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Multivariable Closed-Loop Identification: From Indirect Identification to Dual-Youla Parametrization

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Abstract

Classical indirect methods of closed-loop identification can be applied on the basis of different closed-loop transfer functions. Here the multivariable situation is considered and conditions are formulated under which identified approximative plant models are guaranteed to be stabilized by the present controller. Additionally it is shown in which sense the classical indirect methods are generalized by the recently introduced identification method based on the dual-Youla parametrization. For stable controllers the two methods are shown to be basically equivalent to each other.

1 Introduction

The classical method of indirect identification for handling a closed-loop identification problem is based on the idea of first identifying a closed-loop transfer function, and then calculating the related plant model by using knowledge of the present controller in the loop (see [2, 7]). Attractive properties of this identification scheme are that the method does not suffer from bias effects due to a noise correlation with the input signal, as the input signal for identification is taken to be an external reference signal. The critical part of the indirect identification is the construction of the (open-loop) plant model in the second step, based on the estimated closed-loop transfer. However, if the resulting plant model is not limited in model order, this construction can be done exactly provided that the controller is known and the appropriate closed-loop transfer function has been identified. In this sense the question which transfer is "appropriate" is determined - among other things - by the input/output dimensions of the plant, and the location of the external excitation signal.

In recent years several new ideas concerning closed-loop identification of approximate models have been presented, most of them directed towards the ability to identify approximate models of the open-loop plant on the basis of closed-loop data, while the asymptotic bias distribution is not dependent on the noise and thus explicitly tunable by the designer, see e.g. [3, 4, 8, 9] as summarized in the survey paper [10]. Most of these schemes have been developed in view of the ability to tune the asymptotic bias distribution in order for the identified models to particularly reflect those dynamic aspects of the plant that are most relevant for consecutive model-based control design. One of the newly handled methods is based on a dual-Youla parametrization of the open-loop plant ([3, 6, 4]), and this method is suggested to be particularly attractive because of its guarantee that identified (approximate) plant models are guaranteed to be stabilized by the present controller.

In this paper we start by summarizing some aspects and results related to the classical indirect identification scheme, particularly addressing the question under which conditions multivariable plant models can be identified. Next it will be shown under which conditions identified plant models are guaranteed to be stabilized by the present controller, and in which sense this classical scheme can be considered as a special - simple- case of the recently used identification in the dual-Youla parametrization.

2 System configuration

The system configuration that will be considered in this paper is sketched in figure 1. P_0 and C are linear time-invariant finite-dimensional but not necessarily stable multivariable transfer functions. The input and output dimensions are determined by $u(t), r_1(t) \in \mathbb{R}^m$, $y(t), r_2(t) \in \mathbb{R}^p$. v is a noise disturbance signal, while r_1, r_2 are external signals that can be either reference (tracking) signals or external disturbances, being uncorrelated to eachother and to v.

A particular combination of external signals will be de-

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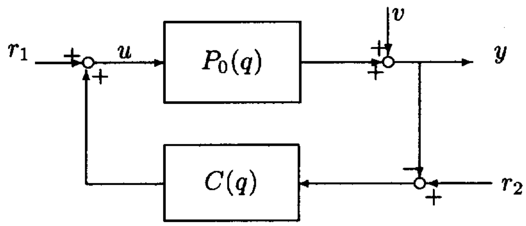

Fig. 1: Closed-loop configuration.

noted by

$$r(t) := r_1(t) + C(q)r_2(t). \qquad (1)$$

The relevant closed-loop transfer functions in the system configuration are reflected by

$$\mathsf{T}(P_0, C) = \begin{bmatrix} P_0 \\ I \end{bmatrix} [I + CP_0]^{-1} \begin{bmatrix} C & I \end{bmatrix} \qquad (2)$$

being the mapping from the signals $\begin{bmatrix} r_2 \\ r_1 \end{bmatrix} \rightarrow \begin{bmatrix} y \\ u \end{bmatrix}$.

