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Approximate Identification in view of LQG Feedback Design

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Abstract

Due to the modelling error a model-based controller generally works better with the model than with the modelled plant. This difference between the performances can be made small by selecting a model that is accurate at the closed loop relevant frequencies. In this paper it is shown that an iterative approach of identification and control design can lead to a model that is much better suited for feedback design than a model resulting from a plain open loop identification. In this iteration each identification is performed such that a certain closed loop criterion function is minimised. Each control design step employs the latest identified model to construct an LQG compensator. The performance requirements are gradually increased during the iteration.

1 Introduction

The design of a linear control system is frequently based on a model of the plant under consideration. A model is never an exact description of the system, and due to the model error the controller will not work as well with the plant as it does with the model. Obviously the model error should be tuned towards the control objective. The need of a high accuracy near the cross-over frequency is well recognised. The open question is how large the model error may be at other frequencies.

An ad hoc solution to this problem is obtained e.g. in [1], which addresses a control problem with a prespecified bandwidth. In [3] system identification is studied in combination with the minimum variance control problem. The model-based controller is compared with the optimal controller that would have been designed, if the plant had been known exactly. The difference between these two controllers can be minimized by an optimal experiment and prediction error identification. In [8] the prediction error method is applied as well, but there a desired sensitivity is used as a weighting function for open-loop identification.

Instead of using plant knowledge or desired feedback transfer function matrices, we intend to identify a model that accurately describes the closed-loop relevant system properties for some given compensator. The resulting model is subsequently used to design a new compensator with slightly increased performance requirements. The rationale is that the model will still be a good representation of the plant for a new compensator, provided that it differs not too much from the previous one. Therefore the performance improvement that is achieved for the model is expected to be achieved for the modelled system as well. Next a new identification is carried out in order to obtain a model that accurately describes the system for the new compensator, and the entire procedure is repeated until a satisfactory controller performance is accomplished. In this paper we show that, at least under favourable circumstances, such an iterative scheme can bring about a model for high-performance control design that cannot be obtained from open loop considerations alone. As exposed in [9] such an iterative scheme is actually necessary for high performance control design.

This paper describes the application of the above scenario to identification of an approximate model for LQG feedback design. We utilize the prediction error identification method, [6], and the concept of a performance criterion as introduced in [3]. A closed loop performance criterion is defined and the design variables of the prediction error method are chosen such that this criterion function is actually minimized by the identification procedure. This makes the identification criterion compatible with the LQG control objective. Similar developments have been established in [13] for LQ control design. The LQG objective has also been addressed in [2], but there the identification procedure minimizes a model error that pertains to robust stability rather than to robust performance. The main results of the current paper have been derived in [4] and are exposed in more detail in [5].

The outline of the paper is as follows. In the next section the prediction error identification procedure is summarised. In section 3 we define the closed loop performance criterion of concern. In section 4 we adjust the prediction error method such that this criterion function is actually minimized. Then in section 5 we consider an example in which the iterative scheme is put into practise for a particular LQG control objective. In section 6 we discuss the results and we make some general observations concerning the interplay between identification and control design. The paper ends with a summary and conclusions.

2 Prediction Error Identification

In this section we adopt the relevant aspects of prediction error identification from [6].

Consider a discrete-time representation of a linear, time-invariant SISO system with additive stochastic disturbances

$$\mathcal{S}: \; y(t) = G_0(q)u(t) + v(t) = G_0(q)u(t) + H_0(q)e(t), \qquad (1)$$

where $G_0(q)$ is the deterministic and $H_0(q)$ the stochastic part of the plant; $u(t)$ and $y(t)$ are respectively the input and output at time t and $e(t)$ is discrete white noise with zero mean value; q is the shift operator: $qu(t) = u(t+1)$.

