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Identification in Dynamic Networks with Known Interconnection Topology

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Abstract— **The problem of identifying dynamical models on the basis of measurement data is usually considered in a classical open-loop or closed-loop setting. In this paper this problem is generalized to dynamical systems that operate in a complex interconnection structure and a particular transfer function in the network needs to be identified. It is shown that classical methods of closed-loop identification in the prediction error context, can be generalized to provide consistent model estimates, under specified experimental circumstances. This applies to indirect methods that rely on external excitation signals like two-stage and IV methods, as well as to the direct method that relies on consistent noise models. Graph theoretical tools are presented to verify the topological conditions under which the several methods lead to consistent estimates of the network transfer functions.**

I. INTRODUCTION

One of the challenges in the systems and control field is to develop effective synthesis methods for distributed control of systems that operate in a network structure. While considerable attention is devoted to this problem from a model-based control perspective, attention for the underlying modelling problem is much more limited. In particular the problem of identifying dynamical models on the basis of measurement data that is obtained from a (complex) dynamic network, and where use can be made of external probing/excitation signals, has not been addressed in much detail yet.

In this paper we will consider identification problems in networks of dynamic systems. From an identification perspective this can be considered as a natural extension of the situation of open-loop data, closed-loop data in a single loop, towards data that is obtained from systems operating in a predefined network connection. While dynamic networks typically contain (feedback) loops, it is expected that methods for closed-loop identification are an appropriate basis for developing generalized tools to deal with complex networks.

In our framework discussed here, a dynamic network is defined as an interconnection of transfer functions where the interconnecting signals are considered as nodes/vertices in the network, and proper transfer functions are considered as links/edges. In this paper it will be assumed that the interconnection topology of the network is known, and the goal is to identify the dynamics of a single transfer or a collection of transfers in the network.

Classical methods for closed-loop identification are addressed in [1], [2]. For a single contribution to the problem of structured systems see also [3]. Whereas nonparametric and parametric methods have been introduced in the problem of network identification (see e.g. [4], [5], [6]), the conditions under which the applied methods work typically include the condition that all disturbance/noise processes should be modelled exactly in conjunction with the dynamic transfers in the network. However in a large-scale dynamic network it can be questioned it this is feasible. In this paper we therefore will particularly focus on identification methods that can consistently identify dynamic transfer functions without relying on exact noise models.

In the closed loop identification literature, the two-stage method [7] and the instrumental variable method [8] can be used to address this problem. In this paper the principles behind these methods are applied to develop generalized methods for identification in dynamic networks.

Notation: $[\cdot]_{ji}$ is matrix element (j,i) of the matrix $[\cdot]$.

II. SYSTEM SETUP - NETWORK ARCHITECTURE

The network structure that we consider in this paper is built up of L nodes, related to L measured scalar signals w_j, $j = 1, \cdots L$. Every node signal w_j can be written as:

$$w_j(t) = \sum_{k\in\mathcal{N}_j} G^0_{jk}(q)w_k(t) + r_j(t) + v_j(t) \qquad (1)$$

with $G^0_{jk}(z)$ a proper rational transfer function, $\mathcal{N}_j$ the set of indices of node signals w_k, $k \neq j$, for which $G^0_{jk} \neq 0$; q^{-1} the delay operator $q^{-1}u(t) = u(t-1)$, v_j a possible unmeasured disturbance term being a realization of a stationary stochastic process with rational spectral density, and r_j a possible external excitation signal, available to and possibly designed by the user.

A single node of the network is sketched in Figure 1, where the transfer function G^0_{ji} has been separately indicated to focus on the transfer function that is supposed to be identified. The topology of a network could then be sketched as in Figure 2, where each node n_i represents a node signal w_i, while the arrows represent causal relationships.