For notational purposes the following notation for the elements of $\mathsf{T}(P_0, C)$ will be employed:

$$\mathsf{T}(P_0, C) = \begin{bmatrix} T_0 & G_0 \\ Q_0 & S_0 \end{bmatrix}. \qquad (3)$$

with

$$\begin{aligned} T_0 &= P_0[I + CP_0]^{-1}C \\ G_0 &= P_0[I + CP_0]^{-1} \\ Q_0 &= [I + CP_0]^{-1}C \\ S_0 &= [I + CP_0]^{-1}. \end{aligned}$$

It is a standard result from stability theory that the considered closed-loop system is internally stable if and only if $\mathsf{T}(P_0, C) \in \mathbb{R}\mathrm{H}_\infty$, with $\mathbb{R}\mathrm{H}_\infty$ the space of real rational transfer functions that are analytic in $z \geq 1$. As additional notation, I_m will refer to the $m \times m$ identity matrix, and $det_{\mathbb{R}(z)}(\cdot)$ is the determinant over the field of rational functions in z.

3 Indirect Identification

3.1 Standard approach - scalar situation

The classical method of indirect identification is composed of two steps. For this moment we will just sketch a particular situation in the scalar case.

(1) Identify the transfer function G_0 from r_1 to y; this can e.g. be done by applying any of the standard prediction error methods ([5]). Note that this identification problem is principally an 'open-loop' type of problem provided that the external signal r_1 is uncorrelated to the noise disturbance term v. The identified model of G_0 is denoted as $\hat{G}$

(2) Reconstruct an open loop plant model from the estimated closed-loop transfer function $\hat{G}$, using knowledge of the controller C.

The second step of this procedure involves the construction of $\hat{P}$ from an available estimate $\hat{G}$, by solving the equation:

$$\hat{G} = \frac{\hat{P}}{1 + C\hat{P}}. \qquad (4)$$

An exact solution for $\hat{P}$ follows by taking

$$\hat{P} = \frac{\hat{G}}{1 - C\hat{G}} \qquad (5)$$

which can be calculated when the controller C is known.
When the model $\hat{G}$ is identified using a least-squares outpur error criterion, i.e.

$$\varepsilon(t, \theta) := y(t) - G(q, \theta)r_1(t)$$

and $\hat{G} = G(q, \hat{\theta})$ with $\hat{\theta} := \arg\min_\theta \bar{E}\varepsilon(t, \theta)^2$, the asymptotic bias-distribution ([5]) in the plant model estimate is characterized by:

$$\hat{\theta} = \arg\min_\theta \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \frac{P_0}{1 + CP_0} - \frac{P(\theta)}{1 + CP(\theta)} \right|^2 \Phi_{r_1} d\omega \qquad (6)$$

provided that the exact relation (5) is used to construct $\hat{P}$ on the basis of $\hat{G}$.

One of the problems that is known to occur in an indirect identification approach, is that the order of the identified plant model is not under control. This means that when calculating (5), the order of $\hat{P}$ will be determined by the order n_G of $\hat{G}$ and the order n_C of C, and will generically equal $n_G + n_C$. Limiting the model order to a prespecified value, requires either an additional model reduction step, or the construction of an approximate solution to the equation (4) where the model order of $\hat{P}$ is fixed. However in this latter situation it is not clear how to "solve" this equation properly.

3.2 Indirect identification from closed-loop transfer functions - multivariable case

Actually all four different transfer functions that are present in $\mathsf{T}(P_0, C)$ can be used for identification in the first step of an indirect identification scheme. Dependent on the particular experimental situation, an identifier may have preferences of identifying a particular transfer. This can e.g. be essentially influenced by the possibility of adding external excitation signals at particular locations in the loop (either on the setpoint or on the output of the controller).

We will now summarize the possibilities of using any of the four transfer functions, while considering the multivariable situation.

Proposition 3.1 *Consider any one of the four transfer functions T_0, G_0, Q_0, or S_0 to be identified in the first stage of an indirect identification scheme, providing identified models $\hat{T}, \hat{G}, \hat{Q}$, or $\hat{S}$. Then*

(a) $\hat{T} = \hat{P}[I_m + C\hat{P}]^{-1}C$ *implies*

$$\hat{P} = \hat{T}(I_p - \hat{T})^{-1}C^{\dagger} \quad (7)$$

under the condition that $p \geq m$ and C has a right inverse $C^{\dagger}$.

