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Dynamics and Modal Control of Piezoelectric Tube Actuators ⋆

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Abstract: In piezoelectric positioning systems the achievable bandwidth is often limited by weakly damped resonant modes. System performance may be improved by avoiding the excitation of these modes. If sufficient mechanical damping is present this can be done by shifting resonant modes towards anti-resonant modes through manipulation of the mechanical boundary conditions. In a second approach the anti-resonances may be shifted towards the resonances by the application of modal actuation. Using this method the excitation of the second and higher order modes is avoided by changing the distribution of actuation forces. This paper investigates and compares the application of modal control techniques in systems based on piezoelectric tube actuators such as atomic force microscopes using finite element analysis and experimental verification.

Keywords: Modal control, Mode analysis, Mechanical properties, Finite element analysis, Model based control, Microscopes

1. INTRODUCTION

Piezoelectric actuators are used in applications where high precision positioning is required over a relatively short range. A typical example of such an application is the atomic force microscope (AFM) described by Binning (1986). A common problem in the design of such systems is the excitation of resonant modes which are close to the controller bandwidth. Possible solutions to this problem were investigated by Balas (1978) and later by Meirovitch (1983) and were based on avoiding excitation of these modes by adapting the control system architecture or by increasing the number of sensors and actuators to enable individual control of resonant modes. This last approach, described in Meirovitch (1982) is a form of distributed control and is commonly referred to as modal actuation. The technique was successfully applied in piezoelectric actuated systems by Lee and Moon (1990) who demonstrated a modal piezoelectric actuator based on electrodes which were shaped by etching. Alternative forms of modal actuators based on porous electrodes or electrodes with a honeycomb motive were described in Preumont (2003) and Preumont (2005). Modal filters based on arrays of individual piezoelectric transducers have been reported by Collins (1994) and by Meirovitch (1985).

A key aspect of modal actuation is that the response to external excitation is changed without altering the natural modes or the natural frequencies. This is achieved by changing the distribution of forces applied to the system. The design of a modal actuator can be based directly on the mode shapes. Implementation of modal actuation is difficult if the mode shapes are very complex, if the flexible structure can only be actuated partially or in cases where the dynamics are uncertain due to changing load conditions or by the introduction of flexible loads. In an alternative approach proposed by Adriaens (2000), the natural modes are shifted towards the anti-resonances by manipulation of the mechanical boundary conditions. Sufficiently damped natural modes which are close to an anti-resonance will appear less pronounced reducing their impact on the design of the feedback control loop.

This paper investigates the application of both forms of modal actuation to positioning systems where the piezoelectric tube actuator is the main flexible component. The objective is to determine which modes are important to the realization of the position control objective and to show that these modes can be controlled through modal actuation. To enable in-situ adaptation to changing load conditions the modal actuator is based on a sectioned electrode which allows tuning of the distribution of the electric field by varying the voltages supplied to each section.

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The paper is organized as follows. In Section 2, piezoelectric actuation and the formulation of finite element models for piezoelectric tube actuators is discussed. In Section 3, manipulation of the dynamical properties using mechanical boundary conditions is discussed. Modal actuation is described in Section 4. In section 5 the limitations to modal control are investigated. Experimental verification of modal control techniques is presented in Section 6. Finally, concluding remarks are given in Section 7.

2. PIEZOELECTRIC ACTUATION

A piezoelectric actuator consists of a stiff ceramic material which expands in the presence of an electric field. Models of piezoelectric tube actuators based on the application of finite element modelling have been proposed by Maess (2008) and Jahromi (2008). Using this framework, the application of modal control through variation of the load as well as the application of modal control based on section electrodes is investigated using a case study of a high speed AFM, (see Abramovitch (2007) for an overview of AFM systems). In essence an AFM consists of a cantilever with a sharp protruding tip which is in contact with a sample. If the dynamical properties of the piezo driver, actuator and sensor are known, the surface of the sample can be reconstructed using the control signal of the feedback regulator controlling the distance between sample and tip.

