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Local module identification in dynamic networks: do more inputs guarantee smaller variance? ⋆

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Abstract: Recent developments in science and engineering have motivated control systems to be considered as interconnected and networked systems. From a system identification point of view, modelling of a local module in such a structured system is a relevant and interesting problem. This work focuses on the quality, in terms of variance, of an estimate of a local module. We analyse which predictor input signals are relevant and contribute to variance reduction, while still guaranteeing the consistency of the estimate. For a targeted local module, a comparison of its estimate variance is made between a full-MISO approach and an immersed network setting, where a reduced number of inputs is used, while still guaranteeing consistency. A case study of a four-node network is considered and it is shown that a smaller set of predictor inputs can, under some conditions, result in a smaller variance compared to the full-MISO approach.

Keywords: System identification, dynamic networks, variance analysis, immerse network, direct identification

1. INTRODUCTION

Data-driven modelling of structured interconnections of dynamic systems (modules), also known as dynamic networks, is gaining considerable attention, see e.g., Gonçalves and Warnick (2008), Materassi and Innocenti (2010), Chiuso and Pillonetto (2012). These interconnected dynamic networks can be found in many scientific and engineering fields such as power systems, biological systems, etc. One of the challenging problems in modelling of these dynamic networks is the identification of a local module when the structure of the network (topology) is known.

In Van den Hof et al. (2013) the closed-loop prediction error framework, including direct and two-stage identification methods, has been extended to the case of dynamic networks and an analysis in terms of consistency properties has been presented. For the case of a known structure in the network, the situation has been considered that *all* signals that directly map into the output node of the target module are taken into account as predictor inputs (full-MISO approach). This condition is relaxed in Dankers et al. (2016), where an *immersed network* is formed using graph theory tools, resulting in a reduced number of signals to be used as predictor inputs, to achieve consistency of a targeted module estimate.

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The selection of inputs not only affects the consistency of the estimate, but also strongly influences its variance. For a large network with many possible sets of predictor inputs, it becomes relevant to analyse which predictor input set results in the smallest variance. In Gevers et al. (2006), a variance analyses of a multi-input single-output system with independent inputs (no correlation) is discussed and it has been shown that a change of the experiment by adding an input signal either increases the accuracy of the estimate or at least does not adversely affect it. In Ramazi et al. (2014), a variance analysis is provided for spatially correlated inputs in a MISO system. For interconnected systems, a variance analysis in a cascade structure has been studied e.g., in Wahlberg et al. (2009), Everitt (2017). A variance reduction technique in dynamic networks has been presented in Gunes et al. (2014).

This work focusses on variance analysis in dynamic networks. We consider the identification and dynamic network setup of (Van den Hof et al. (2013)). Specifically, we compare the variance of a targeted module estimate of the full-MISO approach to that of its immersed network counterpart (Dankers et al. (2016)), which has a reduced number of inputs but increased complexity of some of its modules. Our research contrasts the variance analysis as done e.g., in Gevers et al. (2006) in the sense that we keep the experiment fixed, but vary over the choice of predictor input signals out of the set of available signals, thereby focussing on the choice of available signals in the *modelling* process. We consider the research question: what is the benefit of using an additional predictor input signal on the variance of a target module of interest in a dynamic

network with known topology? We also investigate if it is always preferable to include all information (inputs) in the modelling process. To this end, we consider different modelling processes with various (possible) sets of inputs while the experiment setup remains the same.

Our analysis is primarily based on the observations that for the immersed network settings, with a reduced number of inputs, the number of to-be-identified transfer functions is smaller compared to the full-MISO approach. This may result in a smaller variance of a targeted module in the immersed network settings. On the other hand, process noise at an input node, when that input node is removed from the set of predictor inputs, affects the output directly and can deteriorate the signal-to-noise ratio at the output node. In the full-MISO approach the process noise can play the role of an excitation signal for the predictor input node. Therefore, we investigate the tradeoff between the number and complexity of to-be-estimated transfer functions and the signal-to-noise ratio in the immersed network.

Our approach is to use the parameter covariance (Ljung (1999)) and the asymptotic frequency response variance expressions (Ljung (1985)) for a targeted module in both the full MISO and the immersed network settings. We derive conditions under which the variance of the targeted module in the immersed network setting is higher compared to the full-MISO approach. Other approaches and measures to analyse variance, such as the geometric approach (Hjalmarsson and Martensson (2011)), frequency response variance expression for finite order and sample size (Hjalmarsson and Ninness (2004)), or transfer function variance expression for finite order (Ninness and Hjalmarsson (2004)) may also be used, but are not considered here. In a case study, we consider a four-node dynamic network and show how the signal-to-noise ratio and the number/complexity of the to-be-identified transfer functions affect the quality of estimate in terms of variance.

