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Parameter identification in large-scale models for oil and gas production ⋆

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Abstract: Models used for model-based (long-term) operations as monitoring, control and optimization of oil and gas reservoirs are often first principles models. They are the result of partial differential equations being discretized, leading to nonlinear models that are large-scale in terms of number of states and parameters. Estimating a large number of parameters from measurement data leads to problems of identifiability and consequently to inaccurate identification results. This is problematic since the models are used for model predictions and control strategies. In this paper options are given to deal with the lack of identifiability: approximating the model structure while retaining the physical interpretation of the parameters, redefining the model structure using parameters with e.g. a geological meaning, and thirdly adding additional prior information to the identification problem. These options are illustrated with examples taken from oil and gas reservoir engineering.

Keywords: identifiability; physical model structures; model structure approximation.

1. INTRODUCTION

It becomes more and more challenging to meet the increasing demand for hydrocarbons: production rates of currently producing oil reservoirs decline more rapidly than expected, recently discovered reservoirs are often found in environments that are challenging to operate in, and it is expected that due to retirement of a significant part of the E&P workforce within the coming decade experience and knowledge is lost. On the positive side, in the last years the control possibilities (e.g. valves in the well that can open or close certain segments of the well for flow and can be operated from the the surface) and the measurement possibilities (e.g. multi-phase flow meters that measure the production of each well, pressure sensors that can measure the pressure drop at each segment of the well) have increased.

This demands and enables a more efficient and rationalized operation of petroleum reservoirs, which can be done by using models of subsurface oil and gas reservoirs. The oil and gas reservoir models represent the relevant process dynamics and support the decisions that need to be taken to increase reservoir performance. Reservoir models are typically first principles models that are, after spatial discretization of the partial differential equations, large-scale in terms of number of states and parameters. The models have nonlinear process dynamics that need to be captured in order to make reliable long-term predictions. Due to the discretization the number of uncertain parameters in reservoir models is usually in the order of 10^5 [Jansen et al., 2008]. Identifying extremely large numbers of parameters from measurement data leads to serious problems, and at least it leads to the question which model properties can be reliably estimated from the available measurement data. From a model-based operations point of view (monitoring, control and optimization) it makes sense to limit the complexity of an identified model to a level where the model can be reliably validated from data. If this is not the case then the parameter estimates might be highly determined by the random experiment that is done and the resulting model predictions are unreliable. In identification this problem is addressed by the notion of identifiability.

In this paper options are given to deal with the lack of identifiability: approximating the model structure by reducing the parameter space to a limited dimension while retaining the physical interpretation of the parameters [Van Doren et al., 2009], secondly redefining the model structure using a reduced number of parameters (e.g. parameters with a geological meaning), and thirdly adding additional prior information to the identification problem. These options are discussed and the first two options are illustrated with identification examples taken from oil and gas reservoir engineering.

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2. IDENTIFIABILITY

2.1 The starting point

The notion of identifiability refers -roughly speaking- to the question whether parameter changes in the model can be observed in the model output signal (output identifiability) or in the model's input-output map or transfer function (structural identifiability). The notion of output identifiability has been studied in e.g. Grewal and Glover [1976] and Ljung [1999]. In its essence, identifiability properties are global properties, i.e. holding for the full parameter space. However restricting attention to a local analysis is often the only situation that is feasible in terms of computational complexity. As a result we will focus on local properties of (output) identifiability only.
Consider a nonlinear dynamical model that generates output predictions according to [1]:

$$\hat{\mathbf{y}} = h(\mathbf{u}, \theta; x_0), \tag{1}$$

where $\hat{\mathbf{y}}$ is a prediction of $\mathbf{y} := \begin{bmatrix} y_1^T & \dots & y_N^T \end{bmatrix}^T$ denoting output signal measurements $y_k \in \mathbb{R}^p$ stacked over time, $\theta \in \Theta \subset \mathbb{R}^q$ the parameter vector, $\mathbf{u} := \begin{bmatrix} u_1^T & \dots & u_N^T \end{bmatrix}^T$ the input vector $u_k \in \mathbb{R}^m$ stacked over time, and x_0 the initial state vector. Since the model (1) is parameterized it represents an input/output model structure. The definition of local identifiability now is given as follows(Grewal and Glover [1976]):