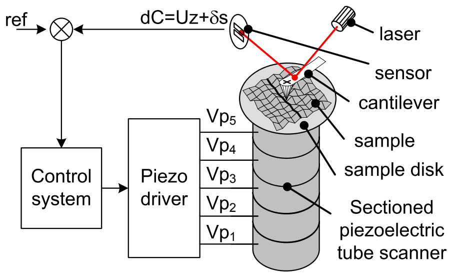

Fig. 1. Feedback control system for AFM vertical axis including modal actuation based on sectioned electrodes.

A finite element model used to describe the dynamics of the system shown in Fig. 1 has been implemented using ANSYS (2007). The model is based on 8-node coupled field elements (SOLID5). To increase symmetry, the elements are arranged in a regular mapped mesh. A relatively high mesh density of 80 by 95 by 2 in axial, circumferential and radial direction has been applied to enable accurate prediction of high order modes with complex mode shapes and to avoid stiffening effects such as shear locking. In low voltage regimes, the direct and indirect piezoelectric effect can be modelled using linear the constitutive equations

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \\ \hline \mathbb{D}_1 \\ \mathbb{D}_2 \\ \mathbb{D}_3 \end{bmatrix} = \left[\begin{array}{cccccc|ccc} c_{11}^E & c_{12}^E & c_{13}^E & 0 & 0 & 0 & 0 & 0 & e_{31} \\ c_{12}^E & s_{11}^E & s_{13}^E & 0 & 0 & 0 & 0 & 0 & e_{31} \\ c_{13}^E & c_{13}^E & s_{33}^E & 0 & 0 & 0 & 0 & 0 & e_{33} \\ 0 & 0 & 0 & c_{44}^E & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & 0 & c_{44}^E & 0 & e_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & & c_{66}^E & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & e_{15} & 0 & \varepsilon_{11} & 0 & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 & 0 & \varepsilon_{11} & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 & 0 & 0 & \varepsilon_{33} \end{array}\right] \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \\ \hline \mathbb{E}_1 \\ \mathbb{E}_2 \\ \mathbb{E}_3 \end{bmatrix}$$

in which σ and ϵ are the mechanical stresses and strains and $\mathbb{E}$ and $\mathbb{D}$ are the electric field strength and displacement. The subscript indices denote the axis of the local element coordinate system using vector notation as shown in Fig. 2 (right). The anisotropic material data relating

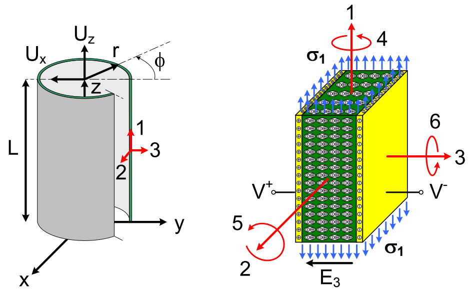

Fig. 2. Overview of piezoelectric tube actuator in an AFM setting (left) modelled locally using a 3D-piezoelectric brick elements (right).

the stress and the strain given in short circuit conditions is denoted by c^E listed in table 1. In this case the tensor notations commonly used in manufacturer data sheets are retained. Rigid boundary conditions are incorporated

Table 1. Material data PIC151 piezoceramic.

Par	Value	Par	Value
c_{11}^E	$1,076 \times 10^{11}\ N/m^2$	e_{31}	$-9,60\ N/Vm$
c_{12}^E	$6,312 \times 10^{10}\ N/m^2$	e_{33}	$15,10\ N/Vm$
c_{13}^E	$6,385 \times 10^{10}\ N/m^2$	e_{15}	$12,00\ N/Vm$
c_{33}^E	$1,004 \times 10^{11}\ N/m^2$	ε_{11}^S	9.8281×10^9
c_{44}^E	$1,962 \times 10^{10}\ N/m^2$	ε_{33}^S	7.5261×10^9
c_{66}^E	$2,224 \times 10^{10}\ N/m^2$	ε_{33}^T	1.8673×10^8
ρ	$7,76 \times 10^3\ Kg/m^3$	Q	88

using coupled sets of constraint equations. The mechanical degrees of freedom of nodes at the interface between the tube and mechanical ground are fixed to zero displacement. The nodes on the upper, free end of the tube are coupled by a set of constraint equations with a rigid mass element (Mass21).

