ANALYSIS OF CLOSED-LOOP IDENTIFICATION WITH A TAILOR-MADE PARAMETRIZATION

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Keywords: closed-loop identification, tailor-made parametrization.

Abstract

An analysis is made of a closed-loop identification scheme in which the parameters of the (open-loop) model are identified on the basis of measurements of input and output signals of a closed-loop transfer function. A parametrization of a closed-loop transfer in terms of the parameters of the open-loop plant model is employed, utilizing knowledge of the implemented feedback controller. This is denoted a tailor-made parametrization as it is tailored to the specific feedback structure at hand. Consistency of the estimate is shown to hold under additional conditions on controller and plant model order. These conditions result from the requirement of a uniformly stable model set. Simulation examples show both the power and the hazard of closed-loop identification with a tailor-made parametrization.

1 Introduction

System identification from closed-loop data has had a lot of attention in literature which has resulted in numerous closed-loop identification schemes. First of all there are the more classical methods like direct identification, indirect identification, instrumental variable methods and joint input/output identification, see e.g. [6]. More recently particular versions of these closed-loop identification schemes have been developed that are directed towards an explicitly tunable bias expression, which is aiming for an identified model that is particularly suitable for use in control design. Examples of such schemes are the two-stage method [7], identification in the dual Youla parametrization [3] and identification of coprime plant factors [9]. An overview of these closed-loop identification schemes can be found in [2] and [8].

In this paper a closed-loop identification method is discussed that has not had a lot of attention in literature: closed-loop identification with a tailor-made parametrization. The basic idea is that the closed-loop transfer function from excitation signal r to output signal y (see figure 1) is identified using an output predictor

$$\hat{y}(t,\theta) = \frac{G(q,\theta)}{1 + C(q)G(q,\theta)} r(t)$$

using the parameters corresponding to the (open-loop) plant model

$$G(q,\theta) = \frac{b_1 q^{-1} + \cdots + b_{n_B} q^{-n_B}}{1 + a_1 q^{-1} + \cdots + a_{n_A} q^{-n_A}}$$

with $\theta = [b_1 \cdots b_{n_B} \ a_1 \cdots a_{n_A}]$.

Using the open-loop plant parameters, and knowledge of the controller C, a prediction error criterion is used to estimate the plant parameters; this requires a nonlinear optimization procedure.

The parametrization is referred to as a tailor-made parametrization, as it is specifically directed towards (tailored to) the closed-loop configuration at hand, including knowledge of the controller.

This identification approach has been mentioned as an exercise in [5]. It is also employed in a recursive version in [4]. In this paper, an analysis will be made of the consistency properties of this method, where in particular we will focus on the connectedness of related parameter sets and the uniform stability of corresponding model sets.

‡The research of Edwin van Donkelaar is supported by the Dutch Technology Foundation (STW) under contract DWT55.3618

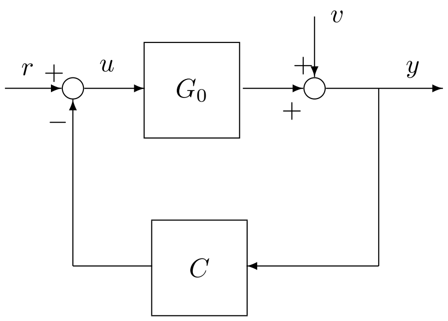

Fig. 1: *Closed-loop configuration*

After preliminary notation and the formulation of the problem, in section 4 it will be made clear that the need for uniform stability of the model set, which is adopted in [5] to obtain consistency results, imposes additional conditions on the parametrization. Sufficient conditions for consistency are derived which results in a condition on controller and model order. In section 5 two simulations are given to illustrate both the possible problems and the power of the application of a tailor-made parametrization. Next, in section 6 the relation between this and other closed-loop identification methods is discussed. Finally, section 7 concludes the paper.

2 Preliminaries

Addressed is the problem of obtaining a model of the linear time-invariant discrete-time single input single output plant $G_0(z)$ from measurements of the closed-loop configuration given in figure 1.

