

# Data-driven analytics and model learning in interconnected systems

Paul Van den Hof

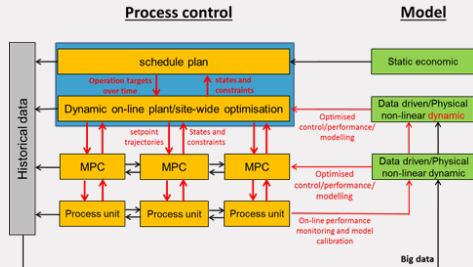
Biomedical imaging and modeling cluster  
TU/e Biomedical Engineering Dept  
23 October 2023

[www.sysdynet.eu](http://www.sysdynet.eu)  
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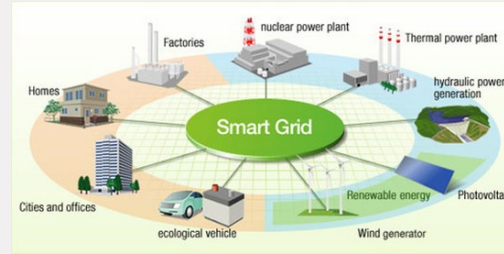


# Introduction – dynamic networks

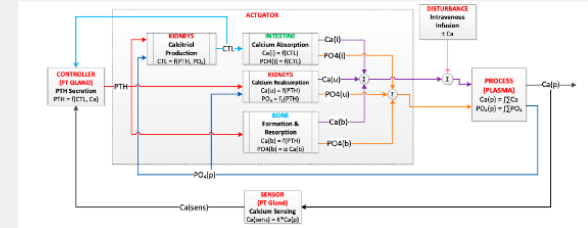
## Decentralized process control



## Smart power grid

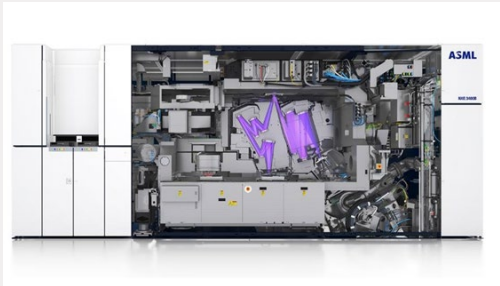


## Physiological models

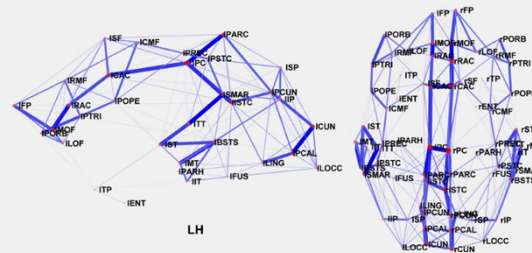


Christie, Achenie and Ogunnaike (2014)

## Complex machines

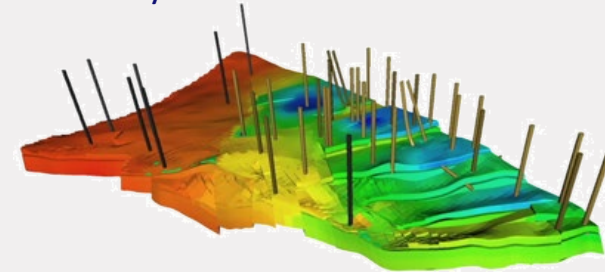


## Brain network



P. Hagmann et al. (2008)

## Hydrocarbon reservoirs



Mansoori (2014)

# Introduction

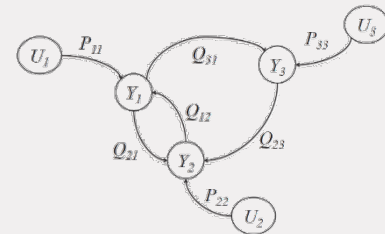
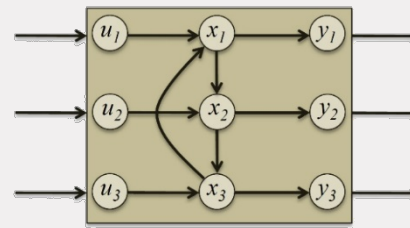
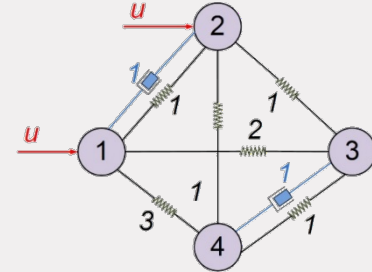
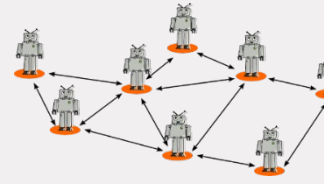
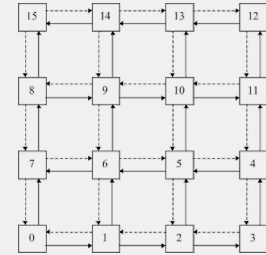
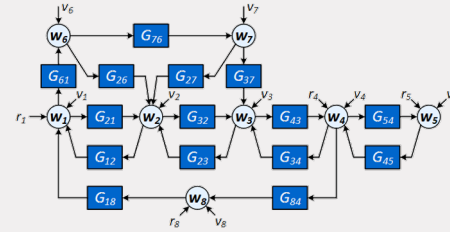
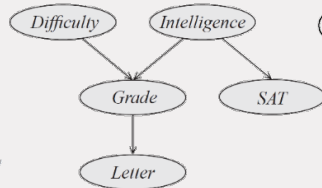
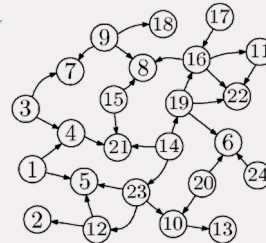
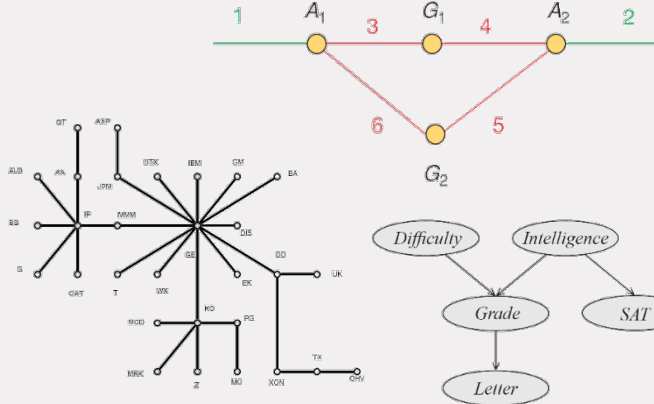
## Overall trend:

- (Large-scale) interconnected dynamic systems
- Many opportunities for sensing / collecting data
- The scope of control/optimization/diagnostics enlarges (larger systems)
- The infrastructure for control/optimization/diagnostics becomes distributed: local actions (multi agents)
- Data is “everywhere”, big data era, AI/machine learning tools
- Model-based operations require accurate/relevant models
- → **Learning and validating models/actions from data**
- → **Exploiting the available physical information (topology, dynamics, disturbances)**

# Network models

# Network models

- scalable, describing the physics
- dynamic elements with cause-effect
- handling feedback loops (cycles)
- combining physical and cyber components
- centered around measured signals
- allow disturbances and probing signals



D. Materassi and M.V. Salapaka (2012)

www.momo.cs.okayama-u.ac.jp  
J.C. Willems (2007)

E.A. Carara and F.G. Moraes (2008)

P.M.J. Van den Hof et al (2013)

R.N. Mantegna (1999)

