Identification in Dynamic Networks with Known Interconnection Topology

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Abstract—The problem of identifying dynamical models on the basis of measurement data is usually considered in a classical open-loop or closed-loop setting. In this paper this problem is generalized to dynamical systems that operate in a complex interconnection structure and a particular transfer function in the network needs to be identified. It is shown that classical methods of closed-loop identification in the prediction error context, can be generalized to provide consistent model estimates, under specified experimental circumstances. This applies to indirect methods that rely on external excitation signals like two-stage and IV methods, as well as to the direct method that relies on consistent noise models. Graph theoretical tools are presented to verify the topological conditions under which the several methods lead to consistent estimates of the network transfer functions.

I. INTRODUCTION

One of the challenges in the systems and control field is to develop effective synthesis methods for distributed control of systems that operate in a network structure. While considerable attention is devoted to this problem from a model-based control perspective, attention for the underlying modelling problem is much more limited. In particular the problem of identifying dynamical models on the basis of measurement data that is obtained from a (complex) dynamic network, and where use can be made of external probing/excitation signals, has not been addressed in much detail yet.

In this paper we will consider identification problems in networks of dynamic systems. From an identification perspective this can be considered as a natural extension of the situation of open-loop data, closed-loop data in a single loop, towards data that is obtained from systems operating in a predefined network connection. While dynamic networks typically contain (feedback) loops, it is expected that methods for closed-loop identification are an appropriate basis for developing generalized tools to deal with complex networks.

In our framework discussed here, a dynamic network is defined as an interconnection of transfer functions where the interconnecting signals are considered as nodes/vertices in the network, and proper transfer functions are considered as links/edges. In this paper it will be assumed that the interconnection topology of the network is known, and the goal is to identify the dynamics of a single transfer or a collection of transfers in the network.

Classical methods for closed-loop identification are addressed in [1], [2]. For a single contribution to the problem of structured systems see also [3]. Whereas nonparametric and parametric methods have been introduced in the problem of network identification (see e.g. [4], [5], [6]), the conditions under which the applied methods work typically include the condition that all disturbance/noise processes should be modelled exactly in conjunction with the dynamic transfers in the network. However in a large-scale dynamic network it can be questioned it this is feasible. In this paper we therefore will particularly focus on identification methods that can consistently identify dynamic transfer functions without relying on exact noise models.

In the closed loop identification literature, the two-stage method [7] and the instrumental variable method [8] can be used to address this problem. In this paper the principles behind these methods are applied to develop generalized methods for identification in dynamic networks.

Notation: $[j]_{ji}$ is matrix element $(j,i)$ of the matrix $[.]$.

II. SYSTEM SETUP - NETWORK ARCHITECTURE

The network structure that we consider in this paper is built up of $L$ nodes, related to $L$ measured scalar signals $w_j, j = 1, \cdots, L$. Every node signal $w_j$ can be written as:

$$w_j(t) = \sum_{k \in \mathcal{N}_j} G_{jk}^0(q)w_k(t) + r_j(t) + v_j(t)$$

with $G_{jk}^0(z)$ a proper rational transfer function, $\mathcal{N}_j$ the set of indices of node signals $w_k, k \neq j$, for which $G_{jk}^0 \neq 0$; $q^{-1}$ the delay operator $q^{-1}u(t) = u(t-1)$, $v_j$ a possible unmeasured disturbance term being a realization of a stationary stochastic process with rational spectral density, and $r_j$ a possible external excitation signal, available to and possibly designed by the user.

A single node of the network is sketched in Figure 1, where the transfer function $G_{ji}^0$ has been separately indicated to focus on the transfer function that is supposed to be identified. The topology of a network could then be sketched as in Figure 2, where each node $n_i$ represents a node signal $w_i$, while the arrows represent causal relationships.