The controller is denoted with $C(z)$ and is assumed to be known. The signal $r(t)$ is an external excitation signal, $u(t)$ and $y(t)$ are respectively the plant input and output. It is assumed that measurements of $r(t)$ and $y(t)$ are available. The output noise $v(t)$ is assumed to be generated by filtering of white noise signal $e(t)$ with variance σ^2 using a stable monic filter $H_0(z)$. The output noise is assumed to be uncorrelated with the excitation signal r. The loop transfer $C(z)G_0(z)$ is assumed to be strictly proper. The closed-loop system is characterized by

$$y(t)=\underbrace{\frac{G_0(q)}{1+C(q)G_0(q)}}_{R_0(q)}r(t)+\underbrace{\frac{1}{1+C(q)G_0(q)}H_0(q)}_{W_0(q)}e(t)$$

where $R_0(q)$ denotes the closed-loop transfer function and $W_0(q)$ the closed-loop noise filter. The sensitivity function is denoted by $S_0(z) = (1 + C(z)G_0(z))^{-1}$ and the parametrized sensitivity is denoted with $S(z,\theta) = (1 + C(z)G(z,\theta))^{-1}$.

3 Closed-loop identification with a tailor-made parametrization

Consider a parametrized model of the plant $G(q,\theta)$ where the parameter vector lies in a parameter set $\theta \in \Theta$. This parametrized plant model together with knowledge of the controller can be used to parametrize the transfer function between the measured signals $r(t)$ and $y(t)$. This yields the following prediction of the output in case the parametrized closed-loop noise filter is set to $W(q,\theta) = 1$ (output error structure)

$$\hat{y}(t,\theta) = \underbrace{\frac{G(q,\theta)}{1 + C(q)G(q,\theta)}}_{R(q,\theta)} r(t),\ \theta \in \Theta \tag{1}$$

The corresponding closed-loop model set is defined as

$$\mathcal{P} := \left\{ R(q,\theta) = \frac{G(q,\theta)}{1 + C(q)G(q,\theta)}, \theta \in \Theta \right\}. \tag{2}$$

The parameter estimate is found by least squares minimization of the prediction error by solving $\hat{\theta}_N = arg\min\limits_{\theta\in\Theta} V_N(\theta)$, in which the criterion function is given by $V_N(\theta) = \frac{1}{N}\sum_{t=1}^{N}\varepsilon^2(t,\theta)$ and the prediction error is defined as $\varepsilon(t,\theta) = y(t) - R(q,\theta)r(t)$. The resulting estimation of the plant model will be denoted by $\hat{G}(q) = G(q,\hat{\theta}_N)$. For this identification method the following consistency result holds [5].

Proposition 3.1 *Let $\mathcal{P}$ be a uniformly stable model set and let the data generating system satisfy the standard conditions in* [5]. *Then $\theta_N \to \theta^*$ w.p. 1 for $N \to \infty$ with*

$$\theta^* = arg\min_{\theta\in\Theta} \frac{1}{2\pi}\int_{-\pi}^{\pi} |R_0(e^{i\omega}) - R(e^{i\omega},\theta)|^2 \Phi_r(\omega) d\omega \tag{3}$$

Whenever there exists a θ such that $G(q,\theta) = G_0(q)$ this choice will be a minimizing argument of the integral expression above which is unique provided that $r(t)$ is persistently exciting of sufficiently high order.

This proposition states that a consistent estimate is obtained with this parametrization under the condition that the model set $\mathcal{P}$ is uniformly stable. This condition is not trivially satisfied in case the tailor-made parametrization given in (2) is used. Therefore, in the next section the conditions under which the model set (2) is guaranteed to be uniformly stable will be investigated.

4 Uniform stability of the model set

Uniform stability of the model set is defined as follows.

Definition 4.1 [5] *A parametrized model set $\mathcal{G}$ is uniformly stable if*

- Θ *is a connected open subset of* $\mathbb{R}^{(n_A+n_B)}$
- $\mu : \Theta \to \mathcal{P}$ *is a differentiable mapping, and*
- *the family of transfer functions* $\{R(z,\theta), \frac{\partial}{\partial\theta}R(z,\theta)\}$ *is uniformly stable.*

In this section it will be made clear that in case a tailor-made parametrization is used, the parameter set Θ is possibly not connected due to the specific parametrization of the closed-loop transfer function $R(z,\theta)$. Also a sufficient condition is derived for guaranteed connectedness of the parameter set.