D. Koller and N. Friedman (2009)

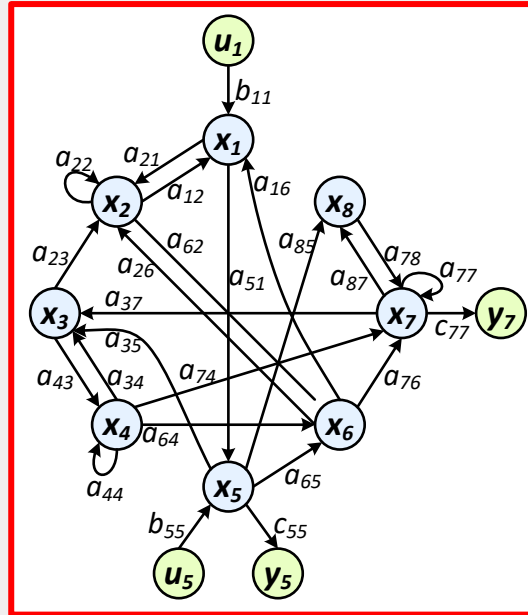
P.E. Paré et al (2013)

X.Cheng (2019)

E. Yeung et al (2010)



# Network models



State space representation

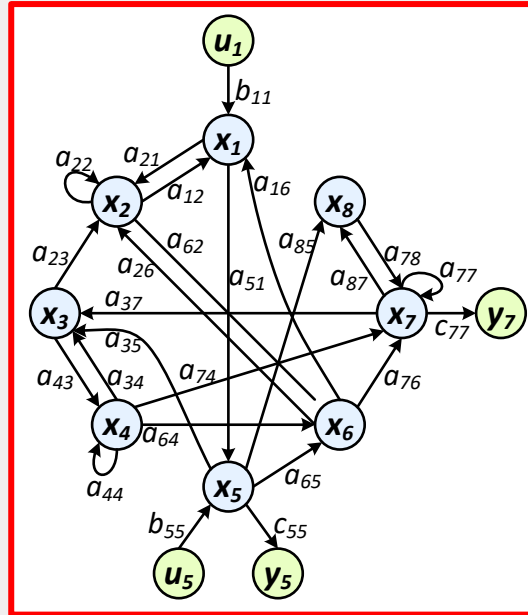
$$\begin{aligned}x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k)\end{aligned}$$

- States as **nodes** in a (directed) graph
- State transitions (1 step in time) reflected by  $a_{ij}$
- Transitions are encoded in links
- Ultimate break-down of system structure
- Actuation ( $u$ ) and sensing ( $y$ ) reflected by separate links

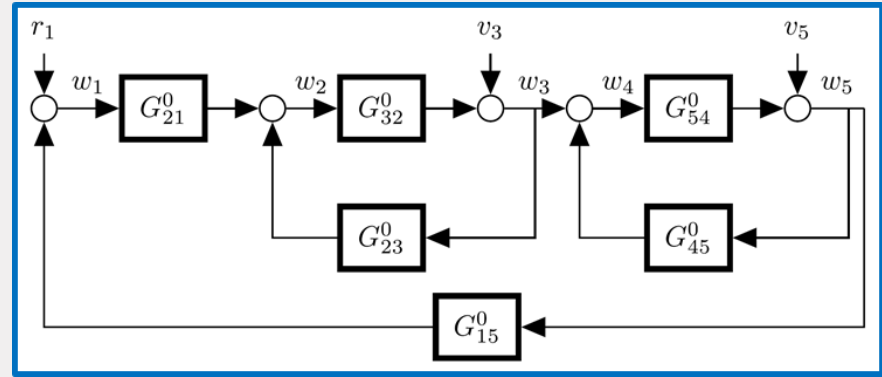
For data analytics problems:

- Lump unmeasured states in dynamic **modules**

# Network models



State space representation [1]

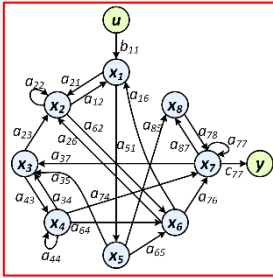


Module representation [2]

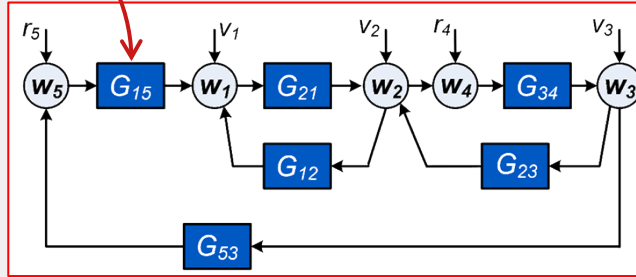
[1] Goncalves, Warnick, Sandberg, Yeung, Yuan, Scherpen,...

[2] VdH, Dankers, Goncalves, Warnick, Gevers, Bazanella, Hendrickx, Materassi, Weerts,...

