2021 European Control Conference (ECC)
June 29 - July 2, 2021. Rotterdam, Netherlands

Controller identification for data-driven model-reference distributed control

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Abstract— **This paper considers data-driven distributed controller synthesis for interconnected linear systems subject to unmeasured disturbances. The considered problem is the optimization of a model-reference control criterion, where the reference model is described by a decoupled system. We provide a method to determine the optimal distributed controller by performing network identification in an augmented network. Sufficient conditions are provided for which the data-driven method solves the distributed model-reference control problem, whereas state-of-the-art methods for data-driven distributed control can only provide performance guarantees in the absence of disturbances. The effectiveness of the method is demonstrated via a simple network example consisting of two interconnected systems.**

I. Introduction

Data-driven methods for controller design are applicable to systems for which a mathematical description is not available. A common property of these methods is that they are based on the model-reference paradigm [1], which employs a user-specified model to describe the desired behavior of the closed-loop system. State-of-the-art data-driven methods that solve the model-reference control problem using a single batch of data are virtual reference feedback tuning (VRFT) [2], optimal controller identification (OCI) [3,4] and asymptotically exact controller tuning [5].

The aforementioned methods are typically applicable to (multi-variable) isolated or small-scale systems. For interconnected systems, the controllers are not implementable due to their lumped nature or cannot be synthesized due to the dimensionality of the system. A sensible approach for interconnected systems would be to synthesize a *distributed* controller directly from data. Data-driven distributed control removes the need for identifying large-scale models, as required by model-based distributed controller synthesis methods [6,7] and provides the scalability that data-driven methods for lumped systems do not provide.

An initial approach to data-driven distributed control based on the model-reference paradigm was developed in [8], based on virtual reference feedback tuning in dynamic networks. By constructing a virtual reference network, the distributed model-reference control problem can be equivalently stated as a network identification problem [8], cf. [9].

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This work is supported by the European Research Council (ERC), Advanced Research Grant SYSDYNET, under the European Unions Horizon 2020 research and innovation programme (grant agreement No. 694504).

The interconnected system considered in [8] is not influenced by any uncontrollable and unmeasured inputs. Even for scalar systems, however, VRFT inherently introduces a bias in the controller estimates when disturbances affect the system [10], leading to degradation of the closed-loop performance. This problem can be solved by using an instrumental variable (IV) approach in case the controller is linearly parametrized. In general, however, it is not straightforward to obtain unbiased estimates. Furthermore, depending on the choice of IV, the introduction of IVs can require additional experiments on the system [10] and increase the parameter variance with a negative effect on the control performance.

In this paper, we solve the distributed model-reference control problem for a decoupled reference model, in the case that the interconnected system is subject to disturbances. By using the direct method for identification in dynamic networks [9], we provide sufficient conditions for the consistent identification of the ideal distributed controller defined in [8], which solves the considered model-reference control problem. The decoupled nature of the identification problems enables a decentralized computation of the distributed controller. The data-driven distributed controller synthesis method developed in this paper generalizes OCI to interconnected systems and provides a solution to the control problem introduced in [8] for a class of interconnected linear systems with disturbances.

Remark: In the literature, various data-driven methods have been developed to solve other (non-model-reference) control problems, including data-enabled predictive control [11] and control enabled by data-based linear matrix inequalities [12]. In recent developments, it has been shown that data-enabled predictive control allows for a distributed implementation [13,14].

II. Preliminaries

A. Dynamic network and distributed controller

Consider a simple and undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with vertex set $\mathcal{V}$ of cardinality L and edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. The neighbour set of vertex $i \in \mathcal{V}$ is defined as $\mathcal{N}_i := \{j \in \mathcal{V} \,|\, (i,j) \in \mathcal{E}\}$. The graph $\mathcal{G}$ describes the structure of a network of linear discrete-time systems, where the dynamics associated with vertex $i \in \mathcal{V}$ are described by

$$y_i(t) = G_i(q)u_i(t) + \sum_{j \in \mathcal{N}_i} G_{ij}(q)y_j(t) + H_i(q)e_i(t), \quad (1)$$

with $u_i : \mathbb{Z} \to \mathbb{R}$ the control input, $y_i : \mathbb{Z} \to \mathbb{R}$ the output, e_i an unmeasured zero-mean white-noise process and q the forward shift defined as $qx(t) = x(t+1)$. The process e_i