Let the strictly proper[1] plant model be parametrized as

$$G(z,\theta) = \frac{B(z,\theta)}{A(z,\theta)} = \frac{b_1 z^{-1} + \ldots + b_{n_B} z^{-n_B}}{1 + a_1 z^{-1} + \ldots + a_{n_A} z^{-n_A}} \qquad (4)$$

where $\theta = [a_1 \ldots a_{n_A}\ b_1 \ldots b_{n_B}]^T$. The controller of order n_c is given by

$$C = \frac{N_c(z)}{D_c(z)} = \frac{n_0 + n_1 z^{-1} + \ldots + n_{n_c} z^{-n_c}}{1 + d_1 z^{-1} + \ldots + d_{n_c} z^{-n_c}} \qquad (5)$$

where $N_c(z), D_c(z)$ are coprime polynomials. With this notation the parametrization of the output predictor is given by

$$\hat{y}(t,\theta) = \frac{D_c(q)B(q,\theta)}{D_c(q)A(q,\theta) + N_c(q)B(q,\theta)} r(t) \qquad (6)$$

All closed-loop models $R(q,\theta)$ are stable if the absolute value of the roots of the denominator $D_c(q)A(q,\theta) + N_c(q)B(q,\theta)$ is strictly less than one. Hence, the parameter set corresponding to closed-loop stable models is given by $\Theta :=$

$$\left\{\theta \in \mathbb{R}^{n_A+n_B} \middle|\ |sol\{D_c(q)A(q,\theta) + N_c(q)B(q,\theta) = 0\}| < 1\right\} \qquad (7)$$

The corresponding set of plant models is denoted by

$$\mathcal{G} := \{G(z,\theta), \theta \in \Theta\}. \qquad (8)$$

[1] For simplicity of notation only the case of a strictly proper plant and a proper controller is regarded. However, the case of a strictly proper controller and a proper plant can be described similarly.

It can be verified that the parameter set for which the polynomial $A(q,\theta)$ is stable, is pathwise connected. As a result, connectedness of the parameter set when using a (standard) numerator-denominator parametrization of the plant in an open-loop setting, will not be a problem. However, in case the tailor-made parametrization (2) is used, with Θ given by (7), Θ may not be pathwise connected as the following simple example shows.

Example 4.2 *Given the 7th order controller defined by the continuous time transfer function*

$$C(s) = \frac{0.499s^5 + 0.715s^4 + 2.577s^3 + 3.397s^2 + 2.155s + 2.620}{s^7 + 1.717s^6 + 5.100s^5 + 8.410s^4 + 4.198s^3 + 6.631s^2}$$

The plant that is to be identified is parametrized by a simple constant $G = \theta$. The parameter space $\Theta \subset \mathbb{R}$ for which the closed-loop system is stable can be simply derived from a root locus plot and is approximately given by $\Theta = \{\theta | \theta \in (0, 1.27) \cup (2.64, 4.69) \cup (9.98, \infty)\}$
This set is a disconnected subset of $\mathbb{R}$. Therefore the corresponding model set $\mathcal{P}$ is not uniformly stable.

A parameter set that is not connected has not only consequences for the formal proof of consistency as was mentioned before, but also for the nonlinear optimization that has to be performed to obtain an estimate. If, for example, a gradient search method is used and an initial estimate is selected in a region of the parameter set that is disconnected from the region where the optimal parameter vector is located, it will be extremely hard if not impossible to reach the optimum.

The denominator of the closed-loop transfer function can be written as a function of the open loop parameter θ as

$$D_c A(z,\theta) + N_c B(z,\theta) = 1 + [z^{-1}\ z^{-2} \ldots z^{-n}]\theta_{cl} \qquad (9)$$

where the closed-loop parameter vector is given by $\theta_{cl} := S\theta + \rho$. The order of the closed-loop polynomial of (9) is given by $n = max(n_A, n_B) + n_c$, $\rho = [p_1 \ldots p_{n_c}\ 0 \ldots 0]^T \in \mathbb{R}^n$ and $S = [P_D\ \ P_N] \in \mathbb{R}^{n\times(n_A+n_B)}$ with $P_D \in \mathbb{R}^{n\times n_A}$, $P_N \in \mathbb{R}^{n\times n_B}$ are matrices given by