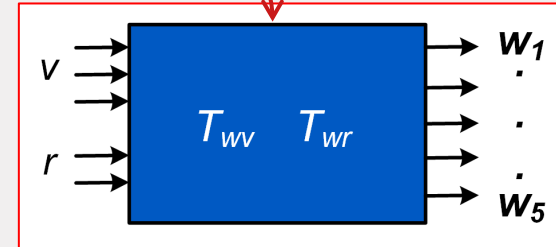
# Dynamic network models - zooming



Increasing level of detail

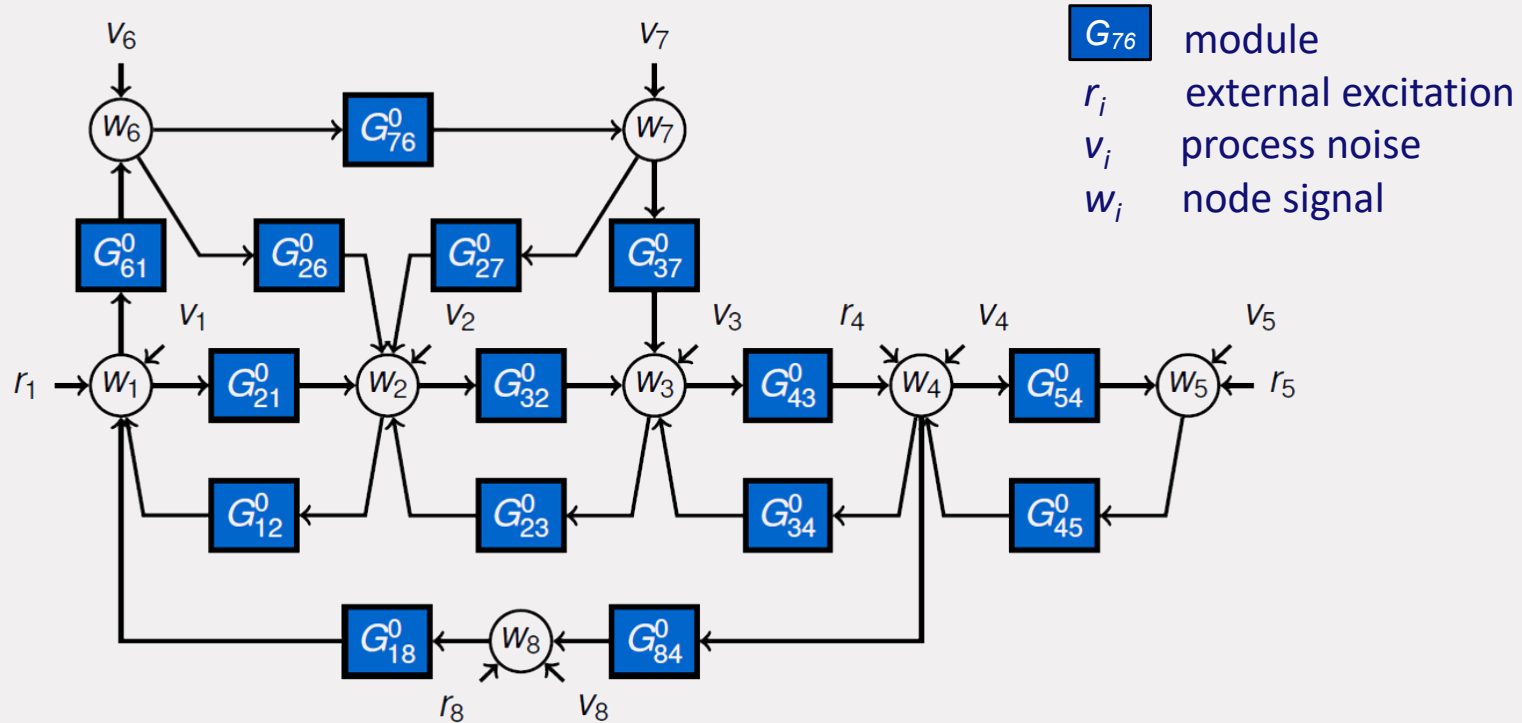


Decreasing structural information

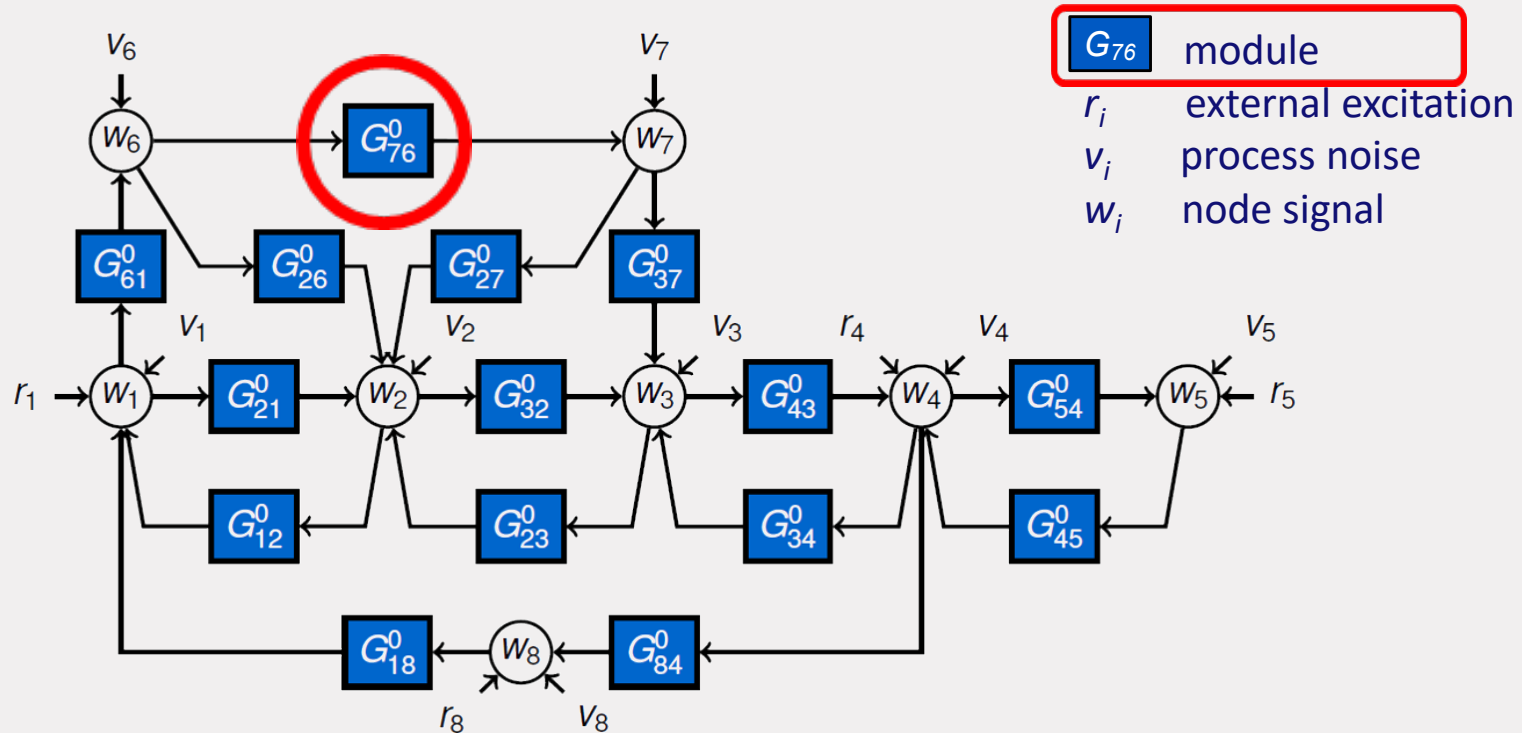




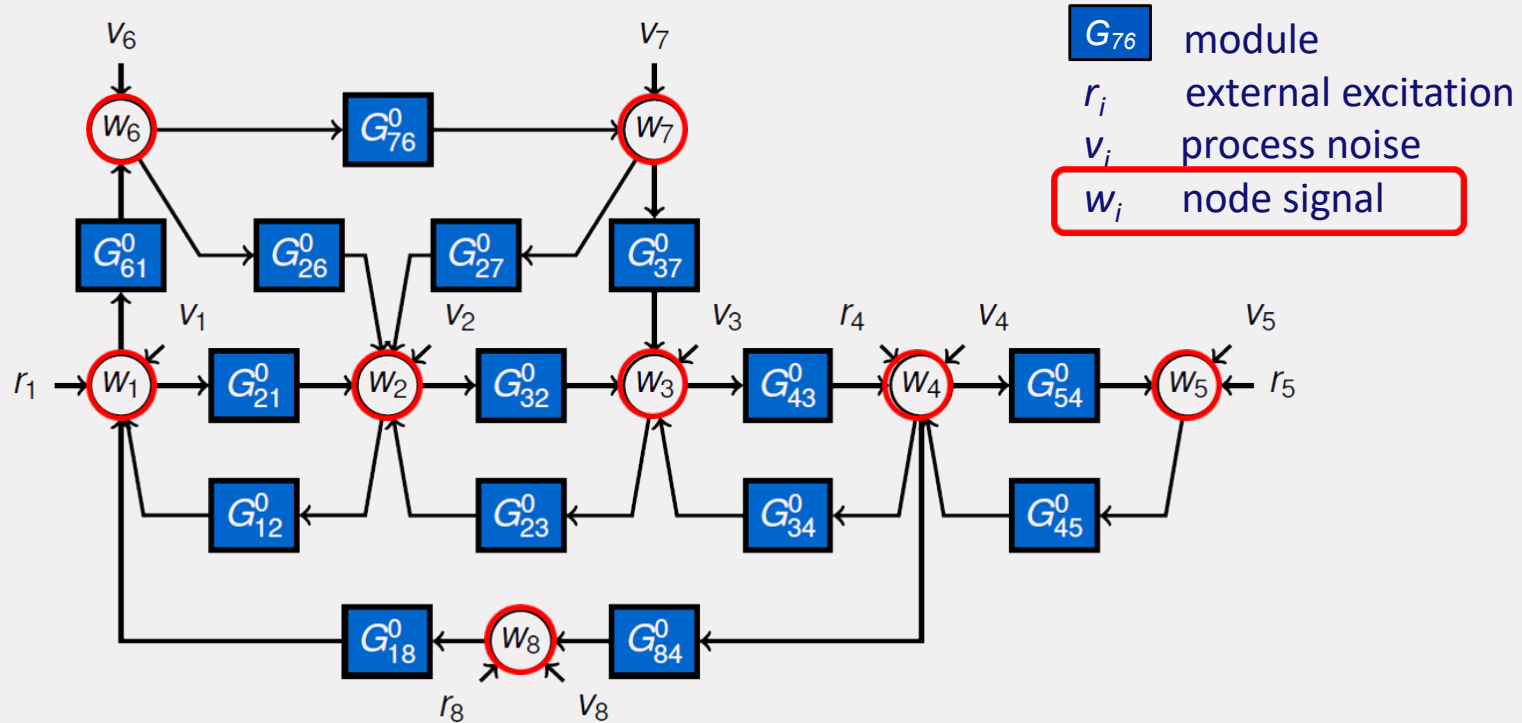
# Dynamic network setup – Module framework



