water-flooding optimization is the non-convexity, and hence the problem of different local optima. For example, in this case, we expect that the worst-case value improves with an increasing value of λ, but at $\lambda = 5$ we observe a different result. One of the possible reasons of this can be that the optimization has ended up in a local optimum and results in a lower worst-case NPV value. The numerical results are summarized in Table 3.

As discussed in the previous section, WCO with economic uncertainty can be simplified to a single optimization with the worst-case oil price value over time and specially in the case of oil price scenarios as shown in Fig. 4(a), it is easy to identify the worst-case oil price scenario. A single optimization with the worst-case oil price is performed. Fig. 8 shows a comparison of the histogram of NPV values resulting from the worst-case formulation as in eq. (7) and from a single optimization with the worst oil price scenario.

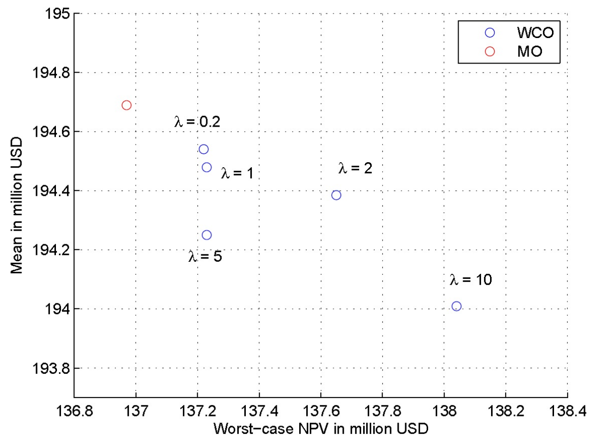


Figure 7: Change in the average and the worst-case NPV values as a function of λ

Table 3: Results for the mean-worst case approach under economic uncertainty

λ	Mean in million USD	Worst-case in million USD	% decrease of mean w.r.t MO	% increase of worst-case w.r.t MO
MO	194.6	136.9	-	-
0.2	194.54	137.22	0.07%	0.18%
1	194.47	137.23	0.10%	0.19%
2	194.38	137.65	0.15%	0.49%
5	194.25	137.23	0.22%	0.18%
10	194.00	138.04	0.34%	0.78%

As expected both optimizations gives the same results. For the oil price realizations, 4(b), there are cross overs between the lowest two realizations, and therefore it is not possible to identify the worst-case realizations.

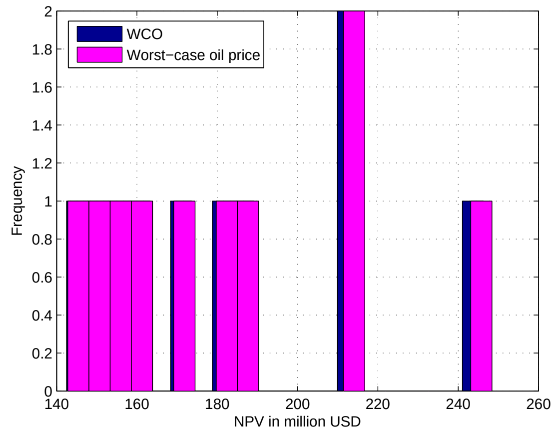


Figure 8: NPV histogram as a result of applying optimal inputs from WCO to the single model realization with each member of the oil price ensemble (economic uncertainty) compared to the input that results from nominal optimization with the worst oil price realization

In conclusion, WCO provides an attractive option of asymmetrically shaping the NPV distribution under both geological and economic uncertainty, specifically when the decision maker is mainly concerned with the worst-case performance. Furthermore, to provide a good balance between the worst-case performance and the improvement in the mean value, the mean-WCO approach is a preferred formulation as the balance can be achieved by tuning λ. As discussed in the previous section, WCO performance also depends upon uncertainty modeling, i.e., in the presence of a large variability in uncertainty set (an outlier in the considered ensemble), it may result in a solution that is very conservative. One possible way to avoid such a conservative solution is to use CVaR as a risk measure, which is motivated in the next section.

4. Conditional Value-at-Risk (CVaR) optimization

CVaR, introduced in [20], is a popular tool for managing risk in finance. It addresses the often overly conservative solution of the worst-case optimization by considering a class of worst cases and improves them without penalizing the best cases.