$$P_D = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ d_1 & 1 & & \\ d_2 & d_1 & \ddots & \vdots \\ \vdots & d_2 & \ddots & 1 \\ d_{n_c} & & \ddots & d_1 \\ 0 & \ddots & & d_2 \\ \vdots & \ddots & & \vdots \\ 0 & \cdots & 0 & d_{n_c} \end{bmatrix}, P_N = \begin{bmatrix} n_0 & 0 & \cdots & 0 \\ n_1 & n_0 & & \\ n_2 & n_1 & \ddots & \vdots \\ \vdots & n_2 & \ddots & n_0 \\ n_{n_c} & & \ddots & n_1 \\ 0 & \ddots & & n_2 \\ \vdots & \ddots & & \vdots \\ 0 & \cdots & 0 & n_{n_c} \end{bmatrix} \qquad (10)$$

The closed loop parameter can vary over a parameter set

$$\Theta_{cl} := \{\theta_{cl} = M\theta + \rho | \theta \in \Theta\}$$

where the allowable closed loop parameters are restricted by the affine relation given above. Now, define a parameter set for stable polynomials of order n as follows

$$\Theta_n := \{\theta_n \in \mathbb{R}^n | \; |sol\{1 + [z^{-1} \; \dots z^{-n}]\theta_n = 0\}| < 1\}$$

From connectedness of the parameter set for stable polynomials it can be concluded that the parameter set Θ_n is also connected. In the following theorem a sufficient condition for connectedness of the parameter space Θ is given using the connected set Θ_n as a starting point.

Lemma 4.3 *Full row rank of the matrix S=$[P_D \; P_N]$ with P_D, P_N given in (10), is a sufficient condition for pathwise connectedness of the parameter set Θ given in (7).*

Proof: The closed-loop parameter θ_n can vary over the connected set Θ_n. Now define the set

$$\bar{\Theta}_{cl} = \{\bar{\theta}_{cl} | \bar{\theta}_{cl} = \theta_n - \rho, \theta_n \in \Theta_n\}$$

This set is a shifted version of Θ_n and is therefore also pathwise connected. An open loop parameter vector $\theta \in \Theta$ and a parameter vector $\bar{\theta}_{cl} \in \bar{\Theta}_{cl}$ are related via $\bar{\theta}_{cl} = S\theta, S \in \mathbb{R}^{n\times(n_A+n_B)}$. If S has full row rank it defines a surjective map, hence $image(S) = \bar{\Theta}_{cl}$. In the connected set $\bar{\Theta}_{cl}$ a continuous path can be constructed between two parameter vectors. This path can be mapped into a continuous path in Θ using the inverse mapping of S. Therefore Θ is also pathwise connected. □

This result implies that the parameter set for which the parametrized transfer function (2) is stable, is only a connected set in specific cases. Therefore it is not guaranteed that the model set defined in (2) is uniformly stable following the definition of uniform stability in Definition 4.1. The following lemma gives an easy test for guaranteed uniform stability of the model set with a tailor-made parametrization.

Proposition 4.4 *Let a model of order n_s be parametrized as in (4) with $n_A = n_B = n_s$ and let the controller of order n_c be given by (5). A sufficient condition for connectedness of the parameter set Θ for a tailor-made parametrization given in (2), is given by $n_s \geq n_c$.*

Proof: From lemma 4.3 it follows that full row rank of S is a sufficient condition for connectedness. By reordering the columns of S a 2×2 upper triangular block matrix can be constructed given by $S = \begin{bmatrix} S_1 & S_{12} \\ 0 & S_2 \end{bmatrix}$ where

$$S_1 = \begin{bmatrix} 1 & 0 & \cdots & 0 & n_0 & 0 & \cdots & 0 \\ d_1 & 1 & & & n_1 & n_0 & & \\ d_2 & d_1 & \ddots & \vdots & n_2 & n_1 & \ddots & \vdots \\ \vdots & d_2 & \ddots & 1 & \vdots & n_2 & \ddots & n_0 \\ d_{n_c} & & \ddots & d_1 & n_{n_c} & & \ddots & n_1 \\ 0 & d_{n_c} & & d_2 & 0 & n_{n_c} & & n_2 \\ \vdots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & d_{n_c} & 0 & \cdots & 0 & n_{n_c} \end{bmatrix}$$

$$S_2 = \begin{bmatrix} d_{n_c} & \dots & d_{2n_c-n_s+1} & n_{n_c} & \dots & n_{2n_c-n_s+1} \\ & \ddots & \vdots & & \ddots & \vdots \\ 0 & & d_{n_c} & 0 & & n_{n_c} \end{bmatrix}$$

