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LOW ORDER CONTROL DESIGN BY FEEDBACK RELEVANT IDENTIFICATION AND CLOSED LOOP CONTROLLER REDUCTION

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Abstract. An approach is presented that can be used to obtain low complexity controllers for an unknown system. In this approach, the identification of a set of models is used to represent the incomplete knowledge of the system. Subsequently, the set is used for the synthesis of a robust controller. In order to design low complexity controllers, the aim is to find a low complexity representation of the set. Additionally, a closed loop reduction tool can be used to decrease the controller complexity further. This approach will be illustrated by an application to a multivariable positioning mechanism present in a wafer stepper.

Keywords. System identification; robust control; servomechanisms; multivariable systems.

1. INTRODUCTION

Industrial systems need feedback control to meet enhanced accuracy or performance requirements. In many applications the plant to be controlled is partly known, whereas limited complexity controllers are required due to hardware limitations. Both the inadequate knowledge of a plant to be controlled and the restriction on the complexity of the controller to be used makes the design of such a feedback controller a challenging task. In this paper, an approach is presented that can be used to obtain such low complexity (low order) linear feedback controllers for an unknown system.

To deal with the lack of information on the plant, the approach in this paper starts with the estimation of a set of models by means of system identification techniques, such that the unknown plant is an element of the set. Such a set of models is unavoidable as the data used for identification purposes only represents a finite time, possibly disturbed, observation of the plant causing the knowledge of the plant to remain incomplete. As a consequence, a set of models consists of all models that are either validated (Ljung, 1987) or cannot be invalidated (Smith *et al.*, 1997) by the observations obtained from the plant.

Subsequently, a robust controller can be designed on the basis of this set of models. For that purpose, the set should be built up from a nominal model along with an allowable model perturbation (Boyd and Barrat, 1991). A general representation of a set of models can be written in terms of linear fractional transformation (LFT) based model perturbation (Boyd and Barrat, 1991). Such an LFT, based on a (dual) Youla-Kucera parametrization, is being estimated in this paper and shown to be particularly useful for both identification and control design purposes (de Callafon and Van den Hof, 1997).

To restrict the complexity (McMillan degree) of the controller, the aim is to estimate a low complexity representation of the LFT via an approximate identification. This is due to the fact that this LFT directly influences the order of a robust controller being computed (Boyd and Barrat, 1991; Zhou *et al.*, 1996). For further re-

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duction of the controller complexity, an additional controller reduction can be employed. In this paper, a closed loop reduction that is based on the work by Ceton *et al.* (1993) is shown to be useful for reducing the complexity of the controller.
The subsequent steps of approximate identification of an LFT and the design of a robust controller followed by a closed loop controller reduction will be illuminated in this paper. To illustrate the approach, the application to a multivariable positioning mechanism present in a wafer stepper has been included.

2. PRELIMINARIES

2.1 *Norm-based feedback design*

Let the notation P and C be used to denote finite dimensional, linear time invariant (FDLTI) (possibly unstable) systems, where C is used to indicate a controller. For notational convenience a control objective function is denoted by $J(P,C) \in \mathbb{R}\mathcal{H}_\infty$ and the notion of performance will be characterized by the value of the norm $\|J(P,C)\|_\infty$: a smaller value of $\|J(P,C)\|_\infty$ indicates better performance (Van den Hof and Schrama, 1995).
A feedback connection of a system P and a controller C is denoted by $\mathcal{T}(P,C)$ and defined as the connection structure depicted in Figure 1. It is assumed that a con-

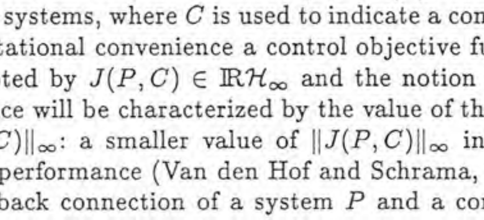

Fig. 1. Feedback connection structure $\mathcal{T}(P,C)$.