(b) $\hat{G} = \hat{P}[I_m + C\hat{P}]^{-1}$ *implies*

$$\hat{P} = \hat{G}[I_m - C\hat{G}]^{-1}. \quad (8)$$

(c) $\hat{Q} = [I_m + C\hat{P}]^{-1}C$ *implies that*

$$\hat{P} = C^{-1}[(\hat{Q}C^{-1})^{-1} - I_m] \quad (9)$$

under the condition that $p = m$ and $det_{\mathbb{R}(z)}C \neq 0$.

(d) $\hat{S} = [I_m + C\hat{P}]^{-1}$ *implies that*

$$\hat{P} = C^{\dagger}[\hat{S}^{-1} - I_m] \quad (10)$$

under the condition that $p \leq m$ and C has a left inverse $C^{\dagger}$.

In the above expressions it is presumed that $\mathsf{T}(\hat{P}, C)$ is well defined.

Proof: Follows by straightforward manipulations of the expressions. ∎

It has to be noted that there is only one transfer function ($\hat{G}$) that provides a unique solution for the related open loop plant model without any conditions on input/output dimensions and controller. For the other transfer functions restrictions apply. Note also that in the scalar case $m = p = 1$, all four transfers can be used without any restrictions.

When taking a look at the relation with available external excitation signals the following can be stated:

- When r_1 is available from measurements (additional to u and y) then one can use $\hat{S}$ ($r_1 \rightarrow u$) or $\hat{G}$ ($r_1 \rightarrow y$) and by choosing $\hat{G}$ no restrictions apply.
- When r_2 is available from measurements, then one can use $\hat{T}$ ($r_2 \rightarrow y$) or $\hat{Q}$ ($r_2 \rightarrow u$) and one has to face the restrictions $p \geq m$ or $p = m$.

In the second situation considered it can be an alternative to first construct the signal $r(t) = C(q)r_2(t)$ and then using $r(t)$ as if it were added to the loop at the location of r_1. In this way, one can avoid the dimensional restrictions as mentioned above.

The fact that a unique plant model $\hat{P}$ can be constructed from either of the equations (7)-(10) does not imply that this plant model will be guaranteed to be proper. This will depend on the properties of the estimated closed-loop transfer and of the controller. Properness of $\hat{P}$ is e.g. guaranteed for (8) whenever $\hat{G}$ is proper and $\lim_{|z|\rightarrow\infty} C\hat{G} = 0$, being the commonly considered situation in indirect identification.

4 Stability of controlled models

In this section the question will be addressed under which conditions a plant model $\hat{P}$ that is identified by an indirect identification as described before, will be - a priori - guaranteed to be stabilized by the controller C. To this end the following standard results from stability theory will be exploited.

Proposition 4.1 *Consider any linear, time-invariant, finite-dimensional plant P and controller C.*

(a) *Let $C \in \mathbb{R}\mathrm{H}_\infty$. Then $\mathsf{T}(P, C) \in \mathbb{R}\mathrm{H}_\infty$ if and only if $P(I + CP)^{-1} \in \mathbb{R}\mathrm{H}_\infty$.*

(b) *Let $m = p$ and let C be invertible and satisfy $C^{-1} \in \mathbb{R}\mathrm{H}_\infty$. Then $\mathsf{T}(P, C) \in \mathbb{R}\mathrm{H}_\infty$ if and only if $(I + CP)^{-1}C \in \mathbb{R}\mathrm{H}_\infty$.*

(c) *Let $rank_{\mathbb{R}(z)}(P) = min(m, p)$, and the Moore-Penrose inverse $P^{\dagger} \in \mathbb{R}\mathrm{H}_\infty$. Then $\mathsf{T}(P, C) \in \mathbb{R}\mathrm{H}_\infty$ if and only if $P(I + CP)^{-1} \in \mathbb{R}\mathrm{H}_\infty$.*

Proof: Part (a) is proven in e.g. [11]. For parts (b) and (c) necessity is obvious. To prove sufficiency for (b), consider $(I + CP)^{-1}C \in \mathbb{R}\mathrm{H}_\infty$, so $(I + CP)^{-1}CC^{-1} = (I + CP)^{-1} \in \mathbb{R}\mathrm{H}_\infty$. As $P(I + CP)^{-1}C + (I + CP)^{-1} = I$ it follows that $P(I + CP)^{-1}C \in \mathbb{R}\mathrm{H}_\infty$ and by postmultiplication of C^{-1} also that $P(I + CP)^{-1} \in \mathbb{R}\mathrm{H}_\infty$. Sufficiency for (c) can be shown along similar lines, distinguishing between the situations $p \geq m$, where $P^{\dagger}$ is a left inverse, and $p \leq m$ when $P^{\dagger}$ is a right inverse. ∎

When applying these results to identified models obtained from indirect identification the following results are direct.