All node signals w_j, $j = 1, \cdots L$ are supposed to be measurable, while at each node a noise signal v_j (non-measurable) and excitation signal r_j (measurable) may or may not be present. Each excitation signal r_j is uncorrelated to all noise signals v_i. Some parts of the network may have

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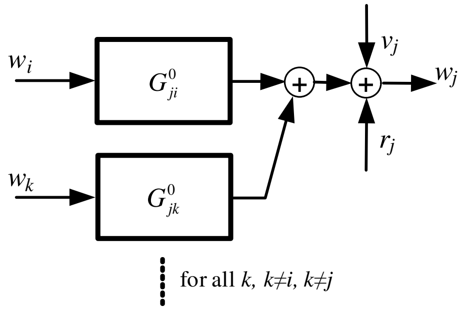

Fig. 1. Single node in a network structure.

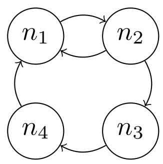

Fig. 2. A dynamic network in the node-and-link style representation of a directed graph. Each node n_i represents a signal w_i, while the arrows (edges) represent causal (transfer function) relationships.

dynamics that are known a priori. This is e.g. the case in a classical closed-loop system with a known controller.

The problem that will be addressed in this paper is: specify conditions under which the transfer function G^0_{ji} can be estimated consistently. For this purpose we specify the next general assumptions on the considered network.

Assumption 1: We consider a dynamic network of which one node is depicted in Figure 1, with the additional properties that

a. The network is stable, in the sense that all node signals w_k, $k = 1, \cdots L$ are bounded, provided that all excitation signals r_j are bounded;
b. The vector noise process $\text{v} := (v_1, \cdots v_L)^T$ has a positive semi-definite spectral density, $\Phi_{\text{v}}(\omega) \geq 0$, not necessarily diagonal.
c. All transfer functions in the network are proper. □

This formulated identification problem will be addressed by analyzing different identification approaches, each of them being closely related to classical prediction error methods for closed-loop identification.

The approaches can be indicated as follows:

1) Indirect identification through external excitation with reference signals r;
2) Indirect identification through reconstructible noise signals v;
3) Direct identification through noise and external excitation, requiring noise modelling; this method requires modelling of all G_{jk} simultaneously;

The first two methods are based on the two-stage method and IV method of closed-loop identification, [7], [8]. The latter method is a natural generalization of the direct method of closed-loop identification [9].

In general each of the several methods will be able to identify a particular subset of the network, dependent on the network topology, the presence of noise and excitation signals, and the presence of a priori known transfers. This will be analyzed in the rest of this paper.

III. Background Information

In this section we will specify the notation that will be used for characterizing the topological structure of the network and some useful results from graph theory and signal theory will be briefly presented.

A. Network topology and graph theory

First of all the topology of the network is characterized by a directed graph that indicates the locations and directions of causal transfers within the network. The topology is represented in the adjacency matrix $A \in \mathbb{R}^{L \times L}$, defined according to:

$$\begin{aligned} A(j,i) &= 0 \qquad \text{if } G^0_{ji}(q) \equiv 0; \\ A(j,i) &= 1 \qquad \text{elsewhere.} \end{aligned}$$

The adjacency matrix characterizes the presence of links (direct causal transfers) in the mapping between node signals. Because of the structures that we consider here (1) it follows that $A(i,i) = 0$, $i = 1, \cdots L$.

One lemma from graph theory will be very useful [10]:

Lemma 1: Consider a directed graph with adjacency matrix A. Then for $k \geq 1$, $[A^k]_{ji}$ indicates the number of different path connections of length k from node i to node j. □

We will further consider the following sets:

- $\mathcal{V}$ denotes the set of indices of node signals to which additive noise sources v are directly connected.
- $\mathcal{R}$ denotes the set of indices of node signals to which external excitation signals r are directly connected.
- $\mathcal{K}_j$ denotes the set of indices of node signals w_k, $k \in \mathcal{N}_j$ where the dynamics G_{jk} are known.
- Let $\mathcal{U}^i_j$ denote the set of indices of node signals w_k, $k \in \mathcal{N}_j, k \neq i$ where the dynamics G_{jk} are unknown.