All node signals $w_j, j = 1, \cdots, L$ are supposed to be measurable, while at each node a noise signal $v_j$ (non-measurable) and excitation signal $r_j$ (measurable) may or may not be present. Each excitation signal $r_j$ is uncorrelated to all noise signals $v_i$. Some parts of the network may have...
dynamics that are known a priori. This is e.g. the case in a classical closed-loop system with a known controller.

The problem that will be addressed in this paper is: specify conditions under which the transfer function \( G_{ji}(q) \) can be consistently estimated from data, provided that the signal \( r \) is persistently exciting for all \( k, k \neq i, k \neq j \).

The adjacency matrix characterizes the presence of links (direct causal transfers) in the mapping between node signals. Because of the structures that we consider here (1) it follows that \( A(i, i) = 0, i = 1, \ldots, L \).

One lemma from graph theory will be very useful [10]:

**Lemma 1:** Consider a directed graph with adjacency matrix \( A \). Then for \( k \geq 1, \|A^k\|_{ji} \) indicates the number of different path connections of length \( k \) from node \( i \) to node \( j \).

We will further consider the following sets:
- \( V \) denotes the set of indices of node signals to which additive noise sources \( v \) are directly connected.
- \( R \) denotes the set of indices of node signals to which external excitation signals \( r \) are directly connected.
- \( K_j \) denotes the set of indices of node signals \( w_k, k \in N_j \) where the dynamics \( G_{jk} \) are known.
- \( \mathcal{U}_k \) denote the set of indices of node signals \( w_k, k \in N_j, k \neq i \) where the dynamics \( G_{jk} \) are unknown.

Note that \( N_j = i \cup K_j \cup \mathcal{U}_j \). Attention will be focussed on identification of the transfer \( G_{ji}^0 \).

**B. Projection of signals - two-stage philosophy**

Finally we add a tool from signal theory that will appear to be very helpful for compactly indicating projection operations on signals. Let \( r \) and \( w \) be quasi-stationary signals ([9]) in a linear dynamic network such that

\[
R_{wr}(k) := \mathbb{E}w(t)r(t-k) = 0 \quad \text{for} \ k < 0,
\]

with \( \mathbb{E} := \lim_{N \to \infty} \frac{1}{N} \sum_{t=0}^{N-1} \mathbb{E} \) and \( \mathbb{E} \) the expectation operator. Then there exists a proper transfer function \( F_{wr}^0 \) such that

\[
w(t) = F_{wr}^0(q)r(t) + z(t)
\]

with \( z \) uncorrelated to \( r \), and leading to a decomposition

\[
w(t) = w^{(r)}(t) + w^{(l,r)}(t)
\]

with \( w^{(l,r)}(t) = z(t) \). If \( r \) and \( w \) are available from measurements then \( F_{wr}^0(q) \) can be consistently estimated from data, provided that the signal \( r \) is persistently exciting.

## III. Background Information

In this section we will specify the notation that will be used for characterizing the topological structure of the network and some useful results from graph theory and signal theory will be briefly presented.

### A. Network topology and graph theory

First of all the topology of the network is characterized by a directed graph that indicates the locations and directions of causal transfers within the network. The topology is represented in the adjacency matrix \( A \in \mathbb{R}^{L \times L} \), defined according to:

\[
A(j, i) = 0 \quad \text{if} \ G_{ji}^0(q) \equiv 0;
A(j, i) = 1 \quad \text{elsewhere}.
\]

The adjacency matrix characterizes the presence of links (direct causal transfers) in the mapping between node signals. Because of the structures that we consider here (1) it follows that \( A(i, i) = 0, i = 1, \ldots, L \).
of a sufficiently high order. This consistent estimation can be done without the necessity to model the noise dynamics of \( z \), namely by either following a two-stage approach ([7]) or an instrumental variable (IV) approach ([8]). Subsequently the projection\
\[
\hat{w}(r)(t) := \hat{F}_{wr}(q)r(t)
\]
can be calculated, with \( \hat{F}_{wr}(q) \) the estimated transfer. This estimate then can serve as an accurate estimate of \( w(r)(t) \). Note that \( w(r) \) is the projection of signal \( w \) onto the space of (causally) time-shifted versions of \( r \).