Definition 1. An input/output model structure $h(\theta, \mathbf{u}; x_0) : \Theta \to \mathcal{H}$ is called locally identifiable in $\theta_m \in \Theta$ for a given $\mathbf{u}$ and x_0, if for all θ_1, θ_2 in the neighborhood of θ_m holds that

$$\{h(\mathbf{u}, \theta_1; x_0) = h(\mathbf{u}, \theta_2; x_0)\} \Rightarrow \theta_1 = \theta_2.$$

If we linearize the nonlinear process dynamics around a chosen operating point or trajectory, a linear dynamical system results. This system can be modelled by an LTI input-output model, represented by the transfer function G, leading to an output predictor

$$\hat{y}_k = G(q, \theta) u_k,$$

with q the shift operator $q u_k = u_{k+1}$.
In general, identifiability questions are considered qualitatively, i.e. deciding whether a model structure is either identifiable or not. The tests required for this evaluation are typically rank evaluations of matrices, as e.g. Fisher's information matrix, around a particular local operating point in the parameter space, see e.g. Dötsch and Van den Hof [1996]. However, when considering parameters in large scale nonlinear physical models it is relevant to raise the question how the notion of identifiability can be quantified. This implies addressing the question which part of the parameter space is best identifiable, and which part of the model structure can be approximated so as to achieve local identifiability, while retaining the interpretation of the physical parameters. This has been addressed in Van Doren et al. [2009]. Assessing identifiability can also be done a posteriori, after the identification of all parameters, by evaluating the parameter variance, see e.g. Hjalmarsson [2005].

[1] Without loss of generality we restrict attention to predictors that are not dependent on output measurements y, which in an LTI-setting is referred to as Output Error predictors.

2.2 Analyzing local identifiability in identification

In a model identification framework we consider parameter estimation methods that are characterized by minimizing a cost function $V(\theta)$:

$$V(\theta) := \frac{1}{2} \boldsymbol{\epsilon}(\theta)^T P_v^{-1} \boldsymbol{\epsilon}(\theta), \tag{2}$$

where the prediction error sequence $\boldsymbol{\epsilon}$ is defined as

$$\boldsymbol{\epsilon}(\theta) = \mathbf{y} - \hat{\mathbf{y}}(\theta) = \mathbf{y} - h(\theta, \mathbf{u}; x_0), \tag{3}$$

where $\mathbf{y}$ denotes the measured output sequence and $\hat{\mathbf{y}}$ the predictor sequence, and P_v is a weighting matrix that could represent (an estimate of) the covariance matrix of the noise sequence $\mathbf{v}$ that is supposed to act on the measured output. In the remainder the shorthand notation $\hat{\mathbf{y}}(\theta)$ is used to indicate $h(\mathbf{u}, \theta; x_0)$. The Hessian of $V(\theta)$ with respect to the parameters is

$$\frac{\partial^2 V(\theta)}{\partial \theta^2} = \frac{\partial \hat{\mathbf{y}}(\theta)^T}{\partial \theta} P_v^{-1} \left(\frac{\partial \hat{\mathbf{y}}(\theta)^T}{\partial \theta} \right)^T + S, \tag{4}$$

where S denotes the second-order information in $\frac{\partial^2 V(\theta)}{\partial \theta^2}$. The Hessian is for a given θ and a given operating point (given by $\mathbf{u}$ and x_0). Parameter estimation now consists in finding the parameter estimate as a minimizing argument of the cost function $V(\theta)$