Note that $\mathcal{N}_j = i \cup \mathcal{K}_j \cup \mathcal{U}^i_j$. Attention will be focussed on identification of the transfer G^0_{ji}.

B. Projection of signals - two-stage philosophy

Finally we add a tool from signal theory that will appear to be very helpful for compactly indicating projection operations on signals. Let r and w be quasi-stationary signals ([9]) in a linear dynamic network such that

$$R_{wr}(k) := \bar{\mathbb{E}} w(t) r(t-k) = 0 \quad \text{for } k < 0,$$

with $\bar{E} := \lim_{N \to \infty} \frac{1}{N} \sum_{t=0}^{N-1} \mathbb{E}$ and $\mathbb{E}$ the expectation operator. Then there exists a proper transfer function F^0_{wr} such that

$$w(t) = F^0_{wr}(q) r(t) + z(t)$$

with z uncorrelated to r, and leading to a decomposition

$$w(t) = w^{(r)}(t) + w^{(\perp r)}(t)$$

with $w^{(\perp r)}(t) = z(t)$. If r and w are available from measurements then $F^0_{wr}(q)$ can be consistently estimated from data, provided that the signal r is persistently exciting of a sufficiently high order. This consistent estimation can be done without the necessity to model the noise dynamics of z, namely by either following a two-stage approach ([7]) or an instrumental variable (IV) approach ([8]). Subsequently the projection

$$\hat{w}^{(r)}(t) := \hat{F}_{wr}(q)r(t)$$

can be calculated, with $\hat{F}_{wr}(q)$ the estimated transfer. This estimate then can serve as an accurate estimate of $w^{(r)}(t)$. Note that $w^{(r)}$ is the projection of signal w onto the space of (causally) time-shifted versions of r.

IV. Indirect identification with external excitation r

A. Basic result - consistent identification

Suppose that there is one external excitation signal r_m present somewhere in the network, that provides excitation for the node signal w_i that is an input to the transfer function G^0_{ji}. Then a reasoning that is similar to the classical two-stage method of closed-loop identification [7] leads to a method that is capable of identifying G_{ji} consistently without the necessity of consistently identifying noise models. The situation is illustrated in Figure 3. The principle identification approach is as follows (in main lines):

Algorithm 1:

1) On the basis of measured signals r_m and w_i, determine $w_i^{(r_m)}$, i.e. the component of w_i that is correlated to r_m (see section III-B);
2) Construct the signal $\tilde{w}_j(t) = w_j(t) - \sum_{k\in\mathcal{K}_j} G^0_{jk}(q)w_k(t)$, i.e. correct w_j with all known terms;
3) Identify the transfer G^0_{ji} on the basis of a predictor model with prediction error

$$\varepsilon_j(t,\theta) \quad = \quad K_j(q)^{-1}[\tilde{w}_j(t) - G_{ji}(q,\theta)w_i^{(r_m)}(t)]$$

using measured signals $\tilde{w}_j$ and $w_i^{(r_m)}$, and by minimizing the (quadratic) prediction error criterion $V_N(\theta) = \frac{1}{N}\sum_{t=1}^{N}\varepsilon_j(t,\theta)^2$; K is a fixed or independently parametrized noise model, and the parametrized model $G_{ji}(q,\theta)$ is chosen flexible enough so as to contain the true transfer $G^0_{ji}(q)$, see [9].

Under the described conditions, the following result can be obtained:

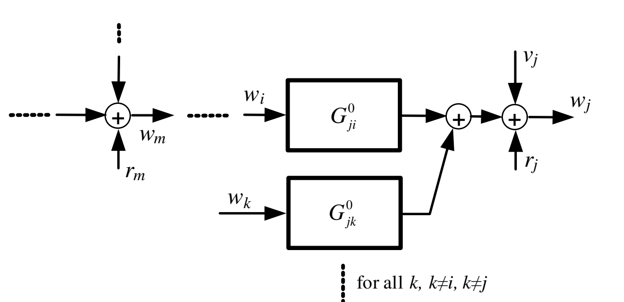

Fig. 3. Single node in a network structure, where the input w_i is excited through an external excitation signal r_m.