The paper is organised as follows: in the next section, preliminaries such as the definition of the dynamic network and variance measures are discussed. The main theoretical results for the variance analysis are given in section 3. Later, in section 4, a case study with a three-node dynamic network is presented. Conclusions of the work are drawn afterwards.

2. PRELIMINARIES AND SYSTEM SETUP

2.1 Dynamic network

We consider the dynamic network setup as given in Van den Hof et al. (2013) and Dankers et al. (2016). The network is built up of L nodes, related to L scalar internal variables $w_j, j = 1, \cdots, L$. It is assumed that each internal variable can be written as:

$$w_j(t) = \sum_{k \in \mathcal{N}_j} G^0_{jk}(q) w_k(t) + r_j(t) + v_j(t), \qquad (1)$$

where $G^0_{jk}(q), k \in \mathcal{N}_j$ is a proper rational transfer function, q^{-1} is the delay operator, i.e., $q^{-1}u(t) = u(t-1)$ and,

- $\mathcal{N}_j$ is the set of indices of internal variables with direct causal connection to $w_j, i.e., i \in \mathcal{N}_j$ iff $G^0_{ji} \neq 0$;
- v_j is an unmeasured disturbance variable with rational spectral density: $v_j = H^0_j(q)e_j$, where e_j is a white noise process, and H^0_j is a monic, stable, and minimum phase transfer function;
- r_j is an external variable that is known and, if it is present at node j, it can be manipulated by the user.

In the matrix form, all measured variables can be written as:

$$\begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{pmatrix} = \begin{pmatrix} 0 & G^0_{12} & \cdots & G^0_{1L} \\ G^0_{21} & 0 & \ddots & G^0_{2L} \\ \vdots & \ddots & \ddots & \vdots \\ G^0_{L1} & G^0_{L2} & \cdots & 0 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{pmatrix} + \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_L \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_L \end{pmatrix}$$
$$= G^0 w + r + v = (I - G^0)^{-1}(r + v).$$

In Van den Hof et al. (2013), it is shown that for a consistent estimate of a local module in a dynamic network G^0_{jk}, it is sufficient to consider all inputs $k \in \mathcal{N}_j$ in the set of predictor inputs. This condition is relaxed in the immersed network setting (Dankers et al. (2016)), where consistency is achieved with a reduced number of signals as predictor inputs. The consistent estimate of G^0_{jk} is possible if:

- w_i is included as predictor input;
- each parallel path from $w_i \to w_j$ passes through a node is chosen as predictor input; and
- each loop from $w_j \to w_j$ passes through a node is also chosen as predictor input,

while for the so-called direct method of identification a condition has to be added on the absence of confounding variables.

In the immersed network settings the new model identification equation becomes:

$$w_j(t) = \sum_{k \in \mathcal{D}_j} \breve{G}_{jk}(q) w_k(t) + r_j + \breve{v}_j, \qquad (2)$$

where $\breve{\cdot}$ indicates terms corresponding to the immersed network, $\mathcal{D}_j$ denote the set of indices of the internal variables chosen as predictor inputs, $\breve{G}_{jk}$ are identified models in immersed network and they can be different from G^0_{jk}, except for the targeted module and the noise $\breve{v}_j(t)$ can be defined as $\breve{v}_j(t) = v_j(t) + \sum_{k \in N_j \setminus D_j} G_{jk} w_k(t)$. Let, for a targeted local module G^0_{j1}, $M \in \mathbb{N}_{\geq 1}$ be the number of inputs that can be removed from the predictor input set, while still guaranteeing consistency, resulting in $L_j - M$ inputs in immersed settings.

2.2 Asymptotic transfer function covariance expression

In Ljung (1985), an expression for the estimated black-box transfer function, being asymptotic in both the number of to-be-estimated parameters of the transfer function n and data points N, is derived and reads:

$$\operatorname{cov} \begin{pmatrix} \hat{G}_N(e^{i\omega}, n) \\ \hat{H}_N(e^{i\omega}, n) \end{pmatrix} \approx \frac{n}{N} \Phi_v(\omega) \cdot \begin{pmatrix} \Phi_u(\omega) & \Phi_{ue}(\omega) \\ \Phi_{ue}(-\omega) & \lambda \end{pmatrix}^{-1}, \qquad (3)$$

where Φ_u and Φ_v are respectively the input and noise spectrum, where the latter has a noise variance λ; Φ_{ue} is the cross spectrum between input u and the noise signal e, and $\hat{G}_N$ and $\hat{H}_N$ are respectively the estimate of the true transfer functions G_0 and H_0.