We choose a model set in which the determistic part is parametrized independently from the stochastic part,

$$\mathcal{M}: \; y(t) = G(q,\rho)u(t) + H(q,\eta)\varepsilon(t). \qquad (2)$$

$G(q,\rho)$ and $H(q,\eta)$ are defined analogously to $G_0(q)$ and $H_0(q)$. We do not assume that the true system $\mathcal{S}$ is in the model set $\mathcal{M}$. For notational convenience we write $\theta = [\rho\ \eta]$ for the parameter vector and we introduce

$$T_0(q) = [\, G_0(q) \;\; H_0(q) \,], \quad T(q,\theta) = [\, G(q,\rho) \;\; H(q,\eta) \,]. \qquad (3)$$

The signal $\varepsilon(t)$ in (2) is the one step ahead prediction error. It can be filtered with a stable linear filter $L(q)$, which yields the filtered prediction error $\varepsilon_f(t,\theta) = L(q)\varepsilon(t,\theta)$. For SISO systems filtering the prediction error is equivalent to filtering the input u and the output y with filter $L(q)$. An estimate of θ is obtained by minimizing the quadratic norm of the filtered prediction error with respect to θ

$$\hat{\theta} = \arg\min_{\theta} \frac{1}{N}\sum_{t=0}^{N-1} \varepsilon_f^2(t,\theta) \qquad (4)$$

with N the number of samples. This yields the estimate $T(q,\hat{\theta})$.

In [6] it is shown that under weak conditions the asymptotic parameter estimate is given by

$$\lim_{N\to\infty} \hat{\theta} = \theta^* = \arg\min_{\theta} E\varepsilon_f^2(t,\theta) \;\; \text{w.p. 1}. \qquad (5)$$

This can be given the frequency domain interpretation

$$\rho^* = \arg\min_{\rho} \int_{-\pi}^{\pi} \tilde{T}(e^{i\omega},\rho,\eta^*)\Phi(\omega)\tilde{T}^T(e^{-i\omega},\rho,\eta^*)\frac{|L(e^{i\omega})|^2}{|H(e^{i\omega},\eta^*)|^2}\,d\omega, \qquad (6)$$

$$\eta^* = \arg\min_{\eta} \int_{-\pi}^{\pi} \tilde{T}(e^{i\omega},\rho^*,\eta)\Phi(\omega)\tilde{T}^T(e^{-i\omega},\rho^*,\eta)\frac{|L(e^{i\omega})|^2}{|H(e^{i\omega},\eta)|^2}\,d\omega, \qquad (7)$$

where $\tilde{T}$ is the model error defined as

$$\tilde{T}(q,\theta) = T(q,\theta) - T_0(q) \qquad (8)$$

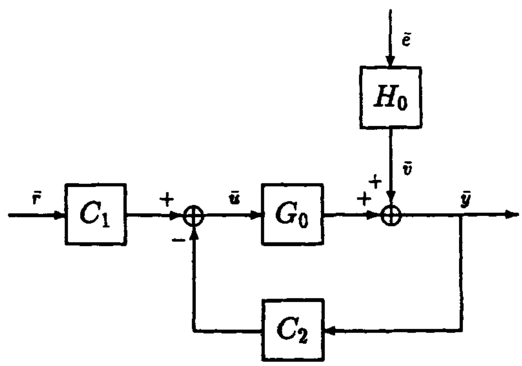

Fig. 1: System in closed loop

and $\Phi(\omega)$ is the spectrum

$$\Phi(\omega) = \begin{bmatrix} \Phi_u(\omega) & \Phi_{ue}(\omega) \\ \Phi_{eu}(\omega) & \Phi_e(\omega) \end{bmatrix}, \tag{9}$$

with $\Phi_u(\omega)$ the spectrum of $u(t)$ and $\Phi_{ue}(\omega)$ the cross-spectrum of $u(t)$ and $e(t)$.

The filter $L(q)$, the input spectrum $\Phi_u(\omega)$ and the cross-spectrum $\Phi_{ue}(\omega)$ dictate the frequency distribution of the model error (see [12] for details). As these design variables are at our disposal, we can choose them such that the model is optimal in view of the intended use. We signify the design variables as

$$\mathcal{D} = \{L(q), \Phi_u(\omega), \Phi_{ue}(\omega)\}. \tag{10}$$

The spectrum $\Phi_u(\omega)$ can be specified by an open loop input design, but a nonzero $\Phi_{ue}(\omega)$ can be realised only by introducing feedback in the identification. Of course one can not assign $\Phi_e(\omega)$ as this would be in contradiction with the nature of a noise. We will not consider the noise model as being a part of the design variables, but as the stochastic part of the model that is to be fitted to the system.