$$\hat{\theta} := \arg\min_{\theta} V(\theta). \tag{5}$$

At $\hat{\theta}$ the cost function $V(\theta)$ is minimized and the Jacobian at $\hat{\theta}$ is zero, i.e. $\frac{\partial V(\theta)}{\partial \theta} = 0$ at $\hat{\theta}$.
Local identifiability in $\hat{\theta}$ is generally evaluated by the test whether the optimization problem (5) has a unique solution in the parameter space. By locally approximating the cost function $V(\theta)$ by a quadratic function [2] (and thus neglecting the second order term S in (4)), uniqueness of $\hat{\theta}$ is guaranteed if the Hessian at $\hat{\theta}$ is positive definite, i.e. $\frac{\partial^2 V(\theta)}{\partial \theta^2} > 0$ at $\hat{\theta}$, which in this case is equivalent to rank $\frac{\partial^2 V}{\partial \theta^2} = q$. This is a sufficient condition for local identifiability in $\hat{\theta}$.
The considered rank test is naturally performed by applying a singular value decomposition (SVD):

$$\frac{\partial^2 V(\theta)}{\partial \theta^2} = U \Sigma V^T = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix},$$

where matrices U and V are unitary matrices, $\Sigma_1 = \mathrm{diag}(\sigma_1, \dots, \sigma_p)$ with $\sigma_1 \geq \dots \geq \sigma_p$.
If $p = q$ then identifiability is confirmed. If $p < q$ then the column space of U_1 represents the subspace of the parameter space that is identifiable, and the column space of U_2 is its orthogonal complement, characterizing the subspace that is not identifiable.

3. DEALING WITH LACK OF IDENTIFIABILITY

3.1 Introduction

In the previous section it has been described how identifiability can be analyzed. Models used for (long-term) optimization of oil and gas reservoir models also have a

[2] This is achieved by approximating $\hat{\mathbf{y}}(\theta)$ with a first-order Taylor expansion around $\hat{\theta}$.

lack of identifiability. As a result the predictions of these models can be unreliable. Lack of identifiability of a model structure and the subsequent non-uniqueness of parameters that are estimated on the basis of measurement data, can be dealt with in different ways, and will be described in the remainder of this section.

3.2 Approximating the identifiable parameter space

When the SVD of the Hessian returns singular values that are (very) small, then the associated singular vectors correspond to directions in the parameter space that have very limited (but nonzero) influence on the cost function V. In identification terms this correspond to directions in the parameter space in which the variance is (very) large. The Hessian evaluated at $\hat{\theta}$ is connected to the variance of $\hat{\theta}$, since for the Gaussian case (and provided that $\hat{\theta}$ is a consistent estimate) it follows that

$$cov\ (\hat{\theta}) = J^{-1}$$

with J the Fisher information matrix

$$J = \mathbb{E}\left[\frac{\partial^2 V(\theta)}{\partial \theta^2}\bigg|_{\hat{\theta}} \right], \tag{6}$$

where $\mathbb{E}$ denotes expectation (Ljung [1999]).
We are interested in specifying that part of the parameter space that is best identifiable by removing the subspace that has only a very small influence on the cost function V. This reasoning would point to removing those parameter (combinations) from the model structure for which the variance is very large, as was also addressed in Vajda et al. [1989] for nonlinear parameter mappings, and in Lund and Foss [2008] for single parameters. To allow for parameter scaling a scaling matrix is introduced with $\Gamma_{\hat{\theta}} = \text{diag}\left(|\hat{\theta}_1| \ \dots \ |\hat{\theta}_q| \right)$ [Van Doren et al., 2009]. The essential information on the SVD of the Hessian is now obtained from:

$$\Gamma_{\hat{\theta}}^{-\frac{1}{2}} \frac{\partial \hat{\mathbf{y}}(\theta)^T}{\partial \theta} P_v^{-\frac{1}{2}} = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}, \tag{7}$$