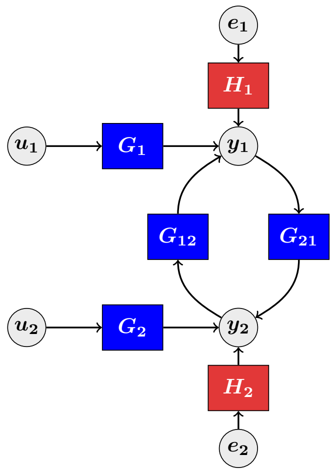


Fig. 1: Two output coupled process with noise represented as a dynamic network.

is assumed to be uncorrelated to all e_j, $j \in \mathcal{V} \setminus \{i\}$ and to all u_j, $j \in \mathcal{V}$. The rational transfer functions G_i, G_{ij} and H_i, $(i,j) \in \mathcal{E}$, describe the local dynamics, coupling dynamics and noise dynamics, respectively. The noise filter H_i is assumed to be monic, stable and minimum phase. The network can be compactly written as

$$y = G_I y + Gu + He, \tag{2}$$

where $G = \text{diag}(G_1, \dots, G_L)$, $H = \text{diag}(H_1, \dots, H_L)$ and

$$G_I = \begin{bmatrix} 0 & G_{12} & \cdots & G_{1L} \\ G_{21} & 0 & \cdots & G_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ G_{L1} & G_{L2} & \cdots & 0 \end{bmatrix}.$$

Furthermore, $y = (I - G_I)^{-1}(Gu + He)$, where it is assumed that $\mathcal{G}_I := (I - G_I)^{-1}$ exists. A two-node network ($L = 2$) is depicted in Figure 1.

Consider a reference tracking problem for the network: each system is equipped with a reference signal r_i and the corresponding tracking error $z_i := r_i - y_i$, i.e.,

$$\mathcal{P}_i : \begin{cases} y_i &= G_i u_i + \sum_{j\in\mathcal{N}_i} G_{ij} y_j + H_i e_i, \\ z_i &= r_i - y_i. \end{cases} \tag{3}$$

The dynamic network is operating in closed-loop with a distributed controller that consists of local controllers

$$\mathcal{C}_i(\rho_i) : \begin{cases} u_i = C_{ii}(q,\rho_i) z_i + \sum_{j\in\mathcal{N}_i} C_{ij}(q,\rho_i) s_{ij}^c, \\ o_{ij}^c = K_{ij}(q,\rho_i) e_i + \sum_{h\in\mathcal{N}_i} K_{ijh}(q,\rho_i) s_{ih}^c,\ j \in \mathcal{N}_i. \end{cases}$$

Each controller is parametrized by a parameter vector ρ_i. The controller class for node i is defined as $\mathscr{C}_i := \{C_i(q,\rho_i) \,|\, \rho_i \in \mathbb{R}^{l_i}\}$. Controllers $\mathcal{C}_i$ and $\mathcal{C}_j$ are interconnected if and only if $(i,j) \in \mathcal{E}$. The interconnection is defined by constraints on the controller interconnection signals s_{ij}^c and o_{ij}^c:

$$s_{ij}^c = o_{ji}^c \quad \text{and} \quad s_{ji}^c = o_{ij}^c, \quad (i,j) \in \mathcal{E}.$$

With the definitions $s_i^c := \text{col}_{j\in\mathcal{N}_i}\, s_{ij}^c$ and $o_i^c := \text{col}_{j\in\mathcal{N}_i}\, o_{ij}^c$, we compactly represent $\mathcal{C}_i$ by

$$\mathcal{C}_i(\rho_i) : \begin{bmatrix} u_i \\ o_i^c \end{bmatrix} = C_i(q,\rho_i) \begin{bmatrix} z_i \\ s_i^c \end{bmatrix}. \tag{4}$$

B. Distributed model reference control

Model reference control in dynamic networks considers the synthesis of a structured controller such that the closed-loop network dynamics are optimal with respect to a structured reference model [8]. We consider here a *decoupled* reference model described by subsystems $\mathcal{K}_i$, $i \in \mathcal{V}$:

$$\mathcal{K}_i : \quad y_i^d = T_i(q) r_i. \tag{5}$$

For well-posedness, we assume that the reference model satisfies $y^d \neq r$ for all non-zero r, i.e., $\det(T - I) \neq 0$, and that T_i is stable for each $i \in \mathcal{V}$.

Given $e_i = 0$ for all $i \in \mathcal{V}$, the distributed model reference control problem is

$$\min_{\rho_1,\dots,\rho_L} J_{\text{MR}}(\rho_1,\dots,\rho_L) = \min_{\rho_1,\dots,\rho_L} \sum_{i=1}^{L} \bar{E}[y_i^d(t) - y_i(t)]^2, \tag{6}$$

where $\bar{E} := \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} E$ and E is the expectation. A distributed controller that solves (6) was developed in [8]. Define the column vector $\mathbf{1} := \text{col}(1,\dots,1)$ and, for $i \in \mathcal{V}$, define $G_{iI} := \text{row}_{j\in\mathcal{N}_i}\, G_{ij}$.

