

# Handling risk of uncertainty in model-based production optimization: a robust hierarchical approach

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**Abstract:** Model-based economic optimization of oil production suffers from high levels of uncertainty. The limited knowledge of reservoir model parameters and varying economic conditions are the main contributors of uncertainty. The negative impact of these uncertainties on production strategy increases and becomes profound with time. In this work, a multi-objective optimization problem is formulated which considers both economic and model uncertainties and aims to mitigate the negative effects i.e., risk of these uncertainties on the production strategy. The improved robustness is achieved without heavily compromising the primary objective of economic life-cycle performance. An ensemble of varying oil price scenarios and geological model realizations are used to characterize the economic and geological uncertainty space respectively. The primary objective is an average NPV over these ensembles. As the risk of uncertainty increases with time, the secondary objective is aimed at maximizing the speed of oil production to mitigate risk. This multi-objective optimization is implemented separately with both forms of uncertainty in a hierarchical or lexicographic way.

*Keywords:* Reservoir engineering, robust optimization, uncertainty handling, multi-objective optimization, lexicographic optimization

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## 1. INTRODUCTION

Risk is a broad concept with various perspectives originating from different fields e.g., health, safety, environment etc. From an economic viewpoint, risk in the model-based optimization of water-flooding can be considered as the unpredicted variability or a potential loss of the expected economic objective. As the model-based optimization suffers from high levels of uncertainty see e.g., Van den Hof et al. (2012), the risk of loosing expected economic objective is also high. Risk management involves various approaches to mitigate the negative consequences of uncertainty e.g., Rockafellar (2007). In water-flooding optimization, robustness to the negative impact of uncertainties can be influenced by changing the production or control strategies. However, this improvement should be obtained without loosing sight of the main objective of maximizing the economic life-cycle performance of the water-flooding process.

Uncertainties are present in the reservoir models as well as in economic conditions. The geological uncertainty is profound because of the limited information contents from the measurement and production data about the true values of the model parameters. Furthermore, economic variables such as oil prices, interest rate etc., that are involved in different ways for quantifying the economic value of oil and gas reserves, fluctuate with time and can not be precisely predicted.

The negative consequences of uncertainties on the production and control strategy increase with time and become more profound with the length of the prediction horizon. By increasing the rate of oil production hence improving short-term gains mitigates risk of uncertainty on production strategy. An indirect or ad-hoc way to increase the speed of oil production by changing economic criteria is proposed in Van Essen et al. (2009b), where a hierarchical multi-objective optimization approach is introduced. NPV with a high discount factor is maximized as a secondary objective to improve short-term gains under the condition that the primary objective i.e., an un-discounted NPV stays close to its optimal value. The optimality of the primary objective in this hierarchical approach is ensured by the availability of redundant degrees of freedom (DOF) with un-discounted NPV optimization. This multi-objective optimization does not consider uncertainty which is the core reason for the risk.

This work aims to address the question: can economic and geological uncertainty be explicitly included in such a hierarchical multi-objective optimization framework and will it provide better risk handling? The main focus will be to improve robustness without heavily compromising the primary objective of maximizing economic performance. An ensemble of varying oil price scenarios and geological model realizations are considered as a discrete approximation of economic and geological parametric un-

certainty space respectively. The primary objective is to improve economic performance by maximizing an average un-discounted NPV over the ensemble of varying oil price scenarios with single geological realization and later with the ensemble of geological realizations with fixed economic conditions. It is shown that in both cases, the optimal solution is non-unique, thus leaving the freedom to optimize a secondary objective without heavily comprising the primary objective in a hierarchical optimization framework. As the negative impacts of uncertainty grow with the time-horizon, the secondary objective function maximizes the rate of oil production by using an identical NPV, as in primary objective, but with a high discount factor. The results for this hierarchical multi-objective optimization are shown with both forms of uncertainties.

The paper is organized as follows: In Section 2, the model-based optimization is explained in detail. Handling risk of economic uncertainty is discussed in Section 3 with subsections on optimization of primary objective function and hierarchical optimization with simulation examples. A similar discussion and simulation examples are presented in Section 4 for handling risk of geological uncertainty. Section 5 presents some conclusions of the work.