where the separation between Σ_1 and Σ_2 is chosen in such a way that the singular values in Σ_2 are considerably smaller than those in Σ_1.
If we now reparameterize the model structure by employing the reduced parameter ρ determined by $\theta = U_1\rho$, we have realized a model structure approximation, in which the parameters to be identified are well identifiable with a limited variance and the physical interpretation of the parameters remains untouched. The singular vectors that occur as the columns in U_1 actually can be seen as basis functions in the parameter space.
With the SVD (7) it follows that the sample estimate of the covariance matrix of $\hat{\theta}$ becomes:

$$cov(\hat{\theta}) = \begin{cases} \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma_1^{-2} & 0 \\ 0 & \Sigma_2^{-2} \end{bmatrix} \begin{bmatrix} U_1^T \\ U_2^T \end{bmatrix} & \text{for trace}(\Sigma_2) > 0 \\ \infty & \text{for } \Sigma_2 = 0 \end{cases} \tag{8}$$

while the sample estimate of the covariance matrix of the reparameterized parameter estimate $U_1\hat{\rho}$ is given by

$$cov(U_1\hat{\rho}) = U_1\Sigma_1^{-2}U_1^T. \tag{9}$$

This shows that if $\Sigma_2 = 0$ there is no benefit of the reparameterization in terms of variance of the estimated parameter $\hat{\theta}$. However if nonzero singular values are discarded in Σ_2, i.e. if trace$(\Sigma_2) > 0$, then

$$cov(\hat{\theta}) > cov(U_1\hat{\rho}),$$

showing a covariance that is reduced by the reparameterization. This reduction is particularly interesting if Σ_2 contains a (very) large number of small singular values.
In Section 4.2 an example is presented in which the model structure is approximated and subsequently a reduced number of parameters is estimated from measurements.

3.3 Using a channel parameterization

Desired features of parameterizations in reservoir engineering applications are that parameters can be reliably estimated, but also that after parameter estimation geological realism is conserved. A potential solution to realize this is to choose a parameterization in terms of a limited number of geological objects (e.g. channels) such that after estimation of the parameters a geologically realistic model is obtained. In Van Doren et al. [2008] a channel parameterization has been introduced. In this parameterization it is assumed that the channels are straight and have a uniform permeability distribution. Each channel is described by six parameters: orientation of the channel body, position in x and y direction, length, width and channel permeability. An additional parameter describes the permeability of the background permeability of the reservoir model. This means that a permeability field with two channels is described by 13 parameters. The channels are generated on a fine-scale grid, and then upscaled to the simulation grid size using the arithmetic mean.
In Section 4.3 an example is presented in which the channel parameters are estimated from measurements.

3.4 A Bayesian approach

In the previous two sections the number of parameters in the model structure has been reduced. Alternatively additional prior information can be added to the identification problem. In those situations where a parameter estimate may not be uniquely identifiable from the data, a regularization term can be added to the cost function that takes account of prior knowledge of the parameters to be estimated. In this setting an alternative (Bayesian) cost function is considered:

$$V_p(\theta) := V(\theta) + \frac{1}{2}(\theta - \theta_p)P_{\theta_p}^{-1}(\theta - \theta_p), \tag{10}$$

where the last term represents the weighted mismatch between the parameter vector and the prior parameter vector θ_p with covariance P_{θ_p}. It has to be noted that this Bayesian approach is typically followed when using sequential estimation algorithms for joint parameter and state estimation, as in Extended Kalman Filters and variations thereof, such as the Ensemble Kalman Filter, see e.g. Evensen [2007].
When again the model output $\hat{\mathbf{y}}(\theta)$ is approximated using a first-order Taylor expansion around θ_p, the Hessian of (10) becomes:

$$\frac{\partial^2 V_p(\theta)}{\partial \theta^2} = \frac{\partial \hat{\mathbf{y}}(\theta)^T}{\partial \theta} P_v^{-1} \left(\frac{\partial \hat{\mathbf{y}}(\theta)^T}{\partial \theta} \right)^T + P_{\theta_p}^{-1}. \tag{11}$$