Proposition 1: Consider a dynamic network that satisfies Assumption 1. Then the transfer function G^0_{ji} can be consistently estimated with algorithm 1 if the following conditions 1)-3) are satisfied:

1) There exists a node signal w_m that contains an additive external reference signal r_m that is uncorrelated to all noise signals $v_k, k \in \mathcal{U}^i_j$ and v_j, and persistently exciting of a sufficiently high order.
2) The node signal w_i is correlated to r_m;
3) All node signals w_k, $k \in \mathcal{U}^i_j, k \neq i$, are uncorrelated to r_m. □

Proof Note that we can write

$$w_j(t) = G^0_{ji}(q)w_i(t) + \sum_{k\in\mathcal{K}_j} G^0_{jk}(q)w_k(t) + \sum_{k\in\mathcal{U}^i_j} G^0_{jk}(q)w_k(t) + r_j(t) + v_j(t)$$
$$= G^0_{ji}(q)w_i(t) + p_j(t) + s_j(t) + v_j(t)$$

where p_j reflects the contributions of all signals $G^0_{jk}(q)w_k$ that are known because of the fact that the dynamics G_{jk} is known, as well as $r_j(t)$; and $s_j(t)$ similarly reflects the contributions of all signals $G^0_{jk}(q)w_k$ that are unknown, because the dynamics G^0_{jk} is unknown.
Subsequently we write

$$w_j(t) - p_j(t) = G^0_{ji}(q)w_i(t) + s_j(t) + v_j(t)$$

with the left hand side being a known signal.
Conditions 1) and 2) guarantee that w_i can be decomposed as $w_i = w_i^{(r_m)} + w_i^{(\perp r_m)}$. Then,

$$w_j(t) - p_j(t) = G^0_{ji}(q)[w_i^{(r_m)}(t) + w_i^{(\perp r_m)}] + s_j(t) + v_j(t).$$

Conditions 3) and 1) guarantee that the signal s_j is uncorrelated to r_m. And by condition 1) the noise v_j is uncorrelated to r_m, while $w_i^{(\perp r_m)}$ is uncorrelated to r_m by construction. Then as a result a prediction error identification on the basis of input $w_i^{(r_m)}$ and output $w_j - p_j$ will provide a consistent estimate of G^0_{ji}, provided that the correlation between r_m and w_i is sufficiently "rich". □

Note that as an alternative for the two-stage algorithm, also an IV estimator could have been used, using r_m as instrument, w_i as input and $w_j - p_i$ as output, leading to the same consistency result, [8].

B. Algorithm for verification of conditions

Next question is whether in a particular network topology, the conditions as formulated in Proposition 1 are satisfied for consistent identification of the transfer function G^0_{ji}. This question can be treated by an algorithm based on the graph of the network. Recall that A is the adjacency matrix of the network.

Algorithm 2:

1) Detect excitation signal:
   - For every index $m \in \mathcal{R}$:

2) Check if there exists a directed path from the reference signal input ($m \to i$):
   - Evaluate element (i, m) of A^ℓ for $\ell = 1, \cdots L-1$;
   - If for any considered power ℓ this element is nonzero then the condition is satisfied, and reference signal r_m qualifies as an excitation source that excites the input w_i.
3) Verify condition (3) on the node j:
   - Evaluate A^ℓ for $\ell = 1, \cdots L-1$;
   - For all $k \in \mathcal{U}_j^i$, $k \neq i$, check whether the entries (k, m) of A^ℓ are zero for all powers ℓ.