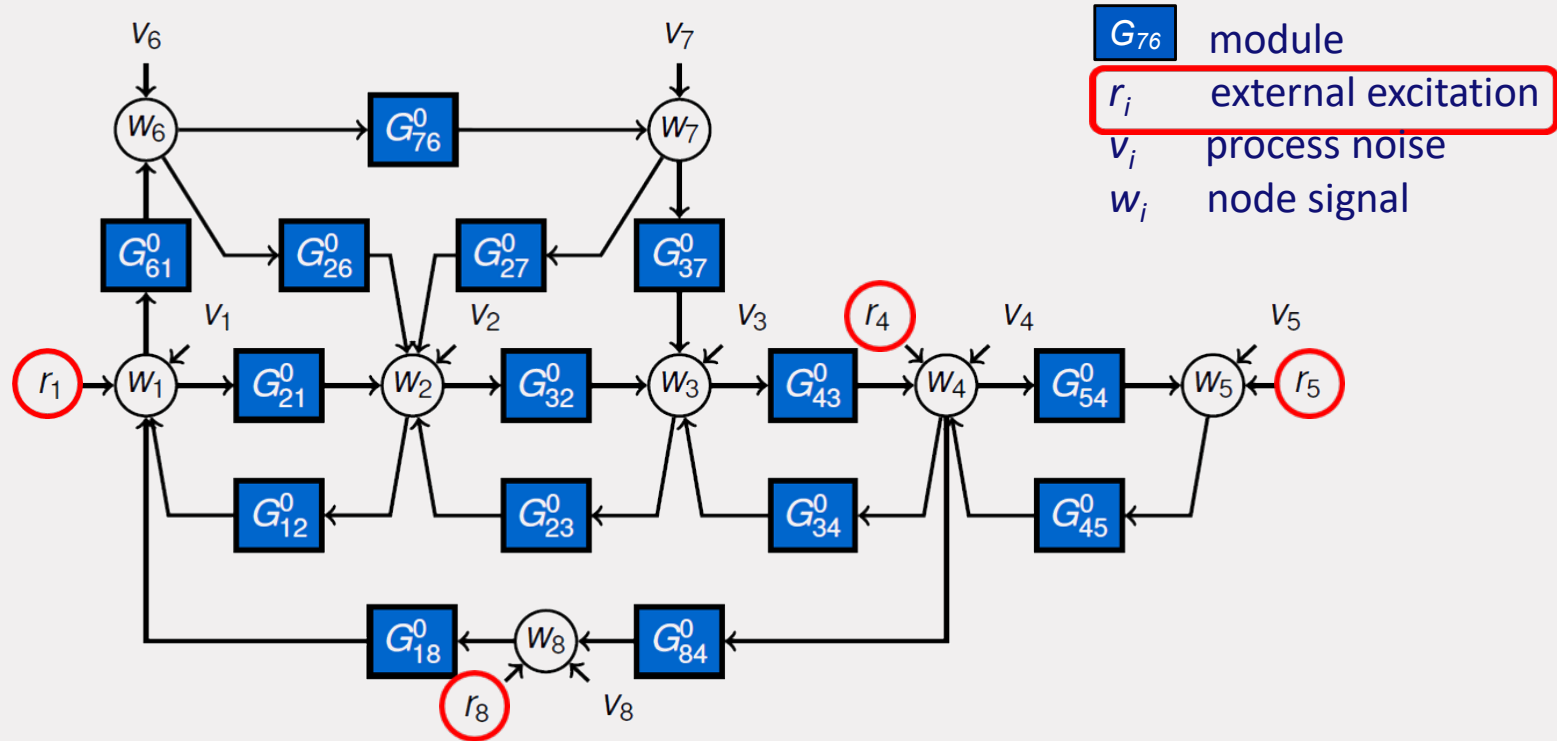
# Dynamic network setup – Module framework



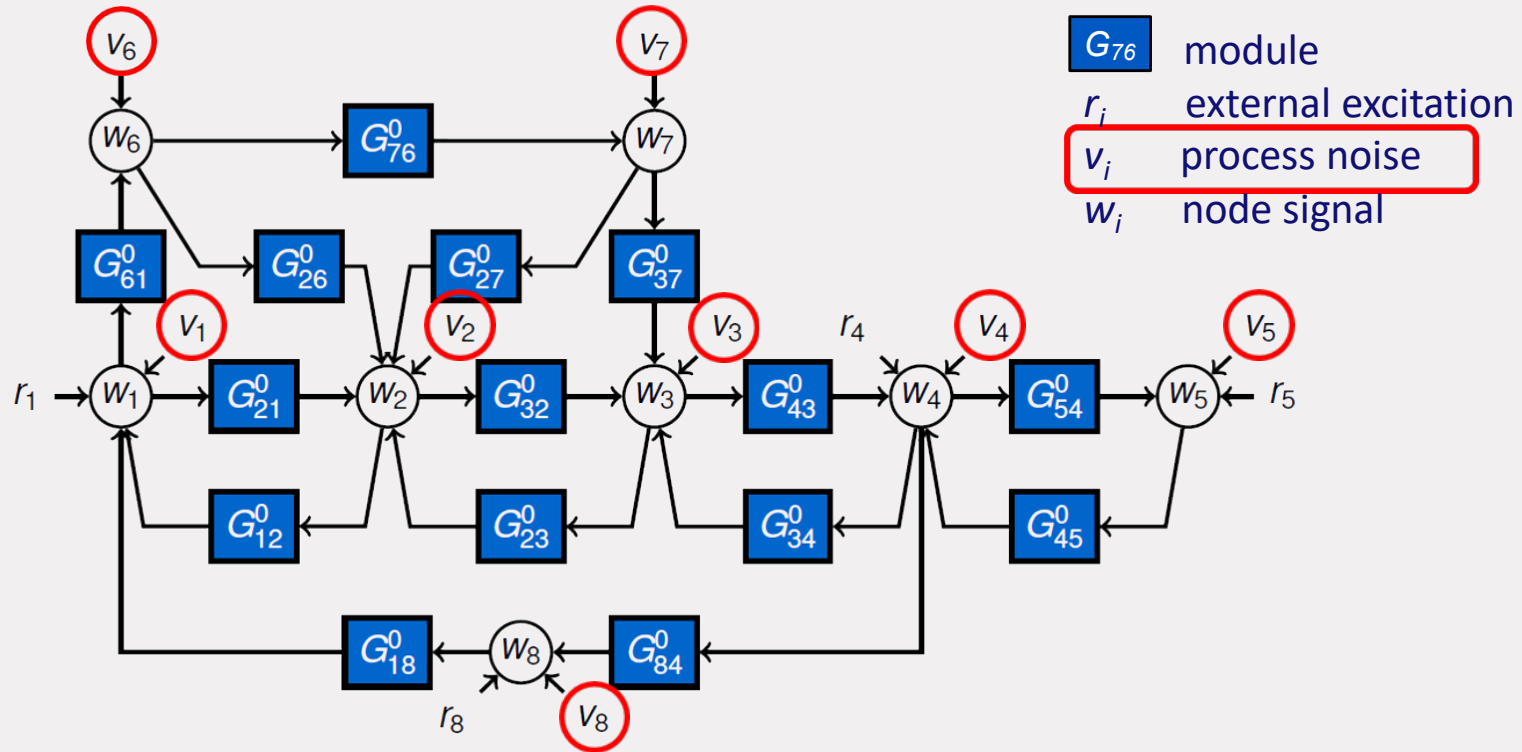
# Dynamic network setup – Module framework



# Dynamic network setup – Module framework



# Dynamic network setup – Module framework



# Dynamic network setup – Module framework

Collecting all equations:

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & G_{12}^0 & \cdots & G_{1L}^0 \\ G_{21}^0 & 0 & \cdots & G_{2L}^0 \\ \vdots & \ddots & \ddots & \vdots \\ G_{L1}^0 & G_{L2}^0 & \cdots & 0 \end{bmatrix}}_{\text{Network matrix } G^0(q)} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + R^0 \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_K \end{bmatrix} + H^0 \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_p \end{bmatrix}$$