nection $\mathcal{T}(P,C)$ is well posed, that is $\det(I+CP) \not\equiv 0$ (Boyd and Barrat, 1991). The mapping from the signals $col(r_2,r_1)$ onto $col(y,u)$ is given by the transfer function matrix $T(P,C)$ with

$$T(P,C) := \begin{bmatrix} P \\ I \end{bmatrix} (I+CP)^{-1} \begin{bmatrix} C & I \end{bmatrix}, \qquad (1)$$

Note that $\mathcal{T}(P,C)$ is internally stable if and only if $T(P,C) \in \mathbb{R}\mathcal{H}_\infty$ (Schrama and Bosgra, 1993). In order to maintain generality, $J(P,C)$ is taken to be a weighted form of $T(P,C)$:

$$\|J(P,C)\|_\infty := \|U_2 T(P,C) U_1\|_\infty \qquad (2)$$

where U_2 and U_1 are (square) weighting functions. The performance characterization (2) is fairly general and will be used for analysis purposes in this paper. In this perspective, the performance objective function $J(P,C)$ as given in (2) will be used to evaluate both the identification of a set of models $\mathcal{P}$ and the additional reduction of a robust controller designed based on the set $\mathcal{P}$. For that purpose, the set of models $\mathcal{P}$ as used in this paper is discussed below.

2.2 *Model uncertainty set*

As indicated in Section 1, the incomplete knowledge of a plant P_o is represented by means of a set of models $\mathcal{P}$. An (upper) LFT

$$\mathcal{F}_u(Q,\Delta) := Q_{22} + Q_{21}\Delta(I - Q_{11}\Delta)^{-1}Q_{12} \qquad (3)$$

provides a general notation to represent all models $P \in \mathcal{P}$ as follows

$$\mathcal{P}_i = \{P \mid P = \mathcal{F}_u(Q,\Delta) \\ \text{with } \Delta \in \mathbb{R}\mathcal{H}_\infty \text{ and } \|\Delta\|_\infty < \gamma^{-1}\}$$

where Δ indicates an unknown (but bounded by γ^{-1}) uncertainty that reflects the incomplete knowledge of the plant P_o. The entries of the coefficient matrix Q in (3) indicate how the set of models $\mathcal{P}$ has been structured, where $\hat{P} := \mathcal{F}(Q,0) = Q_{22}$ denotes the nominal model of the set $\mathcal{P}$.
In this paper, the coefficient matrix Q is formed by employing the knowledge of any (possibly unstable) controller denoted by $\bar{C}$, that is used to form a stabilizing feedback connection $\mathcal{T}(P_o,\bar{C})$. In many practical situations, the presence of such a stabilizing controller $\bar{C}$ is unavoidable due instability of the plant P_o or additional safety requirements during operation.
Employing the knowledge of such a stabilizing feedback controller $\bar{C}$ and using the algebraic theory of fractional model representations (Vidyasagar, 1985), the coefficient matrix Q in (3) is formed by considering a model perturbation that is structured similar to a (dual) Youla-Kucera parametrization:

$$\mathcal{P} = \{P \mid P = (\hat{N} + D_c\bar{\Delta})(\hat{D} - N_c\bar{\Delta})^{-1} \\ \text{with } \bar{\Delta} \in \mathbb{R}\mathcal{H}_\infty \text{ and } \|\hat{V}\bar{\Delta}\hat{W}\|_\infty < \gamma^{-1}\} \qquad (4)$$

where (N_c, D_c) and $(\hat{N},\hat{D})$ respectively denote a right coprime factorization (rcf) of the controller $\bar{C}$ and a nominal model $\hat{P}$, that satisfies $T(\hat{P},\bar{C}) \in \mathbb{R}\mathcal{H}_\infty$. $\hat{V}$, $\hat{W}$ denote stable and stably invertible weighting functions used to normalize the upper bound on $\hat{V}\bar{\Delta}\hat{W}$ to γ^{-1}. It can be verified that the coefficient matrix Q in the LFT of (3) reads as follows.