Since $P_{\theta_p}^{-1}$ is positive definite by construction and the first term is positive semi-definite, the Hessian has full rank and the parameter estimate

$$\hat{\theta}_{Bayes} = \arg\min_{\theta} V_p(\theta)$$

is unique. This uniqueness is guaranteed by the prior information that has been added to the problem. Formally there can still be lack of identifiability, however it is not any more reflected in a non-unique parameter estimate. A consequence of this approach is that the obtained parameter estimate may be highly influenced by the prior information, and less by the measurement data. Although in reservoir engineering it is very common to follow a Bayesian approach on the basis of prior knowledge, here we will particularly focus on the information content of the data, and therefore we will refrain from including detailed prior knowledge.

4. EXAMPLES

4.1 Introduction

Petroleum reservoir engineering is concerned with maximizing the oil and gas production from subsurface reservoirs. A common way to increase the production is to inject water in the reservoir via injection wells to drive the oil via production wells towards the surface. However, due to strong heterogeneities in the porous reservoir rock the resulting oil-water front is not progressing uniformly and a large part of the oil is bypassed and not produced. This can be partly counteracted by manipulating the injection and production settings in the wells. The dynamic control strategy that maximizes the production is calculated based on a model of the reservoir, or preferably a set of models. A reservoir model describes the fluid flow in a porous medium in time and space. The model basically is determined by a non-linear pde, which after discretization in space and time yield the following state-space ordinary differential equation in discrete time

$$\mathbf{x}(k+1) = \mathbf{A}(\theta)\mathbf{x}(k) + \mathbf{B}(\theta)\mathbf{u}(k), \quad \mathbf{x}(0) = \mathbf{x}_0 \tag{12}$$

$$\mathbf{y}(k) = \mathbf{C}(\theta)\mathbf{x}(k). \tag{13}$$

A model typically contains 10^5 states in $\mathbf{x}$, which are composed of the fluid pressure $\mathbf{p}$ and fluid saturations $\mathbf{s}$ in each grid block. The input variables $\mathbf{u} \in \mathbb{R}^m$ denote control settings such as injection or production rates or pressures in grid blocks containing wells. The output variables $\mathbf{y} \in \mathbb{R}^p$ denote flow rates in grid blocks containing wells. In the most simple model, the parameters represent the permeability in each grid block which determines how easily fluids flow through the porous medium. Because the permeability in each grid block directly influences the flow, it is vital to estimate this parameter vector using the available measurements in order to obtain reliable model predictions and control strategies.

4.2 Identification of grid block parameters

The first simple example that we consider here reflects a reservoir with five wells in a characteristic five-spot pattern, indicated in Figure 1 by grey squares. There is one injection well in the center and four production wells in the four corners. The reservoir model is discretized in

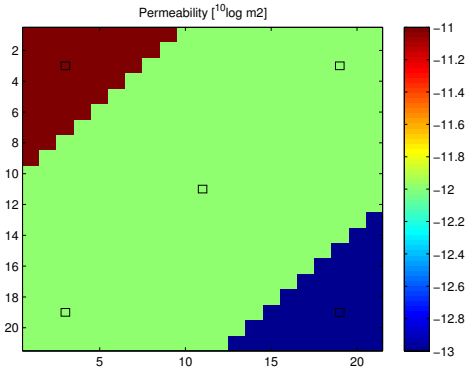

Fig. 1. Permeability distribution (top view) for the example in Section 4.2. Rectangles indicate wells.