2.3 Parameter covariance matrix

In a prediction error identification setting, for a parametric model $G(q,\theta)$, the covariance matrix P_θ is the inverse of the *Fisher information matrix* $\mathcal{M}$ and reads: (Ljung (1999)):

$$P_\theta = \sigma_e^2[E\psi(t,\theta)\psi(t,\theta)^\top]^{-1} = \sigma_e^2\mathcal{M}^{-1}, \tag{4}$$

where σ_e^2 is the noise variance, E is the time expected value and $\psi(t,\theta)$ is the gradient of the one-step ahead prediction error $\epsilon(t/t-1,\theta)$ with respect to the parameters (i.e., the sensitivity of these errors to parameter variations):

$$\psi(t,\theta) := \frac{\partial\epsilon(t/t-1,\theta)}{\partial\theta} = -\frac{\partial\hat{y}(t/t-1,\theta)}{\partial\theta},$$

where $\hat{y}(t/t-1,\theta)$ is one-step ahead predictor.

In the next section, theoretical conditions are derived for the variance analysis of the full-MISO and immersed network approaches.

3. VARIANCE ANALYSIS

In a large network with hundreds of nodes, it is important to analyse which information (inputs) is most relevant for the estimation of a targeted module, e.g., $G_{j1}(q)$. We ponder the question: is it always better to consider *all* possible information sources (inputs) in the modelling process of a targeted module or, instead, only a subset of all inputs? In other words, while still guaranteeing consistency, can we, in the immersed settings, provide a more precise estimate? We address this question below, where we use the asymptotic transfer function variance expression as given in eq. (3).

3.1 Variance analysis based on transfer function aymptotic variance expression

We use the expression of variance as in eq. (3) for the case of MISO system with L_j inputs. Let the targeted module be G_{j1}^0 and define the $(L_j+1)\times 1$ vector of input signals

$$x(t) = [w_1(t)\dots e_j(t)\dots w_{L_j+1}(t)]^\top. \tag{5}$$

Furthermore, let

$$\mathcal{G}_\gamma = [\hat{G}_{j1}(e^{i\omega},n), H_j(e^{i\omega},n), \dots, \hat{G}_{j\gamma}(e^{i\omega},n)]^\top, \tag{6}$$

where $\gamma \in \mathbb{N}_{\geq 1}$ indicates the number of inputs. Then the asymptotic transfer matrix covariance expression is given by:

$$\operatorname{cov}(\mathcal{G}_\gamma) \approx \frac{n_\gamma}{N}\Phi_v(\omega)\cdot\left[\sum_{\tau=-\infty}^{\infty} e^{-i\omega\tau}\bar{E}x(t)x^T(t-\tau)\right]^{-1}, \tag{7}$$

where $\bar{E}$ is the expectation over time. Using a block-matrix inverse, the covariance of the estimated transfer function of interest, $\hat{G}_{j1}(e^{i\omega},n)$, with $\gamma = L_j$ inputs is computed from (7) and reads:

$$\operatorname{cov}(\hat{G}_{j1}(e^{i\omega},n)) \approx \frac{n_{L_j}}{N}\Phi_v(\omega)\cdot\left[\Phi_{w_1}(\omega) - \Gamma_{L_j}\Upsilon_{L_j}^{-1}\Gamma_{L_j}^H\right]^{-1}, \tag{8}$$

where $(\cdot)H$ stands for the Hermitian conjugate,

$$\Gamma_\gamma = \begin{pmatrix}\Phi_{w_{(1)}e}(\omega) & \Phi_{w_{(1)}w_3}(\omega) & \cdots & \Phi_{w_{(1)}w_\gamma}(\omega)\end{pmatrix}, \tag{9}$$

and

$$\Upsilon_\gamma = \begin{pmatrix}\lambda & \Phi_{ew_3}(\omega) & \cdots & \Phi_{ew_\gamma}(\omega)\\ \Phi_{ew_3}(-\omega) & \Phi_{w_3}(\omega) & \cdots & \Phi_{w_3w_\gamma}(\omega)\\ \vdots & \vdots & \ddots & \vdots\\ \Phi_{ew_\gamma}(-\omega) & \Phi_{w_3w_\gamma}(-\omega) & \cdots & \Phi_{w_\gamma}(\omega)\end{pmatrix}. \tag{10}$$