## 2. MODEL-BASED ECONOMIC OPTIMIZATION

A model-based economic optimization approach has shown better economic life-cycle performance compared to the traditional reactive control strategy e.g., see Brouwer and Jansen (2004) and Jansen et al. (2008). The economic objective i.e., Net Present Value (NPV) in these studies can be mathematically represented as follows:

$$J = \sum_{k=1}^K \left[ \frac{r_o \cdot q_{o,k} - r_w \cdot q_{w,k} - r_{inj} \cdot q_{inj,k}}{(1+b)^{\frac{t_k}{\tau_t}}} \cdot \Delta t_k \right] \quad (1)$$

where  $r_o$ ,  $r_w$  and  $r_{inj}$  are the oil price, the water production cost and the water injection cost in  $[\$/m^3]$  respectively.  $K$  represents the production life-cycle i.e., the total number of time steps  $k$  and  $\Delta t_k$  the time interval of time step  $k$  in  $[days]$ . The term  $b$  is the discount rate for a certain reference time  $\tau_t$ . The terms  $q_{o,k}$ ,  $q_{w,k}$  and  $q_{inj,k}$  represent the total flow rate of produced oil, produced water and injected water at time step  $k$  in  $[m^3/day]$ .

In this work, a gradient-based optimization approach is used where the gradients are obtained by solving a system of adjoint equations e.g., Jansen (2011). The gradient information is then used in a steepest ascent algorithm to iteratively converge to the (possible local) optimum.

## 3. HANDLING RISK WITH ECONOMIC UNCERTAINTY

Economic uncertainty has a time-varying dynamic nature and its negative effect on the production strategy increases with the time horizon. Among other economic uncertain variables in NPV, varying oil prices have the most dominant effect. Hence only oil price scenarios are used to characterize economic uncertainty.

### 3.1 Optimization of the primary objective function

In Van Essen et al. (2009a), a so-called robust optimization (RO) approach is introduced. It uses an ensemble of possible geological realizations to determine an average NPV

over that set of realizations. In this work, RO approach is extended to incorporate the economic uncertainty with a single geological realization. The average NPV defined over the ensemble of varying oil price ensemble can be written as:

$$J_1 = \frac{1}{N_{eco}} \sum_{i=1}^{N_{eco}} J^i \quad (2)$$

where  $N_{eco}$  is the number of oil price realizations in an ensemble. Similar to the case of RO with geological uncertainty, from the formulation of the objective function in (2), calculating the gradient of the average NPV involves a linear operation. Hence, the gradient  $\nabla J_1$  can be computed as:

$$\nabla J_1 = \frac{1}{N_r} \sum_{i=1}^{N_r} \nabla J^i. \quad (3)$$

Here we consider  $J_1$  to represent the primary objective of economic life-cycle performance optimization. One important point to consider here is that due to the linearity of the oil price in the NPV with the certainty of a geological model, the average of individual objective functions from each realization is equal to a single objective function with the average value of all oil price realizations as shown below:

$$\frac{1}{N_{eco}} \sum_{i=1}^{N_{eco}} [J(\mathbf{u}_k, \eta_i)] = J(\mathbf{u}_k, \frac{1}{N_{eco}} \sum_{i=1}^{N_{eco}} [\eta_i]) \quad (4)$$

where  $\mathbf{u}_k$  is the input sequence and  $\eta_i$  is the  $i^{th}$  oil price realization in the ensemble.

### 3.2 Simulation example

All simulation experiments are performed using MRST, see Lie et al. (2012), which is a MATLAB based reservoir simulator. The details of simulation example with objective function (2) are given below:

*Reservoir model and economic data:* As the purpose of this simulation example is to show the effect of economic uncertainty on the optimal strategy, a single model realization of the Standard Egg model (Jansen et al. (2014)) is used. The standard egg model is a three-dimensional realization of a channelized reservoir produced under water flooding conditions with eight water injectors and four producers. The life-cycle of this reservoir model is 3600[days]. The absolute-permeability field and well locations of the model realization are shown in Fig. 1.