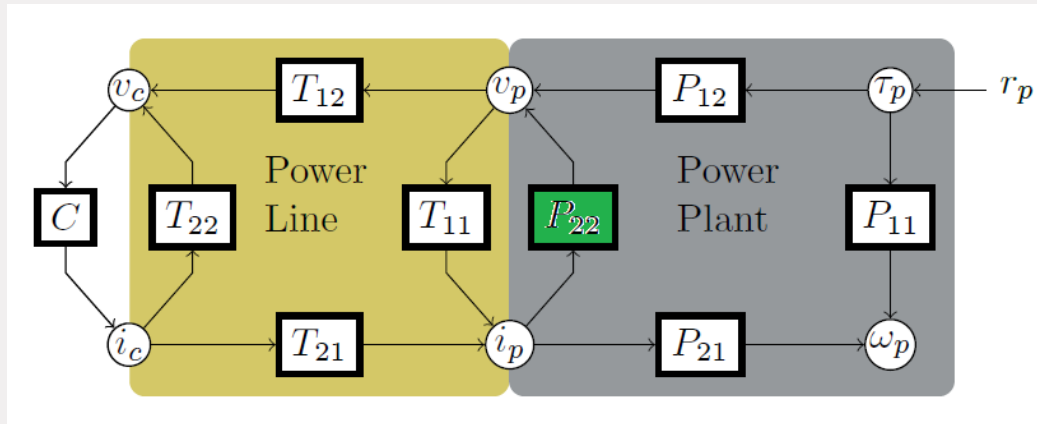
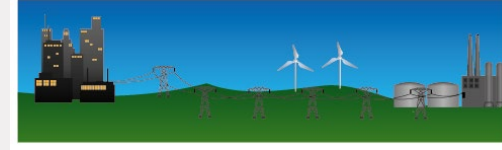
$$w(t) = G^0(q)w(t) + \underbrace{R^0(q)r(t)}_{u(t)} + v(t); \quad v(t) = H^0(q)e(t); \quad \text{cov}(e) = \Lambda$$

- The topology of the network is encoded in the structure (non-zero entries) of  $G^0$ .
- Measured node signals might be subject to **sensor noise**:  $\tilde{w}_k = w_k + s_k$

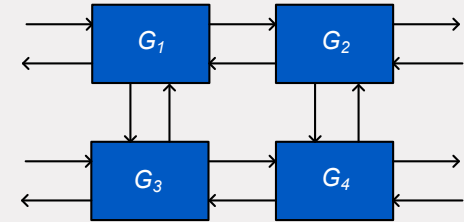


# Alternative models

Bilaterally coupled (two-port) system:

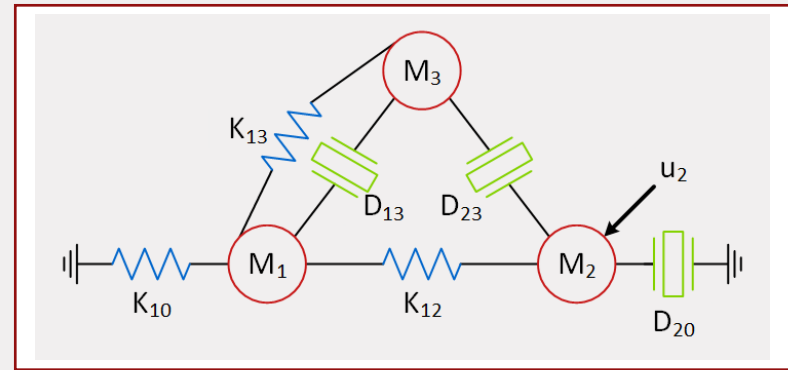


Fully captured in the module-framework



# Alternative models

## Diffusively coupled networks:



$$\begin{bmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{bmatrix} \begin{bmatrix} \ddot{w}_1 \\ \ddot{w}_2 \\ \ddot{w}_3 \end{bmatrix} + \begin{bmatrix} 0 & & \\ & 0 & \\ & & D_{20} \end{bmatrix} \begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \end{bmatrix} + \begin{bmatrix} K_{10} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} + \begin{bmatrix} D_{13} & 0 & -D_{13} \\ 0 & D_{23} & -D_{23} \\ -D_{13} & -D_{23} & D_{13} + D_{23} \end{bmatrix} \begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \end{bmatrix} + \begin{bmatrix} K_{12} + K_{13} & -K_{12} & -K_{13} \\ -K_{12} & K_{12} & 0 \\ -K_{13} & 0 & K_{13} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 0 \\ u_2 \\ 0 \end{bmatrix}$$

$$M_j \ddot{w}_j(t) + D_{j0} \dot{w}_j(t) + \sum_{k \neq j} D_{jk} (\dot{w}_j(t) - \dot{w}_k(t)) + K_{j0} w_j(t) + \sum_{k \neq j} K_{jk} (w_j(t) - w_k(t)) = u_j(t),$$

$$\left[ \underbrace{A(p)}_{\text{diagonal}} + \underbrace{B(p)}_{\text{Laplacian}} \right] w(t) = u(t) \quad A(p), B(p) \text{ polynomial} \quad p = \frac{d}{dt}$$

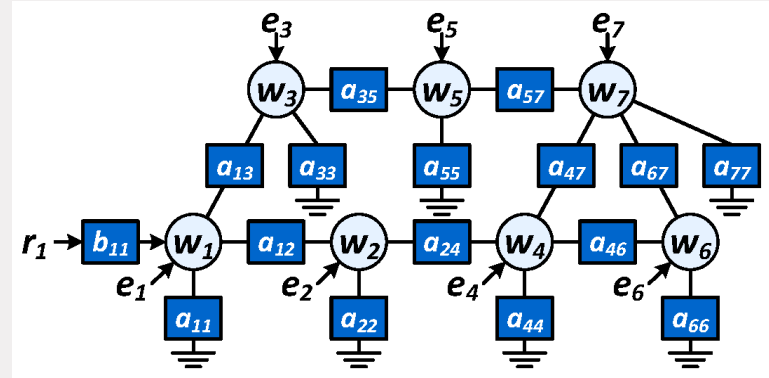
# Alternative models

## Diffusively coupled networks

The related graph is bi-directional:

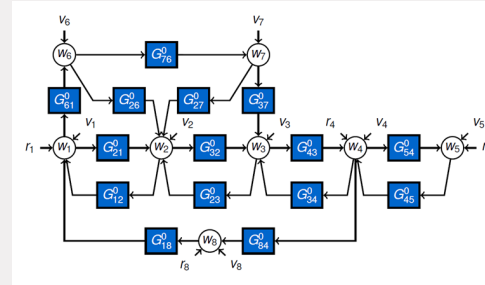
$$\left[ \underbrace{Q(p)}_{\text{diagonal}} - \underbrace{P(p)}_{\text{hollow \& symmetric}} \right] w(t) = u(t)$$

$Q, P$  polynomial



# Dynamic network setup – Module framework

Many data-analytics and data-driven modeling challenges appear



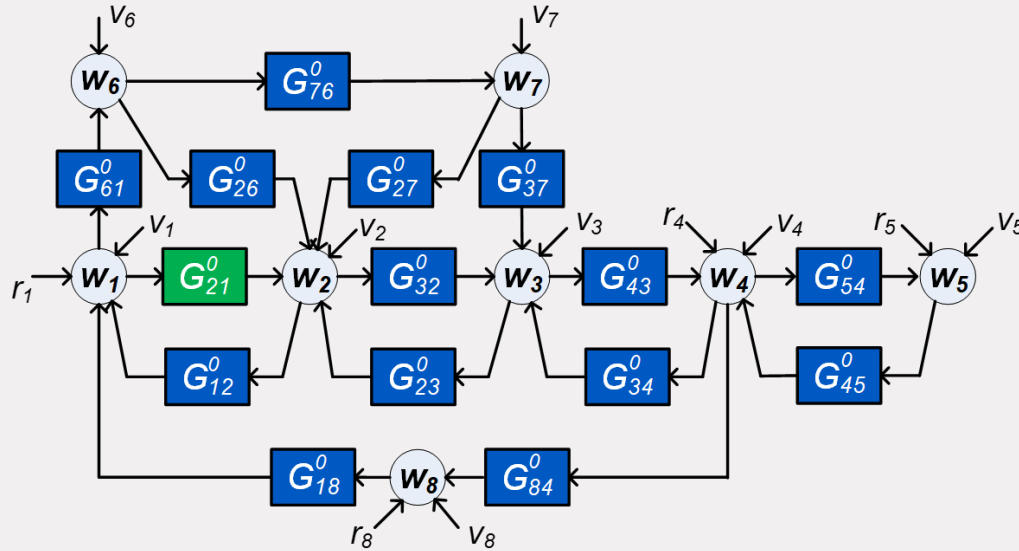
Sensor locations:  $\{w_k(t)\}_{k=\dots};$   
Actuator locations:  $\{r_j(t)\}_{j=\dots};$