3 A Closed Loop Performance Criterion

In this section we will define a closed loop performance criterion to measure the model's capacity to describe the controlled operation of the plant. We will define the criterion function for a fixed controller, irrespective of the applied control design technique. At a later stage this controller will be determined by means of LQG feedback design.

We consider the closed loop configuration of figure 1, in which the plant is controlled by the fixed two-component controller (C_1, C_2). The feedback system is driven by an external disturbance $\bar{v}$ and an external reference signal $\bar{r}$, which are assumed to be mutually uncorrelated. The bars represent the operational conditions under which the model must appropriately describe the plant.

The output $\bar{y}$ satisfies

$$\bar{y}(t) = \frac{G_0(q)C_1(q)}{1+G_0(q)C_2(q)}\bar{r}(t) + \frac{H_0(q)}{1+G_0(q)C_2(q)}\bar{e}(t). \tag{11}$$

A similar equation can be written down for the model by replacing $G_0(q)$ and $H_0(q)$ with $G(q,\hat{\rho})$ and $H(q,\hat{\eta})$. The model is a good closed loop description of the system if the error terms

$$\frac{G(q,\hat{\rho})C_1(q)}{1+G(q,\hat{\rho})C_2(q)} - \frac{G_0(q)C_1(q)}{1+G_0(q)C_2(q)}$$

and

$$\frac{H(q,\hat{\eta})}{1+G(q,\hat{\rho})C_2(q)} - \frac{H_0(q)}{1+G_0(q)C_2(q)}$$

are small in an H_2 sense. Analogously to the minimum variance example in [6], we define a closed loop performance criterion $J_1(\mathcal{D})$ as 2π times the variance of the difference between the system output $\bar{y}(t)$ and the one obtained using the model, i.e.

$$J_1(\mathcal{D}) =$$

$$= \int_{-\pi}^{\pi} \left(\left| \frac{G(e^{i\omega},\hat{\rho}(\mathcal{D}))C_1(e^{i\omega})}{1+G(e^{i\omega},\hat{\rho}(\mathcal{D}))C_2(e^{i\omega})} - \frac{G_0(e^{i\omega})C_1(e^{i\omega})}{1+G_0(e^{i\omega})C_2(e^{i\omega})} \right|^2 \Phi_{\bar{r}}(\omega) + \right.$$
$$\left. + \left| \frac{H(e^{i\omega},\hat{\eta}(\mathcal{D}))}{1+G(e^{i\omega},\hat{\rho}(\mathcal{D}))C_2(e^{i\omega})} - \frac{H_0(e^{i\omega})}{1+G_0(e^{i\omega})C_2(e^{i\omega})} \right|^2 \Phi_{\bar{e}}(\omega) \right) d\omega, \tag{12}$$

where the argument $\mathcal{D}$ has been added to emphasize the dependency of the identification result on the design variables. This criterion function is small if the closed loop of the model is close to the closed loop of the system in respect of the spectra of the signals $\bar{r}$ and $\bar{e}$ that drive the feedback system. The following proposition gives a useful alternative expression for $J_1(\mathcal{D})$.

Proposition 3.1 ([5]) *The performance criterion $J_1(\mathcal{D})$ satisfies*

$$J_1(\mathcal{D}) =$$

$$= \int_{-\pi}^{\pi} \frac{1}{|1+G(e^{i\omega},\hat{\rho}(\mathcal{D}))C_2(e^{i\omega})|^2} \tilde{T}(e^{i\omega},\hat{\theta}(\mathcal{D}))\bar{\Phi}(\omega)\tilde{T}^T(e^{-i\omega},\hat{\theta}(\mathcal{D}))\, d\omega, \tag{13}$$

where $\tilde{T}(q,\theta)$ is given by equation (8), and according to figure 1 the signal $\bar{u}(t)$ satisfies

$$\bar{u}(t) = \frac{C_1(q)}{1+G_0(q)C_2(q)}\bar{r}(t) - \frac{H_0(q)C_2(q)}{1+G_0(q)C_2(q)}\bar{e}(t) \tag{14}$$

and $\bar{\Phi}(\omega)$ satisfies

$$\bar{\Phi}(\omega) = \begin{bmatrix} \Phi_{\bar{u}}(\omega) & \Phi_{\bar{u}\bar{e}}(\omega) \\ \Phi_{\bar{e}\bar{u}}(\omega) & \Phi_{\bar{e}}(\omega) \end{bmatrix}. \tag{15}$$

We want to choose the design variables such that this performance criterion $J_1(\mathcal{D})$ is minimized. More precisely we want to determine the optimal design variables

$$\mathcal{D}_{1,opt} = \arg\min_{\mathcal{D}} J_1(\mathcal{D}). \tag{16}$$

If the identification is carried out according to this optimal choice of design variables, then the resulting model is an optimal closed loop description of the system.