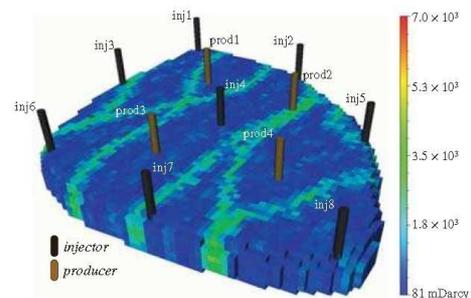


Fig. 1. Permeability field and well locations of the model realization

An un-discounted NPV i.e., with discount factor  $b = 0$  is used. Other economic parameters e.g., water injection cost  $r_{inj}$  and production cost  $r_w$  are chosen and kept fixed at 6  $[\$/m^3]$  and 19  $[\$/m^3]$  respectively. There are various ways to predict the future values of changing oil prices, but for this example a simplified Autoregressive-moving-average model (ARMA) model is used to generate oil price time-series. The ARMA model is shown below:

$$r_{O_k} = a_0 + \sum_{i=1}^6 a_i r_{O_{k-i}} \quad (5)$$

where  $a_i$  are randomly selected coefficients. A total of 10 scenarios i.e.,  $N_{eco} = 10$  with the base oil price of 126  $[\$/m^3]$  are generated as shown with their average value in Fig. 2.

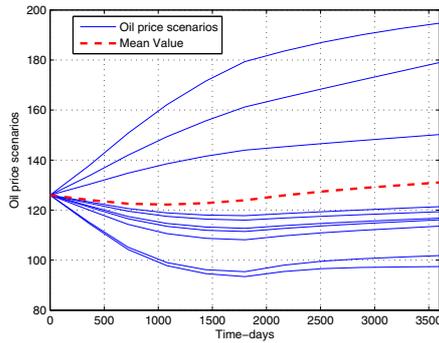


Fig. 2. Oil price scenarios with the mean oil price values

**Control input and control strategies:** The control input  $\mathbf{u}_k$  reflects injection flow rate trajectories for each of the eight injection wells. The minimum and maximum flow rate constraints are 0.2  $[m^3/day]$  and 79.5  $[m^3/day]$ . The production wells operate at a constant bottom-hole pressure of 395[bar]. The control input  $\mathbf{u}_k$  is reparameterized in time using a zero-order-hold scheme with input parameter vector  $\varphi$ . The control input  $\mathbf{u}_k$  is reparameterized into ten time periods of  $t_\varphi$  of 360[days] during which the injection rate is held constant at value  $\varphi_i$ . Thus the input parameter vector  $\varphi$  consists of  $8 \times 10 = 80$  elements.

The extended RO with economic uncertainty is compared to the conventional reactive strategy. In the reactive strategy, water is injected with maximum rate and each production well is simply shut-in when the production is no longer profitable. Here the profitability threshold corresponds to a water-cut of 87%.

### 3.3 Results

The robust optimal control strategy is determined using the same gradient-based optimization procedure as mentioned in Section. 2. A line search is used to find the optimal step size  $\alpha$  along the direction of the greatest ascent. The optimal strategy is applied to the reservoir model with all oil price realizations. The reactive strategy is also applied to the model with the ensemble of oil price scenarios. The time-evolution of NPV with both strategies are compared in Fig. 3. The maximum and minimum values of time-evolution of NPV will form a tube. The first observation is the high width of these tubes that reflects a dominant effect of economic uncertainty on

strategies. Intuitively, optimization should increase the oil production when the oil prices are higher and vice versa. Also considering the averaging property with economic uncertainty given in (4), the NPV time build-up with RO shows expected results. As the mean oil price as shown in Fig. 2 tends to decrease in long-term from the initial value, RO improves production in the early phase.

