A prediction-error identification framework for linear parameter-varying systems

R. Tóth, P.S.C. Heuberger and P.M.J. Van den Hof

1 Introduction

Accurate and efficient control of today's industrial applications requires accurate but low complexity models of the often nonlinear or time-varying behavior of these systems. This raises the need for system descriptions that form an intermediate step between *Linear Time-Invariant* (LTI) systems and nonlinear/time-varying plants. To cope with these expectations, the model class of *Linear Parameter-Varying* (LPV) systems provides an attractive candidate. In LPV systems the signal relations are considered to be linear just as in the LTI case, but the parameters are assumed to be functions of a measurable time-varying signal, the so-called scheduling variable $p : \mathbb{Z} \mapsto \mathbb{P}$, with $\mathbb{P} \subseteq \mathbb{R}^{n_\mathbb{P}}$. The LPV system class has a wide representation capability of physical processes and this framework is also supported by a well worked out and industrially reputed control theory. Despite the advances of the LPV control field, identification of such systems is not well developed.

2 The need for a LPV prediction error framework

Existing LPV approaches are almost exclusively formulated in discrete-time, commonly assuming *static* dependence on p (dependence only on the instantaneous value of p), and they are mainly characterized by the type of LPV model structure used: *Input-Output* (IO) [1], *State-Space* (SS) [4] or *Orthogonal Basis Functions* models [3]. In system identification, IO models are widely used as the stochastic meaning of estimation is much better understood for such models, e.g. via the *Prediction-Error* (PE) setting, than for other model structures. As a consequence, extensions of some classical LTI-PE methods, like *Least-Squares* (LS) approaches, have also been developed in the LPV case (e.g. [1]) and due to their simplicity they become popular in many applications. However, these approaches are usually applied as algorithms, without the understanding of the underlying estimation problem, the represented model structure, or the stochastic properties. In order to establish a mature theory for the identification of LPV systems, first of all it needs to be understood how the classical PE framework can be extended to the LPV case and what the properties of the available LPV approaches are under such a framework.

3 LPV series expansion representations

One of the major gaps in the LPV system theory, which has so far prevented the analysis of the PE methods, has been the lack of a transfer function representation of LPV systems. To overcome this problem, it has been shown in [2] that the dynamic mapping between the input $u : \mathbb{Z} \mapsto \mathbb{R}^{n_\mathbb{U}}$ and the output $y : \mathbb{Z} \mapsto \mathbb{R}^{n_\mathbb{Y}}$ of a LPV system $\mathscr{S}$ can be characterized as a convolution involving p and u. This so called *Impulse Response Representation* (IRR) is given in the form of

$$y(k) = \sum_{i=0}^{\infty} (\mathrm{g}_i \diamond p)(k)\ u(k-i) = \left(\sum_{i=0}^{\infty} (\mathrm{g}_i \diamond p) q^{-i} u \right)(k) = ((F(q) \diamond p)u)(k), \qquad (1)$$

where q is the time-shift operator, i.e. $q^{-1}u(k) = u(k-1)$, and the coefficients g_i, i.e. *Markov parameters*, are functions of $p(k)$ and its time shifted values (i.e. $p(k-1), p(k-2), \ldots$), which is called *dynamic dependence* and expressed by the operator $\diamond$. In identification, we aim to estimate a dynamical model of the system based on measured data, which corresponds to the estimation of each g_i. Equation (1) can also be seen as a series expansion of $\mathscr{S}$ in terms of q and it can be shown that this expansion is convergent if $\mathscr{S}$ is asymptotically stable. Equivalence transformations of LPV-SS and IO representations to IRR are also available.

4 Extension of the prediction-error framework

By using the IRR and the established equivalence relations it becomes possible to extend the PE framework to the LPV case. The data generating LPV system $\mathscr{S}_0$ with an asymptotically stable process and noise part is considered as

$$y(k) = (G_\mathrm{o}(q) \diamond p)(k)\ u(k) + (H_\mathrm{o}(q) \diamond p)(k)\ e_\mathrm{o}(k) \tag{2}$$

where G_o and H_o are LPV IRR's with H_o being monic, i.e. $H_\mathrm{o}(\infty) = 1$, and $e_\mathrm{o}(k)$ is a zero-mean white noise process with a normal distribution. Now if p is deterministic and there exists a convergent adjoint $H_\mathrm{o}^\dagger$ of H_o, then it is possible to show that the *one-step ahead predictor* of y is

$$y(k \mid k-1) = ((H_\mathrm{o}^\dagger(q) G_\mathrm{o}(q)) \diamond p)(k)\ u(k) + ((1 - H_\mathrm{o}^\dagger(q)) \diamond p)(k)\ y(k). \tag{3}$$

With respect to a parameterized model structure, we can define the *one-step ahead prediction error* as $\varepsilon_\theta(k) = y(k) - \hat{y}(k \mid k-1)$ where

$$\hat{y}(k \mid k-1) = ((H^\dagger(q,\theta) G(q,\theta)) \diamond p)(k)\ u(k) + (1 - H^\dagger(q,\theta)) \diamond p)(k)\ y(k)$$

with $G(q,\theta)$ and $H(q,\theta)$ the IRR's of the process and noise part of the model structure respectively and $\theta \in \mathbb{R}^{n_\theta}$ are the parameters to be estimated. Denote $\mathcal{D}_N = \{y(k), u(k), p(k)\}_{k=1}^N$ a data sequence of $\mathscr{S}_\mathrm{o}$. Then, to provide an estimate of θ based on the minimization of ε_θ, an identification criterion $W(\mathcal{D}_N, \theta)$ can be introduced, like the *least squares* criterion $W(\mathcal{D}_N, \theta) = \frac{1}{N} \sum_{k=1}^N \varepsilon_\theta^2(k)$, such that the parameter estimate is $\hat{\theta}_N = \arg\min_{\theta \in \mathbb{R}^{n_\theta}} W(\mathcal{D}_N, \theta)$. The developed PE setting can be seen as the LPV extension of the LTI PE framework and it can be shown that under minor assumptions, the classical results on consistency, convergence, bias and asymptotic variance can be extended for LPV prediction-error models with linear parametrization of the coefficient dependence and the concept of noise models can be clearly understood. Preliminary results on persistency of excitation and identifiability can also be established with respect to particular model structures.

References

[1] B. Bamieh and L. Giarré. Identification of linear parameter varying models. *Int. Journal of Robust and Nonlinear Control*, 12:841–853, 2002.

[2] R. Tóth. *Modeling and Identification of Linear Parameter-Varying Systems, an Orthonormal Basis Function Approach*. PhD thesis, Delft University of Technology, 2008.

[3] R. Tóth, P. S. C. Heuberger, and P. M. J. Van den Hof. Asymptotically optimal orthonormal basis functions for LPV system identification. *Automatica*, 45(6):1359–1370, 2009.

[4] J. W. van Wingerden and M. Verhaegen. Subspace identification of bilinear and LPV systems for open- and closed-loop data. *Automatica*, 45:372–381, 2009.