3. MODAL CONTROL BY LOAD BALANCING

The dynamics of the piezoelectric actuator are influenced by the presence of mechanical loads. In AFM applications, a common type of load is a sample disk. In Fig. 3 the harmonic response of a loaded piezoelectric tube actuator with a length of 30 mm and a diameter of 10 mm is shown. The load is modelled as a rigid steel disk with a radius r_d of 6 mm. Due to the rigid nature of the disk, the displacement U_z of the disk chosen as output is equal to the actuator elongation evaluated at the free end of the actuator. To investigate the effect of a load on the dynamics of the actuator, the thickness h of the load is varied between 0.76 mm and 2.28 mm. The harmonic response depicted in Fig. 3 shows that the low order natural modes of a loaded actuator shift towards lower frequencies due to the increased load mass. From the results shown in Fig. 3 it can be concluded that the second mode is shifted towards the first anti-resonance which is unaffected by the presence of the load. The first natural mode shifts towards a lower frequency. This result is in accordance with Euler-Bernoulli beam theory, see also Adriaens (2000). If sufficient material damping is present, the second mode is effectively cancelled due to its proximity to the first anti-resonance.

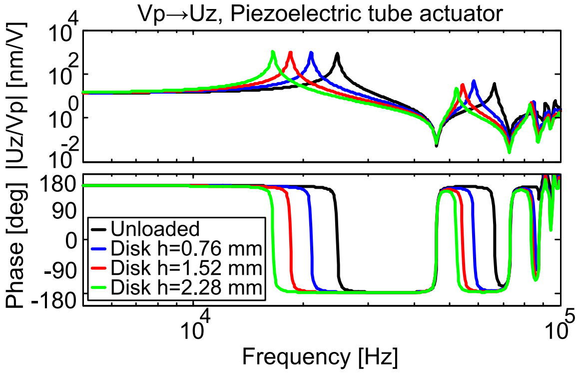

Fig. 3. The dynamics of a piezoelectric tube actuator loaded by a rigid load.

Unfortunately, the assumption that a load is rigid is only reasonable if the frequency range over which the harmonic response is evaluated is limited. The first natural mode of the disk depends on the material properties, the thickness of the disk and the radii of the disk and support. The radius of support is equal to the radius of the piezoelectric tube actuator. For relatively thin loads, the first natural modes of the load may be in the same frequency range as the low order natural modes of the actuator. The harmonic response of the piezoelectric actuator loaded by a flexible disk is shown in Fig. 4. To compare the effect of the load on the dynamics of the actuator, the thickness h of the load is varied between 0.76 mm and 2.28 mm. The displacement of the load is evaluated at the center of the disk. If a thin disk is applied, the low order bending modes of the disk will appear in the same frequency range as the low order extension modes of the tube. As a result, the relative ordering of the low order resonant and anti-resonant modes of the thin disk may deviate from the nominal load case ($h = 1,52$ mm) causing a large mismatch in phase between control system and actuator if the control system is tuned for the nominal load case. Ultimately, this may affect the stability of the feedback loop. To illustrate this effect, the three dimensional mode shapes of the first four modes of the actuator in combination with a thin disk is shown in Fig. 5. From this Figure it is clear that the mode shape of the second mode is dominated by the bending of the disk. This case corresponds to the case shown in blue in Fig. 4. For thicker disks, the low order bending modes appear close to the third and higher order modes in the same frequency range as the radial modes. Modes dominated by radial displacement (see for example Fig. 5, right) are in general hard to control since they tend to appear close together. Therefore, in a practical control design case, the bending modes of the thicker disks (1.52 and 2.28 mm) appear in a frequency range where excitation of modes by the control system is avoided by providing roll off.