C. Algorithm for identification

In section IV-A an identification algorithm is sketched for the situation that one single external excitation signal r_m is present in the network. This situation is the basis for the consistency condition of the resulting estimate of G_{ji}^0. However if our aim is not only consistency but also to reduce the estimator variance, as well as to handle the situation of possibly more than one excitation signals that satisfy the conditions, as formulated in Proposition 1, then the identification algorithm needs to be reconsidered.
Suppose that for input node i the node numbers of external excitation signals r that are correlated to w_i are collected in the set $\mathcal{R}_i$. The identification algorithm as sketched in section IV-A can then be adapted as follows:

Algorithm 3:

1) For each $m \in \mathcal{R}_i$ construct the prediction error:

   $$\varepsilon_m(t, \theta) = H(q)^{-1}[w_i(t) - p_{i,m}(t) - F_{ir_m}(q, \theta_m) r_m(t)],$$

   with $p_{i,m}(t) = \sum_{k \in \mathcal{K}_i} G_{ik}(q) w_k^{(\perp r_m)}$, $H(q)$ a fixed noise model, and estimate the parameter $\theta_{N,m}$ with a quadratic prediction error criterion: $\hat{\theta}_{N,m} = \arg\min \frac{1}{N} \sum_{t=1}^N \varepsilon_m(t, \theta_m)^2$.
2) Simulate the noise free inputs: $w_i^{(r)} = \sum_{m \in \mathcal{R}_i} F_{ir_m}(q, \hat{\theta}_{N,m}) r_m$.
3) Identify the transfer function from $w_i^{(r)}$ to w_j, through LS minimization of the prediction error

   $$\begin{aligned}\varepsilon_j(t, \theta) \quad &= \quad K_j(q)^{-1}[w_j(t) - G_{ji}(q, \theta) w_i^{(r)}(t) + \\ &\quad - \sum_{k \in \mathcal{K}_j} G_{jk}(q) w_k(t)],\end{aligned}$$

   with $K_j(q)$ a fixed noise model.

Comment 1: When writing

$$w_j = G_{ji}^0 w_i + v_j + \sum_{k \in \mathcal{K}_j} G_{jk}^0 w_k + \sum_{k \in \mathcal{U}_j^i} G_{jk}^0 w_k^{(\perp r)}$$

it follows that

$$\begin{aligned}\varepsilon_j(t, \theta) \quad &= \quad K_j(q)^{-1} \Big[[G_{ji}^0 - G_{ji}(q, \theta)] w_i^{(r)}(t) + v_j(t) + \\ &\quad + G_{ji}^0 w_i^{(\perp r)}(t) + \sum_{k \in \mathcal{U}_j^i} G_{jk}^0 w_k^{(\perp r)}(t)],\end{aligned}$$

where use is made of condition 3 of Proposition 1, that requires that all inputs to unknown transfers G_{jk}^0, $k \neq i$, are uncorrelated to r_m. For consistent estimation of G_{ji}^0 it suffices that the latter three terms on the right hand side of the equation are uncorrelated to r. For reducing the variance of the estimate additional tools may be applied, such as e.g. estimating the unknown transfers G_{jk}^0, $k \in \mathcal{U}_j^i$ simultaneously with G_{ji}^0.
Example 1: An example of a dynamic network is depicted in Figure 4. When applying the conditions of Proposition 1 it appears that the blue-colored transfers, G_{32}, G_{54}, G_{15} and G_{45} can be consistently identified with the two-stage approach presented in this section. These four transfers satisfy the conditions that their inputs are correlated to r, while their outputs are not disturbed by unknown terms that are correlated with r.