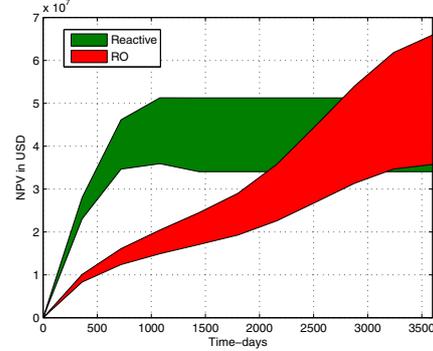


Fig. 3. Max and min (tube) for time-evolution of NPV

To make this point clear, consider another oil price scenario as shown in Fig. 4. Here in this case, the mean value of oil price realizations is increasing. The results obtained from this ensemble are also shown in Fig. 4. It can be seen that as the oil price realizations have an increasing mean value and it is highest at the end of life-cycle time, RO focuses to improve production at the end of life-cycle. The above two cases show a dominant effect of varying oil

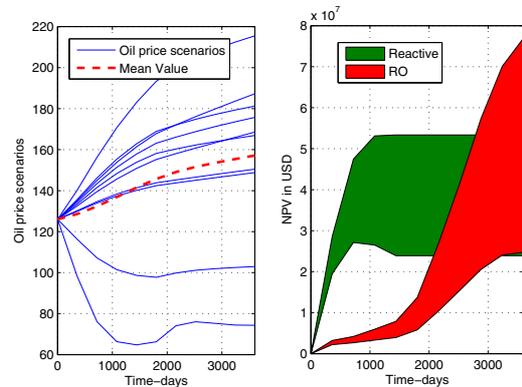


Fig. 4. Oil price scenario with resulting time-evolution of NPV

prices on optimal strategy with the direct dependence of NPV time build-up on chosen ensemble. Another important observation is that though RO incorporates economic uncertainty but it does not aim to reduce the sensitivity of the optimal solution to uncertainty. The uncertainty in the ensemble is mapped to the higher variance of the obtained NPV, which reflects higher risk and hence poor risk handling. As the negative effect of uncertainty grows with time, an indirect way of mitigating risk is to improve short-term gains by adapting the criterion. The main question of incorporating economic uncertainty in the hierarchical framework to handle risk without heavily compromising the primary objective of life-cycle performance will be discussed in the next section. A qualitative discussion on the risk handling of this approach with both forms of uncertainty will be presented at the end.

### 3.4 Hierarchical multi-objective optimization

Multi-objective optimization involves the optimization of more than one (possibly contradictive) objective functions simultaneously. Haimes and Li (1988), describes a hierarchical or lexicographic method that requires to prioritize the multiple objectives, such that optimization of a secondary objective is constrained by the condition that the primary objective should remain close to its optimal value. The structure of hierarchical optimization can be explained as follows:

$$\begin{aligned} & \max_{\mathbf{u}_k} J_2(\mathbf{u}_k), \\ & \text{s.t. } g(\mathbf{u}_k, \mathbf{x}) = 0, \quad \bar{\mathbf{x}}_0 = \mathbf{x}_0, \\ & c(\mathbf{u}_k) \leq 0, \\ & J_1^* - J_1(\mathbf{u}_k) \leq \epsilon. \end{aligned}$$

where  $g(\mathbf{u}_k, \mathbf{x})$  is the system model with states  $\mathbf{x}$  and initial condition  $\mathbf{x}_0$ ,  $c(\mathbf{u}_k)$  are constraints,  $\epsilon$  is a real-value scalar,  $J_1(\mathbf{u}_k)$  is the primary objective function defined and used in Section 3.1, and  $J_1^*$  its value in the optimum. The secondary objective  $J_2(\mathbf{u}_k)$  aims to address the risk of uncertainty by maximizing the speed of oil production, without heavily loosing or compromising  $J_1(\mathbf{u}_k)$ .  $J_2(\mathbf{u}_k)$  is defined as identical to the primary objective  $J_1(\mathbf{u}_k)$  but with the addition of a very high annual discount rate  $b$  of 0.25. This strategy has been introduced in the reservoir domain by Van Essen et al. (2009b) using a single model realization, and followed by Fonseca et al. (2014) who has applied it in a robust hierarchical approach with geological uncertainty and with an ensemble approach to approximate gradients. In this strategy the short-term production is weighted more heavily than the future predictions. In Van Essen et al. (2009b) it has been shown that if the solution of the primary objective optimization is non-unique, there exist degrees of freedom (DOF) such that the multi-objective optimization can be performed with  $\epsilon = 0$ . The information about the DOF can be obtained by the sensitivity relation i.e., the gradients  $\frac{\partial J_1}{\partial \mathbf{u}_k}$ , but at the optimal  $\mathbf{u}_k^*$  the above sensitivity (gradients) from the optimality condition is zero. Hence no information on the possible DOF can be obtained. Second-order derivatives of  $J_1$  with respect to  $\mathbf{u}_k$  are collected in the Hessian matrix  $H = \frac{\partial^2 J_1}{\partial \mathbf{u}_k^2}$ . If  $H$  is negative semi-definite the considered optimal solution  $\mathbf{u}_k^*$  contains redundant DOF.