The dynamics of the piezoelectric actuator are also influenced by the dynamics of a flexible actuator mount. The effect of solid steel base on the dynamics of the tube is shown in Fig. 6. The presence of the base introduces a series of resonant and anti-resonant modes which appear

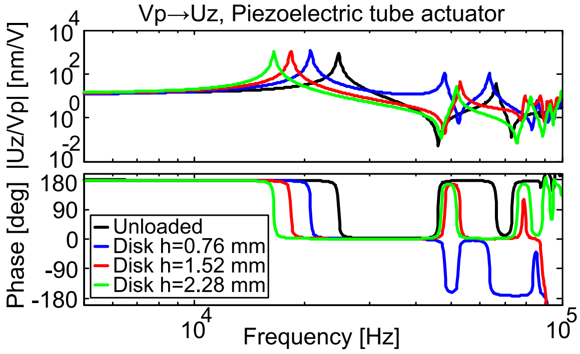

Fig. 4. The dynamics of a piezoelectric tube actuator loaded by a flexible load.

in pairs. This effect occurs due to the fact that the forces at the fixed end of the tube are no longer constrained by the base. The additional anti-resonances are introduced at frequencies where all energy generated by the excitation of the actuator results in deformation of the base. If the base is very short, a similar effect as the effect of a flexible load occurs. To minimize the influence of the base, the ratio between the length of the actuator and the length of the base should be as high as possible. The higher this ratio, the closer the introduced resonant modes are to the anti-resonant modes.

4. MODAL ACTUATION

A modal actuator is based on the redistribution of actuation forces and can be regarded as a form of distributed actuation. In the case of a piezoelectric actuator this redistribution of forces can be achieved by modification of the electrodes. In the approach described by Lee and Moon (1990) this was achieved using an etching process where portions of the live electrode were removed. Modal actuators based on sectioned electrodes are constructed by dividing the live electrode into a discrete number of sections. The voltages applied to each section varies have a fixed amplitude ratio depending on a set of section gains g_i. The design of a sectioned electrode modal actuator is based on summation of the dynamics of each section. In Fig. 7, a modal actuator based on five sections is shown. The number of sections applied in a given situation depends on the width of the inactive border between two sections. Also in applications with a large number of sections, the electric wiring and the soldered connections may have a

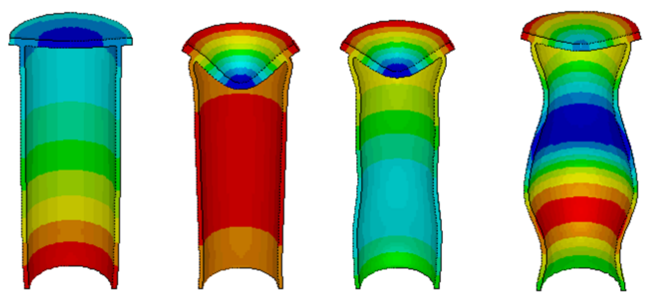

Fig. 5. Three-dimensional mode shapes of the first four modes of the vertical dynamics of the piezoelectric actuator including a thin disk flexible load.

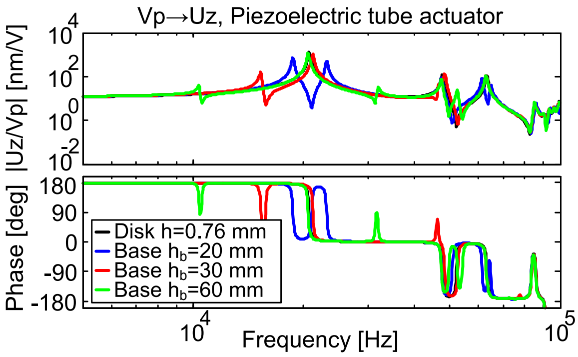

Fig. 6. The dynamics of a piezoelectric tube actuator supported by a flexible mount.