For a random variable Ψ with cumulative distribution function (CDF) $F_\Psi(z) = P\{\Psi \leq z\}$, the Value-at-Risk (VaR) (α_β) and CVaR (ϕ_β) of Ψ with confidence level $\beta \in]0, 1[$ are given as:

$$\alpha_\beta(\Psi) = \max\{z | F_\Psi(z) \leq \beta\},$$

$$\phi_\beta(\Psi) = \mathbb{E}[\Psi | \Psi \leq \alpha_\beta],$$

where VaR (α_β) or chance constrained optimization [40] is a $\beta-$percentile of an objective function distribution. CVaR (ϕ_β) is the expected value of all those points in the distribution which fall below the VaR value. Fig. 9 illustrates the concepts of the worst case, VaR and CVaR for a given cumulative distribution function.

For a function $f(u, \theta)$ that represents a loss distribution, where $u \in U \subseteq \mathbb{R}^m$ is the decision vector and $\theta \in \mathbb{R}^n$ is a random vector representing uncertainties, a simple auxiliary function F_β on $U \times \mathbb{R}$ for the computation of ϕ_β has been introduced in [20], defined as follows:

$$F_\beta(u, \alpha) = \alpha + \frac{1}{1 - \beta} \int_{\theta \in \mathbb{R}^n} [f(u, \theta) - \alpha]^+ p(\theta) d\theta, \tag{10}$$

where $[t]^+ := \max\{t, 0\}$. It has also been shown in [20] that the ϕ_β of the loss associated with any $u \in U$ can be determined as follows:

$$\phi_\beta(u) = \min_{\alpha \in \mathbb{R}} F_\beta(u, \alpha). \tag{11}$$

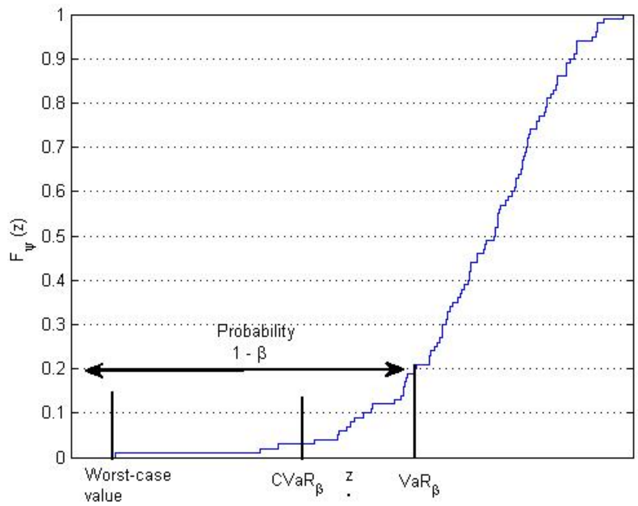


Figure 9: Worst case, VaR and CVaR for a given CDF $F_\Psi(z)$

Furthermore, minimizing the ϕ_β of the loss associated with u is equivalent to minimizing $F_\beta(u, \alpha)$ over all $(u, \alpha) \in U \times \mathbb{R}$, in the sense that

$$\min_{u \in U} \phi_\beta(u) = \min_{(u,\alpha) \in U \times \mathbb{R}} F_\beta(u, \alpha). \tag{12}$$

For the water-flooding optimization problem, we can write the mean-CVaR approach as:

$$J_{MCVaR} = J_{MO} - \omega J_{\phi_\beta}, \tag{13}$$

where J_{ϕ_β} represents the CVaR objective and $\omega \in \mathbb{R}$ is a weighting parameter. This approach aims to maximize the mean value and minimize CVaR. As the sampling of the uncertainty space generates a collection of scenarios $\boldsymbol{\theta}_1, \cdots, \boldsymbol{\theta}_{N_{geo}}$, the integral in the CVaR optimization formula in eq. (10) has to be approximated by a sum. For the NPV distribution, the CVaR is then given by:

$$J_{\phi_\beta}(\mathbf{u},\alpha) = -\alpha - \frac{1}{N_{geo}(1-\beta)}\sum_{i=1}^{N_{geo}} \min\{J_i(\mathbf{u},\boldsymbol{\theta}_i) - \alpha, 0\}.$$

J_{ϕ_β} is non-differentiable and it can not be directly used with a gradient-based optimization method. A common approach to solve this problem, like in the max-min problem considered before, is to reformulate the problem using slack variables t_i and additional constraints as follows:

$$\begin{aligned} J_{\phi_\beta}(\mathbf{u},\alpha) &= \{-\alpha - \frac{1}{N_{geo}(1-\beta)}\sum_{i=1}^{N_{geo}} t_i\}, \\ \text{s.t.} &\quad \begin{cases} t_i \leq J_i(\mathbf{u},\boldsymbol{\theta}_i) - \alpha \\ t_i \leq 0 \end{cases} \qquad \forall i. \end{aligned}$$