Corollary 4.2 *Consider identified models $\hat{G}$ and $\hat{Q}$ of the related closed-loop transfer functions G_0 and Q_0.*

(a) *If C is stable then the plant model estimate (8) is stabilized by C if and only if $\hat{G}$ is stable.*

(b) *If $m = p$ and C^{-1} is stable then the plant model estimate (9) is stabilized by C if and only if $\hat{Q}$ is stable.*

Particularly, a plant model obtained by indirect identification from estimating the closed-loop transfer function G_0, will be guaranteed to be stabilized by C in

the case that C is stable. The only restriction that the estimate $\hat{G}$ has to satisfy for this result to hold, is that $\hat{G}$ should be stable. Since the closed-loop system is stable, this condition wil be naturally satisfied by any sensible identification method.
It would be tempting to formulate a result similar to (a) without any condition on the stability of C or on input/output dimensions. However this will lead to more complex restrictions on $\hat{G}$ as shown next.

Corollary 4.3 *Consider a model $\hat{G}$ of the related closed-loop transfer function G_0, with $rank_{\mathbb{R}(z)}(\hat{G}) = min(m,p)$, and satisfying*

$$[I_m - C\hat{G}]\hat{G}^{\dagger} \text{ is stable} \tag{11}$$

where $\hat{G}^{\dagger}$ is the Moore-Penrose inverse. Then the plant model estimate (8) *is stabilized by C if and only if $\hat{G}$ is stable.*

Proof: The result follows by manipulation of the expressions in Proposition 4.1(c). ■

When the controller is not stable an additional restriction (11) has to be considered. This constraint on $\hat{G}$ can not simply be incorporated in a parametrization of the closed-loop transfer G_0 to be used during identification. A solution to this problem does exist, as shown in the forthcomming sections.

The stability results shown above, suggest that there is a relationship between these indirect identification methods, and the approach of using a dual-Youla parametrization of all plants that are stabilized by the given controller. This relation is pursued in the next sections.

5 Identification in the dual-Youla form

The Youla-parametrization parametrizes for a given plant $P_0 \in \mathbb{R}\mathrm{H}_\infty$ the set of all controllers $C \in \mathbb{R}\mathrm{H}_\infty$ that stabilize P_0. In the dual-Youla parametrization, a similar mechanism is used, but now the set of all plants is considered that is stabilized by a given controller.
In order to formulate this parametrization, the concept of coprime factorizations over $\mathbb{R}\mathrm{H}_\infty$ is required.
A pair of stable transfer functions $N, D \in \mathbb{R}\mathrm{H}_\infty$ is a right coprime factorization (rcf) of P_0 if $P_0 = ND^{-1}$ and there exist stable transfer functions $X, Y \in \mathbb{R}\mathrm{H}_\infty$ such that $XN + YD = I$. This implies that two factors are coprime if there are no unstable canceling zeros in the factorization.

Proposition 5.1 ([1]) *Let P_x with* rcf (N_x, D_x) *be any auxiliary model that is stabilized by the controller C with* rcf (N_c, D_c). *Then a plant P_0 is stabilized by C if and only if there exists an $R \in \mathbb{R}\mathrm{H}_\infty$ such that*

$$P_0 = [N_x + D_cR][D_x - N_cR]^{-1}. \tag{12}$$

For a given plant P_0, the related dual-Youla parameter $R = R_0$ is given by

$$R_0 = D_c^{-1}[I + P_0C]^{-1}(P_0 - P_x)D_x. \tag{13}$$

With this parametrization the original system configuration can be resketched into the alternative form as presented in figure 2. In this dual-Youla form the signals $x(t)$ and $z(t)$ are determined by

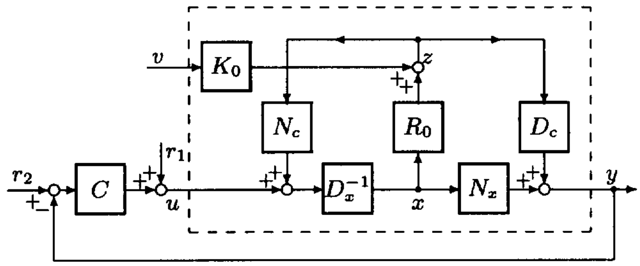

Fig. 2: Dual Youla-representation of the data generating system.