$$Q = \left[\begin{array}{c|c} \hat{W}^{-1}\hat{D}^{-1}N_c\hat{V}^{-1} & \hat{W}^{-1}\hat{D}^{-1} \\ \hline (D_c + \hat{P}N_c)\hat{V}^{-1} & \hat{P} \end{array}\right] \qquad (5)$$

It should be noted that in order to guarantee that $P_o \in \mathcal{P}$, additional prior information on the plant P_o must be introduced. This is due to the fact that $P_o \in \mathcal{P}$ cannot be validated solely on the basis of finite time, possibly disturbed, observations coming from the plant P_o (Mäkilä *et al.*, 1995; Ninness and Goodwin, 1995). Such information is in accordance with the uncertainty modelling procedure of Hakvoort (1994), that is used in this paper to bound the uncertainty $\bar{\Delta}$ in (4).

2.3 *Evaluation of performance*

The theory of fractional model representations provides a unified approach to handle both stable and unstable models and controllers within the set $\mathcal{P}$ of (4). Additionally, the set $\mathcal{P}$ has some favourable properties that can be illuminated by evaluating the performance objective function $J(P, C)$ for all $P \in \mathcal{P}$.

Lemma 2.1 *Consider the set $\mathcal{P}$ defined in (4) and a controller C such that the map $J(P,C) = U_2 T(P,C) U_1$ is well-posed for all $P \in \mathcal{P}$. Then*

$$J(P,C) = \mathcal{F}_u(M, \Delta) \ \forall P \in \mathcal{P}$$

where the entries of M are given by

$$\begin{aligned} M_{11} &= -\hat{W}^{-1}(\hat{D} + C\hat{N})^{-1}(C - \bar{C})D_c\hat{V}^{-1} \\ M_{12} &= \hat{W}^{-1}(\hat{D} + C\hat{N})^{-1}\begin{bmatrix} C & I \end{bmatrix} U_1 \\ M_{21} &= -U_2 \begin{bmatrix} -I \\ C \end{bmatrix} (I + \hat{P}C)^{-1}(I + \hat{P}\bar{C})D_c\hat{V}^{-1} \\ M_{22} &= U_2 \begin{bmatrix} \hat{N} \\ \hat{D} \end{bmatrix} (\hat{D} + C\hat{N})^{-1}\begin{bmatrix} C & I \end{bmatrix} U_1 \end{aligned} \tag{6}$$

Proof: By algebraic manipulation, see de Callafon and Van den Hof (1997). □

It can be observed from (6) that substitution of $C = \bar{C}$ yields $M_{11} = 0$. This implies that when a controller C (equal to the controller $\bar{C}$ used in the construction of the set $\mathcal{P}$ in (4)) is applied to the set $\mathcal{P}$, stability robustness is satisfied, regardless of the value of γ in (4). This advantage, observed also by Sefton *et al.* (1990), is not shared by alternative uncertainty characterizations such as an open loop additive uncertainty description. Moreover, for $C = \bar{C}$ the upper LFT $\mathcal{F}_u(M, \Delta)$ modifies into

$$M_{22} + M_{21}\Delta M_{12} \tag{7}$$

which is an affine expression in Δ. As a result, when the controller $\bar{C}$ is applied to the plant P_o, finding the smallest possible allowable model perturbation Δ such that $P_o \in \mathcal{P}$ (via system identification techniques) will effectively minimize the worst case performance (de Callafon and Van den Hof, 1997). This property can be exploited to formulate a (control relevant) identification problem to estimate a set of models by employing the knowledge of a stabilizing controller $\bar{C}$ that is currently being implemented on the (unknown) plant P_o.