IV. INDIRECT IDENTIFICATION WITH EXTERNAL EXCITATION \( r \)

A. Basic result - consistent identification

Suppose that there is one external excitation signal \( r_m \) present somewhere in the network, that provides excitation for the node signal \( w_i \) that is an input to the transfer function \( G_{ji}^0 \). Then a reasoning that is similar to the classical two-stage method of closed-loop identification [7] leads to a method that is capable of identifying \( G_{ji} \) consistently without the necessity of consistently identifying noise models. The situation is illustrated in Figure 3. The principle identification approach is as follows (in main lines):

**Algorithm 1:**

1) On the basis of measured signals \( r_m \) and \( w_i \), determine \( w_{ij}(r_m) \), i.e. the component of \( w_i \) that is correlated to \( r_m \) (see section III-B);
2) Construct the signal\
\[
\tilde{w}_j(t) = w_j(t) - \sum_{k \in K_j} G_{jk}(q)w_k(t),
\]
i.e. correct \( w_j \) with all known terms;
3) Identify the transfer \( G_{ji}^0 \) on the basis of a predictor model with prediction error\
\[
\hat{e}_j(t, \theta) = K_j(q)^{-1}[\tilde{w}_j(t) - G_{ji}(q, \theta)w_t(r_m)(t)]
\]
using measured signals \( \tilde{w}_j \) and \( w_{ij}(r_m) \), and by minimizing the (quadratic) prediction error criterion \( V_N(\theta) = \frac{1}{N}\sum_{t=1}^{N} \hat{e}_j(t, \theta)^2 \); \( K \) is a fixed or independently parametrized noise model, and the parametrized model \( G_{ji}(q, \theta) \) is chosen flexible enough so as to contain the true transfer \( G_{ji}^0(q) \), see [9].

Under the described conditions, the following result can be obtained:

**Proposition 1:** Consider a dynamic network that satisfies Assumption 1. Then the transfer function \( G_{ji}^0 \) can be consistently estimated with algorithm 1 if the following conditions 1)-3) are satisfied:

1) There exists a node signal \( w_m \) that contains an additive external reference signal \( r_m \) that is uncorrelated to all noise signals \( v_k, k \in U_j \) and \( v_j \), and persistently exciting of a sufficiently high order.
2) The node signal \( w_i \) is correlated to \( r_m \).
3) All node signals \( w_k, k \in U_j, k \neq i \), are uncorrelated to \( r_m \). □

**Proof** Note that we can write\
\[
w_j(t) = G_{ji}^0(q)w_i(t) + \sum_{k \in K_j} G_{jk}(q)w_k(t) + \sum_{k \in U_j} G_{jk}(q)v_k(t) + r_j(t) + v_j(t)
\]
where \( p_j \) reflects the contributions of all signals \( G_{jk}(q)w_k \) that are known because of the fact that the dynamics \( G_{jk} \) is known, as well as \( r_j(t) \); and \( s_j(t) \) similarly reflects the contributions of all signals \( G_{jk}(q)w_k \) that are unknown, because the dynamics \( G_{jk} \) is unknown. Subsequently we write\
\[
w_j(t) - p_j(t) = G_{ji}^0(q)w_i(t) + s_j(t) + v_j(t)
\]
with the left hand side being a known signal.