4 Optimal Identification Strategy

In this section we will derive the optimal choice of design variables such that the closed loop performance criterion $J_1(\mathcal{D})$ defined in (12) is minimized. First we recapitulate some theory presented in [3], where the general scalar criterion $J_G(\mathcal{D})$,

$$J_G(\mathcal{D}) = \int_{-\pi}^{\pi} \tilde{T}(e^{i\omega},\hat{\theta}(\mathcal{D}))C(\omega)\tilde{T}^T(e^{-i\omega},\hat{\theta}(\mathcal{D}))\, d\omega, \tag{17}$$

has been introduced as a measure for the model quality. In here $C(\omega)$ is a 2×2 Hermitian weighting matrix that describes the relative importance of a good fit at different frequencies depending on the intended use of the model.

For the number of samples increasing to infinity, it is shown in [3] that w.p. 1,

$$\lim_{N\to\infty} J_G(\mathcal{D}) = J_B(\mathcal{D}) = \int_{-\pi}^{\pi} \tilde{T}(e^{i\omega},\theta^*(\mathcal{D}))C(\omega)\tilde{T}^T(e^{-i\omega},\theta^*(\mathcal{D}))\, d\omega, \tag{18}$$

where J_B is a bias-contribution to the performance criterion J_G. We consider the optimization problem

$$\mathcal{D}_{opt} = \arg\min_{\mathcal{D}} J_B(\mathcal{D}). \tag{19}$$

In [3] this optimization problem is solved by matching the criterion function (18) to the criterion that is minimised in the identification procedure (6). In this way it is achieved that the identification (6) actually performs the desired minimisation (18). It should be noted that the minimization (7) with respect to the stochastic part does not contribute to the minimisation of the above performance criterion J_B. In [3] this is accounted for by adopting a fixed-noise model such that the parameter vector η is empty and (7) vanishes. Here we formulate a slightly more general result for the case of an independently parametrized noise model.

Theorem 4.1 ([5]) *The optimal choice of design variables (19) with respect to the deterministic part of the model (6) is given by*

$$\frac{|L_{opt}(e^{i\omega})|^2}{|H(e^{i\omega},\eta^*)|^2}\Phi_{u,opt}(\omega) = c\,C_{11}(\omega),\ \frac{|L_{opt}(e^{i\omega})|^2}{|H(e^{i\omega},\eta^*)|^2}\Phi_{ue,opt}(\omega) = c\,C_{12}(\omega), \tag{20}$$

where C_{ij} is the ith row, jth column entry of C and c is an arbitrary positive constant.

We intend to apply this result to the situation of the performance criterion (13). This can however not be done straightforwardly. The reason is that the criterion function J_G in (17) is quadratic in the model error, while J_1 is not a quadratic criterion function as the corresponding weight $C(\omega)$ would depend on $G(q,\hat{\rho}(\mathcal{D}))$. We proceed by first introducing the auxiliary quadratic performance criterion J_2 as

$$J_2(\mathcal{D}) =$$

$$= \int_{-\pi}^{\pi} \frac{1}{|1+\hat{G}_f(e^{i\omega})C_2(e^{i\omega})|^2}\tilde{T}(e^{i\omega},\hat{\theta}(\mathcal{D}))\bar{\Phi}(\omega)\tilde{T}^T(e^{-i\omega},\hat{\theta}(\mathcal{D}))\,d\omega, \tag{21}$$

where $\hat{G}_f$ is some fixed model. This criterion function is quadratic in the model error, with the (constant) weighting matrix $C(\omega)$ given by

$$C(\omega) = \frac{\bar{\Phi}(\omega)}{|1+\hat{G}_f(e^{i\omega})C_2(e^{i\omega})|^2}. \tag{22}$$