where $S_1 \in \mathbb{R}^{2n_c\times 2n_c}$ and $S_2 \in \mathbb{R}^{(n_s-n_c)\times 2(n_s-n_c)}$. The matrix S has full row rank if S_1 and S_2 have full row rank. The first is a Sylvester matrix which has full row rank if and only if the numerator and denominator of the controller are coprime [1] . The second has full row rank if $d_{n_c} \neq 0$ or $n_{n_c} \neq 0$. This is always the case for a controller of order n_c. The number of rows of S is smaller than or equal to the number of columns if $n_A + n_B \geq max(n_A, n_B) + n_c$. This reduces to $2n_s \geq n_s + n_c$ or equivalently $n_s \geq n_c$. □

From this it can be concluded that connectedness of the parameter set Θ causes no problem if the order of the controller is smaller than the model order. So for identification of a simple model based on experiments with a complex controller connectedness of the parameter set may be a problem. Note that this is the case in example 4.2.

5 Simulation examples

In this section two simulation examples are given. One in the case where $G_0 \in \mathcal{G}$ and the parameter set is not connected and the other where $G_0 \notin \mathcal{G}$ with a connected parameter set but with a very bad signal to noise ratio. In the first example the tailor-made parametrization induces an optimization problem which is difficult to solve while in the second example it is demonstrated that closed-loop identification with this parametrization can be very powerful.

Simulation 1
In figure 2 the three separate branches of the cost function $V_N(\theta)$ for the system from Example 4.2 is depicted for a system $G_0 = 3.5.$ and $G(\theta) = \theta$. The output disturbance $v(t)$ in figure 1 is white noise with variance $\sigma = 0.1$. The excitation signal $r(t)$ is white noise with variance 1. Note that the parameter regions $(-\infty, 0], [1.27, 2.64]$ and $[4.69, 9.98]$ induce an unstable closed-loop system. The criterion function has several local minima that are located at the boundary of the stability area if the iterative search for the optimal parameter vector is confined to those parameters θ for which the closed-loop system is stable. This makes it difficult to find the optimum with gradient search methods.

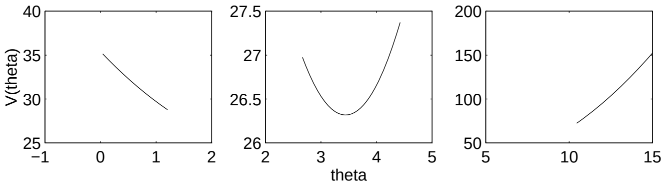

Fig. 2: *Criterion function for the identification problem in Example 4.1 with $G_0 = 3.5$.*

The global optimum will generally only be found if an initial estimate is selected from the middle of the three branches of the criterion function. If the number of data points goes to infinity, these local minima are not at the boundary of the stability area because in that case the value of the criterion function goes to infinity if the parameter approaches the closed-loop instability area.

Simulation 2
A simulation is made with a fifth order system, which is given by the transfer function $G_0(z) =$

$$10^{-5}\frac{5.278z^{-1}+126.7z^{-2}+299.3z^{-3}+110.8z^{-4}+404.2z^{-5}}{1-4.391z^{-1}+7.879z^{-2}-7.247z^{-3}+3.430z^{-4}-4.391z^{-5}}$$

which is an integrator with two resonant modes. The controller used in the simulation is a PI-controller which stabilizes the system. The excitation signal $r(t)$ is Gaussian white noise with standard deviation $\sigma_r = 1$ and the output noise $v(t)$ is Gaussian white noise with a standard deviation of $\sigma_v = 1.5$. The data length is $N = 500$. The open loop and closed-loop transfer functions are given in figure 3.
For this system a third order model is estimated with a tailor-made parametrization. For the nonlinear optimization a Gauss-Newton method is applied where the initial estimate is obtained with use of direct identification with

