Data-driven model learning in linear dynamic networks

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Introduction – dynamic networks

Decentralized process control

Smart power grid

Autonomous driving

Brain network

Hydrocarbon reservoirs

www.envidia.com

P. Hagmann et al. (2008)

Mansoori (2014)
Introduction

Overall trend:

• (Large-scale) interconnected systems
• With hybrid dynamics
• Distributed / multi-agent type monitoring, control and optimization problems
• Data is “everywhere”, big data era
• Model-based operations require accurate/relevant models
• \(\rightarrow\) Learning models from data (including physical insights when available)
Introduction

Distributed / multi-agent control:

With both physical and communication links between systems $G_i$ and controllers $C_i$

How to address data-driven modelling problems in such a setting?
Introduction

The classical (multivariable) identification problems[^1]:

Identify a model of $G$ on the basis of measured signals $u, y$ (and possibly $r$), focusing on continuous LTI dynamics.

We have to move from a simple and fixed configuration to deal with *structure* in the problem.

[^1]: Ljung (1999), Söderström and Stoica (1989), Pintelon and Schoukens (2012)
Early contributors

**Topology detection**: Materassi, Innocenti, Salapaka, Yuan, Stan, Warnick, Goncalves, Sanandaji, Vincent, Wakin, Chiuso, Pillonetto
exploring Granger causality, Bayesian networks, Wiener filters

Subspace algorithms for **spatially distributed systems** with
identical modules (Fraanje, Verhaegen, Werner), or
non-identical ones (Torres, van Wingerden, Verhaegen, Sarwar, Salapaka, Haber)

Here: focus on **structural aspects** in identification setups.
Contents

- Introduction and motivation
- How to model a dynamic network?
- Single module identification
- Global network identification
- Physical networks
- Extensions - Discussion
Dynamic networks for data-driven modeling
Network models

D. Materassi and M.V. Salapaka (2012)
R.N. Mantegna (1999)

www.momo.cs.okayama-u.ac.jp
J.C. Willems (2007)
D. Koller and N. Friedman (2009)

E.A. Carara and F.G. Moraes (2008)
P.E. Paré et al (2013)

X.Cheng (2019)
Network models

**State space representation** \([1]\)

**Module representation** \([2]\)

[1] Goncalves, Warnick, Sandberg, Yeung, Yuan, Scherpen, ...

[2] VdH, Dankers, Goncalves, Warnick, Gevers, Bazanella, Hendrickx, Materassi, Weerts, ...
Dynamic network setup

$G_{76}$  module
$r_i$  external excitation
$v_i$  process noise
$w_i$  node signal
Dynamic network setup

- **$G_{76}$** module
- $r_i$ external excitation
- $v_i$ process noise
- $w_i$ node signal
Dynamic network setup

$G_{76}$ module
$r_i$ external excitation
$v_i$ process noise
$w_i$ node signal
Dynamic network setup

- $G_{76}$ module
- $r_i$ external excitation
- $v_i$ process noise
- $w_i$ node signal
Dynamic network setup

\[ G_{76} \] module

\[ r_i \] external excitation

\[ v_i \] process noise

\[ w_i \] node signal
Dynamic network setup

Assumptions:
- Total of $L$ nodes
- Network is well-posed and stable
- Modules are dynamic LTI, may be unstable
- Disturbances are stationary stochastic and can be correlated

\[
\begin{bmatrix}
    w_1 \\
    w_2 \\
    \vdots \\
    w_L
\end{bmatrix} =
\begin{bmatrix}
    0 & G_{12}^0 & \cdots & G_{1L}^0 \\
    G_{21}^0 & 0 & \cdots & G_{2L}^0 \\
    \vdots & \vdots & \ddots & \vdots \\
    G_{L1}^0 & G_{L2}^0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
    w_1 \\
    w_2 \\
    \vdots \\
    w_L
\end{bmatrix} + R^0
\begin{bmatrix}
    r_1 \\
    r_2 \\
    \vdots \\
    r_K
\end{bmatrix} +
\begin{bmatrix}
    v_1 \\
    v_2 \\
    \vdots \\
    v_L
\end{bmatrix}
\]

\[
w(t) = G^0(q)w(t) + R^0(q)r(t) + v(t)
\]
Dynamic network setup

Setup covers the situation of bilaterally coupled (physical) systems:
Dynamic network setup

Many new data-driven modeling questions can be formulated

Measured time series:
\[ \{w_i(t)\}_{i=1,...,L}; \quad \{r_j(t)\}_{j=1,...,K} \]
Model learning problems

Under which conditions can we estimate the topology and/or dynamics of the full network?
How/when can we learn a local module from data (with known/unknown network topology)? Which signals to measure?
Model learning problems