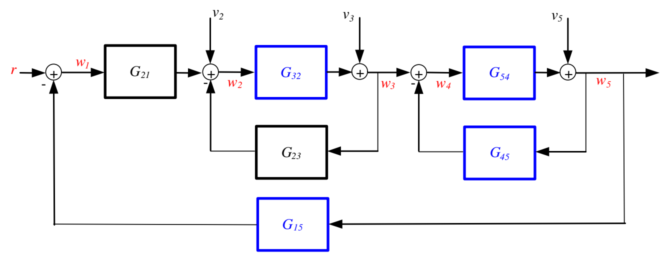

Fig. 4. Dynamic network with 5 node signals, of which 4 (blue-colored) transfer functions can be consistently identified with the two-stage method presented in section IV.

Note that the transfers G_{21} and G_{23} do not satisfy the conditions of the Proposition because there are unknown contributions to w_2 that are correlated to r_m. However if we extend the Proposition to also allow simultaneous identification of different transfers, i.e. estimating G_{21} and G_{23} in a multi-input single-output model, then also these transfers could be identified consistently provided appropriate excitation conditions are satisfied; e.g. this may require more than one external excitation signal.

V. Indirect identification with reconstructible noise signal v

A. Basic result - consistent identification

A second identification approach for consistently identifying the transfer G_{ji}^0 is obtained when there is one noise signal v_m present somewhere in the network, that can be reconstructed on the basis of measured signals and known transfers, and that provides excitation for the node signal w_i that is an input to the transfer function G_{ji}^0. Then a reasoning that is closely related to the classical two-stage method of closed-loop identification [7] leads to a method that is capable of identifying G_{ji}^0 consistently without the necessity of consistently identifying noise models. The principle approach is as follows (in main lines):

Algorithm 4:

- Suppose that there is a node m in the network for which holds that every transfer G_{mk}^0, $k \in \mathcal{N}_m$ is known. Then $x_m := \sum_{k \in \mathcal{N}_j} G_{mk}^0(q) w_k$ is known, and $v_m = w_m -$

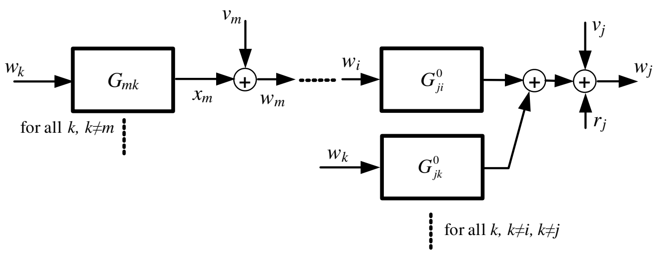

Fig. 5. Single node in a network structure, where the input w_i is excited through a reconstructed noise signal v_m.

x_m can be reconstructed.
If w_i is correlated to v_m then one can construct $w_i^{(v_m)}$.

- In the second step, the transfer from $w_i^{(v_m)}$ to w_j is identified using the prediction error

$$\varepsilon(t,\theta) = K(q)^{-1}[w_j(t) - G_{ji}(q,\theta)w_i^{(v_m)}(t) + - \sum_{k\in\mathcal{K}_j} G_{jk}(q)w_k(t)],$$

similar to the situation in the previous section, where we have replaced the external excitation signal r_m by a reconstructible noise signal v_m.

On the basis of the reasoning above, the following result can be formulated:

Proposition 2: Consider a dynamic network that satisfies Assumption 1. Then the transfer function G^0_{ji} can be consistently estimated through Algorithm 4 if the following conditions (1)-(3) are satisfied:

1) There exists a node w_m, that satisfies the property that
   - $w_m - v_m$ is known,
   - noise source v_m has a variance > 0, and is uncorrelated to v_j if this latter signal is present.
2) The node signal w_i is correlated to v_m;
3) All node signals w_k, $k \in \mathcal{U}_j^i, k \neq i$, are uncorrelated to v_m. □

Proof

Consider the noise signal $v_m = w_m - x_m$ with $x_m := \sum_k G^0_{mk}(q)w_k$. Since by condition 1 the signal x_m is known, it follows that v_m can be reconstructed. And since by condition 2 w_i is correlated to v_m we can construct $w_i^{(v_m)}$. One can then write

$$w_i = w_i^{(v_m)} + w_i^{(\perp v_m)} \quad (2)$$

$$w_j - r_j - \sum_{k\in\mathcal{K}_j} G^0_{jk}(q)w_k = G^0_{ji}w_i^{(v_m)} + \eta_j \quad (3)$$

where the left hand side represents a known signal, and by condition 3 η_j is uncorrelated to v_m.