We can perform the same calculations for the immersed network that has a reduced set of $L_j - M$ inputs. Using the breve notation as applied in (2), we define the vector of transfer functions $\breve{\mathcal{G}}_\gamma$ (cf. 6). Expression (7) and (8) then follow straightforwardly for the case of the immersed network, where the latter becomes:

$$\operatorname{cov}(\hat{G}_{j1}(e^{i\omega},n)) \approx \frac{n_{L_j-M}}{N}\Phi_{\breve{v}}(\omega)\cdot\left[\Phi_{w_1}(\omega) - \breve{\Gamma}_{L_j-M}\breve{\Upsilon}_{L_j-M}^{-1}\breve{\Gamma}_{L_j-M}^H\right]^{-1}, \tag{11}$$

in which $\breve{\Gamma}$ and $\breve{\Upsilon}$ are the immersed network equivalents of respectively (9) and (10) (the noise $e(t)$ is replaced with it's respective immersed network counterparts $\breve{e}(t)$).

3.2 Main results

Theorem 1. If

$$\frac{\Phi_{\breve{v}}(\omega)}{\Phi_v(\omega)} > \frac{n_{L_j}.\Phi_{w_{(1)}}(\omega) - \breve{\Gamma}_{(L_j-M)}(\omega)\breve{\Upsilon}^{-1}_{(L_j-M)}(\omega)\breve{\Gamma}^H_{(L_j-M)}(\omega)}{n_{L_j-M}.\Phi_{w_{(1)}}(\omega) - \Gamma_{L_j}(\omega)\Upsilon^{-1}_{L_j}(\omega)\Gamma^H_{L_j}(\omega)} \tag{12}$$

then $\operatorname{cov}^{(L_j-M)}(\hat{G}_{j1}(e^{i\omega},n)) > \operatorname{cov}^{(L_j)}(\hat{G}_{j1}(e^{i\omega},n))$.

Proof 1. See Appendix.

Theorem 1 provides a result for each frequency in the frequency response of the targeted module.

3.3 Variance analysis based on parameter covariance expression

As an alternative for the frequency-based variance analysis, we briefly cover variance analysis based on a scalar measure on the covariance matrix of the to-be-identified parameters. The covariance matrices of both original and immersed networks are respectively given by $P_\alpha^{L_j}$ and $P_\alpha^{L_j-m}$; see (4). Below, we will consider two useful measures: the E- and D-optimality measures.

E-optimality and D-optimality The D-optimality criterion measures the informativeness of an experiment based on the volume of the confidence ellipsoid that can be constructed with the covariance matrix of the to-be-identified parameters (4), i.e., when

$$\det((P_\theta^{L_j})^{-1}) > \det((P_\theta^{L_j-M})^{-1}), \tag{13}$$

the overall accuracy of the parameters in the module G_{j1} is higher than those in the immersed network. E-optimality can indicate whether one ellipsoid lies completely inside the other. Mathematically, this conditions holds when

$$(P_\theta^{L_j})^{-1} \succeq (P_\theta^{L_j-M})^{-1}, \tag{14}$$

Hence, contrasting the D-optimality measure, all variances of the to-be-identified parameters in the full-MISO approach are smaller than in its equivalent immersed network representation if the above condition is satisfied.

4. CASE STUDY

To study the effect of the number and complexity of the to-be-estimated transfer functions and the poor signal-to-noise ratio on the variance of a targeted module, we consider a four-node dynamic network as shown in Fig. 1. The dynamic network representation is given by:

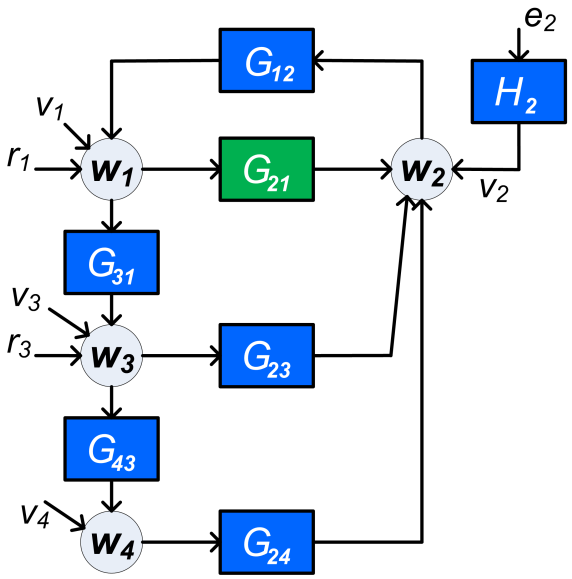

Fig. 1. 4-node dynamic network

$$w = Gw + r + v, \tag{15}$$

In the matrix form,

$$\begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{pmatrix} = \begin{pmatrix} 0 & G_{12} & 0 & 0 \\ G_{21} & 0 & G_{23} & G_{24} \\ G_{31} & 0 & 0 & 0 \\ 0 & 0 & G_{43} & 0 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{pmatrix} + \begin{pmatrix} r_1 \\ 0 \\ r_3 \\ 0 \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} \tag{16}$$