- Estimate or validate a single module/subnetwork (known topology)
- Estimate or validate the full network
- Estimate or validate the topology
- Identifiability
- Detect a fault and diagnose its location
- Exploit active probing (experiment design)
- User prior knowledge of modules/topology
- Scalable algorithms

# Single module identification

# Single module identification

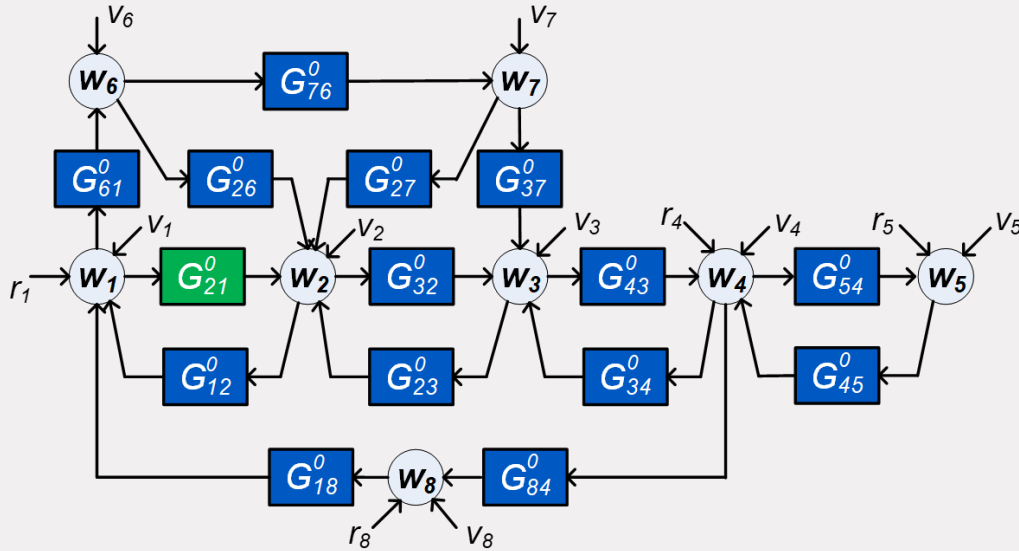


For a network with  
known topology:

- Identify  $G^0_{21}$  on the basis of measured signals
- Which signals to measure?  
Preference for local measurements
- When is there enough excitation / data informativity?



# Single module identification



Different types of methods:

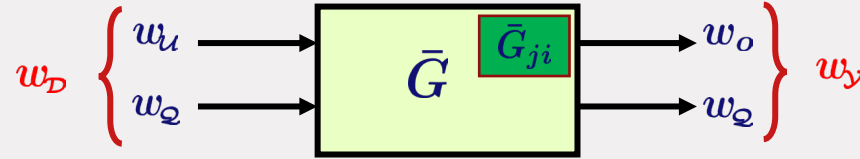
## Indirect methods:

- Rely on mappings  $r \rightarrow w$  and on sufficient excitation signals  $r$

## Direct methods:

- Rely on mappings  $w \rightarrow w$  and use excitation from both  $r$  and  $v$  signals

# Single module identification

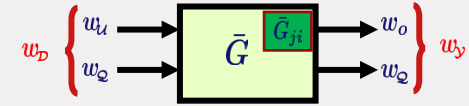


Conditions for arriving at a consistent model estimate:

1. Module invariance:  $\bar{G}_{ji} = G_{ji}^0$
2. Handling of confounding variables
3. Data-informativity
4. *Technical conditions on presence of delays*

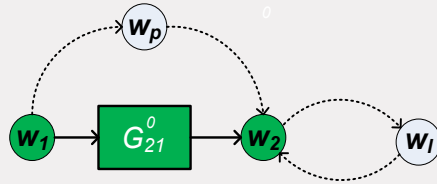
**Path-based conditions on the selected signals and the network graph**

# Single module identification



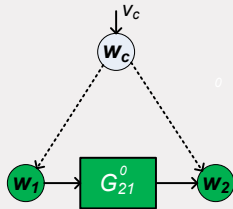
Conditions for arriving at a consistent model estimate:

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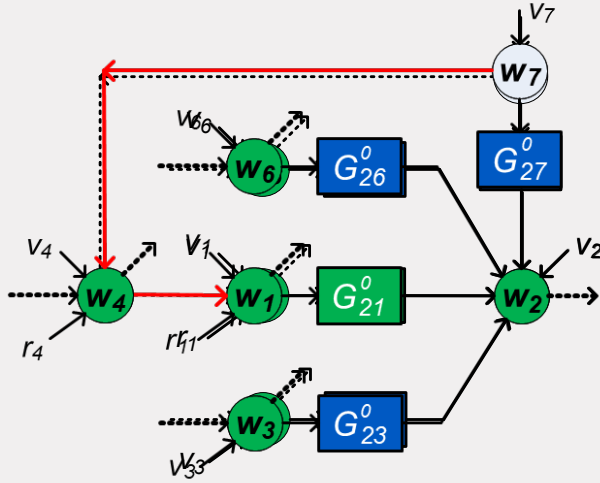
PPL condition: all parallel paths and loops around the output should be blocked by a measured node that is present in  $w_D$

2. **Handling of confounding variables**



No correlated disturbances between  $w_y$  and signals in  $w_u$  that are in-neighbors of  $w_y$

# Confounding variables – solutions



Non-measurable  $w_7$  is a confounding variable

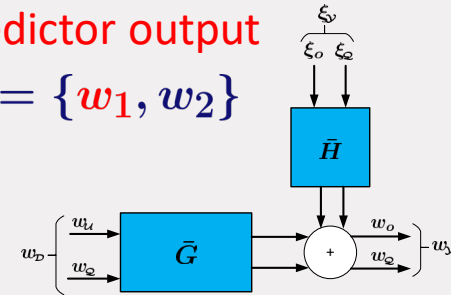
Two possible solutions:

1. Include  $w_4$   $\Rightarrow$  add predictor input

$$w_D = \{w_1, w_3, w_4, w_6\} \quad w_y = \{w_2\}$$

2. Predict  $w_1$  too  $\Rightarrow$  add predictor output

$$w_D = \{w_1, w_3, w_6\} \quad w_y = \{w_1, w_2\}$$

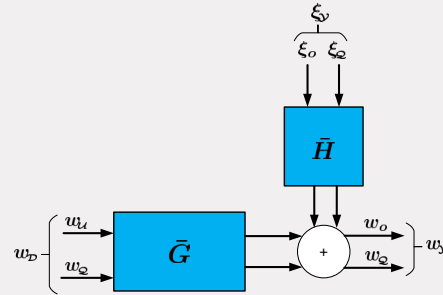


- There are degrees of freedom in choosing the predictor model

[1] A.G. Dankers et al., *Proc. IFAC World Congress*, 2017.

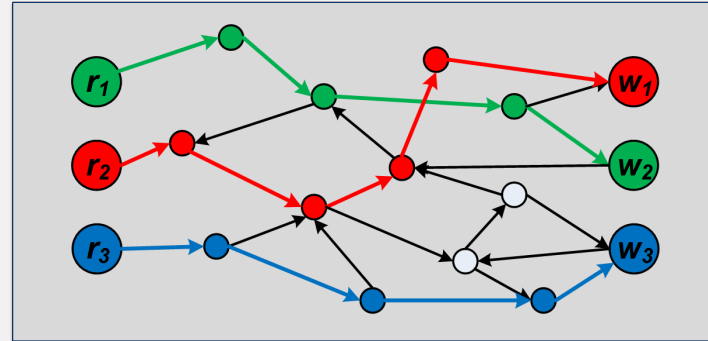
[2] K.R. Ramaswamy, *IEEE TAC*, 2021.