Conditions 1) and 2) guarantee that \( w_i \) can be decomposed as \( w_i = w_{ij}(r_m) + w_i^{(\perp r_m)} \). Then,
\[
w_j(t) - p_j(t) = G_{ji}^0(q)[w_i(r_m)(t) + w_i^{(\perp r_m)}] + s_j(t) + v_j(t).
\]
Conditions 3) and 1) guarantee that the signal \( s_j \) is uncorrelated to \( r_m \). And by condition 1) the noise \( v_j \) is uncorrelated to \( r_m \), while \( w_i^{(\perp r_m)} \) is uncorrelated to \( r_m \) by construction. Then as a result a prediction error identification on the basis of input \( w_i^{(r_m)} \) and output \( w_j - p_j \) will provide a consistent estimate of \( G_{ji}^0 \), provided that the correlation between \( r_m \) and \( w_i \) is sufficiently "rich". □

Note that as an alternative for the two-stage algorithm, also an IV estimator could have been used, using \( r_m \) as instrument, \( w_i \) as input and \( w_j - p_i \) as output, leading to the same consistency result, [8].

**B. Algorithm for verification of conditions**

Next question is whether in a particular network topology, the conditions as formulated in Proposition 1 are satisfied for consistent identification of the transfer function \( G_{ji}^0 \). This question can be treated by an algorithm based on the graph of the network. Recall that \( A \) is the adjacency matrix of the network.

**Algorithm 2:**

1) Detect excitation signal:
   - For every index \( m \in \mathcal{R} \):
2) Check if there exists a directed path from the reference signal input \((m \rightarrow i)\):
- Evaluate element \((i, m)\) of \(A^\ell\) for \(\ell = 1, \cdots, L-1\);
- If for any considered power \(\ell\) this element is nonzero then the condition is satisfied, and reference signal \(r_m\) qualifies as an excitation source that excites the input \(w_i\).
3) Verify condition (3) on the node \(j\):
- Evaluate \(A^\ell\) for \(\ell = 1, \cdots, L-1\);
- For all \(k \in \mathcal{U}^j, k \neq i\), check whether the entries \((k, m)\) of \(A^\ell\) are zero for all powers \(\ell\).

C. Algorithm for identification

In section IV-A an identification algorithm is sketched for the situation that one single external excitation signal \(r_m\) is present in the network. This situation is the basis for the consistency condition of the resulting estimate of \(G_{ji}^0\). However if our aim is not only consistency but also to reduce the estimator variance, as well as to handle the situation of possibly more than one excitation signals that satisfy the conditions, as formulated in Proposition 1, then the identification algorithm needs to be reconsidered.

Suppose that for input node \(i\) the node numbers of external excitation signals \(r\) that are correlated to \(w_i\) are collected in the set \(\mathcal{R}_i\). The identification algorithm as sketched in section IV-A can then be adapted as follows:

**Algorithm 3:**
1) For each \(m \in \mathcal{R}_i\) construct the prediction error:

\[ \varepsilon_m(t, \theta) = H(q)^{-1}[w_i(t) - p_{i,m}(t) - F_{ir_m}(q, \theta_m)r_m(t)], \]

with \(p_{i,m}(t) = \sum_{k \in \mathcal{K}_i} G_{ik}(q)w_{k}^{(\perp r_m)}(t)\), \(H(q)\) a fixed noise model, and estimate the parameter \(\theta_{N,m}\) with a quadratic prediction error criterion: \(\hat{\theta}_{N,m} = \arg \min \frac{1}{N} \sum_{t=1}^N \varepsilon_m(t, \theta_m)^2\).
2) Simulate the noise free inputs:

\[ w_i^{(r)}(t) = \sum_{m \in \mathcal{R}_i} F_{ir_m}(q, \hat{\theta}_{N,m})r_m(t). \]
3) Identify the transfer function from \(w_i^{(r)}\) to \(w_j\), through LS minimization of the prediction error

\[ \varepsilon_j(t, \theta) = K_j(q)^{-1}[w_j(t) - G_{ji}(q, \theta)w_i^{(r)}(t) + \sum_{k \in \mathcal{K}_j} G_{jk}(q)w_k(t)], \]

with \(K_j(q)\) a fixed noise model.