In the example, the targeted module is G_{21}. For the direct identification method as explained in Van den Hof et al. (2013), the full-MISO approach requires $\{w_1, w_3, w_4\}$ as predictor inputs to obtain a consistent estimate of G_{21}, as shown in the equation below.

$$w_2 = G_{21}w_1 + G_{23}w_3 + G_{24}w_4 + H_2 e. \tag{17}$$

Using the conditions presented in Dankers et al. (2016), a consistent estimate of G_{21} is obtained with a reduced number of predictor inputs $\{w_1, w_3\}$, i.e., w_4 can be removed. The immersed network is shown in Fig. 2. In the immersed network settings, the following equation is derived for the estimation of G_{21}:

$$w_2(t) = G_{21}(q)w_1(t) + \breve{G}_{23}(q)w_3(t) + \breve{H}_2(q)\breve{e}(t), \tag{18}$$

where $\breve{G}_{23} = G_{23} + G_{24}G_{43}$ and $\breve{H}_2$ can be obtained via a spectral decomposition, see e.g., Dankers et al. (2016).

Two aspects, i.e. poor signal-to-noise ratio and the number/complexity of the to-be-estimated transfer functions in the immersed network setting, affecting the quality of estimate of G_{21}^0 are discussed next:

4.1 Poor signal-to-noise ratio in immersed network

Process noise v at input nodes can provide external excitation in dynamic network. For the considered example, the noise v_4 on the node signal w_4 can induce excitation when the node signal w_4 is considered in the set of predictor inputs (full-MISO case). In the immersed network settings, when w_4 is removed from the set of predictor inputs, the noise v_4 directly affects node w_2, causing a poor signal-to-noise ratio at the output. The important point to consider is that the transfer function G_{24} determines how the noise v_4 affects output node w_2. A low gain G_{24} results in a minimal effect while a high gain transfer function G_{24} worsens the signal-to-noise ratio. Therefore, the gain of G_{24} becomes an important variable to consider for the variance analysis of targeted module G_{21}.

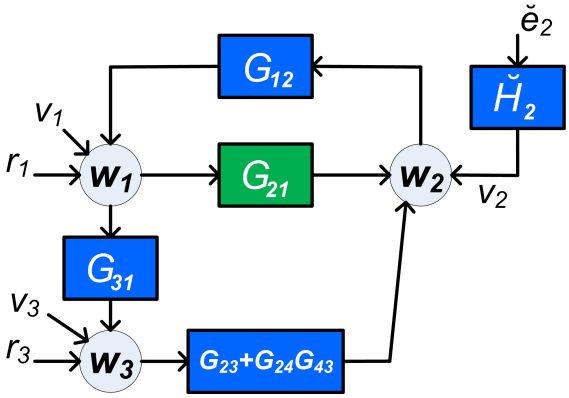

Fig. 2. Immersed dynamic network with reduced number of nodes

4.2 Number and/or complexity of to-be-estimated transfer functions in immersed network

Another important aspect is the number/complexity of to-be-estimated transfer functions. It can be observed that in the immersed network, the number of transfer functions is reduced compared to full-MISO approach. In the considered example, only two transfer functions, G_{21} and $\breve{G}_{23}$ are to be estimated. However, $\breve{G}_{23}$ can be more complex than G_{23}. For example, $\breve{G}_{23} = G_{23} + G_{24}G_{43}$ becomes complex if the number of parameters in G_{43} are high or if there is no pole-zero cancellation in G_{24} and G_{43} and hence there are more to-be-estimated parameters in the immersed network settings compared to the full-MISO approach. To study this effect on the variance of the targeted module G_{21}, we consider G_{43} with different orders.

In the following simulation example, we analyse the effect of i)- the gain of G_{24} and ii)- the complexity of G_{43} on the quality of estimation of G_{21} in terms of variance.

4.3 Simulation experiment

For the variance analysis, we only consider the conditions derived for the asymptotic transfer function covariance expression and compare the results with the sample covariance measure. It would be attractive if the theoretical analysis could lead to generic conclusions but, due to the complexity of the problem at hand, it is difficult to achieve. As an alternative these theoretical results are used to a posteriori compare the variance expressions in a particular case. The simulation details and results are presented in the next subsections.