negative impact on the dynamics of the tube. The 30 mm tube used in the experiments was divided into five sections. If all sections are supplied with equal voltages the response shown in the lower right frame of Fig. 7 is obtained. To achieve modal actuation the amplitude of the voltages are adjusted using a static gain g_i. The gain set g needed to achieve modal actuation can be optimized using a constrained least squares optimization which is formulated as

$$\min_{g\in\mathbb{R}^n} \|H(\omega_0)g - b\|_2\,, \quad H(\omega_0) \in \mathbb{C}^{n\times m}, \quad b \in \mathbb{C}^m$$

with $H(\omega_0)$, gain set g and objective b defined as

$$H(\omega_0) = \begin{bmatrix} H_1(\omega_1) & \cdots & H_n(\omega_1) \\ H_1(\omega_2) & \cdots & H_n(\omega_2) \\ \vdots & \ddots & \vdots \\ H_1(\omega_m) & \cdots & H_n(\omega_m) \end{bmatrix}, g = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{bmatrix}, b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

where $\omega_0 = \{\omega_1, ..., \omega_m\}$ represent a set of frequencies chosen to fit a particular problem. The frequency set may include the natural frequencies of the system but may also include the gain at low frequencies. The constrained least squares problem can be formulated as a set of linear matrix inequalities (Boyd (1994)) in the form

$$\begin{bmatrix} -\gamma & (H(\omega_0)g - b)^H \\ H(\omega_0)g - b & -\gamma I \end{bmatrix} < 0, \; g > 0$$

where the constrained $g > 0$ is added to assure that the solution contains only positive gains. This constraint is added to preserve static range of the piezoelectric actuator and may be relaxed if the static gain of the system is less important.

In a practical design, the modes of interest are typically the low order modes. The objective would be to avoid excitation of modes in the cross over region of the feedback control system. This is reflected in the case study shown in Fig. 8 where optimal gains are calculated for four cases involving the first five modes of the actuator loaded by a flexible disk as shown in Fig. 7. The frequency set ω_0 is chosen to match the resonant modes of the first five resonances of the sections shown in Fig. 7. In the first case shown in Fig. 8 (frame 1), the constraint enforcing positive gain sets is relaxed and the modal actuator is tuned to fit the first mode and suppress modes 2-5. This is done by setting the first component of the objective to $b_1 = H_1(\omega_1) + ... + H_5(\omega_1)$ and $b_2, ...b_5$ to zero. In the second case the same objective is set, however in this case the solution is constrained to have only positive gains. In the third case the positive gain constrained is enforced but the constraint on mode 3-5 is relaxed. In the forth set sections 1-3 and sections 4-5 are coupled to form two electrodes. In this case the objective contains only the first and the second mode. The gain sets obtained are summarized in table 2. The results shown in Fig. 8 indicate that restricting the gains to be positive has a large impact on the results obtained. The load case with the thin flexible disk is the most effected. The reason for this is that the second mode is dominated by deformation of the disk. The mode shape does not resemble a beam type extension mode shape and is therefore not fully orthogonal to the mode shape of the first mode. This problem does not occur in the case with the thicker disks. In these cases the load can be regarded as rigid in the frequency region of interest. In both cases the coupling between radial and axial modes is apparent in the high frequency region. These modes are difficult to control using load balancing or modal actuation. For this reason a feedback control system needs to provide sufficient roll off in this region to avoid the accidental excitation of these modes.

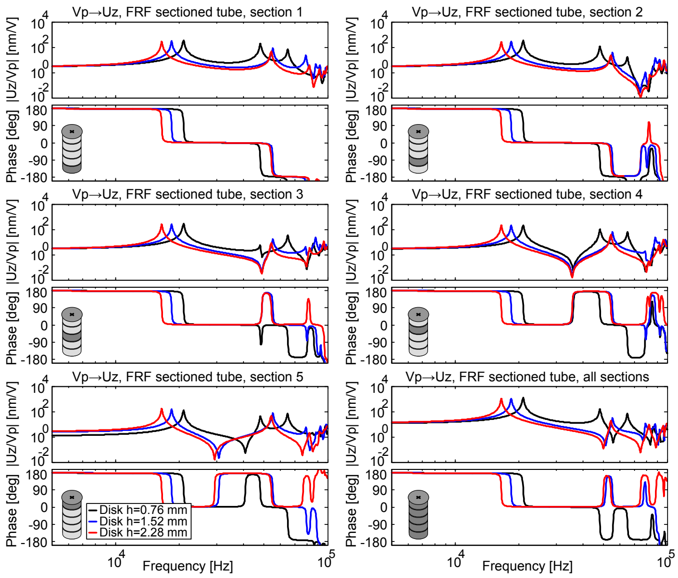

Fig. 7. Harmonic response of a piezoelectric actuator with sectioned electrodes.