So theorem 4.1 can be straightforwardly applied to find

$$\mathcal{D}_{2,opt} = \arg\min_{\mathcal{D}} J_2(\mathcal{D}). \tag{23}$$

Next we define the discrepancies $\delta_1(\omega)$ and $\delta_2(\omega)$ as

$$\delta_1(\omega) = \frac{1}{|1+\hat{G}_f(e^{i\omega})C_2(e^{i\omega})|^2} - \frac{1}{|1+G(e^{i\omega},\hat{\rho}(\mathcal{D}_{1,opt}))C_2(e^{i\omega})|^2}, \tag{24}$$

$$\delta_2(\omega) = \frac{1}{|1+G(e^{i\omega},\hat{\rho}(\mathcal{D}_{2,opt}))C_2(e^{i\omega})|^2} - \frac{1}{|1+\hat{G}_f(e^{i\omega})C_2(e^{i\omega})|^2}, \tag{25}$$

which are of use in the next theorem.

Theorem 4.2 ([5]) *Consider the performance criterion defined by (12). If the number of samples tends to infinity then the choice of design variables*

$$\mathcal{D}_{2,opt} = \begin{cases} L_{opt}(q) &= \frac{H(q,\eta^*)}{1+\hat{G}_f(q)C_2(q)} \\ \Phi_{u,opt}(\omega) &= \Phi_{\bar{u}}(\omega) \\ \Phi_{ue,opt}(\omega) &= \Phi_{\bar{u}\bar{e}}(\omega) \end{cases} \tag{26}$$

converges to the optimal solution $\mathcal{D}_{1,opt}$ if $\delta_1(\omega)$ and $\delta_2(\omega)$ converge to zero, i.e.

$$\lim_{\delta_1,\delta_2\to 0} J_1(\mathcal{D}_{2,opt}) - J_1(\mathcal{D}_{1,opt}) = 0. \tag{27}$$

This means that the choice of the design variables (26) generally is a good choice, and it is even the best possible design (in a quadratic error sense) if both δ_1 and δ_2 vanish. From equation (25) it follows that δ_2 is small if $\hat{G}_f(q)$ is close to $G(q,\hat{\rho}(\mathcal{D}_{2,opt}))$, which is the result of the identification conducted according to theorem 4.2; more specifically, the corresponding sensitivity functions have to be similar. This discrepancy δ_2 can be calculated afterwards. Moreover it can be reduced to an arbitrarily small value by an iterative procedure. In each step of this iteration $\hat{G}_f$ is chosen as the identification result of the previous step. This means that the filter $L_{opt}(q)$ is determined iteratively. From equation (24) it follows that δ_1 is small if $\hat{G}_f(q)$ is close to the (unknown) optimal identification result $G(q,\hat{\rho}(\mathcal{D}_{1,opt}))$. This discrepancy δ_1 cannot be determined precisely, but it is small if for example the modelling error is made sufficiently small, i.e. if both $G(q,\hat{\rho}(\mathcal{D}_{1,opt}))$ and $G(q,\hat{\rho}(\mathcal{D}_{2,opt}))$ (or equivalently $\hat{G}_f$) are close to the real system $G_0(q)$.

Let us now discuss the optimal design given by theorem 4.2. It states that the input spectrum (and the cross-spectrum of noise and input) in the identification experiment should be the same as those in the operational conditions (figure 1), which means identification in closed loop. The data collected under operational conditions have