4.4 Experiment details

We have implemented the network shown in Fig. 1. A white noise signal with power 0.1 is used as external excitation signals, i.e., $\{r_1, r_3\}$ and is applied at nodes 1 and 3. All the white noises e have variance 0.1, i.e., $\lambda = 0.1$. The local module of interest is G_{21}. An (simulation) experiment is performed and the data set of length $N = 10^4$ with sampling time $T_s = 1$sec is collected. In the first modelling process setup, direct identification for dynamic networks (Van den Hof et al. (2013) is performed using all inputs (full-MISO approach), i.e., $\{w_1, w_3, w_4\}$. A second modelling setup is considered with immersed network (Dankers et al. (2016)) where the predictor inputs include $\{w_1, w_3\}$. Consistency results for both modellings setups are presented in Van den Hof et al. (2013) and Dankers et al. (2016). A Box-Jenkins (BJ) model structure is used for the identification of each module $G_{21}, G_{23}, G_{43}, \breve{G}_{23}$ and noise filters $H_2, \breve{H}_2$ in both setups.

4.5 Results

Firstly, the transfer function G_{43} is considered to have only one parameter, i.e., delay with a gain. One hundred Monti-Carlo simulations are performed for different gains of G_{24}, i.e., $\{0.005, 0.05, 0.5, 1\}$. Fig. 3 shows sample covariance per frequency over the 100 Monti-Carlo simulations, where the sample covariance is defined as:

$$\text{cov}(G_{j1}(e^{i\omega}, n) = \frac{1}{\mathbf{n}} \sum_{i=1}^{\mathbf{n}} ((G^i_{j1}(e^{i\omega}, n) - E[(G^i_{j1}(e^{i\omega}, n)])^2,$$

where $\mathbf{n}$ is the number of Monti-Carlo simulations and E represents the average value.

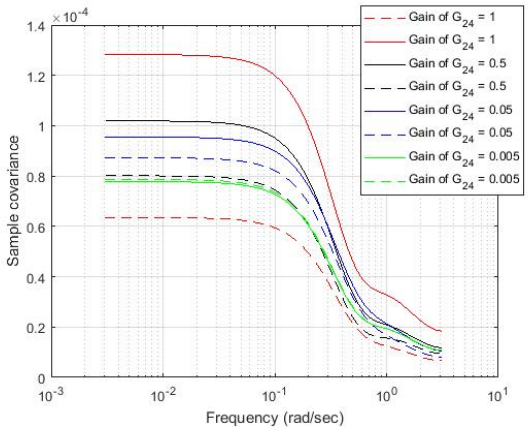

Fig. 3. Sample covariance per frequency of G_{21} when G_{43} has only one parameter. In this case, the number of to-be-estimated parameters in the immersed n_{L_j-M} and the full-MISO approaches n_{L_j} are the same, dotted line shows full-MISO approach, solid line shows immersed network

It can be observed in Fig. 3 that as the gain of G_{24} is reduced the covariance of G_{21} reduces, as expected. For 0.005, the covariance of the immersed network becomes smaller compared to the full-MISO approach, resulting in a better estimate even with a reduced number of information sources (inputs) used in the estimation.

Fig. 4 shows the covariance per frequency when G_{43} has two parameters. In this case, the complexity of the to-be-identified transfer functions in the immersed network settings is higher compared to the full-MISO approach. The first observation is that, as can also be observed in fig. 3, the covariance of G_{21} reduces with the gain of G_{24}. Due to an increased complexity the covariance of G_{21}, in this case, is always higher in the immersed network settings irrespective of the gain of G_{24}.

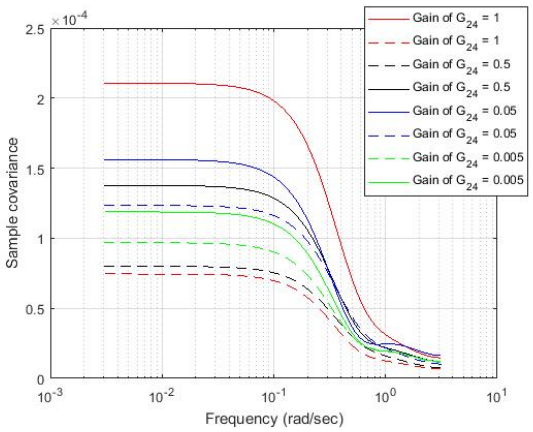

Fig. 4. Sample covariance per frequency of G_{21} when G_{43} has two parameters. In this case, the number of to-be-estimated parameters in the immersed settings are more than the full-MISO approach, i.e., $n_{L_j-M} > n_{L_j}$, dotted line shows full-MISO approach, solid line shows immersed network

The theoretical conditions using the asymptotic transfer function expression as derived in section 3.1 are also implemented. Fig 5 and 6 show the conditions when G_{43} has one and two parameters respectively. For the case when G_{43} has one parameter and with gain 0.005 for G_{24}, the condition becomes negative, indicating that the covariance of full-MISO approach is higher compared to the immersed network settings. The theoretical results confirms the observation obtained via the sample covariance.