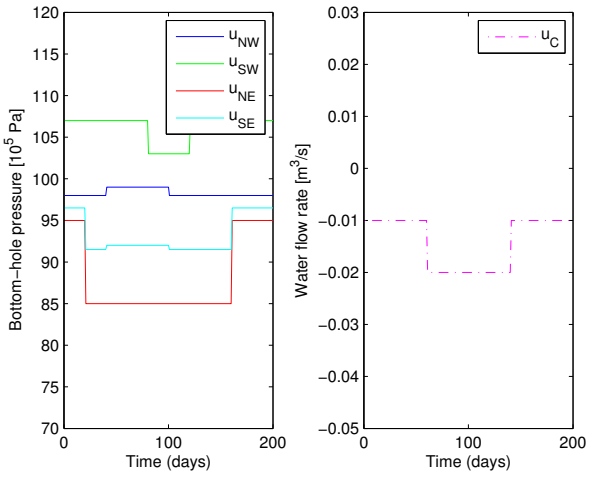

Fig. 2. Input signals as function of time that are used to excite the two-phase reservoir model. Liquid flow rates (right) and bottom-hole pressures (left).

441 grid blocks and the permeability in each grid block is regarded as uncertain. The permeability distribution consists of three zones: the upper left corner has a high permeability, the lower right corner a low permeability, and the intermediate zone an intermediate permeability. As inputs we use the injection flow rate in the injection well and four bottom hole pressures in the production wells. The input signals $\mathbf{u}$ are depicted in Figure 2. As measurements we have used the oil and water flow rates in the four production wells, where we note that water breakthrough has occurred in all wells. The measurement signals are depicted in Figure 5 with thick lines. In our identifiability analysis we have chosen $\mathbf{\Gamma}_{\hat{\theta}} = \mathrm{diag}(^{10}\log\theta)$ and $\mathbf{P}_v^{-\frac{1}{2}} = \mathbf{I}$ in (7). As can be clearly seen the singular values in Figure 3 drop steeply. Since the identifiable parameterization partly depends on the parameter value we use an iterative procedure. Starting from an initial parameter vector an (locally) identifiable parameterization is determined. With this parameterization a new parameter vector is estimated, and the procedure is repeated. The iteration is stopped when no substantial improvement of the cost function is obtained. Perfect measurements $\mathbf{y}$ are generated by simulating the two-phase reservoir model for 200 days in the in-house reservoir simulator with the

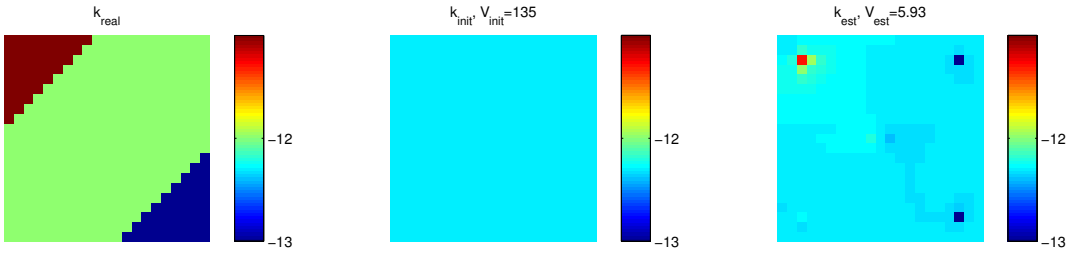

Fig. 4. Real (left), initial (middle) and estimated permeability distribution (right) obtained with the identifiable parameterization in Section 4.2.