Proposition II.1 *Consider $e_i = 0$ for all $i \in \mathcal{V}$ and consider a distributed controller described by the subsystems*

$$\mathcal{C}_i^d : \begin{bmatrix} u_i \\ o_i^c \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{T_i}{G_i(1-T_i)} & -\frac{1}{G_i} G_{iI} \\ \frac{T_i}{1-T_i}\mathbf{1} & 0 \end{bmatrix}}_{=:C_i^d(q)} \begin{bmatrix} z_i \\ s_i^c \end{bmatrix}, \tag{7}$$

for $i \in \mathcal{V}$ and the controller interconnections described by

$$s_{ij}^c = o_{ji}^c \quad \text{and} \quad s_{ji}^c = o_{ij}^c, \tag{8}$$

for $(i,j) \in \mathcal{E}$. The network (2) *in closed-loop with the distributed controller* (7)-(8) *satisfies*

$$y_i = y_i^d, \quad i \in \mathcal{V}.$$

We refer the reader to [8] for the proof and the corresponding result [8, Theorem III.1]. The controller described by (7)-(8) provides a solution to (6), by showing the existence of a minimizing argument $(\rho_1^d, \dots, \rho_L^d)$. Details on how to obtain $(\rho_1^d, \dots, \rho_L^d)$ from C_i^d, $i \in \mathcal{V}$, can be found in [8, Section III].

C. Problem formulation

The problem that we consider in this paper is to determine the ideal controller described by (7)-(8) in the case that the network described by (2) is unknown, i.e., in the case that the transfer functions G_i, G_{ij} and H_i, $(i,j) \in \mathcal{E}$, are unknown. The local controller modules $\mathcal{C}_i^d$ contain known modules,

$$K_{ij} := \frac{T_i}{1 - T_i}, \quad j \in \mathcal{N}_i,$$

depending solely on the reference model dynamics $\mathcal{K}_i$ and unknown modules describing the top row in (7):

$$u_i = C^d_{ii}(q)z_i + \sum_{j\in\mathcal{N}_i} C^d_{ij}(q)s^c_{ij}. \tag{9}$$

Given data collected from the network (2), the transfer functions C^d_{ii} and C^d_{ij}, $j \in \mathcal{N}_i$, can be determined through VRFT in dynamic networks [8], in the case that no noise is present, i.e., $e_i = 0$. The application of VRFT for systems affected by noise leads to biased estimates, both for the single-process case [10] and the network case [8].

Due to the bias, controller parameter estimates will in general not minimize the cost function (6) and hence lead to a reduced closed-loop performance if the process is subject to noise. The problem that is considered in this paper is: how to obtain consistent distributed controller estimates in the presence of noise?

Optimal controller identification (OCI) [3] and VRFT in conjunction with instrumental variables (IVs) [10,15] are two methods for obtaining consistent estimates of controllers for lumped systems. The use of IVs can lead to estimators with a high variance, however, which in turn affects the controller performance (illustrated for an example network in Section IV). As such, in what follows we consider the identification of auxiliary controller modules in order to obtain consistent estimates of the ideal controllers modules C^d_i, $i \in \mathcal{V}$, by extending the OCI method to dynamic networks.

III. Main result: Indirect distributed controller identification

In order to formulate an identification problem, we start by rewriting the network dynamics in terms of the ideal distributed controller dynamics. The approach of rewriting the dynamics of a single-input single-output system in terms of an ideal controller for prediction-error identification was introduced in [3].

Let us consider the decoupled reference model, i.e., for each $i \in \mathcal{V}$,

$$\mathcal{K}_i : \quad y^d_i = T_i(q)r_i.$$

By (7), we observe that the transfer functions in (9) are

$$C^d_{ii} = \frac{T_i}{G_i(1-T_i)}, \quad C^d_{ij} = -\frac{G_{ij}}{G_i}, \quad (i,j) \in \mathcal{E}.$$

Hence, we can write the network dynamics (1) in terms of the ideal distributed controller and the reference model as

$$G_i = \frac{1}{C^d_{ii}}\frac{T_i}{1-T_i} \quad \text{and} \quad G_{ij} = -C^d_{ij}G_i = -\frac{C^d_{ij}}{C^d_{ii}}\frac{T_i}{1-T_i}.$$