Therefore, the optimization problem in eq. (13) (excluding the system dynamics, initial conditions and input bound constraints) can be re-written as:

$$\begin{aligned} &\max_{u,\alpha,t}\{\frac{1}{N_{geo}}\sum_{i=1}^{N_{geo}} J_i(\mathbf{u},\boldsymbol{\theta}_i) + \omega\alpha + \omega\frac{1}{N_{geo}(1-\beta)}\sum_{i=1}^{N_{geo}} t_i\}, \\ &\text{s.t.} \begin{cases} t_i \leq J_i(\mathbf{u},\boldsymbol{\theta}_i) - \alpha \\ t_i \leq 0 \end{cases} \qquad \forall i. \end{aligned}$$

4.1. Simulation example under geological uncertainty

The geological model uncertainty ensemble (the standard egg model), economic parameters and the control inputs are the same as used in the previous simulation example. The confidence interval β is chosen as 80%. Hence the CVaR equals the negative of an average of the 20% of the worst-case values. In the case of an ensemble size of 100, it is the average of the worst 20 NPV values. The CVaR problem is optimized for different values of $\omega \in \{0.5, 2, 2.5\}$. As there are no well-defined rules on how to choose ω, these values are chosen in an ad-hoc way. Furthermore, as discussed before, in case of a non-convex optimization such as NPV optimization where many local optima can be attained, the selection of ω requires some trials to see the effect of CVaR reduction. These trials are also computationally very expensive, and hence a systematic selection of ω is important and will be considered as a future research topic. The obtained CVaR-optimal strategies with different ω are applied to the ensemble of 100 reservoir model realizations, resulting in 100 different NPVs. The corresponding approximate PDFs are obtained by applying KDE with MATLAB routine '*ksdensity*' to these NPV values, with results as shown in Fig. 10. It can be observed that the NPV distribution

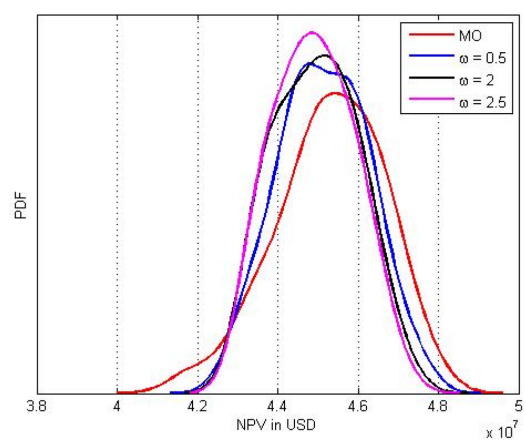


Figure 10: NPV distribution by applying optimal inputs from MO and CVaR for different ω to each ensemble member under geological uncertainty

resulting from MO has a longer and lower tail than the other distributions.. All CVaR strategies provide an improvement in the worst-case values but this improvement is achieved at the expense of compromising the best-case values. The results also depend on the weighting parameter ω. In this case, $\omega = 0.5$ provides better results in terms of improving the worst-case values with a minimum decrease of best-case values. The average value is also decreased with increasing ω. MVO solutions as shown in Fig. 3 do not offer an attractive solution because the worst case NPVs are reduced compared to both MO and mean-CVaR.

The CVaR optimization problem is highly non-convex and computationally very demanding. A server computer having 20 physical cores has been used with the MATLAB parallel computing toolbox to reduce the time of computation. Furthermore, the approximation of the CVaR measure of eq. (10) provides a numerically stable estimate of CVaR for the situation of a high number of uncertainty samples [20]. However, due to the non-convexity of the optimization problem, many local optima are attained with different values of ω. In water-flooding optimization, increasing the number of geological realizations will give a better CVaR approximation but at the cost of increasing the computational complexity.