**Comment 1:** When writing

\[ w_j = G_{j1}^0w_i + v_j + \sum_{k \in \mathcal{K}_j} G_{jk}^0w_k + \sum_{k \in \mathcal{U}_j^i} G_{jk}^0w_k^{(\perp r)} \]

it follows that

\[ \varepsilon_j(t, \theta) = K_j(q)^{-1}\left[G_{j1}^0 - G_{ji}(q, \theta)w_i^{(r)}(t) + v_j(t) + \sum_{k \in \mathcal{U}_j^i} G_{jk}^0w_k^{(\perp r)}(t)\right], \]

where use is made of condition 3 of Proposition 1, that requires that all inputs to unknown transfers \(G_{jk}, k \neq i,\) are uncorrelated to \(r_m\). For consistent estimation of \(G_{ij}^0\), it suffices that the latter three terms on the right hand side of the equation are uncorrelated to \(r\). For reducing the variance of the estimate additional tools may be applied, such as e.g. estimating the unknown transfers \(G_{jk}^0, k \in \mathcal{U}_j^i\) simultaneously with \(G_{ji}^0\).

**Example 1:** An example of a dynamic network is depicted in Figure 4. When applying the conditions of Proposition 1 it appears that the blue-colored transfers, \(G_{32}, G_{54}, G_{15}\) and \(G_{45}\) can be consistently identified with the two-stage approach presented in this section. These four transfers satisfy the conditions that their inputs are correlated to \(r\), while their outputs are not disturbed by unknown terms that are correlated with \(r\).

![Dynamic network with 5 node signals](image)

Fig. 4. Dynamic network with 5 node signals, of which 4 (blue-colored) transfer functions can be consistently identified with the two-stage method presented in section IV.

Note that the transfers \(G_{21}\) and \(G_{23}\) do not satisfy the conditions of the Proposition because there are unknown contributions to \(w_2\) that are correlated to \(r_m\). However if we extend the Proposition to also allow simultaneous identification of different transfers, i.e. estimating \(G_{21}\) and \(G_{23}\) in a multi-input single-output model, then also those transfers could be identified consistently provided appropriate excitation conditions are satisfied; e.g. this may require more than one external excitation signal.

V. INDIRECT IDENTIFICATION WITH RECONSTRUCTIBLE NOISE SIGNAL \(v\)

A. Basic result - consistent identification

A second identification approach for consistently identifying the transfer \(G_{ji}^0\) is obtained when there is one noise signal \(v_m\) present somewhere in the network, that can be reconstructed on the basis of measured signals and known transfers, and that provides excitation for the node signal \(w_i\) that is an input to the transfer function \(G_{ji}^0\). Then a reasoning that is closely related to the classical two-stage method of closed-loop identification [7] leads to a method that is capable of identifying \(G_{ji}^0\) consistently without the necessity of consistently identifying noise models. The principle approach is as follows (in main lines):

**Algorithm 4:**
- Suppose that there is a node \(m\) in the network for which holds that every transfer \(G_{mk}^0, k \in \mathcal{N}_m\) is known. Then

\[ x_m := \sum_{k \in \mathcal{N}_j} G_{mk}^0(q)w_k \text{ is known, and } v_m = w_m - \]
Fig. 5. Single node in a network structure, where the input \( w_i \) is excited through a reconstructed noise signal \( v_m \).

\( x_m \) can be reconstructed.

If \( w_i \) is correlated to \( v_m \), then one can construct \( w_i^{(v_m)} \).

- In the second step, the transfer from \( w_i^{(v_m)} \) to \( w_j \) is identified using the prediction error

\[
\varepsilon(t, \theta) = K(q)^{-1}[w_j(t) - G_{ji}(q, \theta)w_i^{(v_m)}(t)] + \sum_{k \in K_j} G_{jk}(q)w_k(t),
\]

similar to the situation in the previous section, where we have replaced the external excitation signal \( r_m \) by a reconstructible noise signal \( v_m \).