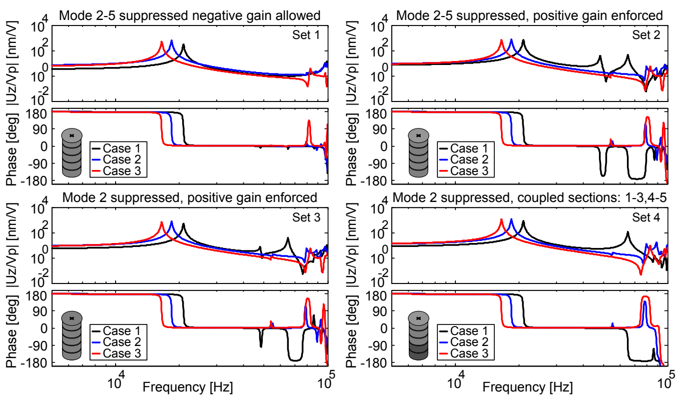

Fig. 8. Modal actuation case study with gain sets based on constrained least squares optimization.

The harmonic responses plotted in Fig. 7 show that in the case of sections 4-5 there is an anti-resonance between the first two modes which is not present in the case of sections 1-3. This allows the electric coupling of the first three sections in one group and the last two sections in the second group. Then, by balancing the voltages supplied to both groups the second mode can be reduced. This is the simplest way to achieve modal actuation with positive voltages which leads to a reduction of the second mode.

Table 2. Gain set of modal actuation case study

set	case	g_1	g_2	g_3	g_4	g_5
1	1	-0.3198	1.0000	-0.0312	0.8527	-0.8058
	2	0.1395	1.0000	0.4338	0.9771	-0.2691
	3	0.0540	0.6120	1.0000	0.4878	-0.1983
2	1	0.5501	0.3756	0.4293	1.0000	0.2108
	2	0.4413	1.0000	0.5132	0.9249	0.0962
	3	0.3702	0.7552	1.0000	0.5230	0.1767
3	1	0.4647	0.1877	0.1660	1.0000	0.0589
	2	0.4413	1.0000	0.5132	0.9249	0.0962
	3	0.3702	0.7552	1.0000	0.5230	0.1767
4	1	0.4671	0.4671	0.4671	1.0000	1.0000
	2	1.0000	1.0000	1.0000	0.7473	0.7473
	3	1.0000	1.0000	1.0000	0.7505	0.7505

5. LIMITATIONS TO MODAL CONTROL

In high speed designs, the achievable bandwidth can be optimized by the application of relatively short piezoelectric tube actuators which have resonant modes appearing at higher frequencies than those of long tube actuators. A complicating factor in designs based on short tubes is the relative low ratio between length and radius of the piezoelectric tube. As a result the coupling between the low order radial and axial modes is stronger for short tube actuators.

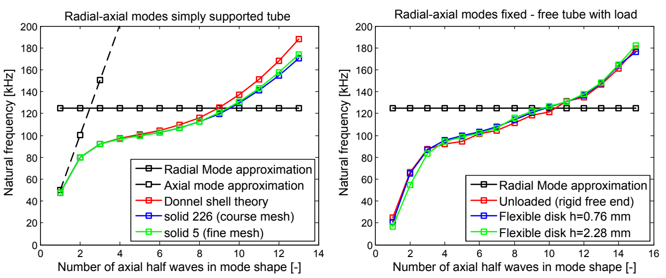

Fig. 9. Natural frequencies of the dominant modes in vertical direction for a piezoelectric tube ($L \times R = 30 \times 4.75\ mm$) which is simply supported (left) and fixed-free including load (right)

In Fig. 9, two sets of boundary conditions are compared. In the left frame the natural frequencies of radial axial modes for a simply supported tube are shown [1] . In this case both ends of the tube are constrained in circumferential and radial directions but are free to move in axial direction. For the case shown here only the first mode resembles a beam type extension mode. The second and higher order modes are fully coupled. And since all radial modes have the same natural frequency, coupling causes a large number of modes to appear in a narrow frequency range.