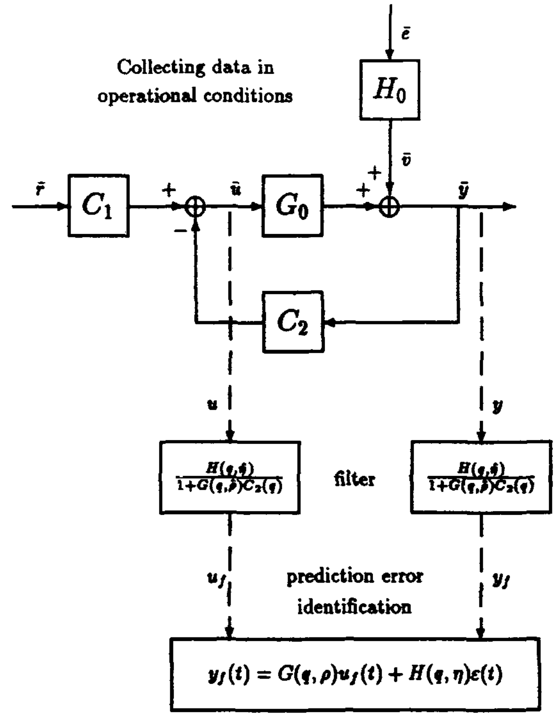

Fig. 2: Optimal identification strategy

to be properly filtered in order to obtain the optimal model. The interpretation of the optimal identification is that it includes a weight at those frequencies where the closed loop of the plant is close to the stability margin ($\bar{u}$ contains much energy) and/or where the closed loop of the model is close to the stability margin ($L_{opt}(q)$ has a large gain).

The complete identification procedure is visualized in figure 2. Notice that no knowledge is required of the external reference signal $\bar{r}(t)$ nor of the noise $\bar{v}(t)$ in order to carry out this identification. We also notice that in the identification procedure no perfect knowledge of the true system $T_0(q)$ is required, which is a very attractive property.

5 Application to LQG Feedback Design

The theory of the previous sections has been developed without making assumptions about a specific controller design method. In the example of this section we will employ one particular controller design procedure, viz. LQG feedback design. We shortly summarise the relevant topics. For a more detailed discussion the reader is referred to e.g. [7].

Consider the continuous time SISO model $\mathcal{M}$ for which a controller has to be designed

$$\mathcal{M}: \begin{array}{rcl} \dot{x} &=& Ax + Bu_m + \Gamma w \\ y_m &=& Cx + v \end{array} \tag{28}$$

where w and v are zero-mean white noises with covariance matrices

$$E\{ww^T\} = W \geq 0,\quad E\{vv^T\} = V > 0,\quad E\{wv^T\} = 0. \tag{29}$$

The signal u_m is the control signal to the model and y_m is the output of the model. Now the LQG problem is to devise a feedback control law which minimizes the cost function

$$J_{LQG} = \lim_{T\to\infty} E\left\{\frac{1}{T}\int_0^T (x^TQx + u_m^TRu_m)\,dt\right\}, \tag{30}$$

with Q a positive semi-definite weighting matrix and R a positive definite weighting matrix.

There are several weighting matrices that we can freely choose. We want to investigate the impact of the identification procedure on the quality of the resulting controller and not the impact of the design weight. Therefore we pragmatically fix the weighting matrices,

$$\Gamma = B,\ W = 1,\ V = c,\ Q = C^T C,\ R = c. \tag{31}$$

Then the LQG criterion function becomes

$$J_{LQG} = \lim_{T\to\infty} E\left\{\frac{1}{T}\int_0^T (y_m^2(t) + cu_m^2(t))\,dt\right\} \tag{32}$$

and it implies that the white noise w is assumed to be additive at the input u_m. The latter is equivalent to stating that a white noise external reference signal r enters at the input. This actually determines the operational conditions in the figures 1 and 2, i.e. $C_1(q) = 1$ and $\bar{r}$ is white noise.

The parameter c is the only design variable that is left and we will use it to establish the performance requirements on the controller. A relatively small value of c gives less weight to u_m in the criterion function, and the output is assumed to be disturbed less, which gives rise to a tighter feedback loop. This will generally also lead to less robustness, even though no LQG controller optimizes robustness at all.

Now we apply the optimal identification procedure derived in the previous sections in combination with this fixed controller design procedure, i.e. we perform an iteration of identification and feedback design. We use low order models and noise–free simulations in order to emphasize the effects due to undermodelling. We compare the outcome of the iteration with the result of a direct open loop identification.

We consider the fifth order system shown in figure 3. In the same figure the result of the open loop identification of a strictly proper third order output error model is given, for a white noise input signal. The low–frequency fit appears to be very good. Next we design an LQG controller for the model, choosing $c = 0.0002$. In figure 4 the resulting closed loops are given of the controller implemented on the model and on the system. It turns out that the controller destabilizes the system! Apparently the model identified in open loop does not describe the relevant closed loop properties of the system sufficiently well.