3. ESTIMATION OF A SET OF MODELS

3.1 *Control relevant identification*

In order to design an enhanced performing robust controller, it is preferable to use a set of models $\mathcal{P}$ for which

$$\sup_{P \in \mathcal{P}} \|J(P, C)\|_\infty$$

is minimized. Clearly, this makes the modelling of a set of models $\mathcal{P}$ and the design of a robust controller interrelated (Skelton, 1989). To deal with the interrelation between modelling and control design, knowledge of a controller $\bar{C}$ that is implemented on the unknown plant P_o, similar as in (4), can be exploited to estimate a set of models $\mathcal{P}$. In that case, a set of models $\mathcal{P}$ subjected to the condition $P_o \in \mathcal{P}$ should be estimated such that

$$\sup_{P \in \mathcal{P}} \|J(P, \bar{C})\|_\infty \tag{8}$$

is minimized. In this way, a set of models is found for which the worst case performance for the controller $\bar{C}$ is minimized.
As the controller $\bar{C}$ is assumed to be known, the unknown variables in the coefficient matrix Q of (5) are the factorization $(\hat{N}, \hat{D})$ of a nominal model and the weighting functions $(\hat{V}, \hat{W})$. Minimizing (8) using these variables simultaneously is (as yet) unfeasible. Therefore, minimization of (8) is tackled by estimating the *rcf* $(\hat{N}, \hat{D})$ and the pair $(\hat{V}, \hat{W})$ separately. In this way, (standard) tools for the identification of a nominal factorization and an uncertainty bound can be employed.

3.2 *Estimation of a nominal model*

Estimation of a nominal model involves the estimation of $\hat{P} = \hat{N}\hat{D}^{-1}$, subjected to internal stability of the feedback connection $\mathcal{T}(\hat{P}, \bar{C})$, such that (8) is being minimized. At this stage, the variables $\hat{V}$ and $\hat{W}$ are unknown and assumed to vary freely in order to satisfy $P_o \in \mathcal{P}$. Consequently, the set $\mathcal{P}$ is still unknown and (8) cannot be computed. However, for any $P \in \mathcal{P}$ the following upper bound for $\|J(P, \bar{C})\|_\infty$ can be given.

$$\|J(P_o, \bar{C})\|_\infty + \|J(P, \bar{C}) - J(P_o, \bar{C})\|_\infty$$

As $\|J(P_o, \bar{C})\|_\infty$ in (3.2) does not depend on the nominal model $\hat{P}$, a *rcf* $(\hat{N}, \hat{D})$ of a nominal model can be found by minimizing

$$\|J(P, \bar{C}) - J(P_o, \bar{C})\|_\infty \tag{9}$$

Estimation of a *rcf* of a nominal model of limited complexity by minimizing (9) on the basis of closed loop experiments obtained from the connection $\mathcal{T}(P_o, \bar{C})$, has been studied in Van den Hof *et al.* (1995). An approach to minimize (9) on the basis of frequency domain data can be found in de Callafon and Van den Hof (1995).

3.3 *Estimation of uncertainty bounds*

Estimation of an allowable model perturbation involves the characterization of an upper bound on $\bar{\Delta}$ in (4) via $(\hat{V}, \hat{W})$ such that (8) is being minimized and $P_o \in \mathcal{P}$. For that purpose, first a frequency dependent upper bound on the allowable model perturbation $\bar{\Delta}$ in (5) is determined such that $P_o \in \mathcal{P}$. For that purpose, any uncertainty estimation procedure can be used, as the input and output data of the allowable model perturbation $\bar{\Delta}$ can be accessed simply by a filtering of the input u and output y signals present in the feedback connection $\mathcal{T}(P_o, \bar{C})$ (de Callafon and Van den Hof, 1997).

Similar to the approach presented in (Lee *et al.*, 1993), the availability of the input and output signals of $\bar{\Delta}$ gives rise to an open loop identification problem of the stable dual Youla-Kucera parameter. However, the estimation is being used here to find an upper bound on $\bar{\Delta}$. An uncertainty estimation routine such as the procedure described by Hakvoort (1994) can be used to obtain a frequency dependent upper bound for Δ

$$\|\Delta_i(\omega)\| \leq \delta(\omega) \text{ with probability} \geq \alpha \qquad (10)$$

where α is a prechosen probability. In the multivariable case, the upper bound (10) can be obtained for each transfer function. Subsequently, stable and stably invertible weighting filters $\hat{V}(\omega)$ and/or $\hat{W}(\omega)$ of limited complexity can be constructed to over bound $\delta(\omega)$ (Hakvoort, 1994).