Models for the transfer functions within the network can then be written in terms of the controller parameters as

$$G_i(\rho_i) := \frac{1}{C_{ii}(\rho_i)}\frac{T_i}{1-T_i} \text{ and } G_{ij}(\rho_i) := -\frac{C_{ij}(\rho_i)}{C_{ii}(\rho_i)}\frac{T_i}{1-T_i}.$$

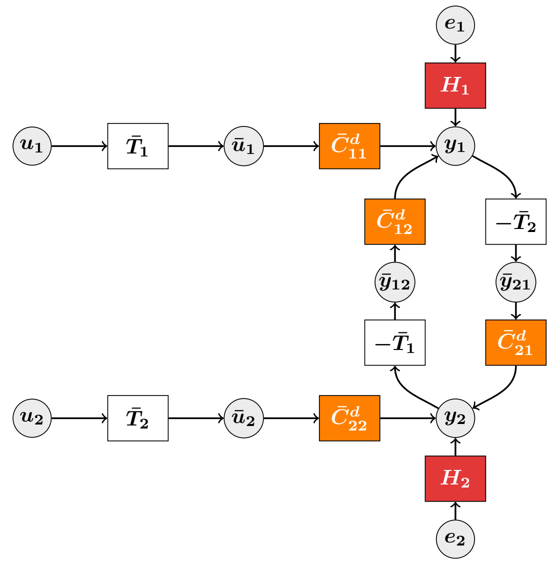


Fig. 2: The dynamic network represented by modules of the ideal distributed controller and reference model.

We can thus rewrite the network dynamics (1) as

$$y_i = \frac{1}{C^d_{ii}}\frac{T_i}{1-T_i}u_i - \sum_{j\in\mathcal{N}_i}\frac{C^d_{ij}}{C^d_{ii}}\frac{T_i}{1-T_i}y_j + H_ie_i,$$

or

$$y_i = \bar{C}^d_{ii}\bar{u}_i + \sum_{j\in\mathcal{N}_i}\bar{C}^d_{ij}\bar{y}_{ij} + H_ie_i, \tag{10}$$

with $\bar{C}^d_{ii} := \frac{1}{C^d_{ii}}$, $\bar{C}^d_{ij} := \frac{C^d_{ij}}{C^d_{ii}}$, and the signals

$$\bar{u}_i := \frac{T_i}{1-T_i}u_i = \bar{T}_iu_i, \tag{11}$$

$$\bar{y}_{ij} := -\frac{T_i}{1-T_i}y_j = -\bar{T}_iy_j. \tag{12}$$

In the case that T_i is proper, $\bar{T}_i$, $i \in \mathcal{V}$, will be proper and the dynamical relations (10)-(12) can be interpreted as an (augmented) dynamic network:

$$\begin{bmatrix} y \\ \bar{y} \\ \bar{u} \end{bmatrix} = \begin{bmatrix} 0 & \bar{C}^d_I & \bar{C}^d \\ \bar{T}_\mathcal{N} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} y \\ \bar{y} \\ \bar{u} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \bar{T} \end{bmatrix}u + \begin{bmatrix} H \\ 0 \\ 0 \end{bmatrix}e, \tag{13}$$

with the matrices $\bar{C}^d_I := \text{diag}_{i\in\mathcal{V}}\,\text{row}_{j\in\mathcal{N}_i}\,\bar{C}^d_{ij}$, $\bar{C}_d := \text{diag}(\bar{C}^d_{11},\dots,\bar{C}^d_{LL})$, $\bar{T} := \text{diag}(\bar{T}_1,\dots,\bar{T}_L)$, $\bar{T}_\mathcal{N} := \text{col}_{i\in\mathcal{V}}\,\text{col}_{j\in\mathcal{N}_i} -\bar{T}_i\mathbf{e}_j^\top$, with $\mathbf{e}_i$ the i-th unit vector, and $\bar{T} := \text{diag}(\bar{T}_1,\dots,\bar{T}_L)$. This augmented network is visualized in Figure 2 for $L=2$, with the first row visualized in Figure 3.