On the basis of the reasoning above, the following result can be formulated:

**Proposition 2:** Consider a dynamic network that satisfies Assumption 1. Then the transfer function \( G_{ji}^{0} \) can be consistently estimated through Algorithm 4 if the following conditions (1)-(3) are satisfied:

1. There exists a node \( w_m \), that satisfies the property that

   - \( w_m - v_m \) is known,
   - noise source \( v_m \) has a variance > 0, and is uncorrelated to \( v_j \) if this latter signal is present.

2. The node signal \( w_i \) is correlated to \( v_m \).

3. All node signals \( w_k, k \in U_j, k \neq i \), are uncorrelated to \( v_m \).

**Proof**

Consider the noise signal \( v_m = w_m - x_m \) with \( x_m := \sum_k G_{mk}^{0}(q)w_k \). Since by condition 1 the signal \( x_m \) is known, it follows that \( v_m \) can be reconstructed. And since by condition 2 \( w_i \) is correlated to \( v_m \) we can construct \( w_i^{(v_m)} \).

One can then write

\[
w_j - r_j - \sum_{k \in K_j} G_{jk}(q)w_k = G_{ji}^{0}w_i^{(v_m)} + \eta_j
\]

where the left hand side represents a known signal, and by condition 3 \( \eta_j \) is uncorrelated to \( v_m \).

This situation mimics the situation of the classical two-stage method for closed-loop identification, where based on (3) it follows that consistent identification of \( G_{ji}^{0} \) is possible.

**B. Algorithm for verification of conditions**

The conditions of Proposition 2 can be verified by evaluating the topological properties of the network represented in its graph, and in the graph theoretical tools. Since topological conditions can not verify whether different noise sources are correlated, we add the assumption here that all noise sources are uncorrelated, i.e. that \( \Phi_v(\omega) \) is diagonal.

**Algorithm 5:**

1. Detect reconstructible noise signals:

   - For every index \( m \in V \): check if \( K_m = N_m \). If so, \( v_m \) qualifies as a reconstructible noise signal.

2. Check if there exists a directed path from the reference signal input \( (m \rightarrow i) \):

   - Evaluate element \((i, m)\) of \( A^\ell \) for \( \ell = 1, \cdots L - 1 \).
   - If for any considered power \( \ell \) this element is nonzero then the condition is satisfied, and the noise signal \( v_m \) qualifies as an excitation source that excites the input \( w_i \).

3. Verify condition (3) on the node \( j \):

   - Evaluate \( A^\ell \) for \( \ell = 1, \cdots L - 1 \).
   - For all \( k \in U_j, k \neq i \), check whether the entries \((k, m)\) of \( A^\ell \) are zero for all powers \( \ell \).

**C. Algorithm for identification**

Analogous to the situation in section IV, the situation can be handled of multiple noise signals that are reconstructible and that can serve as an excitation to the node input signal \( w_i \), while additionally the variance of the estimated transfer function \( G_{ji}^{0} \) needs to be reduced.

Suppose that for input node \( i \) the node numbers \( m \) of noise signals \( v_m \) that can be reconstructed and that are correlated to \( w_i \) are collected in the set \( E_i \). The identification algorithm as sketched in section V-A can then be adapted in a way that is completely analogous to the algorithm provided in section IV-C, by replacing \( K_i \) by \( E_i \) and replacing signals \( r_m \) by \( v_m \).

Note that Comment 1 also applies to this situation.

**Example 2:** If we consider the network example of Figure 4, it appears that both \( v_3 \) and \( v_5 \) qualify as a reconstructible noise signal, provided that the transfers \( G_{32} \) and \( G_{54} \) are known, e.g. through consistent identification via the method of section IV. However in the considered situation none of the remaining transfer functions satisfies the other condition of Proposition 2 that the outputs should not be disturbed by unknown terms that are correlated to the (reconstructible) noise source.