We now want to identify a third order model that gives an optimal closed loop description of the system, using the identification scheme of figure 2. We do this in an iteration of identification and feedback design. First we design a low–performance controller ($c = 0.0004$) for the model identified in open loop. Then we identify in closed loop with a proper filtering, using the designed controller and the open loop identification result. Next we design a new controller for the resulting model with increased performance requirement ($c = 0.0003$). Then we conduct a new identification and we design a controller with $c = 0.0002$. We repeat this last step till there is no significant change in controller or model.

Altogether four iterations were sufficient to reach the final result. Bode plots of the resulting controllers are shown in figure 5, which displays the increasing control action. Figure 6 reveals that the resulting optimal model has a poor open loop fit. The closed loops of the final controller implemented on the optimal model (designed loop) and on the system are depicted in figure 7. The controller designed for the optimal model gives a satisfactory, stable performance for the system. Finally in figure 8 open loop stepresponses are shown of the true system, of the model identified in open loop and of the optimal model. We remark that the optimal model has a bad open loop step response fit, but it is nevertheless more suited for feedback design than the model identified in open loop.

6 Discussion

In the example of the previous section it has been shown that for LQG controller design the optimal identification scheme of figure 2 yields a model that is superior to a model obtained by a simple open loop identification. This means that a combined iterative approach of identification and controller design can lead to results that are better than those obtained from open loop considerations alone. The iterative aspect is essential, because a model is needed for controller design and knowledge of the controller is needed in order to identify a good model.

The motivation for the applied iterative approach is, as already has been argued, that a model optimal for a certain controller will be close to optimality for a slightly different controller. This explains why the procedure converged in the example of the previous section. However it also means that the procedure might very well diverge if in each iteration the performance requirement is increased too much. For in that case optimality is completely lost for the new controller. Presently it is unknown under what conditions convergence can be guaranteed. In the example of the previous section the controller update has simply been carried out by trial and error.

We now take a closer look at the criteria that are minimized in the identification and the controller design procedure. In the LQG controller design procedure the quadratic criterion J_{LQG} in (32) is minimized. For a high performance controller ($c = 0.0002$ for example) the contribution of $\int y_m^2(t)\,dt$ dominates this criterion function. The external reference signal is white noise so that the LQG controller design procedure actually (approximately) minimizes

$$J_{LQG}(\hat{G}) = \left\|\hat{G}(I + C_2\hat{G})^{-1}\right\|_2. \tag{33}$$

In the identification procedure the quadratic criterion J_1 in (12) is minimized. As the external reference during identification is white noise, this means that the identification procedure minimizes

$$J_1 = \left\|G(I + C_2G)^{-1} - \hat{G}(I + C_2\hat{G})^{-1}\right\|_2. \tag{34}$$

Using the triangle inequality we obtain

$$J_{LQG}(G) = \left\|G(I + C_2G)^{-1}\right\|_2 \le J_{LQG}(\hat{G}) + J_1, \tag{35}$$

which means that the criterion value $J_{LQG}(G)$ is bounded. Moreover, if the model is a good description of the system, $J_{LQG}(G)$ will be close to $J_{LQG}(\hat{G})$, which implies that in that case the controller C_2 is nearly optimal for the system. This topic of matching criteria in identification and control design is further elaborated in [9].

Finally we remark that identification in closed loop may be troublesome if there is noise present in the loop, as is practically always the case. If the noise model is too simple to represent the noise, then the deterministic part of the model can not be estimated consistently, see [10]. There are two ways to tackle this problem. The first approach is to use a sufficiently parametrized noise model (using e.g. Box–Jenkins instead of output error models). The second approach is to "decouple" the deterministic and noise contribution for instance by the two–step procedure proposed in [11].

7 Conclusions

Based on asymptotic results for prediction error identification a scheme has been developed to identify a model that gives an optimal closed loop description of the controlled system under investigation. The procedure consists of data collection in operational conditions and after that the data are filtered properly. The identified model can be used for feedback design. This is carried out in an iterative procedure of identification and controller design. In each iteration step a new model is identified, which is then used to design a new controller for increased performance requirements. In an example the procedure has succesfully been applied to design a high–performance LQG feedback controller. The identification procedure turns out to be superior to straightforward open loop identification. This arises from the fact that the identification minimizes a criterion that is compatible with the LQG objective.