In the case shown in the right frame of Fig. 9, the piezoelectric actuator is fixed on one end and loaded by a mass on the other. Compared to the simply supported case, the axial modes appear at a lower frequency. In contrast, the radial modes are essential the same as in the simply supported case. The reason for this is that radially, the boundary conditions are similar due to the high in-plane stiffness of load and actuator mount. There is a small change in natural frequency which is caused by the introduction of bending moments at the boundary.

It can be concluded that the radial-axial coupling is essentially independent of the boundary conditions and depends only on the ratio between the length of the actuator and its radius. Because modes which are close together are inherently difficult to control, the coupling of radial and axial modes may complicate the design of a control system in cases where the load conditions are subject to change. Effective control of these modes using either modal actuation or by increasing the boundary conditions is difficult due to the complex nature of the modes and their insensitivity to loading.

6. EXPERIMENTAL VERIFICATION

Experimental verification of the modal actuation approach was based on a commercially available piezoelectric tube actuator (PT130.20, Physik Instrumente, Karlsruhe, Germany) with a special outer electrode geometry. The outer electrode was separated into 5 segments with a length of 5.6 mm by circumferential electrode removal. The tube is constructed using PIC151 ceramic. The sectioned tube was excited using a set of five custom piezo amplifiers with a configurable gain.

In Fig. 10 an overview of the experimental setup is presented. The actuator has a length $L = 30\ mm$, an inner diameter of $d_i = 9\ mm$ and an outer diameter $d_o = 10\ mm$. The tube is bonded to a steel base with a length of 160 mm using a cyanoacrylate adhesive. Loads in the form of standard AFM sample carriers are held into place using a small magnet bonded to a M4 bold which is connected to the base. The magnet and the load are in close proximity of each other but not in contact. The displacement of the load is measured using a capacitive sensor (MicroSense II 6810 with 6504-01 probe, MicroSense LLC, Lowell, Massachusetts, USA). The actuator is loaded using a set of steel ($\rho = 7.8 \times 10^3\ kg/m^3$, $E = 200\ GPa$) disks with

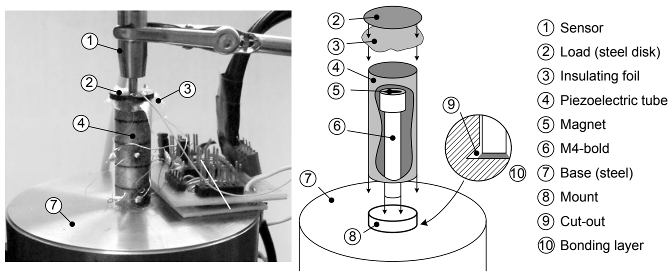

Fig. 10. Experimental setup with modal actuator and capacitive sensor (left) and overview showing the internal components (right).

[1] The natural modes may be calculated directly using shell theory (see Blevins (2001) for an overview) by substituting $c_{11}^E(1-\nu^2)$ for the elasticity modulus E and c_{12}^E/c_{11}^E for the Poisson's ratio ν.

radius $r_d = 6\ mm$, thickness $h_d = 0.72\ mm$ and $h_d = 1.56\ mm$. The disks have a mass m_l of $0.64\ g$ and $1.436\ g$. The piezoelectric tube was excited using a set of up to five independent piezo amplifiers.