However if we remove the outer loop connection \( G_{15} \), as depicted in Figure 6, then \( G_{23} \) can be identified consistently through reconstructible noise signal \( v_3 \). In Figure 6 this transfer is indicated in red.
VI. DIRECT IDENTIFICATION

In contrast to the previous methods for identification of closed-loop systems, the classical direct method of prediction error identification requires the consistent identification of noise models in order to be able to identify consistent plant models [9]. The direct method identifies a transfer function $G_{ji}$ in a standard feedback configuration, by applying a parametrized model $(G_{j1}(q, \theta), H_j(q, \theta))$ leading to the prediction error

$$
\varepsilon(t, \theta) = H_j(q, \theta)^{-1}[w_j(t) - G_{ji}(q, \theta)w_i(t)].
$$

A quadratic prediction error criterion then leads to consistent estimates of $G_{ji}^0$ and $H_j^0$ under particular excitation requirements. For further details, see [9]. Here we formulate a generalization of this classical method to the situation of dynamic networks. For the proof, the reader is referred to the companion paper [11].

**Proposition 3:** Consider a dynamic network that satisfies Assumption 1. Then the transfer functions between $\{w_j\}_{j \in \mathcal{N}_j}$ and $w_j$ can be consistently estimated with the direct prediction error method if the following conditions (1)-(4) are satisfied:

1) Noise source $v_j$ is present with variance $> 0$;
2) $\Phi_v$ is diagonal;
3) Every loop in the network that runs through node $j$ has at least one time step delay;
4) The parametrized model is sufficiently flexible to contain the true system $G_{jk}^0$, $k \in \mathcal{N}_j$, and $H_j^0$;
5) For both the real network and its parametrized model holds that every loop that runs through node $j$ has at least one time step delay;
6) The spectral density of the composed signal $z := \{w_j \ w_{n_1} \ldots w_{n_m}\}^T$, $n_a \in \mathcal{N}_j$, satisfies $\Phi_z(\omega) > 0$ for all $\omega \in [0, \pi]$.

Note that in the considered situation all transfers $G_{ji}^0$, $i \in \mathcal{N}_j$ need to be estimated simultaneously in order for the result to hold, and that the dynamics of noise source $v_j$ needs to be modeled correctly through a noise model $H_j$. Note also that both the noise signal $v_j$ and the probing signal $r_j$ provide excitation to the loop that is going to be identified. The excitation condition 6) is rather generic. A further specification for particular finite dimensional model structures can possibly be made along the results for classical feedback loops as discussed in [12]. If particular transfers $G_{kj}^0$ are known a priori, the result of the Proposition can simply be formulated for the set of identified transfers between $\{w_k\}_{k \in \{i, i^T\}}$ and $w_j$.

**Example 3:** If we apply the result of the direct method to the network example of Figure 4, it appears that the direct method can be applied to node $w_2$, by identifying a model between inputs $w_1$ and $w_3$, and output $w_2$. Under the condition that a delay is present in the loops $(G_{22}G_{21})$ and $(G_{25}G_{34}G_{32}G_{21}G_{15})$ and by the use of an appropriate model set that includes accurate noise modelling, the trans- fers $G_{21}$ and $G_{23}$ can be estimated consistently. In Figure 7 they are indicated in red.

VII. CONCLUSIONS

Several methods for closed-loop identification have been generalized to become applicable to systems that operate in a general network configuration. Complex networks can be handled and effective use can be made of external excitation signals. These excitation signals limit the necessity to perform exhaustive consistent modelling of all noise sources in the network. The several methods presented (indirect methods based on either excitation signals or on reconstructible noise signals, and the direct method) are able to estimate particular subparts of the network, while they can be applied in combination with each other to cover the full network. The methods can also be applied subsequently in an iterative way. It opens questions as to where and how many external probing/excitation signals are required to identify the full network. A more detailed analysis of the direct method of identification is included in a companion paper [11].

REFERENCES