Where to optimally locate sensors and actuators?
Model learning problems

Same questions for a subnetwork
Model learning problems

How can we benefit from known modules?
Model learning problems

Fault detection and diagnosis; detect/handle nonlinear elements
Model learning problems

Can we distribute the computations?
Dynamic network setup

Identification of a local module (known topology)
Identification of the full network
Topology estimation
Identifiability
Sensor and excitation allocation
Fault detection
User prior knowledge of modules
Distributed identification
Scalable algorithms

Many new data-driven modeling questions can be formulated

Measured time series:
\[ \{w_i(t)\}_{i=1,...,L}; \ \{r_j(t)\}_{j=1,...,K} \]
Dynamic network setup - graph

Nodes are vertices; modules/links are edges

Extended graph:
including the external signals and disturbance correlations
Application: Networks of (damped) oscillators

- Power systems, vehicle platoons, thermal building dynamics, ...
- Spatially distributed
- Bilaterally coupled
- No central coordination $\implies$ local identification problems
Decentralized MPC
2 interconnected MPC loops

Target:
Identify interaction dynamics
\(G_{21}, G_{12}\)

Addressed by Gudi & Rawlings (2006) for the situation \(G_{12} = 0\) (no cycles)
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Single module identification
Single module identification

For a network with known topology:

- Identify $G_{21}^0$ on the basis of measured signals
- Which signals to measure? Preference for local measurements
- When is there enough excitation / data informativity?
Naïve approach: identify based on $w_1$ and $w_2$: in general does not work.
Single module identification

Identifying $G_{21}^0$ is part of a 4-input, 1-output problem

If noises $v_k$ are correlated it may even be part of a MIMO problem
Single module identification

Input signals will be correlated:
similar as in a closed-loop situation

What is required for
identifiability / data informativity?

Ability to distinguish between models
independent of id-method

Information content of signals
dependent on id-method

Identifying \( G_{21}^0 \) is part of a
4-input, 1-output problem
Single module identification

Identifiying $G_{21}^0$ is part of a 4-input, 1-output problem

Generic identifiability:

All parallel paths, and loops around the output, plus input $w_1$ should have an independent external signal $r$ or $v$

[1] Weerts et al., Automatica 2018, CDC 2018
Single module identification

Which node signals to measure?
Dependent on

- $v$ signals uncorrelated or not
- Excitation conditions satisfied through $r$- and/or $v$-signals

Typical solution:
- One additional measured signal for each parallel path/loop
- Additional signals if excitation is through $v$ signals
- Variation in available algorithms / options
Single module identification

one signal per parallel path/loop:
With a 3-input, 1 output model we can consistently identify $G_{21}^0$

When excitation is through disturbance signals $\nu$:

- dealing with confounding variables, $^{[1][2]}$ i.e. correlated disturbances on inputs and outputs
- can be addressed by adding inputs/outputs to the estimation problem $^{[3]}$

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[3] PVdH et al, CDC 2019
Single module identification

Typical solution:

- MISO (sometimes MIMO) estimation problem
- to be solved by any (closed-loop) identification algorithm, e.g. direct/indirect method
Machine learning in local module identification

- MISO identification with all modules parameterized
- Brings in two major problems:
  - Large number of parameters to estimate
  - Model order selection step for each module (CV, AIC, BIC)
- For 5 modules, combinations = 244,140,625
  - Increases variance
  - Computationally challenging
- We need only the target module. No NUISANCE!
Machine learning in local module identification

### Strategy

- **Parametric model**: $G_{ji}(\theta)$
- **Gaussian process + TC Kernel**: $G_j(\theta)$

### Model

- $g_{ji}(\theta)$
- $s_j \sim \mathcal{N}(0, \lambda_j K_{\beta j})$
- $s_{k_1} \sim \mathcal{N}(0, \lambda_{k_1} K_{\beta k_1})$
- $s_{k_p} \sim \mathcal{N}(0, \lambda_{k_p} K_{\beta k_p})$

Maximize marginal likelihood of output data: 

$$
\hat{\eta} = \text{argmax } p(w_j; \eta) \\
\eta := [\theta \ \lambda_j \ \lambda_{k_1} \ ... \ \lambda_{k_p} \ \beta_j \ \beta_{k_1} \ ... \ \beta_{k_p} \ \sigma_j^2]^T
$$

- smaller no. of parameters
- simpler model order selection step
- scalable to large dynamic networks
- simpler optimization problems to estimate parameters

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Numerical simulation

- Identify $G_{31}$ given data
- 50 independent MC simulation
- Data = 500
Summary single module identification

- Path-based conditions for **network identifiability** (where to excite?)