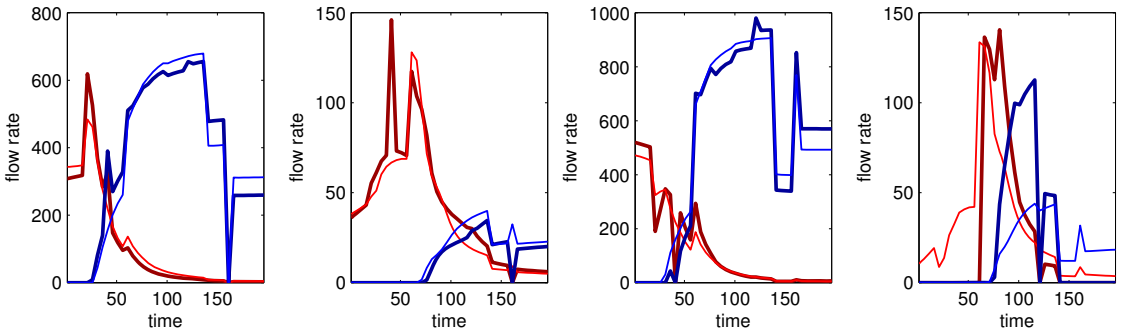

Fig. 5. Measured and predicted water (blue) and oil (red) flow rates for the first example.

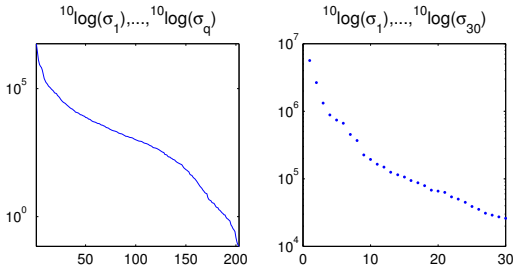

Fig. 3. All singular values (left) and 30 largest singular values (right).

so-called real permeability distribution (see left plot in Figure 4), initial pressure $\mathbf{p}_0 = 100 \times 10^5$Pa and initial oil saturation $\mathbf{s}_0 = 0.2$ in every grid block. As input we have used the pressures in the production wells and the injection flow rate in the injection well (see Figure 2 for the signals). As measurements we have used the oil and water flow rates in the four production wells (see thick lines in Figure 5 for the signals).

As initial guess a homogeneous permeability distribution is chosen with the value $\theta_{\text{init}} = -13.3$ which is equivalent to a permeability value of $5 \times 10^{-13}\text{m}^2$. This permeability distribution is depicted in the middle of Figure 4. The corresponding value of the objective function is $V(\theta_{\text{init}}) = 135$. Based on θ_{init} the model structure is approximated using the SVD of (7) keeping only the first 15 singular values. To estimate the grid block permeability we have used the Gauss-Newton update rule. In this example the best result is obtained in case the model structure is approximated after each update. The estimate has converged after 30 iterations to the permeability distribution depicted in the right of Figure 4. The value of the objective function has decreased to $V = 5.93$. From the estimated permeability distribution we see that the largest changes have occurred in the grid blocks which are penetrated by production wells. Although the real permeability distribution is not recognizable anymore, the flow relevant features are apparently estimated, since the objective function has decreased significantly and the fluid flow rates predicted by the model according to Figure 5 are very well matching the measurements. The example shows that the permeability is only identifiable in the grid blocks that are in the direct neighborhood of wells and in the grid blocks in between the wells.

4.3 Identification of channel parameters

Next, channel parameters are estimated from the water and oil flow rate measurements in the producers depicted in Figure 7 with the thick lines. The initial permeability distribution resulting from the initial channel parameters is depicted in the middle of Figure 6. The corresponding value of the cost function before estimation is $V = 25.8$. Note that channels are not penetrated by the wells.

Before estimation this model structure is analyzed calculating

$$\left.\frac{\partial \mathbf{y}(\theta_{gb})^T}{\partial \theta_{ch}}\right|_{\theta_{ch}} = \frac{\partial \theta_{gb}^T}{\partial \theta_{ch}} \frac{\partial \mathbf{y}(\theta_{gb})^T}{\partial \theta_{gb}}, \tag{14}$$

where θ_{gb} denotes the grid block permeability and θ_{ch} denotes the channel parameters. Subsequently, the scaled identifiability matrix is analyzed by applying an SVD on

$$\mathbf{\Gamma}_{\theta_{ch}} \frac{\partial \mathbf{y}(\theta_{gb})^T}{\partial \theta_{ch}} \boldsymbol{P}_v = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T, \tag{15}$$