This situation mimics the situation of the classical two-stage method for closed-loop identification, where based on (3) it follows that consistent identification of G^0_{ji} is possible. □.

B. Algorithm for verification of conditions

The conditions of Proposition 2 can be verified by evaluating the topological properties of the network represented in its graph, and in the graph theoretical tools. Since topological conditions can not verify whether different noise sources are correlated, we add the assumption here that all noise sources are uncorrelated, i.e. that $\Phi_v(\omega)$ is diagonal.

Algorithm 5:

1) Detect reconstructible noise signals:
   - For every index $m \in \mathcal{V}$: check if $\mathcal{K}_m = \mathcal{N}_m$. If so, v_m qualifies as a reconstructible noise signal.
2) Check if there exists a directed path from the reference signal input ($m \to i$):
   - Evaluate element (i,m) of A^ℓ for $\ell = 1, \cdots L-1$;
   - If for any considered power ℓ this element is nonzero then the condition is satisfied, and the noise signal v_m qualifies as an excitation source that excites the input w_i.
3) Verify condition (3) on the node j:
   - Evaluate A^ℓ for $\ell = 1, \cdots L-1$;
   - For all $k \in \mathcal{U}_j^i$, $k \neq i$, check whether the entries (k,m) of A^ℓ are zero for all powers ℓ.

C. Algorithm for identification

Analogous to the situation in section IV, the situation can be handled of multiple noise signals that are reconstructible and that can serve as an excitation to the node input signal w_i, while additionally the variance of the estimated transfer function G_{ji} needs to be reduced.

Suppose that for input node i the node numbers m of noise signals v_m that can be reconstructed and that are correlated to w_i are collected in the set $\mathcal{E}_i$. The identification algorithm as sketched in section V-A can then be adapted in a way that is completely analogous to the algorithm provided in section IV-C, by replacing $\mathcal{R}_i$ by $\mathcal{E}_i$ and replacing signals r_m by v_m. Note that Comment 1 also applies to the this situation.

Example 2: If we consider the network example of Figure 4, it appears that both v_3 and v_5 qualify as a reconstructible noise signal, provided that the transfers G_{32} and G_{54} are known, e.g. through consistent identification via the method of section IV. However in the considered situation none of the remaining transfer functions satisfies the other condition of Proposition 2 that the outputs should not be disturbed by unknown terms that are correlated to the (reconstructible) noise source.

However if we remove the outer loop connection G_{15}, as depicted in Figure 6, then G_{23} can be identified consistently through reconstructible noise signal v_3. In Figure 6 this transfer is indicated in red.

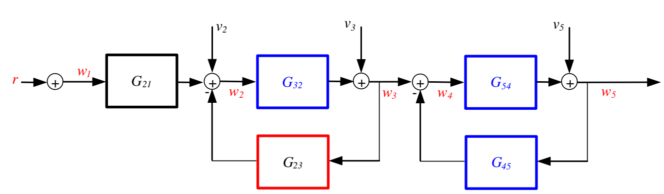

Fig. 6. Dynamic network with 5 node signals, of which 1 (red-colored) transfer function can be consistently identified with the two-stage method presented in this section V based on reconstructed noise signals.