# Data informativity (path-based condition)



Data-informativity for estimating  $(\bar{G}, \bar{H})$  is obtained if  $\Phi_{\kappa}(\omega) > 0$  for almost all  $\omega$ , with  $\kappa = \begin{bmatrix} w_{\mathcal{D}} \\ \xi_{\mathcal{Y}} \end{bmatrix}$ .

This is satisfied **generically** if there are  $\dim(\kappa)$  **vertex disjoint paths** from all external network signals to  $\kappa$



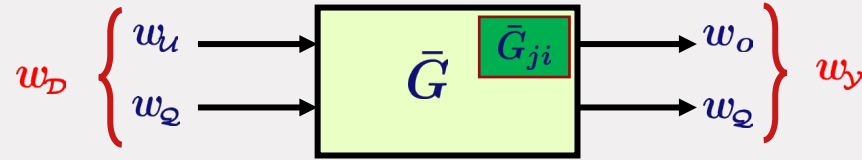
$$b_{\mathcal{R} \rightarrow \mathcal{W}} = 3$$

[1] Van der Woude, 1991

[2] Hendrickx, Gevers & Bazanella, CDC 2017, TAC 2019.

[3] VdH et al., CDC 2020.

# Single module identification



Conditions for arriving at a consistent model estimate:

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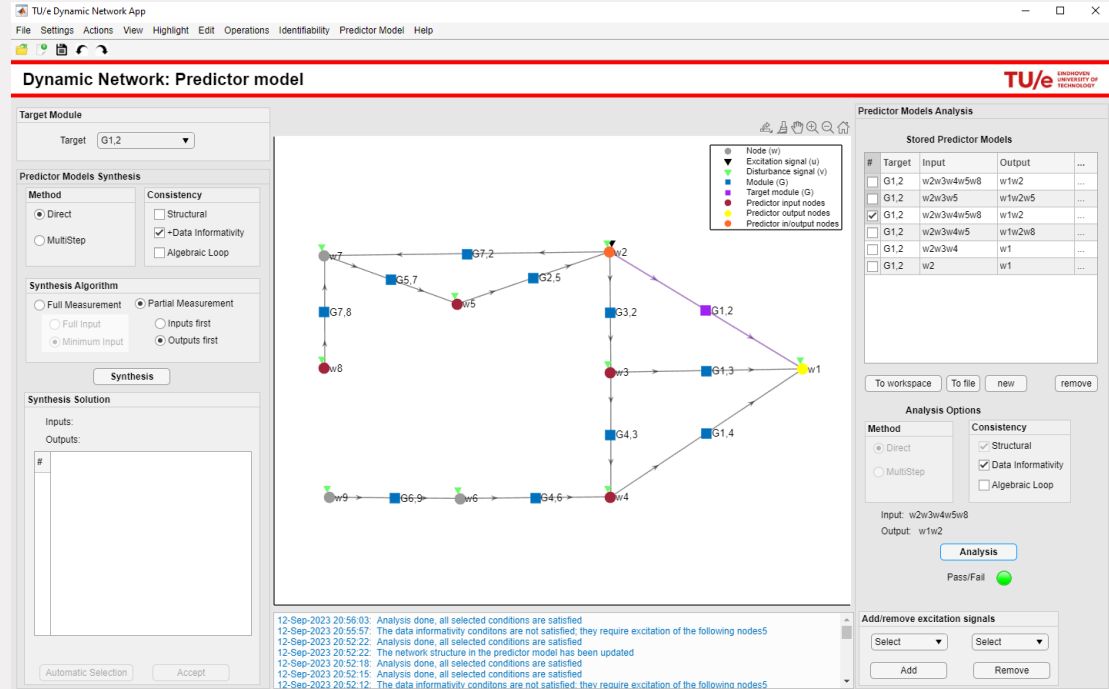
**Path-based conditions on the selected signals and the network graph**



# Single module identification

Different synthesis algorithms can provide predictor models that satisfy the conditions

Multiple solutions  
for either full/partial  
measurement



[1] K.R. Ramaswamy et al., IEEE-TAC, 2021.

[2] S.J.M. Fonken et al., CDC, 2023.

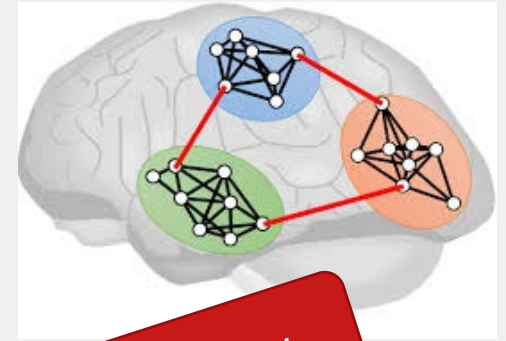
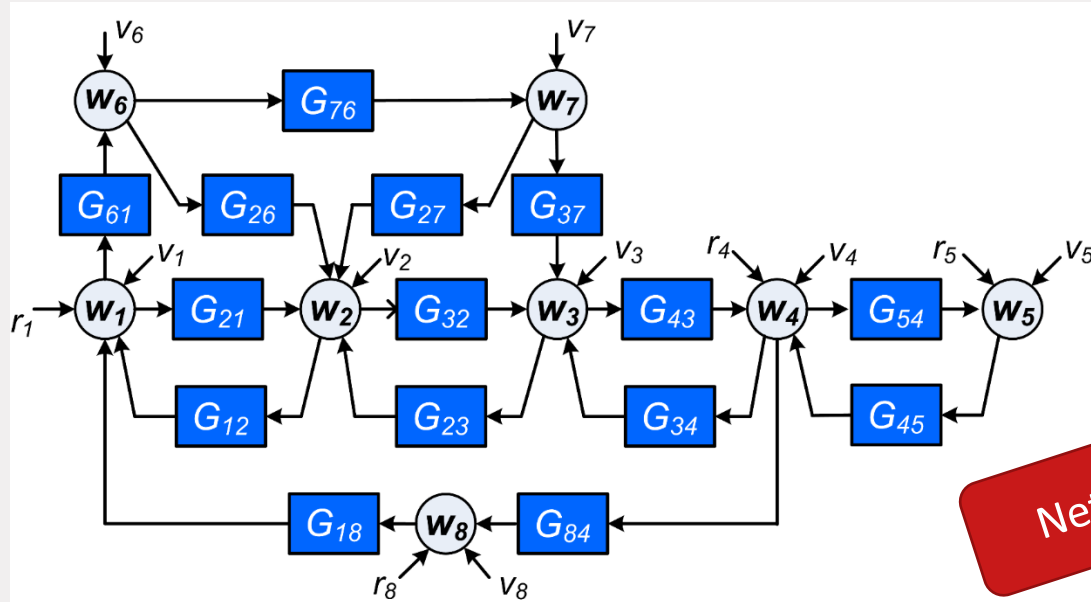
[3] Control Systems Group TU/e, SYSYNET Toolbox for MATLAB, 2023, [www.sysynet.net](http://www.sysynet.net).