Unfortunately, no reservoir simulator is currently equipped with second-order derivatives. A forward-difference scheme to approximate the Hessian  $H$  is implemented with primary objective function optimization. In total  $N_u + 1$  simulations (function evaluations) are required to obtain the approximate Hessian matrix  $H$  at a particular optimal solution  $\mathbf{u}_k^*$ , where  $N_u$  is the number of input elements. If Hessian is negative semi-definite, it does not have full rank. The zero singular values  $\sigma_i = 0$  in a singular value decomposition of  $H$  given as follows:

$$H = U \Sigma V^T \quad (6)$$

determines the non-uniqueness of the solution.  $U$  and  $V$  are matrices with orthogonal columns. In this example, as a numerical model and an approximation of the Hessian is used, the condition of redundant DOF as  $\sigma_i = 0$  is relaxed to  $\sigma_i/\sigma_1 < 0.02$ . 67 out of 80 input elements  $N_u$ , are found to be redundant.

As the computation of the Hessian is computationally a very expensive operation, an alternate switching method for hierarchical multi-objective optimization has been proposed in Van Essen et al. (2009b) that does not require an explicit knowledge of redundant DOF. This proposed method and its implementation with the results are discussed in the next sub-sections.

### 3.5 Switching method for multi-objective optimization

An alternative method to solve the hierarchical optimization problem without explicitly calculating the redundant DOF is through the use of a balanced objective function as follows:

$$J_{bal} = \Omega_1 J_1 + \Omega_2 J_2 \quad (7)$$

where  $\Omega_1$  and  $\Omega_2$  are switching function of  $J_1$  and  $J_1^*$  that take on values of 1 and 0 as follows:

$$\Omega_1(J_1) = \begin{cases} 1 & \text{if } J_1 - J_1^* > \epsilon \\ 0 & \text{if } J_1 - J_1^* \leq \epsilon \end{cases} \quad (8a)$$

$$\Omega_2(J_1) = \begin{cases} 0 & \text{if } J_1 - J_1^* > \epsilon \\ 1 & \text{if } J_1 - J_1^* \leq \epsilon \end{cases} \quad (8b)$$

where  $\epsilon$  shows the allowable decrease of  $J_1^*$ . This switching method is implemented with the economic uncertainty in the next sub-section.

### 3.6 Simulation example

The reservoir model realization, the economic data and the control inputs in this example are the same as used in the previous simulation example in Section 3.1. The oil price scenarios shown in Fig. 2 are used for this example.

**Results** The optimal solution of the primary objective function optimization  $\mathbf{u}_k^*$  serves as an initial input guess for the switching multi-objective optimization, the values of primary and secondary objective functions with optimization iteration numbers are given in Fig. 5. The value

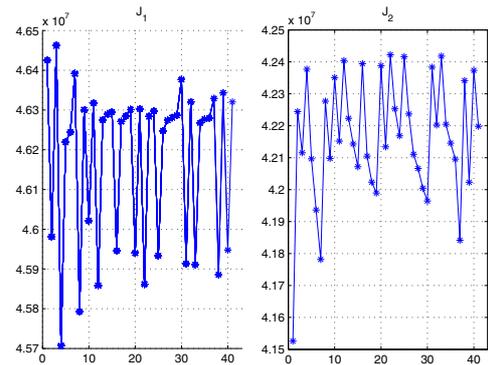


Fig. 5. Primary  $J_1$  and secondary  $J_2$  objectives

of the primary function decreases as the value of secondary function increases and vice versa. As this is a non-convex optimization problem with (possible) many local maxima, it can be observed that the primary objective function obtains another (possibly) local maxima in the second iteration. The selection of  $\epsilon$  is a user's choice and shows how much deviation of  $J_1$  is allowed from the optimal value  $J_1^*$ . In this example chosen  $\epsilon$  shows a decrease of 0.3% from the optimal value of the primary objective function.