VI. Direct identification

In contrast to the previous methods for identification of closed-loop systems, the classical *direct* method of prediction error identification requires the consistent identification of noise models in order to be able to identify consistent plant models [9]. The direct method identifies a transfer function G_{ji} in a standard feedback configuration, by applying a parametrized model $(G_{ji}(q,\theta), H_j(q,\theta))$ leading to the prediction error

$$\varepsilon(t,\theta) = H_j(q,\theta)^{-1}[w_j(t) - G_{ji}(q,\theta)w_i(t)].$$

A quadratic prediction error criterion then leads to consistent estimates of G_{ji}^0 and H_j^0 under particular excitation requirements. For further details, see [9]. Here we formulate a generalization of this classical method to the situation of dynamic networks. For the proof, the reader is referred to the companion paper [11].

Proposition 3: Consider a dynamic network that satisfies Assumption 1. Then the transfer functions between $\{w_i\}_{i\in\mathcal{N}_j}$ and w_j can be consistently estimated with the direct prediction error method if the following conditions (1)-(4) are satisfied:

1) Noise source v_j is present with variance > 0;
2) $\Phi_{\rm v}$ is diagonal;
3) Every loop in the network that runs through node j has at least one time step delay;
4) The parametrized model is sufficiently flexible to contain the true system G_{jk}^0, $k \in \mathcal{N}_j$, and H_j^0;
5) For both the real network and its parametrized model holds that every loop that runs through node j has at least one time step delay;
6) The spectral density of the composed signal $\mathrm{z} := [w_j \; w_{n_1} \; \cdots w_{n_n}]^T$, $n_* \in \mathcal{N}_j$, satisfies $\Phi_{\rm z}(\omega) > 0$ for all $\omega \in [0,\pi]$. □

Note that in the considered situation all transfers G_{ji}^0, $i \in \mathcal{N}_j$ need to be estimated simultaneously in order for the result to hold, and that the dynamics of noise source v_j needs to be modeled correctly through a noise model H_j. Note also that both the noise signal v_j and the probing signal r_j provide excitation to the loop that is going to be identified. The excitation condition 6) is rather generic. A further specification for particular finite dimensional model structures can possibly be made along the results for classical feedback loops as discussed in [12]. If particular transfers G_{kj}^0 are known a priori, the result of the Proposition can simply be formulated for the set of identified transfers between $\{w_k\}_{k\in\{i,\mathcal{U}_j^i\}}$ and w_j.

Example 3: If we apply the result of the direct method to the network example of Figure 4, it appears that the direct method can be applied to node w_2, by identifying a model between inputs w_1 and w_3, and output w_2. Under the condition that a delay is present in the loops $(G_{32}G_{23})$ and $(G_{25}G_{54}G_{32}G_{21}G_{15})$ and by the use of an appropriate model set that includes accurate noise modelling, the transfers G_{21} and G_{23} can be estimated consistently. In Figure 7 they are indicated in red.

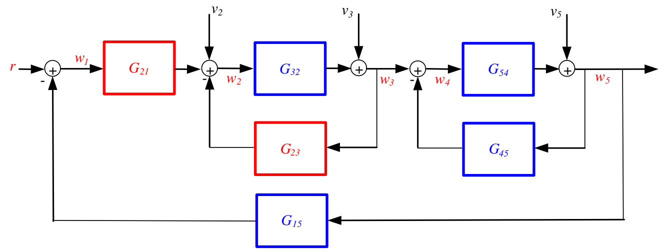

Fig. 7. Dynamic network with 5 node signals, of which 2 (red-colored) transfer functions G_{21} and G_{23} can be consistently identified with the direct method.

VII. Conclusions

Several methods for closed-loop identification have been generalized to become applicable to systems that operate in a general network configuration. Complex networks can be handled and effective use can be made of external excitation signals. These excitation signals limit the necessity to perform exhaustive consistent modelling of all noise sources in the network. The several methods presented (indirect methods based on either excitation signals or on reconstructible noise signals, and the direct method) are able to estimate particular subparts of the network, while they can be applied in combination with each other to cover the full network. The methods can also be applied subsequently in an iterative way. It opens questions as to where and how many external probing/excitation signals are required to identify the full network. A more detailed analysis of the direct method of identification is included in a companion paper [11].

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