4.2. Simulation example under economic uncertainty

In order to investigate CVaR optimization with economic uncertainty, an ensemble of 100 scenarios of oil prices, as shown in Fig. 4(b), is chosen. The confidence interval β is chosen as 80%, and the penalty parameter ω is $\omega \in \{1, 2, 5\}$. These values are chosen in an ad-hoc way. The optimal strategies obtained from MO and mean-CVaR are applied to the reservoir model realization with each oil price scenario resulting in 100 different NPVs. As in the previous simulation examples, the corresponding approximate PDFs are obtained by applying a non-parametric KDE to these NPV values, with results as shown in Fig. 11. It can be observed that economic uncertainty does not have a profound effect on the optimized strategies compared to geological uncertainty. The improvement in the worst-case values under economic uncertainty is less than expected. The CVaR risk measure, depending upon the confidence interval β, offers an asymmetric shaping of the NPV distribution by focusing only on the worst cases. The increase in the 20% worst-case values can be observed in the figure. Fig. 12 shows the change in

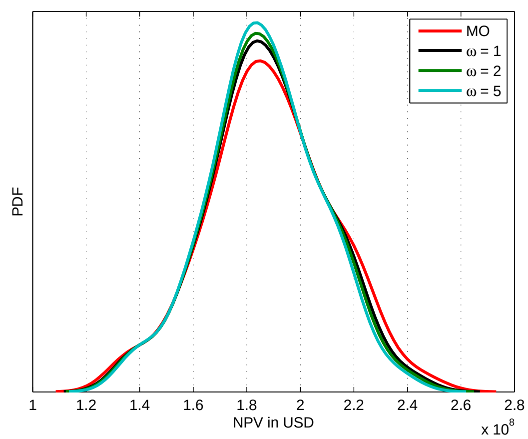


Figure 11: NPV distribution by applying optimal inputs from MO and CVaR for different ω to model realization with each member of oil price ensemble (economic uncertainty)

the CVaR values (the average of the 20% worst-case values) and the corresponding change in the average NPV as a function of ω. MO has the highest return with the lowest CVaR value. With an increasing value of ω, as the CVaR risk measure is the negative of CVaR value, risk is reduced at the cost of compromising return. At $\omega = 10$, an 'outlier' is observed because probably the optimization has gotten stuck in a local optimum. The numerical results are summarized in Table 4.

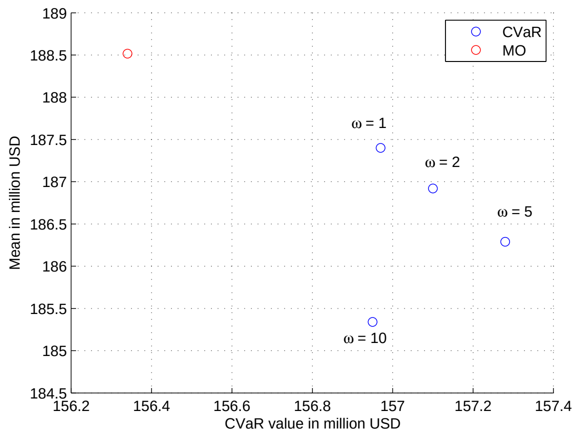


Figure 12: Change in average and CVaR NPV values as a function of ω

Table 4: Results for the mean-CVaR approach under economic uncertainty

ω	Mean in million USD	CVaR value in million USD	% decrease of mean w.r.t MO	% increase of CVaR value w.r.t MO
MO	188.51	156.34	-	-
1	187.40	156.97	0.58%	0.40%
2	186.92	157.10	0.84%	0.48%
5	186.29	157.28	1.17%	0.60%
10	185.34	156.95	1.68%	0.39%

To compare the mean-CVaR and MVO approaches in terms of improving the worst-case value without heavily penalizing the best-case value, MVO is also implemented for the oil price scenarios as shown in Fig. 4(b) for different values of the weighting parameter, i.e., $\gamma \in \{1 \times 10^{-8}, 0.5 \times 10^{-8}, 1 \times 10^{-7}\}$. The results for the MVO and MO optimal strategies are shown in Fig. 13. It can be seen that, due to the reduction of variance, the best cases are heavily penalized with a small improvement in the worst-case values compared to a mean-CVaR approach as shown in Fig. 11.