The time-evolution of NPV with all the three strategies i.e., RO, reactive and multi-objective optimization are compared in Fig. 6. As the secondary objective is aimed at maximizing the oil production rate, the short-term gains are heavily weighted, which can be observed in the figure. But this improving short-term gains are achieved with compromising only 0.24% on the long-term NPV. At this

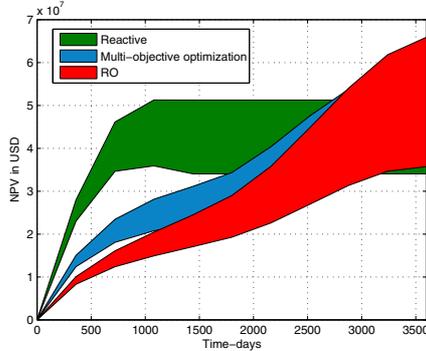


Fig. 6. Max and min (tube) for time-evolution of NPV

stage, it is very difficult to give any convergence proof of the multi-objective optimization. The optimization routine is stopped when a sufficient increase i.e., 1.61% in the secondary objective function is obtained.

#### 4. HANDLING RISK WITH GEOLOGICAL UNCERTAINTY

Reservoir models used in model-based optimization, do not capture all the dynamics of the underlying system, and are equipped with substantial geological uncertainty. In Van Essen et al. (2009a) an averaging approach has been presented with geological uncertainty. An average objective function with an ensemble of geological realizations is considered as the primary objective. In this section we reproduce the simulation example from Van Essen et al. (2009a) to show that on average, RO performs better than all nominal and reactive strategies.

The time-evolution of NPV with all three strategies i.e., nominal (NO), robust and reactive are compared in Fig. 7. The 100 NO strategies from each model is applied to themselves, while the RO and reactive strategies are applied to the set of 100 models. The maximum and minimum values of time-evolution of NPV will form a tube as shown in Fig. 7.

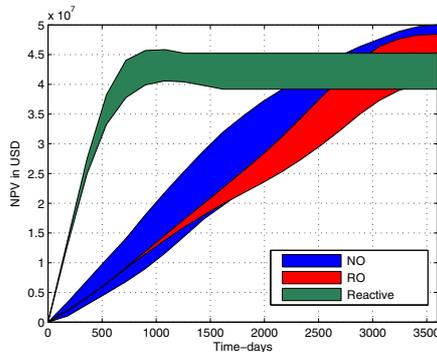


Fig. 7. Time-evolution of NPV for RO with geological uncertainty

The forward-difference scheme is implemented with the primary objective function optimization. Similar to the example of economic uncertainty, the condition of redundant DOF as  $\sigma_i = 0$  is relaxed to  $\sigma_i/\sigma_1 < 0.02$ . In this example, the total control input elements  $N_u$  are 160, where 121 out of 160 input elements are found to be redundant.

The secondary objective  $J_2(\mathbf{u}_k)$  is defined as identical to the primary objective  $J_1(\mathbf{u}_k)$  but with the addition of a very high annual discount rate  $b$  of 0.25. The switching method for hierarchical multi-objective optimization as discussed in the previous section is implemented with the case of geological uncertainty with the results given in next sub-sections. The ensemble of reservoir models, the economic data and the control inputs in this example are the same as used in the previous simulation example, a fixed oil price value, i.e.,  $r_o = 126 [$/m^3]$  is used. The optimal solution of the primary objective function optimization  $\mathbf{u}_k^*$  serves as an initial input guess for the switching multi-objective optimization, the values of primary and secondary objective functions with optimization iteration numbers are given in Fig. 8. The value of the