5. CONCLUSIONS

We analysed the quality of a local module estimate in dynamic networks, in terms of its variance, for two different modelling setups: the full-MISO and immersed networks. The research question addresses the effect of using an additional input for the estimation problem. The variance analysis for a targeted module in dynamic networks is a complex problem where many factors, e.g. the dynamics of the targeted and other transfers in the network, different excitation conditions etc., can play an important role. We identify two dominant factors that affect the quality of estimate and it is shown that it is possible to obtain a better quality in terms of a reduced variance with smaller set of inputs compared to considering all inputs as predictor inputs.

REFERENCES

Chiuso, A. and Pillonetto, G. (2012). A Bayesian approach to sparse dynamic network identification. *Automatica*, 48(8), 1553–1565.

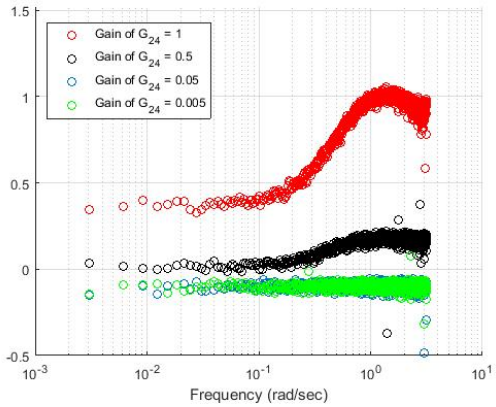

Fig. 5. An implementation of the condition $\frac{\Phi_{\breve{v}}(\omega)}{\Phi_v(\omega)} - \frac{n_L.\Phi_{w_{(1)}}(\omega) - \breve{\Gamma}_{(L_j-M)}(\omega)\breve{\Upsilon}^{-1}_{(L_j-M)}(\omega)\breve{\Gamma}^{H}_{(L_j-M)}(\omega)}{n_{L_j-M}.\Phi_{w_{(1)}}(\omega) - \Gamma_{L_j}(\omega)\Upsilon^{-1}_{L_j}(\omega)\Gamma^{H}_{L_j}(\omega)}$ per frequency, When G_{43} has one parameter. According to theorem 1, if > 0, then it implies covariance of the immersed network is higher compared to the full-MISO approach

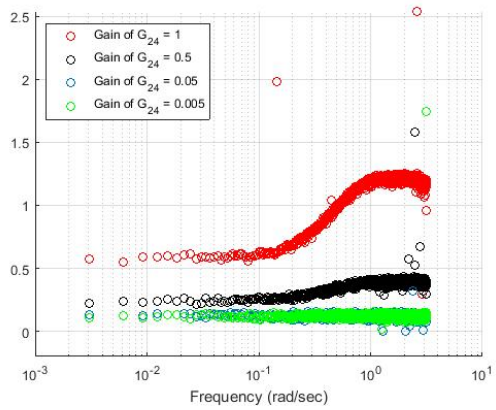

Fig. 6. An implementation of the condition $\frac{\Phi_{\breve{v}}(\omega)}{\Phi_v(\omega)} - \frac{n_L.\Phi_{w_{(1)}}(\omega) - \breve{\Gamma}_{(L_j-M)}(\omega)\breve{\Upsilon}^{-1}_{(L_j-M)}(\omega)\breve{\Gamma}^{H}_{(L_j-M)}(\omega)}{n_{L_j-M}.\Phi_{w_{(1)}}(\omega) - \Gamma_{L_j}(\omega)\Upsilon^{-1}_{L_j}(\omega)\Gamma^{H}_{L_j}(\omega)}$ per frequency, When G_{43} has two parameter. According to theorem 1, if > 0, then it implies covariance of the immersed network is higher compared to the full-MISO approach

Dankers, A., Van den Hof, P.M.J., Bombois, X., and Heuberger, P.S.C. (2016). Identification of dynamic models in complex networks with prediction error methods: Predictor input selection. *IEEE Transactions on Automatic Control*, 61(4), 937–952.

Everitt, N. (2017). *Module identification in dynamic networks: parametric and empirical Bayes methods*. Ph.D. thesis, KTH Royal Institute of Technology.

Gevers, M., Mišković, L., Bonvin, D., and Karimi, A. (2006). Identification of multi-input systems: variance analysis and input design issues. *Automatica*, 42(4), 559–572.