- Graph tools for checking conditions

- Degrees of freedom in selection of measured signals – sensor selection

- Methods for **consistent** and **minimum variance** module estimation, and effective (scalable) algorithms

- A priori known modules can be accounted for
Contents

• Introduction and motivation
• How to model a dynamic network?
• Single module identification
• Global network identification
• Diffusively coupled physical networks
• Extensions - Discussion
Under which conditions can we estimate the topology and/or dynamics of the full network?
Network identifiability

Question: Can different dynamic networks be distinguished from each other from measured signals $w$, $r$?
Network identifiability

The identifiability problem:

The network model:

\[ w(t) = G(q)w(t) + R(q)r(t) + \underbrace{H(q)e(t)}_{v(t)} \]

can be transformed with any rational \( P(q) \):

\[ P(q)w(t) = P(q)\{G(q)w(t) + R(q)r(t) + H(q)e(t)\} \]

to an equivalent model:

\[ w(t) = \tilde{G}(q)w(t) + \tilde{R}(q)r(t) + \tilde{H}(q)e(t) \]

Nonuniqueness, unless there are structural constraints on \( G, R, H \).
Network identifiability

Consider a network model set:

\[ \mathcal{M} = \{ (G(\theta), R(\theta), H(\theta)) \}_{\theta \in \Theta} \]

representing structural constraints on the considered models:

• modules that are fixed and/or zero (topology)
• locations of excitation signals
• disturbance correlation

Generic identifiability of \( \mathcal{M} \):

- There do not exist distinct equivalent models
- for almost all models in the set.

[1] Weerts et al., SYSID2015; Weerts et al., Automatica, March 2018;
Example 5-node network

Conditions for identifiability: rank conditions on transfer function

For the generic case, the rank can be calculated by a graph-based condition\[1,2\]:

**Generic rank = number of vertex-disjoint paths**

- 2 vertex-disjoint paths $\rightarrow$ full row rank 2

The rank condition has to be checked for all nodes.

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\[1\] Van der Woude, 1991
\[2\] Hendrickx, Gevers & Bazanella, CDC 2017, TAC 2019
Synthesis solution for network identifiability

Allocating external signals for generic identifiability:

1. Cover the graph of the network model set by a set of **disjoint pseudo-trees**
   - Pseudo-trees:
     - Tree with root in green
     - Cycle with outgoing trees; Any node in cycle is root
   - Edges are **disjoint** and all out-neighbours of a node are in the same pseudo-tree

2. Assign an independent external signal (\( r \) or \( e \)) at a root of each pseudo-tree.
   - This guarantees generic identifiability of the model set.

Where to allocate external excitations for network identifiability?

All indicated modules are parametrized

Two disjoint pseudo-trees
Two independent excitations guarantee generic network identifiability

Where to allocate external excitations for network identifiability?

- Nodes are signals $w$ and external signals $(r, e)$ that are input to parametrized link.
- Known (nonparametrized) links do not need to be covered.

Summary identifiability of full network

Identifiability of network model sets is determined by

- Presence and location of external signals, and
- Correlation of disturbances
- Topology of parametrized modules

- Graphic-based tool for synthesizing allocation of external signals

Extensions:
- Situations where not all node signals are measured [1]

Algorithms for identification of full network

(Prediction error) identification methods will typically lead to large-scale non-convex optimization problems

Convex relaxation algorithms are being developed\(^1\) as well as machine learning tools

\(^1\) Weerts, Galrinho et al., SYSID 2018
Topology identification

- Topology resulting from full dynamic model

- Alternative: non-parametric models (Wiener filters \([1]\)) or kernel-based approaches \([2][3]\)

- Modeling module dynamics by Gaussian processes,
  
  kernel with 2 parameters for each dynamic module

- Optimizing likelihood of the data as function of parameters and topology:

\[
p(\{w(t)\}_{t=1}^{N} | \theta, \mathcal{G})
\]

- Forward-backward search over topologies + empirical Bayes (EM) for parameters

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[3] Shi, Bottegal, PVdH, ECC 2019
Topology identification

50 MC realizations of network with 6 nodes.

[1] Shi, Bottegal, PVdH, ECC 2019
Neurodynamic effect of listening to Mozart music

Identifying changes in network connections in the brain, after intensely listening for one week

Figure 2: Spatial maps of the 30 active brain networks found through the ICA decomposition. Each image consists of 3 relevant horizontal slices of the brain, where the spatial map is indicated by the red color scale.

Algorithms for identification of full network

Particular feature for larger networks:

Modeling disturbances as a **reduced rank process**: (cf dynamic factor analysis\(^1\))

Consequences for **estimation**\(^3\):

- Optimization becomes a **constrained quadratic problem** with ML properties for Gaussian noise
- Reworked Cramer Rao lower bound
- Some parameters can be estimated variance free \(\rightarrow\) regularization effect

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\(^1\) Deistler et al., EJC, 2010.