Let $\bar{C}_{ii}(\rho_i) := \frac{1}{C_{ii}(\rho_i)}$ and $\bar{C}_{ij}(\rho_i) := \frac{C_{ij}(\rho_i)}{C_{ii}(\rho_i)}$, $(i,j) \in \mathcal{E}$, define the controller models for the augmented network. The predictor is defined as

$$\hat{y}_i(t,\theta_i) := H_i^{-1}(\theta_i)\left(\bar{C}_{ii}(\rho_i)\bar{u}_i + \sum_{j\in\mathcal{N}_i}\bar{C}_{ij}(\rho_i)\bar{y}_{ij}\right) + \left(1 - H_i^{-1}(\theta_i)\right)y_i, \tag{14}$$

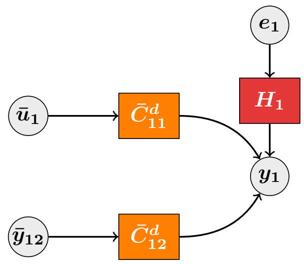


Fig. 3: Building block of the augmented network (13) for identification of the controller modules of $\mathcal{C}^d_1$, $L = 2$.

with $\theta_i = \mathrm{col}(\rho_i, \eta_i)$, where η_i is a parameter vector for independent parametrization of the noise model $H_i(\theta_i)$.

By using the direct method for dynamic network identification [9], the estimates are obtained as

$$\hat{\theta}_i = \arg\min_{\theta_i} V_i(\theta_i), \quad V_i(\theta_i) = \frac{1}{N}\sum_{t=1}^{N} \varepsilon_i^2(t,\theta_i), \tag{15}$$

with the prediction error defined by $\varepsilon_i(t,\theta_i) := y_i(t) - \hat{y}_i(t,\theta_i)$. By definition of the auxiliary controller models, the controller estimates are then

$$C_{ii}(\hat{\rho}_i) = \frac{1}{\bar{C}_{ii}(\hat{\rho}_i)}, \quad C_{ij}(\hat{\rho}_i) = \bar{C}_{ij}(\hat{\rho}_i)C_{ii}(\hat{\rho}_i), \ (i,j) \in \mathcal{E}.$$

Under standard assumptions [9], the estimator $\hat{\theta}_i$ converges asymptotically [16], i.e.,

$$\hat{\theta}_i \to \theta_i^* \quad \text{w.p. 1 as} \quad N \to \infty,$$

where $\theta_i^* = \arg\min_{\theta_i} \bar{V}_i(\theta_i)$ with $\bar{V}_i(\theta_i) := \bar{E}[\varepsilon_i^2(t,\theta_i)]$.

Theorem III.1 *Let $i \in \mathcal{V}$ and assume that the following conditions hold true:*

- *the spectral density of $\bar{z}_i := \mathrm{col}(\bar{u}_i, \bar{y}_{i1}, \ldots, \bar{y}_{iL})$, $\Phi_{\bar{z}_i}(\omega)$, is positive definite for almost all $\omega \in [-\pi, \pi]$,*
- *there exists a θ_i^d such that $C_i(\theta_i^d) = C_i^d$ and $H_i(\theta_i^d) = H_i$,*
- *$[\mathcal{G}_I]_{ji}$ contains a delay for every $j \in \mathcal{N}_i$.*

Then it holds that $C_{ii}(\theta_i^*) = C_{ii}^d$, $H_i(\theta_i^*) = H_i$ and $C_{ij}(\theta_i^*) = C_{ij}^d$ for all $j \in \mathcal{N}_i$.

Proof: We will first show that the minimum of the objective function $\bar{V}_i$ is $\sigma_{e_i}^2 = Ee_i^2$. By the definition of the predictor (14) and the prediction error, $\bar{V}_i(\theta_i)$ is equal to

$$\bar{E}\bigg(H_i(\theta_i)^{-1}\Big(v_i + \sum_{j\in\mathcal{N}_i}\Delta\bar{C}_{ij}(\theta_i)\bar{y}_{ij} + \Delta\bar{C}_{ii}(\theta_i)\bar{u})_i\Big)\bigg)^2.$$

Then, by (13) it follows that $\bar{V}_i(\theta_i)$ is equal to

$$\bar{E}\bigg(H_i(\theta_i)^{-1}\Big(v_i - \sum_{j\in\mathcal{N}_i}\Delta\bar{C}_{ij}(\theta_i)\bar{T}_i y_j + \Delta\bar{C}_{ii}(\theta_i)\bar{T}_i u_i\Big)\bigg)^2,$$

which, by (2), is equal to

$$\begin{aligned}
&\bar{E}\bigg(H_i(\theta_i)^{-1}\Big(v_i + \Delta\bar{C}_{ii}(\theta_i)\bar{T}_i u_i \\
&\quad - \sum_{j\in\mathcal{N}_i}\Delta\bar{C}_{ij}(\theta_i)\bar{T}_i\sum_{k\in\mathcal{V}}[\mathcal{G}_I]_{jk}(G_k u_k + H_k e_k)\Big)\bigg)^2 \\
&= \bar{E}\bigg(H_i(\theta_i)^{-1}\Big(\Delta H_i(\theta_i)e_i + \Delta\bar{C}_{ii}(\theta_i)\bar{T}_i u_i \\
&\quad - \sum_{j\in\mathcal{N}_i}\Delta\bar{C}_{ij}(\theta_i)\bar{T}_i\sum_{k\in\mathcal{V}}[\mathcal{G}_I]_{jk}(G_k u_k + H_k e_k)\Big) + e_i\bigg)^2.
\end{aligned}$$