4. CONTROLLER DESIGN

The set of models $\mathcal{P}$ represents the incomplete knowledge on the plant P_o and can be used for subsequent control design. Again taking into account the performance specification (2), a controller C can be designed by minimizing

$$\sup_{P \in \mathcal{P}} \|J(P, C)\|_\infty \qquad (11)$$

where $\mathcal{P}$ denotes the set of models being estimated. For $J(P,C) = U_2 T(P,C) U_1$, (11) constitutes a (standard) $\mathcal{H}_\infty$-norm based control design, wherein the worst case performance is being optimized. For that purpose, a μ-synthesis via a so-called D-K iteration (Zhou *et al.*, 1996) can be used. In order to use the available techniques on μ-synthesis, the transfer function M in (6) should be represented as a lower fractional transformation $\mathcal{F}_l(G, C)$, where the controller C to be designed has been extracted. An expression for G can be found by standard algebraic manipulations.

5. CLOSED LOOP REDUCTION

The design of a controller as mentioned in Section 4 generally leads to full order controllers, although limited complexity of the coefficient matrix Q in (5) can be enforced by the approximate identification of a *rcf* $(\hat{N}, \hat{D})$ and the weighting filters $(\hat{V}, \hat{W})$.

In light of the performance objective function $J(P, C)$ given in (2), a reduction of the controller may be required, that takes account of this performance function. For that purpose, a closed loop balanced reduction, as proposed by Ceton *et al.* (1993), is well suited. In Ceton *et al.* (1993), a similarity transformation that balances the states of a stable feedback connection is used for partial balancing of the (unstable) controller states (Wortelboer, 1993). As a result, an (unstable) controller can be reduced in closed loop, taking into account the closed loop operation of the controller.

The closed loop configuration in Ceton *et al.* (1993) is slightly different from the one used in this paper. However, the results of Ceton *et al.* (1993) can be readily carried over to perform closed loop reduction of the controller C in the feedback connection $\mathcal{T}(P, C)$, incorporating the performance weightings U_2 and U_1.

6. APPLICATION TO WAFERSTEPPER

6.1 *Description of the positioning mechanism*

The approach outlined in this paper has been applied to a multivariable positioning mechanism, denoted by the wafer stage, present in a wafer stepper.

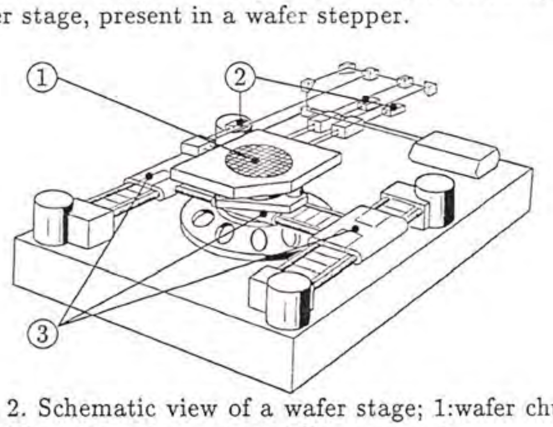

Fig. 2. Schematic view of a wafer stage; 1:wafer chuck, 2:laser interferometers, 3:linear motors.

A wafer stepper is a fast and high accuracy positioning machine, used in chip manufacturing processes; a schematic view is depicted in Figure 2. The position of the wafer chuck on the horizontal surface of a granite block is measured by means of three laser interferometry measurements, whereas three linear motors are used to position the wafer chuck. The three currents to the linear motors denote the input u, whereas the three position measurements denote the output y of the system. A diagonal PID controller is used as an initial controller $\bar{C}$ to stabilize and position the wafer chuck for experimental purposes. External references signals r_1 and r_2 are used to excite the closed loop similar to Figure 1. Time and frequency domain data where gathered for identification purposes. The aim is to design a low complexity controller that is able to attain a high bandwidth, tracking and suppression of residual vibrations. For that purpose, only relatively simple (diagonal) weighting functions U_2 and U_1 are used to enforce a controller with high gain at low frequencies.