Gonçalves, J. and Warnick, S. (2008). Necessary and sufficient conditions for dynamical structure reconstruction of LTI networks. *IEEE Transactions on Automatic Control*, 53(7), 1670–1674.

Gunes, B., Dankers, A., and Van den Hof, P.M.J. (2014). A variance reduction technique for identification in dynamic networks. In *Proc. IFAC World Congress, Cape Town, South Africa*, 2842–2847.

Hjalmarsson, H. and Martensson, J. (2011). A geometric approach to variance analysis in system identification. *IEEE Transactions on Automatic Control*, 56(5), 983–997.

Hjalmarsson, H. and Ninness, B. (2004). An exact finite sample variance expression for a class of frequency function estimates. In *Decision and Control, 2004. CDC. 43rd IEEE Conference on*, volume 1, 370–375. IEEE.

Ljung, L. (1985). Asymptotic variance expressions for identified black-box transfer function models. *IEEE transactions on Automatic Control*, 30(9), 834–844.

Ljung, L. (1999). *System Identification – Theory for the User*. Prentice-Hall.

Materassi, D. and Innocenti, G. (2010). Topological identification in networks of dynamical systems. *IEEE Transactions on Automatic Control*, 55(8), 1860–1871.

Ninness, B. and Hjalmarsson, H. (2004). Variance error quantifications that are exact for finite-model order. *IEEE Transactions on Automatic Control*, 49(8), 1275–1291.

Ramazi, P., Hjalmarsson, H., and Mårtensson, J. (2014). Variance analysis of identified linear miso models having spatially correlated inputs, with application to parallel hammerstein models. *Automatica*, 50(6), 1675–1683.

Van den Hof, P.M.J., Dankers, A., Heuberger, P.S.C., and Bombois, X. (2013). Identification of dynamic models in complex networks with prediction error methodsbasic methods for consistent module estimates. *Automatica*, 49(10), 2994–3006.

Wahlberg, B., Hjalmarsson, H., and Mårtensson, J. (2009). Variance results for identification of cascade systems. *Automatica*, 45(6), 1443–1448.

Appendix A. APPENDIX

Proof theorem 1: We need to show that:

$$\text{cov}^{(L_j-M)}(\hat{G}_{j1}(e^{i\omega},n)) > \text{cov}^{(L_j)}(\hat{G}_{j1}(e^{i\omega},n)),$$

which can also be written as:

$$\text{cov}^{(L_j-M)}(\hat{G}_{j1}(e^{i\omega},n)) - \text{cov}^{(L_j)}(\hat{G}_{j1}(e^{i\omega},n)) > 0 \quad \text{(A.1)}$$

Substituting eq. (11) and (8) in above eq. (A.1), we obtain

$$\frac{n_{L_j-M}}{N}\Phi_{\breve{v}}(\omega)\cdot\left(\Phi_{w_{(1)}}(\omega) - \breve{\Gamma}_{L_j-M}\breve{\Upsilon}^{-1}_{L_j-M}\breve{\Gamma}^{H}_{L_j-M}\right)^{-1} - \frac{n_{L_j}}{N}\Phi_v^{(L_j)}(\omega)\cdot\left(\Phi_{w_1}(\omega) - \Gamma_{L_j}\Upsilon^{-1}_{L_j}\Gamma^{H}_{L_j}\right)^{-1} > 0.$$

Dividing the above expression by $\frac{N}{\Phi_v^{(L_j)}(\omega)}$, we obtain

$$\frac{\Phi_{\breve{v}}(\omega)}{\Phi_v(\omega)}\cdot\left(n_{L_j-M}\Phi_{w_{(1)}}(\omega) - \breve{\Gamma}_{L_j-M}\breve{\Upsilon}^{-1}_{L_j-M}\breve{\Gamma}^{H}_{L_j-M}\right)^{-1} - \left(n_{L_j}\Phi_{w_{(1)}}(\omega) - \Gamma_{L_j}\Upsilon^{-1}_{L_j}\Gamma^{H}_{L_j}\right)^{-1} > 0,$$

which can be rearranged into the inequality

$$\frac{\Phi_{\breve{v}}(\omega)}{\Phi_v(\omega)} > \frac{n_{L_j}.\Phi_{w_{(1)}}(\omega) - \breve{\Gamma}_{(L_j-M)}(\omega)\breve{\Upsilon}^{-1}_{(L_j-M)}(\omega)\breve{\Gamma}^{H}_{(L_j-M)}(\omega)}{n_{L_j-M}.\Phi_{w_{(1)}}(\omega) - \Gamma_{L_j}(\omega)\Upsilon^{-1}_{L_j}(\omega)\Gamma^{H}_{L_j}(\omega)} \quad \text{(A.2)}$$