$$\begin{aligned} z(t) &= (D_c + P_xN_c)^{-1}[y(t) - P_x(q)u(t)] && (14)\\ x(t) &= (D_x + CN_x)^{-1}[r_1(t) + C(q)r_2(t)] && (15)\end{aligned}$$

while K_0 is given by

$$K_0 = D_c^{-1}(I + P_0C)^{-1} \tag{16}$$

see e.g. [10]. In view of the identification problem, one is dealing with the relation

$$z(t) = R_0(q)x(t) + K_0(q)v(t) \tag{17}$$

where the important mechanism is that both signals z and x can be reconstructed from available data y, u, r and by using knowledge of the controller C and of just *any* auxiliary model P_x that is stabilized by C. Moreover as it appears from (15) the signal x is uncorrelated with the noise v, and so relation (17) points to an "open-loop" identification problem of identifying R_0 on the basis of measurement data z, x.

One of the properties of this identification approach is that any identified stable model $\hat{R}$ of R_0 will yield an open-loop plant model

$$\hat{P} = [N_x + D_c\hat{R}][D_x - N_c\hat{R}]^{-1} \tag{18}$$

that is guaranteed to be stabilized by C, because of the dual-Youla parametrization.
A property of this dual-Youla identification method is - similar to the situation of the indirect approach - that the model order of the identified open-loop plant model

is not under control. Because of the relation (18), an identified transfer $\hat{R}$ with a specific model order, will lead to an open-loop plant model that has an increased model order, that incorporates the order of the controller and the order of the auxiliary model P_x.

6 Indirect identification as a special case of the dual-Youla method

The question occurs whether the identification of R_0 in the dual-Youla situation is equivalent to the identification of a closed-loop transfer function as present in the first step of an indirect identification scheme. A number of special cases will be pointed out.

Proposition 6.1 *If C is stable then there exists a choice for P_x and right coprime factorizations of C and P_x such that in the dual-Youla form:*

$$\begin{aligned} R_0 &= G_0 \\ z(t) &= y(t) \\ x(t) &= r(t) \end{aligned}$$

and consequently identification of the dual-Youla parameter is identical to identification according to the indirect method (8) *on the basis of $\hat{G}$.*

Proof: Since C is stable, one may choose $N_c = C$, $D_c = I$, $N_x = 0$ and $D_x = I$, taking into account that the model $P_x = 0$ is stabilized by a stable controller. The result follows by substitution in the appropriate expressions. ∎

It appears that for stable controllers, the dual-Youla identification method is actually equivalent to an indirect identification on the basis of the transfer $r_1 \to y$ (G_0). A similar result can be formulated for the indirect identification through the transfer $r_2 \to y$ (T_0).

Proposition 6.2 *If C is stable then there exists a choice for P_x and right coprime factorizations of C and P_x such that in the dual-Youla form:*

$$\begin{aligned} R_0 &= T_0 \\ z(t) &= y(t) \\ x(t) &= r(t) \end{aligned}$$

and consequently identification of the dual-Youla parameter is identical to identification according to the indirect method (7) *on the basis of $\hat{T}$.*

Proof: The result follows by choosing $N_c = C$, $D_c = I$, $N_x = 0$ and $D_x = C$, and by substituting this in the appropriate expressions. ∎

The closed-loop transfer functions considered in the two propositions above are transfers towards the closed-loop output signal $y(t)$. The question now occurs whether the two other transfer function (Q_0 and S_0) can be considered in a similar way. This appears to be less trivial than expected, most importantly because they are transfers towards the closed-loop input signal $u(t)$. As a consequence, the choices of particular factorizations should be made in such a way that this results in $z(t) = u(t)$. Considering the general expression for $z(t)$ in (14) this seems not possible. A solution for this problem appears to be in considering a dual-Youla parametrization based on the controllers inverse, which is discussed in the next section.