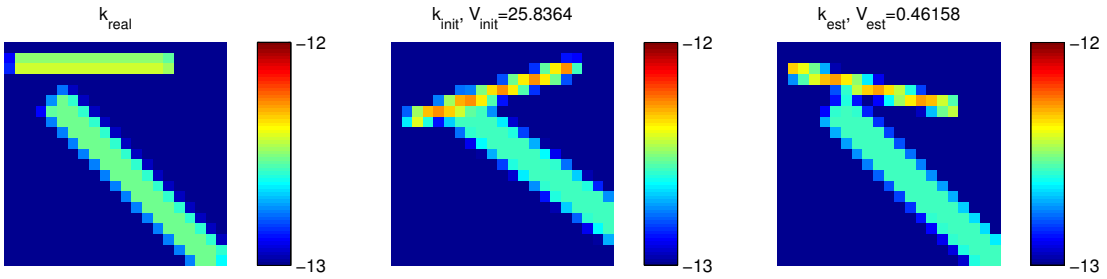

Fig. 6. Real (left), initial (middle) and estimated permeability distribution (right) obtained with the channel parameterization in Section 4.3.

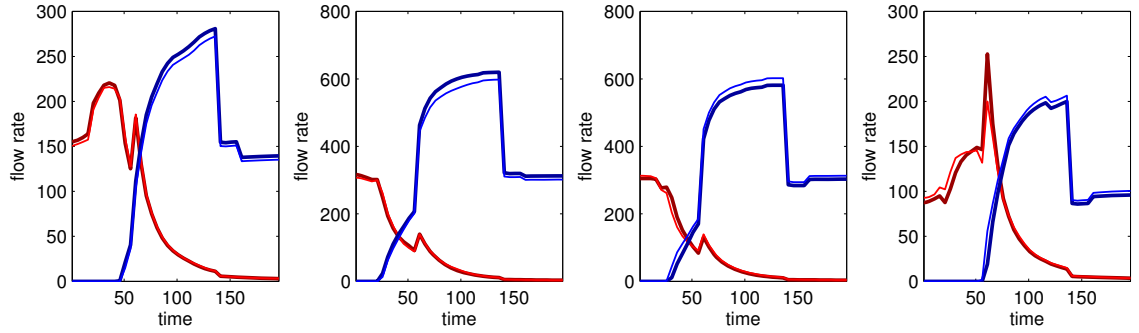

Fig. 7. Measured and predicted water (blue) and oil (red) flow rates for the channel example.

evaluated at θ_{ch}. After inspection of the singular values we conclude for this example that the orientations of both channels are best identifiable and that the position in y direction is least identifiable from water and oil flow rate measurements.

With the same inputs as in the previous example the 13 channel parameters are estimated. This results after convergence in the permeability distribution depicted in the right of Figure 6. The value of the cost function has decreased to $V = 0.46$. For this situation the input-output behavior with this permeability distribution is similar to the behavior with the real permeability distribution. Also the estimated permeability distribution is similar to the real permeability distribution, and the channels are in this example correctly positioned at the well locations.

5. CONCLUSIONS

The large-scale (nonlinear) physical reservoir models as used for model-based operations (monitoring, control and optimization) of oil and gas production are not identifiable. As a result the parameters cannot be reliably estimated, which is problematic since the models are used for long-term model predictions and control strategies.

In this work options have been presented to achieve identifiability of reservoir models: the model structure can be approximated while retaining the interpretation of the physical parameters, the model structure can be redefined using a parameterization with a reduced number of geological parameters, and a regularization term can be added to the cost function (i.e. Bayesian estimation). Examples have been given to illustrate how parameters in large-scale oil and gas reservoir models can be reliably estimated.

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