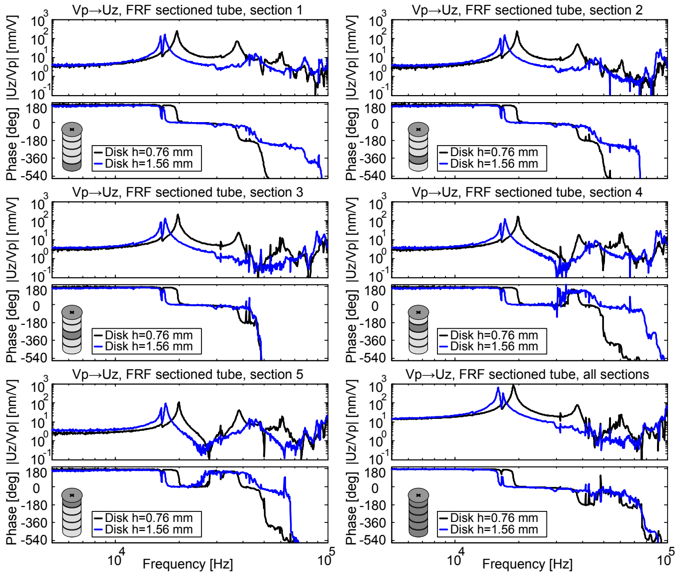

Fig. 11. Measured response of sectioned tube loaded by disks of 0.76 and 1.56 *mm*.

6.1 *Dynamics of full tube*

The measurements show that the frequency of the high order resonant modes is overestimated by the model. This effect is caused by the assumption that the disk and the tube actuator are perfectly bonded. In reality the deformation of the disk is not counteracted by a bending moment between the free end of the piezoelectric actuator and the load. A more accurate estimation of the low order modes of the disk may be obtained by assuming a simply supported load.

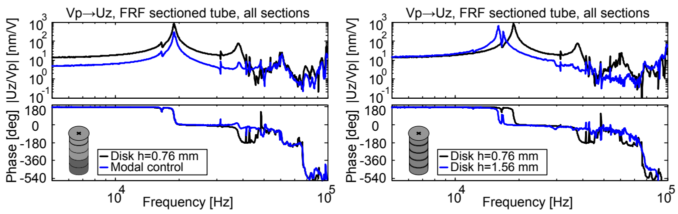

Fig. 12. Measured harmonic response of a modal actuator with gains set for suppression of the second mode (left) compared to an approach where a thick load is applied (right).

6.2 *Modal actuator tuning*

The ordering of the low order resonant and anti-resonant modes are predicted correctly by the model. The design of a modal actuator can be based on five segments or on a reduced set by combining the segments in groups. The approach shown in frame 4 of Fig. 8 implemented on the experimental set-up resulting in an almost perfect cancellation of the second mode, see Fig. 12 (left).

The load case with the thicker disk, shown in blue in Fig. 12 (right) indicates that the application of the load causes the second mode to disappear. This effect is caused by the mass which shifts the second extension mode towards the first anti-resonance in combination with the increased damping of the second mode compared to the thin disk load case. In addition, the extra mode included by the flexibility of the thin disk is not present in this case. The first mode of the thick load case is at a lower frequency than the thin disk load case. This disadvantage is offset by the advantage of mode two and three of the thin disk load case not being present. Also the static gain of the piezoelectric amplifier is unaffected since all sections are driven at the same voltage

Both approaches lead to the reduction of the second mode. This fact may be exploited as an additional degree of freedom facilitating the design of feedback control systems. Ultimately the reduction in peak gain may allow an increase in bandwidth without the need for (additional) notch filters.

7. CONCLUDING REMARKS

In this paper it is shown that the dynamics of a piezoelectric tube actuator can be shaped using modal actuation. In addition the dynamics may be shaped by increasing the load mass. The latter approach depends on the presence of sufficient damping in the modes to be reduced. The advantage of modal actuation using a sectioned electrodes is that the method can be tuned in-situ and is in principle independent of damping. The advantage of this flexibility is offset by the reduction caused by the reduced static gain and the increased complexity of the control circuitry. Therefore, if sufficient damping can be introduced, the load balancing approach may lead to a better trade-off between range and controllable bandwidth.

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