Since both H_i and $H_i(\theta_i)$ are monic, $\Delta H_i(\theta_i)$ is strictly proper. Hence, $\Delta H_i(\theta_i)e_i$ is uncorrelated with e_i. Also $-\sum_{j\in\mathcal{N}_i}\Delta\bar{C}_{ij}(\theta_i)\bar{T}_i\sum_{k\in\mathcal{V}}[\mathcal{G}_I]_{jk}G_k u_k$ is uncorrelated with e_i, since it is a filtered linear combination of u_k, $k \in \mathcal{V}$, which are uncorrelated with e_i by assumption. Moreover, $-\sum_{j\in\mathcal{N}_i}\Delta\bar{C}_{ij}(\theta_i)\bar{T}_i\sum_{k\in\mathcal{V}}[\mathcal{G}_I]_{jk}H_k e_k$ is uncorrelated with e_i, since $\Delta\bar{C}_{ij}(\theta_i)$ is proper and $[\mathcal{G}_I]_{ji}$ is strictly proper for all $j \in \mathcal{N}_i$. Hence, $\bar{V}_i(\theta_i)$ is equal to

$$\begin{aligned}
&\bar{E}\bigg(H_i(\theta_i)^{-1}\Big(\Delta H_i(\theta_i)e_i + \Delta\bar{C}_{ii}(\theta_i)\bar{T}_i u_i \\
&\quad - \sum_{j\in\mathcal{N}_i}\Delta\bar{C}_{ij}(\theta_i)\bar{T}_i\sum_{k\in\mathcal{V}}[\mathcal{G}_I]_{jk}(G_k u_k + H_k e_k)\Big)\bigg)^2 + \bar{E}e_i^2 \\
&\geq \sigma_{e_i}^2.
\end{aligned}$$

It remains to show that $\bar{V}_i(\theta_i) = \sigma_{e_i}^2 \Rightarrow \theta_i = \theta_i^d$. This follows from the first two assumed conditions; for this part of the proof, the reader is referred to [9, Appendix B], due to space limits. ■

Positive definiteness of $\Phi_{\bar{z}_i}$ is implied by sufficient excitation of the filtered input $\bar{u}_i = \bar{T}_i u_i$ and the signals $\bar{y}_{ij}$, $j \in \mathcal{N}_i$. The condition on $\Phi_{\bar{z}_i}$ can be translated to conditions on external signals u_j, e_j, $j \in \mathcal{V}$, and conditions on the augmented network topology, as described in [17]. While the excitation conditions are sufficient, they can be conservative. We point out the results on data informativity for centralized control problems in [18], which open the way for the formulation of sufficient excitation conditions that are also *necessary* for the distributed model-reference control problem.

The data-driven distributed controller synthesis method in this paper amounts to solving L multi-input-single-output (MISO) identification problems, i.e., optimizing V_i in (15) for each $i \in \mathcal{V}$. The optimization is distributed in the sense that the optimization problems (15) are not coupled and is, therefore, scalable with respect to the size L of the network. The MISO identification problems can lead to non-convex optimization schemes, depending on the parametrization. We note that regularization, as employed by the method developed in [19], can reduce the compuational complexity of each MISO problem through iteratively solving linear least-squares problems. Comparatively, model-based distributed controller synthesis as in, e.g., [6], [7], in conjunction with dynamic network identification of the subsystems $\mathcal{P}_i$, requires to solve L MISO identification problems [9] plus one *centralized* LMI for synthesis of the controller. The data-driven method in this paper, respectively model-based distributed controller synthesis with network identification, achieve stability of the closed-loop network for $N \to \infty$, if the estimates of the controller (Theorem III.1), respectively the estimates of $\mathcal{P}_i$ ([9, Proposition 2]), are consistent; the stability is dictated by the reference model (Proposition II.1), respectively by the imposed dissipation inequality ([7, Proposition 1]).