6.2 *Estimation of a nominal factorization*

First a MIMO nominal *rcf* $col(\hat{N}, \hat{D})$ having 6 outputs and 3 inputs is estimated. For that purposes, frequency measurements are used to curve fit a factorization $(\hat{N}, \hat{D})$ of 30th order using the procedure described in de Callafon and Van den Hof (1995). This procedure requires an initial estimate for the non-linear optimization which is found by a MIMO least squares curve fitting (de Callafon *et al.*, 1996).

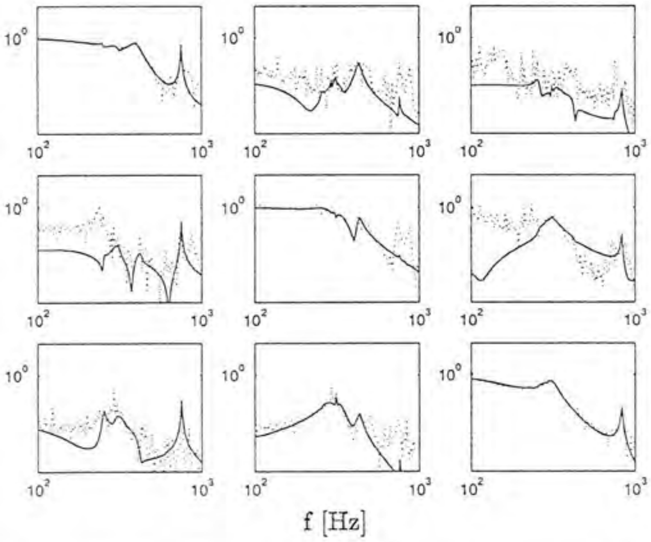

Fig. 3. Amplitude Bode plot of nominal numerator factor $\hat{N}$ (–), and the corresponding frequency domain data ($\cdots$)

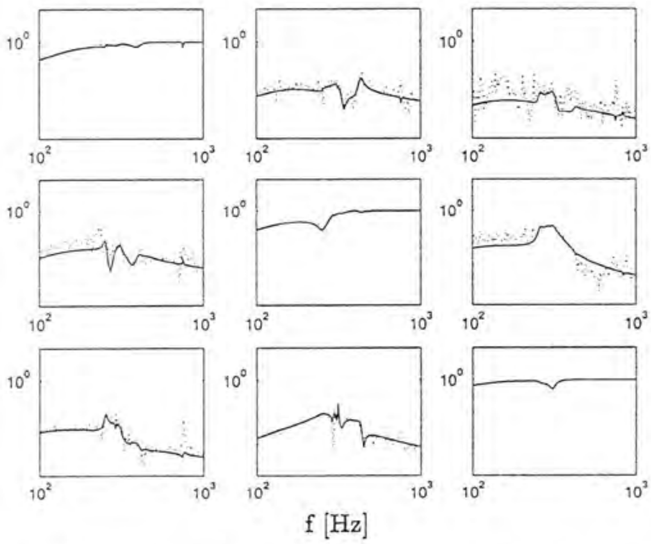

Fig. 4. Amplitude Bode plot of nominal denominator factor $\hat{D}$ (–) and the corresponding frequency domain data ($\cdots$)

An amplitude Bode plot of the result is presented in Figures 3 and 4. It can be observed from these figures that the frequency domain data has only been approximated by the factorization $(\hat{N}, \hat{D})$, as more accurate modelling would require a much higher order model.

6.3 *Estimation of model uncertainty*

Given the nominal factorization $(\hat{N}, \hat{D})$ and a normalized *rcf* (N_c, D_c) of the controller $\bar{C}$, an estimation of the allowable model perturbation $\bar{\Delta}$ in (4) is performed. For that purpose, the uncertainty estimation as presented in (Hakvoort, 1994) has been applied to estimate a frequency dependent upper bound on $\bar{\Delta}$. Due to space limitations only the result is presented in Figure 5.