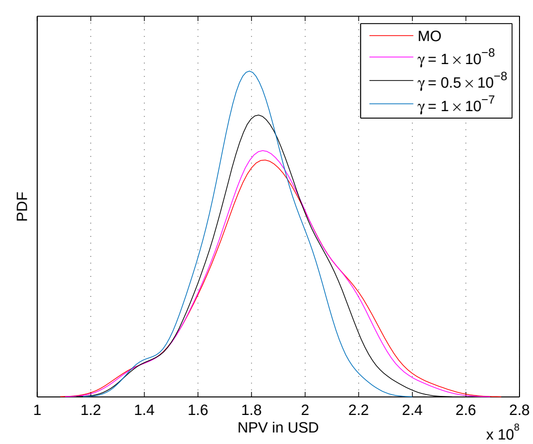


Figure 13: NPV distribution by applying optimal inputs from MO and MVO for different γ to the single model realization with each member of the oil price ensemble (economic uncertainty)

In conclusion, CVaR with either geological or economic uncertainty provides an attractive opportunity for asymmetrically shaping the NPV distribution. Compared to WCO, it also reduces the conservatism of the solution by considering a class of worst-cases as defined with the confidence interval β. With the parameter β, the level of conservatism can be controlled.

One of the main challenges with CVaR in water-flooding optimization is the computational complexity and non-convexity of the original water-flooding optimization. As a consequence, finding a proper weight, ω, in mean-CVaR becomes a difficult task.

5. Semi-variance optimization

Standard semi-deviation or semi-variance has been originally introduced in [9]. Semi-variance is a measure of dispersion, defined as the expected squared deviation from the mean, for only those points that fall below or above the mean. Therefore, it measures the down/up-side risk of an objective function distribution and provides an asymmetric treatment of the NPV distribution. For the random variable Ψ, the semi variance is defined as:

$$\mathrm{Var}_{+}(\Psi) = \mathbb{E}[\max\{\Psi - \mathbb{E}\Psi, 0\}]^2, \tag{14}$$

$$\mathrm{Var}_{-}(\Psi) = \mathbb{E}[\max\{\mathbb{E}\Psi - \Psi, 0\}]^2, \tag{15}$$

where Var_{+} characterizes the spread of the values of Ψ greater than the mean $\mathbb{E}\Psi$, and Var_{-} the spread of the lower tail. With an NPV objective the worst cases are represented by the lower tail and the maximization of these worst-case values is the main concern. Therefore, Var_{-} is minimized in a weighted mean-semi-variance (MSV) formulation as follows:

$$J_{MSV} = J_{MO} - \gamma J_{\mathrm{Var}_{-}}, \tag{16}$$

where J_{MO} is the average objective, $J_{\mathrm{Var}_{-}}$ represents the semi variance objective and $\gamma \in \mathbb{R}$ is the weighting parameter.

Consider the case of geological uncertainty with an ensemble of N_{geo} model realizations. The sample semi-variance is then given as:

$$J_{\text{Var}_-} = \frac{1}{(N_{geo}-1)} \sum_{i=1}^{N_{geo}} \max\{J_{MO} - J_i, 0\}^2. \tag{17}$$

Therefore, eq. (16) becomes:

$$J_{MSV} = J_{MO} - \gamma \frac{1}{(N_{geo}-1)} \sum_{i=1}^{N_{geo}} \max\{J_{MO} - J_i, 0\}^2.$$

The above J_{MSV} is non-differentable due to the 'max' operator. To use a gradient-based optimization routine, a common approach, as applied earlier in the worst-case and CVaR formulations, is to replace the 'max' operator by introducing slack variables t_i and additional constraints. The mean-semi variance optimization problem can then be written as follows:

$$\begin{aligned} \max_{\mathbf{u},t} J_{MSV} &= J_{MO} - \gamma \frac{1}{(N_{geo}-1)} \sum_{i=1}^{N_{geo}} t_i^2, \\ s.t. &\quad \begin{cases} t_i \geq J_{MO} - J_i \\ t_i \geq 0 \end{cases} \quad \forall i. \end{aligned} \tag{18}$$

5.1. Simulation example under geological uncertainty

The geological model ensemble (standard egg model), economic parameters and the control inputs are the same as used in the previous simulation examples. The mean-semi variance problem is optimized for different values of $\gamma \in \{2 \times 10^{-8}, 3 \times 10^{-8}, 5 \times 10^{-8}\}$. As discussed before, the selection of these values is a difficult problem and requires some trials to see the effect of semi-variance optimization. The corresponding approximate PDFs are obtained by applying KDE with MATLAB routine '*ksdensity*' and are displayed