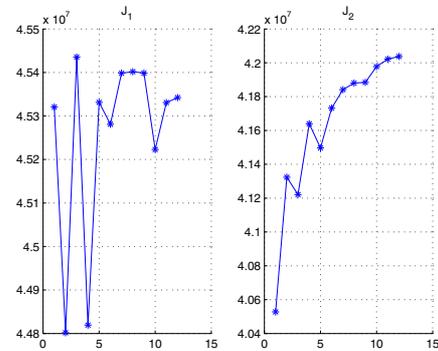


Fig. 8. Primary  $J_1$  and secondary  $J_2$  objectives

primary function decreases as the value of the secondary function increases and vice versa. Similar to the economic uncertainty case, it can be observed that the primary function obtains another (possibly) local maxima in the second iteration, which can be a pure co-incident and is not investigated further. In this example, chosen  $\epsilon$  shows a decrease of 0.3% from the optimal value of the primary objective function.

The optimal input trajectories are applied to each member of the set and the corresponding Probability distribution function (PDF) of the resulting NPV are shown in Fig. 9. The PDF is compared with the PDF resulting from applying RO and reactive strategies to the ensemble. It can be observed that due to the availability of redundant DOF, the average NPV which shows the economic life-cycle performance of the water-flooding process is almost the same with the multi-objective optimization.

The time-evolution of NPV with all the three strategies are compared in Fig. 10. As the secondary objective is aimed at maximizing the oil production rate, the short-term gains are heavily weighted, which can be observed in the figure. But this improving short-term gains are achieved with almost no compromise on the long-term NPV. Similar to the economic uncertainty case, the convergence is difficult to prove. The optimization routine is stopped when a sufficient increase i.e., 3.73% in the secondary

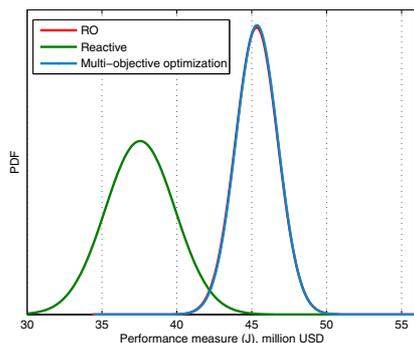


Fig. 9. PDF based on robust, multi-objective and reactive.

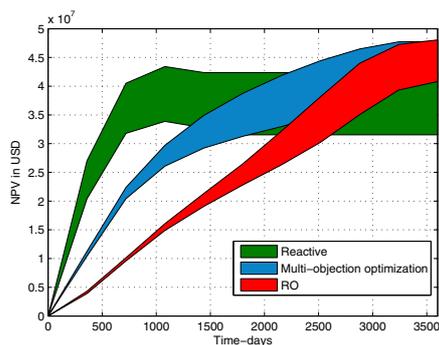


Fig. 10. Time-evolution of NPV for RO hierarchical with geological uncertainty

objective function is obtained.

Changing the economic criteria in such robust hierarchical optimization is still an indirect way of handling risk in model-based optimization. Like RO, the robust hierarchical optimization does incorporate economic and model uncertainties but does not attempt to reduce the sensitivity of the solution to these uncertainties. Better characterization and quantification of risk with the reduction of the sensitivity of solution to uncertainty will provide better risk handling e.g., in Capolei and Jørgensen (2013), the variance of NPV distribution is used to quantify risk and it is reduced with the maximization of mean NPV for the case of geological uncertainty. The economic criteria should not change in the presence of uncertainty but the solution should change to mitigate risk in optimization framework, we addressed the issue and it is shown in Siraj et al. (2015) that the rate of oil production naturally increases hence better risk handling by explicitly incorporating both forms of uncertainties with a sensitivity reduction of solution to uncertainty.

## 5. CONCLUSION

A multi-objective optimization is presented that improves robustness of the control strategy to the uncertainty without compromising the primary objective of model-based optimization i.e., improving life-cycle performance of water-flooding process. The approach explicitly incorporates economic and geological uncertainties by considering an average NPV over an ensemble of varying oil price and model realizations. The rate of oil production is maximized by a secondary objective to mitigate the negative impact of uncertainties. The results with both forms of uncertainty show that with such hierarchical multi-optimization

approach, the secondary objective is optimized without heavily compromising the primary objective.

## 6. ACKNOWLEDGEMENTS

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