IV. Illustrative Example

Consider a two-node network ($L = 2$) described by (3) for $i \in \{1, 2\}$, with transfer functions

$$G_1(q) = \frac{c_1}{q - a_1}, \quad G_{12}(q) = \frac{d_1}{q - a_1}, \quad H_1 = 1,$$
$$G_2(q) = \frac{c_2}{q - a_2}, \quad G_{21}(q) = \frac{d_2}{q - a_2}, \quad H_2 = 1,$$

where $a_1 = 0.5$, $a_2 = 0.2$, $c_1 = c_2 = 1$ and $d_1 = d_2 = 0.1$. The objective is to let the closed-loop interconnected system behave as two decoupled processes with first-order dynamics, according to

$$y_i^d(t) = T_i(q)r_i(t), \quad T_i(q) = \frac{1-\gamma_i}{q-\gamma_i}, \quad i \in \{1, 2\}, \quad (16)$$

with $\gamma_1 = \gamma_2 = 0.8$.

As presented in Section II-B, the ideal distributed controller is described by (7) for $i \in \{1, 2\}$, with the interconnections $s_1^c = o_2^c$, $s_2^c = o_1^c$, and

$$C_{11}^d(q) = \frac{1-\gamma_1}{c_1}\frac{q-a_1}{q-1}, \quad C_{12}^d(q) = -\frac{d_1}{c_1},$$
$$C_{22}^d(q) = \frac{1-\gamma_2}{c_2}\frac{q-a_2}{q-1}, \quad C_{21}^d(q) = -\frac{d_2}{c_2},$$
$$K_{12}^d(q) = \frac{1-\gamma_1}{q-1}, \qquad K_{21}^d(q) = \frac{1-\gamma_2}{q-1}.$$

For the experiment, consider that u_1 and u_2 are Gaussian white-noise signals with unit variance and e_1 and e_2 are (unmeasured) Guassian white-noise signals with variance $\sigma_e^2 = 0.25$. As discussed in Section II-C, the noise will cause a bias in the controller parameter estimates when the distributed virtual reference feedback tuning (DVRFT) method is applied directly. For the distributed optimal controller identification (DOCI) method, described in Section III, we expect consistent estimates and hence an improved closed-loop performance.

We first represent the network as shown in Figure 2, where

$$\bar{C}_{11}^d = \frac{c_1}{1-\gamma_1}\frac{q-1}{q-a_1}, \quad \bar{C}_{12}^d = -\frac{d_1}{c_1}\frac{q-1}{q-a_1}.$$

The modules are therefore parametrized as $H_1(\theta_1) = 1$ and

$$\bar{C}_{11}(\theta_1) = \theta_{1a}\frac{1-q^{-1}}{1-\theta_{1b}q^{-1}}, \quad \bar{C}_{12}(\theta_1) = \theta_{1c}\frac{1-q^{-1}}{1-\theta_{1b}q^{-1}},$$

so that there exists θ_1^d such that $\bar{C}_{11}^d = \bar{C}_{11}(\theta_1^d)$, $\bar{C}_{12}^d = \bar{C}_{12}(\theta_1^d)$ and $H_1 = H_1(\theta_1)$. By forming the predictor

$$\hat{y}_1(t, \theta_1) := \bar{C}_{11}(\rho_1)\bar{u}_1 + \bar{C}_{12}(\rho_1)\bar{y}_2$$

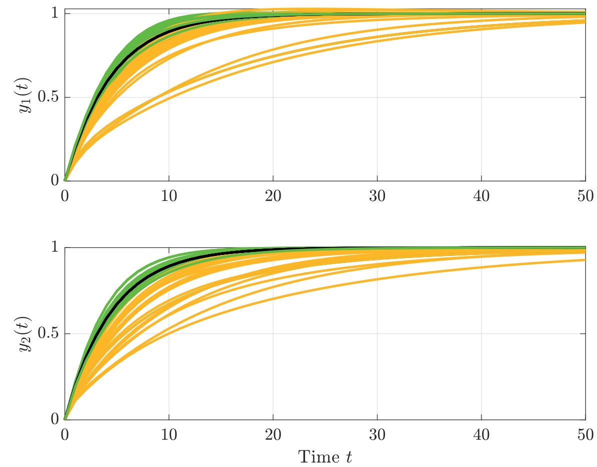


Fig. 4: Step response of the closed-loop network for 20 experiments with DOCI (green), DVRFT (yellow) and the desired closed-loop network (black).