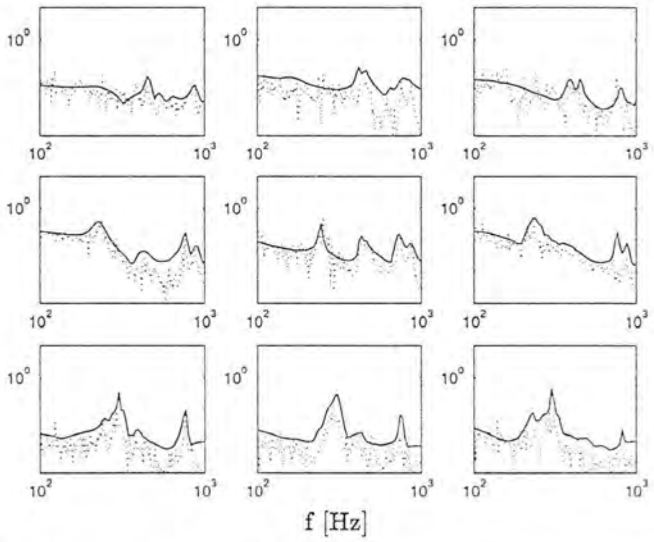

Fig. 5. Amplitude Bode plot of uncertainty bound $\delta(\omega)$ (–) for each transfer function of $\bar{\Delta}$ and frequency domain estimate of $\bar{\Delta}$ ($\cdots$)

It can be observed from Figure 5 that the upper bound of the frequency domain estimation of $\bar{\Delta}$ is crossing the upper bound $\delta(\omega)$. Partly, this is due to the fact the upper bound only holds within a prespecified probability of 95%.

6.4 *Full order controller and reduction*

On the basis of the nominal factorization $(\hat{N}, \hat{D})$ and (only) a single stable and stable invertible weighting filter $\hat{V}$ that over-bounds the upper bounds $\delta(\omega)$ depicted in Figure 5, a robust controller has been designed by means of a μ-synthesis. An amplitude Bode plot of the controller has been depicted in Figure 6.
Despite of the low complexity modelling, the full order controller being designed still has a McMillan degree of 74. Additional reduction of the controller as descrined in section 5 enables the controller to be reduced to 32nd order. The additional closed loop reduction deteriorates the performance robustness only by 2.12 %. The 32nd

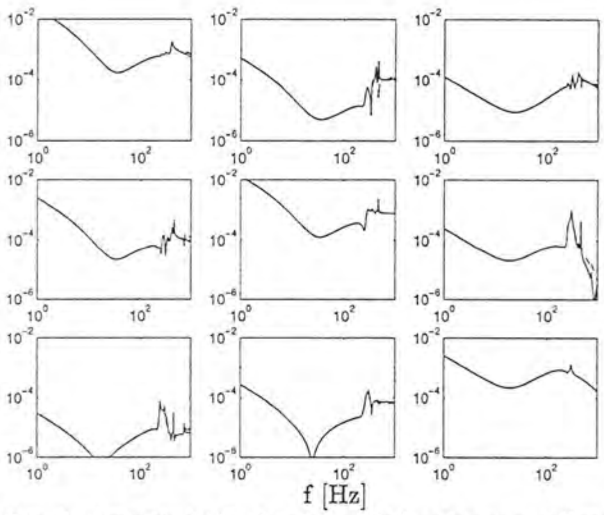

Fig. 6. Amplitude Bode plot of full order controller (- -) and closed loop reduced controller (–)

order controller has been applied to the wafer stepper mechanism successfully.

7. CONCLUSIONS

In this paper a systematic approach to find a low complexity controller for a unknown system has been presented. The approach consists of a system identification technique to estimate a model uncertainty set, followed by a robust controller design and an additional controller reduction. In all these steps, the performance and the closed loop operation of both the uncertainty set and the low complexity controller being constructed is taken into account.

The approach has been illustrated on a highly complex multivariable mechanical servo system present in a wafer stepper. This has resulted in a relatively low order controller that successfully has been applied.

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