\(^2\) Zorzi and Chiuso, Automatica 2017.

\(^3\) Weerts et al., Automatica dec 2018.
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Physical networks
Back to the basics of physical interconnections

In connecting physical systems, there is often no predetermined direction of information \[1\]

Example: resistor / spring connection in electrical / mechanical system:

\[ I = \frac{1}{R} (V_1 - V_2) \]

\[ F = K (x_1 - x_2) \]

Difference of node signals drives the interaction: **diffusive coupling**

Diffusively coupled physical network

Equation for node $j$:

$$M_j \ddot{w}_j(t) + D_{j0} \dot{w}_j(t) + \sum_{k \neq j} D_{jk} (\dot{w}_j(t) - \dot{w}_k(t)) + K_{j0} w_j(t) + \sum_{k \neq j} K_{jk} (w_j(t) - w_k(t)) = u_j(t),$$
Mass-spring-damper system

- Masses $M_j$
- Springs $K_{jk}$
- Dampers $D_{jk}$
- Input $u_j$

\[
\begin{bmatrix}
M_1 & M_2 & M_3
\end{bmatrix}
\begin{bmatrix}
\dot{\ddot{w}}_1 \\
\dot{\ddot{w}}_2 \\
\dot{\ddot{w}}_3
\end{bmatrix}
+ \begin{bmatrix}
0 & D_{20} & 0 \\
0 & 0 & 0 \\
-D_{13} & -D_{23} & D_{12} + D_{23}
\end{bmatrix}
\begin{bmatrix}
\ddot{w}_1 \\
\ddot{w}_2 \\
\ddot{w}_3
\end{bmatrix}
+ \begin{bmatrix}
K_{12} + K_{13} & -K_{12} & -K_{13} \\
-K_{12} & K_{12} & 0 \\
-K_{13} & 0 & K_{13}
\end{bmatrix}
\begin{bmatrix}
w_1 \\
w_2 \\
w_3
\end{bmatrix}
= \begin{bmatrix}
0 \\
u_2
\end{bmatrix}
\]

\[
\begin{bmatrix}
A(p) & B(p)
\end{bmatrix}
\begin{bmatrix}
w(t)
\end{bmatrix}
= \begin{bmatrix}
u(t)
\end{bmatrix}
\]

$A(p)$, $B(p)$ polynomial

$p = \frac{d}{dt}$
Mass-spring-damper system

\[
\begin{bmatrix}
A(p) + B(p)
\end{bmatrix} w(t) = u(t) \quad A(p), B(p) \text{ polynomial}
\]

\[
\begin{bmatrix}
Q(p) - P(p)
\end{bmatrix} w(t) = u(t)
\quad \text{diagonal hollow & symmetric}
\]

This fully fits in the earlier module representation:

\[
w(t) = Gw(t) + \underbrace{Rr(t) + He(t)}_{Q^{-1}(p)u(t)}
\]

with the additional condition that:

\[
G(p) = Q(p)^{-1}P(p) \quad Q(p), P(p) \text{ polynomial}
\]

\[
P(p) \text{ symmetric, } Q(p) \text{ diagonal}
\]
Module representation

Consequences for node interactions:

• Node interactions come in pairs of modules
• Where numerators are the same

Framework for network identification remains the same

• Symmetry can simply be incorporated in identification
Local network identification

Identification of **one** physical interconnection

Identification of **two** modules $G_{jk}$ and $G_{kj}$
Immersion conditions

For simultaneously identifying two modules in one interconnection:

The parallel path and loops-around-the-output condition, now simplifies to:

Measuring/exciting all neighbouring nodes of $w_2$ and $w_3$ leads to a solution

E.E.M. Kivits et al., CDC 2019.
Summary physical networks

- Physical networks fit within the module framework (special case)
  - no restriction to second order equations
- Earlier identification framework can be utilized
- Local identification is well-addressed (and stays really local)
- Framework is fit for representing **cyber-physical systems**
  (combining physical bi-directional links, and cyber uni-directional links).
Extensions - Discussion
Extensions - Discussion

• **Including sensor noise** [1]
  - Errors-in-variabels problems can be more easily handled in a network setting

• **Distributed estimation (MISO models)** [2]
  - Communication constraints between different agents
  - Recursive (distributed) estimator converges to global optimizer (more slowly)

• **Experiment design** [3],[4]
  - Design of least costly experiments

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Summary

• **Dynamic network modeling:**
  intriguing research topic with many open questions
• The (centralized) LTI framework is only just the beginning
• Further move towards data-aspects related to distributed control
• and large-scale aspects
• and bring it to real-life applications
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Further reading


Papers available at www.pvandenhof.nl
The end