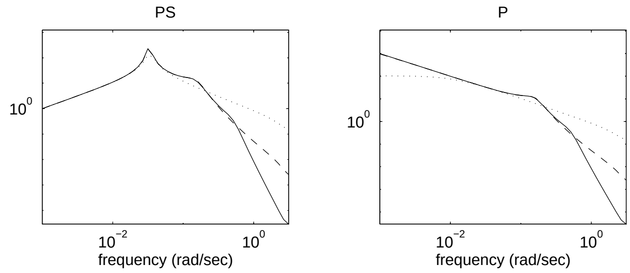

Fig. 3: *Amplitude plot closed-loop transfer from $r(t)$ to $y(t)$ (left) and open-loop transfer (right): plant (solid), estimation with tailor-made parametrization (dashed) and direct identification (dotted).*

an ARX(3,3,1) model structure. The speed of the nonlinear optimization routine is improved by using the explicit gradient expression derived in [11]. The estimated model is given in figure 3. Also the initial model is given. From this it can be seen that the estimation with a tailor-made parametrization gives a good fit for the integrator and the first resonant mode, despite the bad signal-to-noise ratio and the bad initial estimate.

6 Relation to other closed-loop identification methods

In this section the relation between closed-loop identification with a tailor-made parametrization and other closed-loop identification methods is discussed.
An obvious parametrization making further use of knowledge of the closed-loop structure, is given by

$$R(q,\theta) = \frac{G(q,\theta)}{1 + C(q)G(q,\theta)}, W(q,\theta) = \frac{H(q,\theta)}{1 + C(q)G(q,\theta)}$$

Least squares minimization of the corresponding prediction error yields a criterion function $V_N(\theta) = \frac{1}{N}\sum_{t=1}^{N} H^{-1}(q,\theta)(y(t) - G(q,\theta)(r(t) - C(q)y(t)))$ which is equal to the cost function for direct identification from $u(t) = (r(t) - C(q)y(t))$ to $y(t)$ which is known to be only consistent in case both the plant G_0 and the noise model H_0 can be modelled exactly within the chosen model set. It is important to note that this inconsistency is due to the dependent parametrization of the closed-loop transfer and the closed-loop noise filter. If $R(q,\theta)$ and $W(q,\theta)$ are parametrized independently, the consistency result given in Proposition 3.1 still holds in case $G_0 \in \mathcal{G}$.
The specific approximative properties of closed-loop identification with a tailor-made parametrization can be ob-

tained from (3). This expression can be further specified as $\theta^* =$

$$arg \min_{\theta \in \Theta} \frac{1}{2\pi} \int_{-\pi}^{\pi} |S_0(e^{i\omega})[G_0(e^{i\omega}) - G(e^{i\omega},\theta)] S(e^{i\omega},\theta)|^2 \Phi_r(\omega) d\omega.$$

From this it can be seen that the estimation error is weighted by both the sensitivity function and the estimated sensitivity function which puts an emphasis on the crossover region. This implies that in the case of approximative modelling, $G_0 \notin \mathcal{G}$, the undermodelling error is particularly small in this frequency region which is favourable in case the identified model is used in control design as is pointed out in [8]. In many control-relevant identification schemes this type of weighting is pursued but can there only be approximated by use of specific filtering strategies, while by using a tailor-made parametrization this weighting is inherent.

Identification using a tailor-made parametrization resembles the indirect identification method where first the closed-loop transfer function $R(q)$ is identified with a standard numerator-denominator parametrization. Next, a plant model is calculated using knowledge of the controller with $\hat{G}(q) = R(q, \hat{\theta})(1 - R(q, \hat{\theta})C(q))^{-1}$. Estimation of a plant model with a prespecified model order is not a trivial task here. This same mechanism holds true also for identification in the dual Youla parametrization, which is a direct generalization of the indirect method [10]. Using a tailor-made parametrization a plant model can be estimated with prespecified complexity.

7 Conclusions

In this paper identification of a model from closed-loop data with a tailor-made parametrization is discussed. Special attention is given to the possible occurrence of a non-connected parameter set which is induced by the structure of the parametrization.

Sufficient conditions are derived for the model order in terms of the controller complexity such that the parameter set is connected. These conditions indicate that the parameter set may not be a connected set in case a low complexity model is identified from data with a high complexity controller.

Additionally it is shown that for a specific parametrization of the noise model, the method reduces to closed-loop identification with the classical direct method.

From simulations it follows that the approach can yield very accurate models also in case of approximative modelling with a bad signal-to-noise ratio.

Acknowledgements

The authors wish to thank Carsten Scherer for fruitful discussions on the results presented.

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