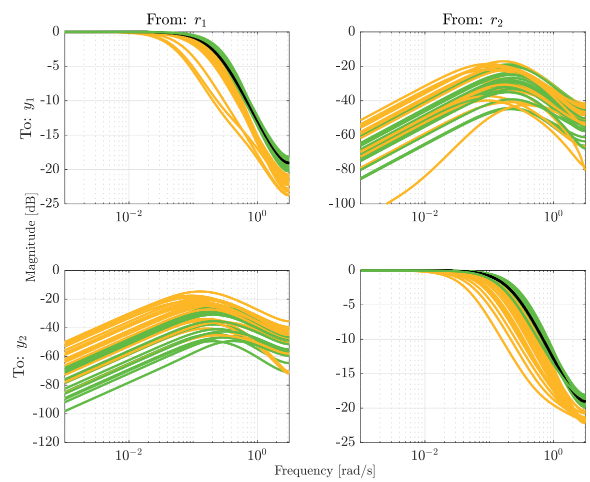
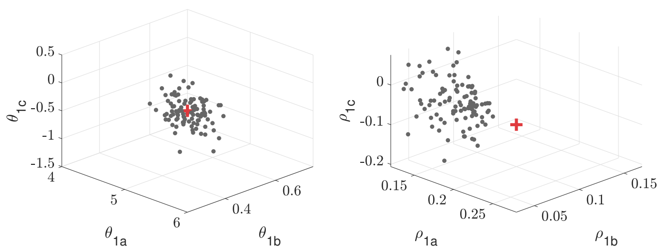


Fig. 5: Frequency response of the closed-loop network for 20 experiments with DOCI (green), DVRFT (yellow) and the desired closed-loop network (black). Notice that the desired transfers $r_i \to y_j^d$, $i \neq j$, are identical to zero.

and minimizing $V_1(\theta_1)$ in (15) for $N = 100$ samples, we find the estimate $\hat{\theta}_1$. The estimate $\hat{\theta}_2$ for controller 2 is obtained by following an analogous procedure. Note that V_1 and V_2 are not quadratic functions in the parameters. The corresponding DVRFT cost functions are quadratic and the optimization problems have explicit solutions.

The distributed controller resulting from DOCI leads to a closed-loop network with a step response shown in Figure 4 and a frequency response shown in Figure 5 in green of the transfer $r \to y$, for 20 experiments. For comparison, we synthesize a DVRFT controller using the same data via the method described in [8, Section IV]. The corresponding responses are shown in Figure 4 and 5 in yellow. Note that the controller classes are chosen such that the ideal controller



(a) DOCI (b) DVRFT

Fig. 6: Parameter estimates for 100 experiments (gray) and the true parameter (red) for $\mathcal{C}_1^d$.

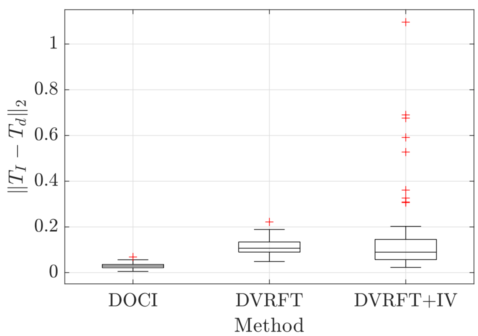


Fig. 7: Distribution of the achieved performance for DOCI, DVRFT and DVRFT with IVs, where T_I and T_d denote the transfers $r \to y$ and $r \to y_d$, respectively.

belongs to the controller class for each method, which leads to a quadratic cost and non-quadratic cost function for DVRFT and DOCI, respectively. We observe that the distributed controller synthesized via DOCI leads to a closed-loop network with a response that is closer to the reference model compared to the controller synthesized via VRFT.

As discussed in Section II-C, DVRFT leads to biased controller estimates when the noise terms e_1 and e_2 are non-zero. This bias is illustrated in Figure 6b, where the parameter estimates for controller 1 are plotted for 100 experiments. The parameter estimates for controller 1 with DOCI are plotted in Figure 6a. Finally, Figure 7 shows the distribution of the achieved performance for DOCI and DVRFT in the presence of noise. As described in the introduction, VRFT can yield consistent estimates when instrumental variables (IVs) are used. The construction of IVs for the example network was performed using an additional experiment [10], *mutatis mutandis*. The mean value of the performance of DVRFT with IVs is significantly lower compared to DVRFT, while the variance is significantly higher. We observe that the mean value as well as the variance of the performance are considerably lower for DOCI. Hence, although both DOCI and DVRFT with IVs yield consistent estimates, the increased variance due to IVs in DVRFT yields an overall worse performance compared to DOCI.

V. Conclusions

We have developed a reference-model based data-driven method for the construction of a distributed controller for an interconnected system subject to disturbances. Sufficient conditions for obtaining consistent estimates of controller modules in a dynamic network have been given. The estimated distributed controller therefore solves the model-reference control problem asymptotically in the number of data. By a simple network consisting of two interconnected systems, we have shown the effectiveness and the improvement over biased or high-variance alternative methods on the closed-loop performance.

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