With respect to the asymptotic bias distribution, as indicated in (6) for the indirect method, it is shown in [4, 10] that for the dual-Youla method, the corresponding expression is (for the SISO-case):

$$\hat{\theta} = \arg\min_{\theta} \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \frac{P_0}{1+CP_0} - \frac{P(\theta)}{1+CP(\theta)} \right|^2 \frac{\Phi_{r_1}}{|D_c|^2} d\omega$$

which is similar to (6), except for an additional weighting with D_c. In case C is stable, one can always choose $D_c = 1$ leading to equal expressions for both methods. Note that for unstable C the model sets in the two approaches will be slightly different if in the indirect method one does not take account of the parametrization constraint (11).

7 A dual-Youla parametrization on the basis of C^{-1}

In this section attention will be limited to the situation that $m = p$ and controller and plant can be inverted, i.e. they have full rank over $\mathbb{R}(z)$.

Lemma 7.1 *Consider the situation $m = p$ and P_0 and C invertible. Then $\mathsf{T}(P_0, C) \in \mathbb{R}\mathrm{H}_\infty$ if and only if $\mathsf{T}(P_0^{-1}, C^{-1}) \in \mathbb{R}\mathrm{H}_\infty$.*

Proof: By simple manipulations it can be shown that $\mathsf{T}(P_0^{-1}, C^{-1})$ is equal to a permuted version of the original $\mathsf{T}(P_0, C)$. ∎

A dual-Youla parametrization can now be formulated on the basis of the inverse controller C^{-1}.

Proposition 7.2 *Let P_x with* rcf (N_x, D_x) *be any auxiliary model that is stabilized by the controller C^{-1} with* rcf $D_c N_c^{-1}$. *Then a plant P_0 is stabilized by C if and only if there exists an $R \in \mathbb{R}\mathrm{H}_\infty$ such that*

$$P_0 = [D_x - D_c R][N_x + N_c R]^{-1}. \qquad (19)$$

Proof: The proof follows by parametrizing P_0^{-1} in a dual-Youla parametrization, and applying lemma 7.1.

Under the conditions of the proposition, it follows that for a given plant P_0, the related R is given by

$$R = R_0 = D_c^{-1}(I + P_0C)^{-1}(D_x - P_0N_x) \qquad (20)$$

and the system's equations become:

$$\begin{aligned} y(t) &= (D_x - D_cR_0)x(t) + (I + P_0C)^{-1}v(t) \\ u(t) &= (N_x + N_cR_0)x(t) - C(I + P_0C)^{-1}v(t). \end{aligned}$$

Based on these latter equations one can extract R_0 by:

$$z(t) = R_0(q)x(t) + K_0(q)v(t) \qquad (21)$$

with

$$z(t) = (N_c + P_xD_c)^{-1}[u(t) - P_x(q)y(t)]. \qquad (22)$$

This alternative structure, will allow to choose particular factorizations in the scheme in order to realize $z(t) = u(t)$. This is reflected in the following two results.

Proposition 7.3 *Let $p = m$ and let C^{-1} be stable. Then there exist choices for P_x and right coprime factorizations of C and P_x such that in the dual-Youla form of this section:*

$$\text{either } R_0 = Q_0 \quad \text{or} \quad R_0 = S_0$$
$$z(t) = u(t)$$
$$x(t) = r(t)$$

and consequently identification of the dual-Youla parameter is identical to identification according to the indirect method (9) *on the basis of $\hat{Q}$ or* (10) *on the basis of $\hat{S}$.*

Proof: The result follows by choosing $N_c = I$, $D_c = C^{-1}$, $N_x = 0$ and either $D_x = I$ (for the case of Q_0) or $D_x = C^{-1}$ (for the case of S_0), and by substituting this in the appropriate expressions. ∎

This shows that the two closed-loop transfer functions that are related to the input signal u can also be directly estimated in a dual-Youla framework, provided that we restrict attention to the square situation ($p = m$) and to a stably invertible controller.

8 Conclusions

The classical indirect method for closed-loop identification and the recently discussed approach based on the dual-Youla parametrization appear to be closely related to each other. In the situation of a stable controller, the two methods are algebraically equivalent. In the situation of an unstable controller, the dual-Youla method provides models that are guaranteed to be stabilized by the controller, which goes beyond the capabilities of a simple indirect method. Several relations are given between the two approaches, showing that the dual-Youla method is actually a generalization of the classical indirect approach.
Both approaches share the problem that it is not simply possible to control the model order of the identified plant model.

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