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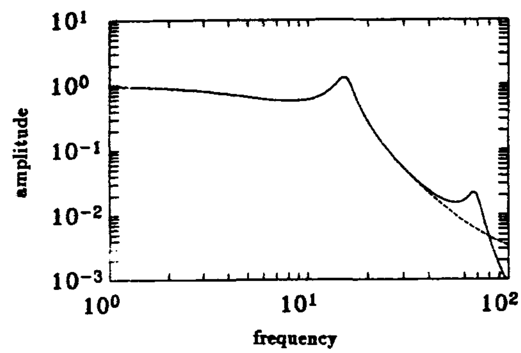

Fig. 3: Bode diagram 5th order system (solid) and 3rd order output–error model (dashed) obtained from open loop experiments

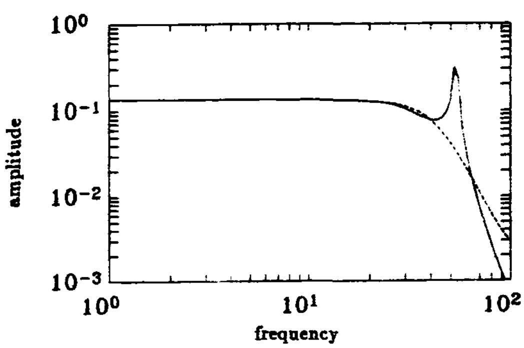

Fig. 4: Bode diagram closed loops of output–error model identified in open loop (dashed) and system (solid) both controlled by a model–based LQG controller

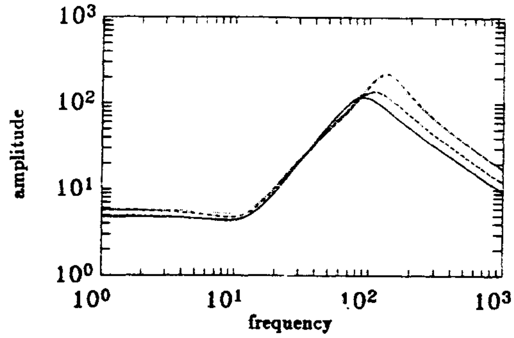

Fig. 5: Bode diagram LQG controllers determined in an iterative way for $c = 0.0004$ (solid), $c = 0.0003$ (dashed) and $c = 0.0002$ (dotted, dash–dotted)

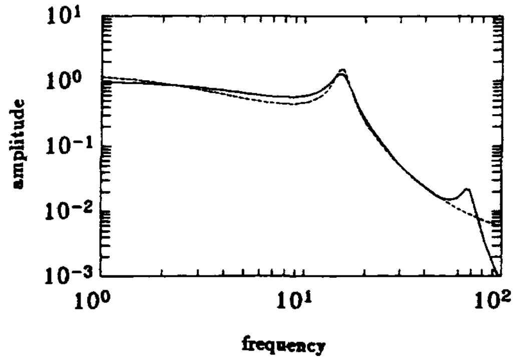

Fig. 6: Bode diagram system (solid) and 3rd order optimal model (dashed)

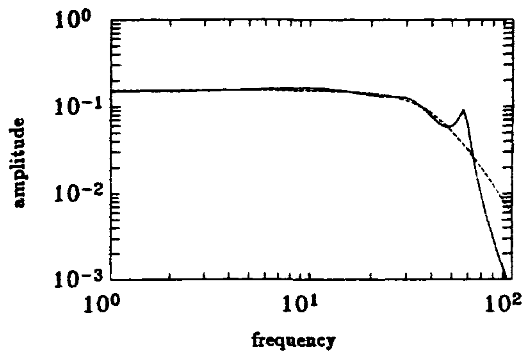

Fig. 7: Bode diagram closed loops of optimal model (dashed) and system (solid) both controlled by a model–based LQG controller

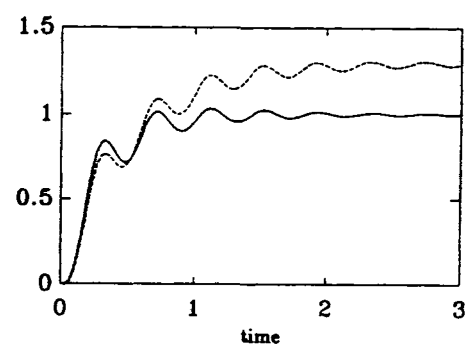

Fig. 8: Step–responses system (solid), output–error model identified in open loop (solid) and optimal model (dashed)