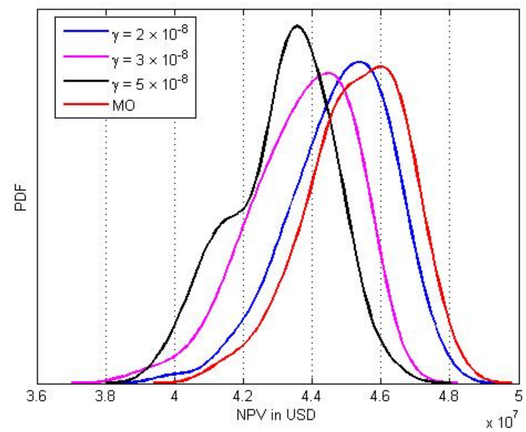


Figure 14: NPV distribution by applying optimal inputs from MO and mean-semi variance for different γ to each ensemble member under geological uncertainty

in Fig. 14. Similarly to MVO, in MSV the reduction of semi-variance can be achieved irrespective of the worst-case and the best-case performance. In this case, MSV results in a undesirable decrease of both worst-case values and mean values. Hence this does not provide an attractive solution with the given geological uncertainty. One possible way to improve the result is by using an additional constraint on the worst-case value such that the it remains closed to the one obtained from MO approach.

5.2. Simulation example under economic uncertainty

The oil price scenarios as shown in Fig. 4(b) are used. The remaining economic parameters and the control inputs are the same as in the previous examples under economic uncertainty. A single realization of the standard egg model as shown in Fig. 1 is used. The mean-semi variance problem with economic uncertainty is optimized for different values of $\gamma \in \{5 \times 10^{-8}, 8 \times 10^{-8}, 1 \times 10^{-7}\}$, which are selected in an ad-hoc way. The obtained strategies are applied to the single reservoir model with 100 oil price scenarios, resulting in 100 NPV values. The corresponding PDFs are shown in Fig. 15. In

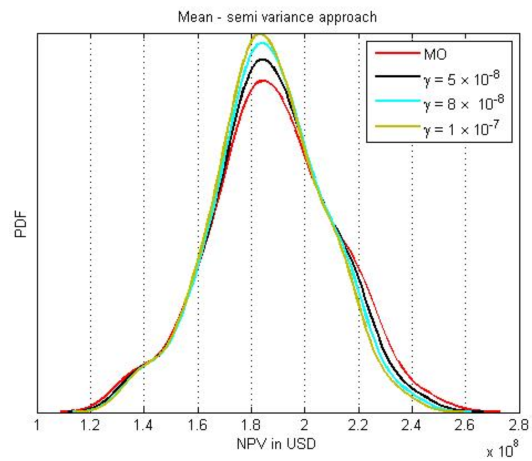


Figure 15: NPV distribution by applying optimal inputs from MO and mean-semivariance for different γ to model realization with each member of oil price ensemble (economic uncertainty)

this case, it improves the worst-case values unlike in the case of geological uncertainty. The improvement is achieved at the cost of compromising the mean and the best-case values.

6. Conclusion

We have evaluated the use of downside risk and deviation measures to provide an asymmetric shaping of the NPV distribution. For this purpose, worst-case optimization, CVaR and semi-variance have been considered. Application of these measures with geological uncertainty leads to attractive results by improving the worst-case performance without heavily compromising the best-case values as compared to a symmetric mean-variance optimization. For the case of economic uncertainty, the improvement is less than expected. With the mean semi-variance (MSV) approach, and for the example considered, the reduction of semi-variance may not necessarily improve the worst-case value. To avoid this effect, a constraint on the minimal achieved worst-case value can be added. Similarly, if the decision maker is not willing to compromise the best-case or the mean value, additional constraints on these values can also be added. Selection of a preferred risk and deviation measure depends on the risk-return attitude of the decision maker, i.e., how much he is willing to compromise the returns for a given level of risk. Furthermore, it also depends upon other factors, such as conservatism of the solution and computational complexity. In presence of high-computing power, CVaR is a preferred risk measure as it provides a less conservative solution and, for a given confidence, considers a set of worst-case values.

Acronyms

ARMA = Auto-Regressive Moving Average

CDF = Cumulative Distribution Function

CVaR = Conditional Value-at-Risk

EIA = Energy Information Administration

KDE = Kernel Density Estimation

MO = Mean Optimization

MPC = Model Predictive Control

MRST = Matlab Reservoir Simulation Toolbox

MSV = Mean Semi Variance

MVO = Mean Variance Optimization

MWCO = Mean Worst Case Optimization

NEMS = National Energy Modeling Systems

NPV = Net Present Value

POLES = Prospective Outlook on Long-term Energy Systems

TE = Time Explicit

VaR = Value-at-Risk

WCO = Worst Case Optimization

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