# Summary single module identification

- Path-based conditions that the predictor model should satisfy
- Different algorithms for synthesizing predictor model
- Degrees of freedom in sensor / actuator placement
- Once a predictor model is constructed, estimation comes down to a “classical” MISO/MIMO estimation problem

# Generic network identifiability

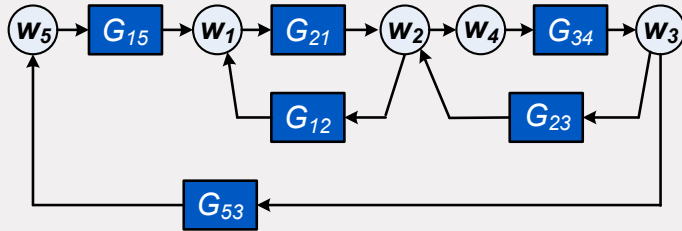
# Network identifiability



Network identifiability

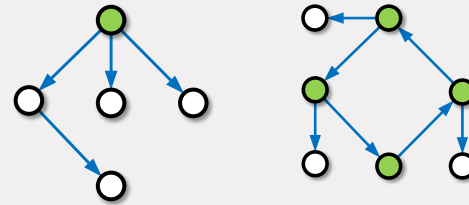
Under which conditions can we estimate the topology and/or dynamics of the full network?

# Where to allocate external excitations for network identifiability?

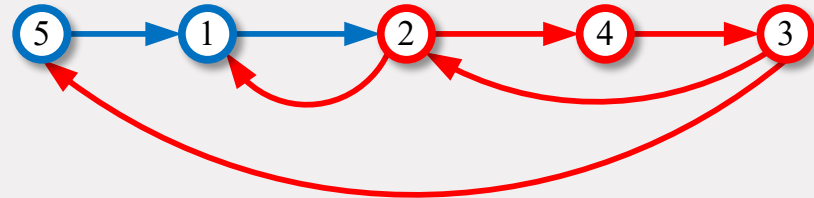


All indicated modules are parametrized

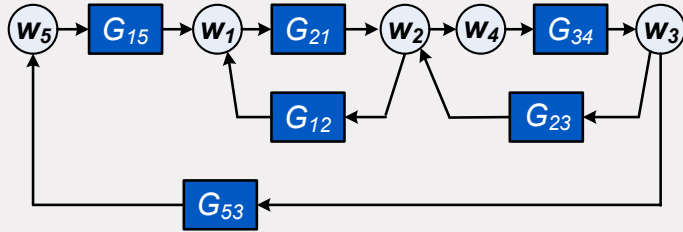
Decompose the network graph in pseudo-trees:



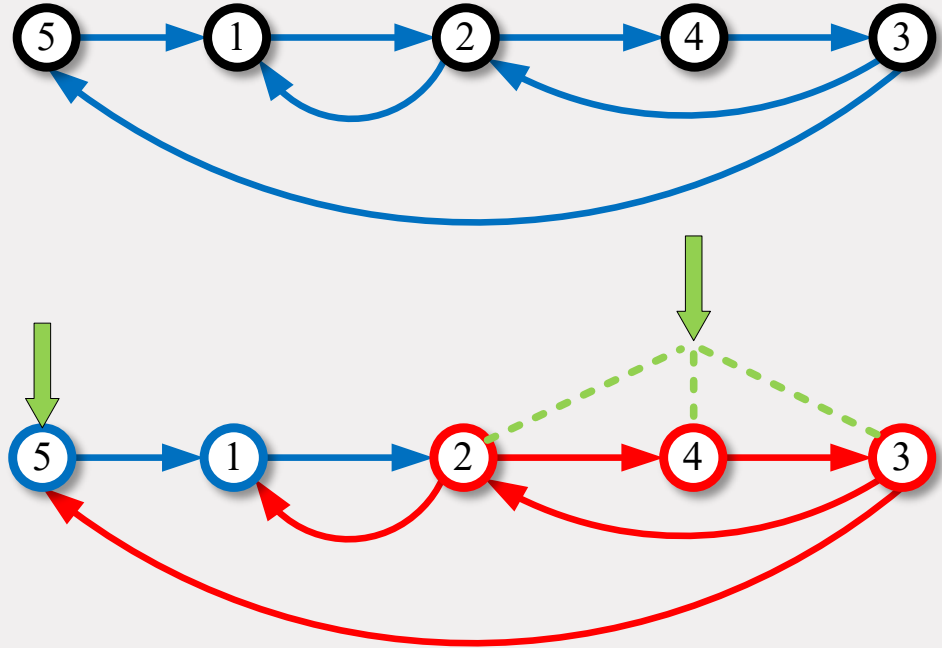
Two disjoint pseudo-trees



# Where to allocate external excitations for network identifiability?

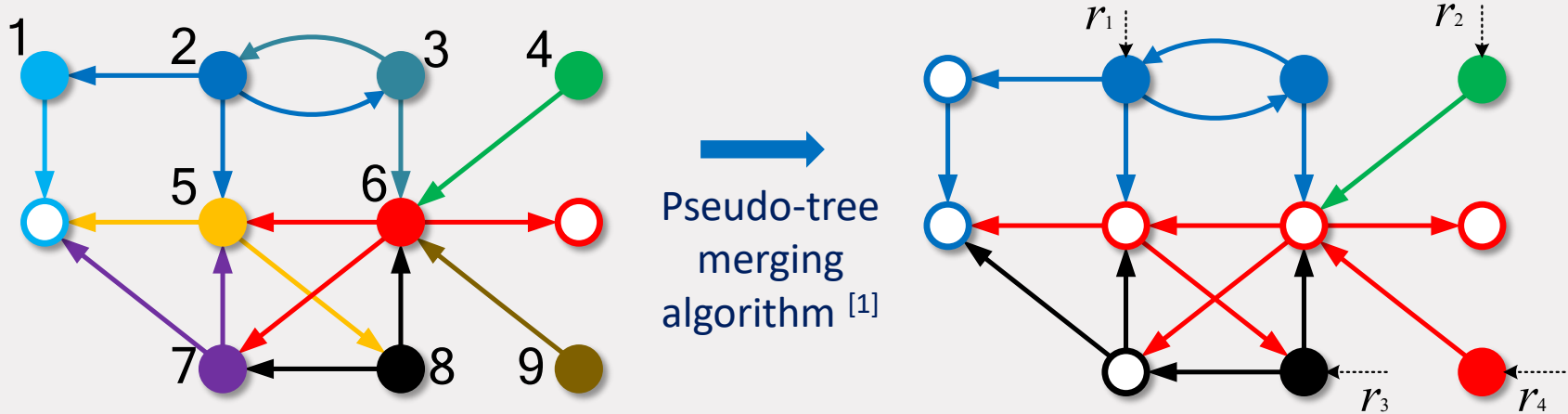


Two independent excitations  
guarantee  
generic network identifiability





# Where to allocate external excitations for network identifiability?

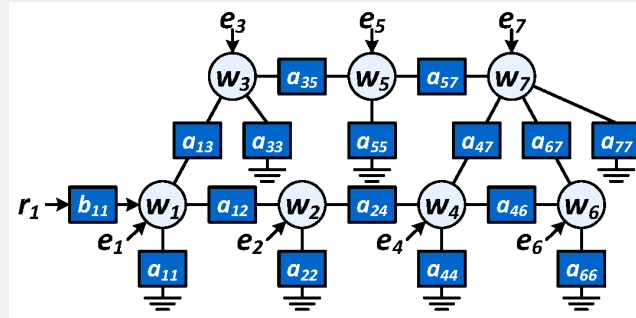


- Nodes are signals  $w$  and external signals  $(r, e)$  that are input to parametrized link
- Known (nonparametrized) links do not need to be covered

# Modeling considerations

# Modeling considerations

- **Module framework** for dynamic networks (with directed graphs) most extensively developed.
- **Diffusively coupled** modelling framework is highly attractive for physical systems<sup>[1]</sup>



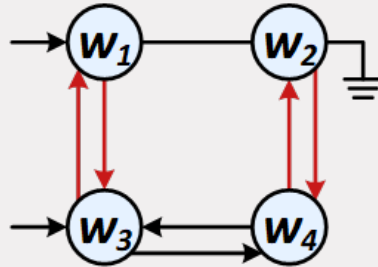
- Algorithms available for full network and single connection identification
- PPL condition simplifies to: **measure all neighbours**

[1] E.M.M. Kivits and PVdH, TAC, June 2023; CDC 2022, IFAC 2023

# Modeling considerations

## Ultimate challenge

- Combine both frameworks for cyber (directed) – physical (non-directed) systems



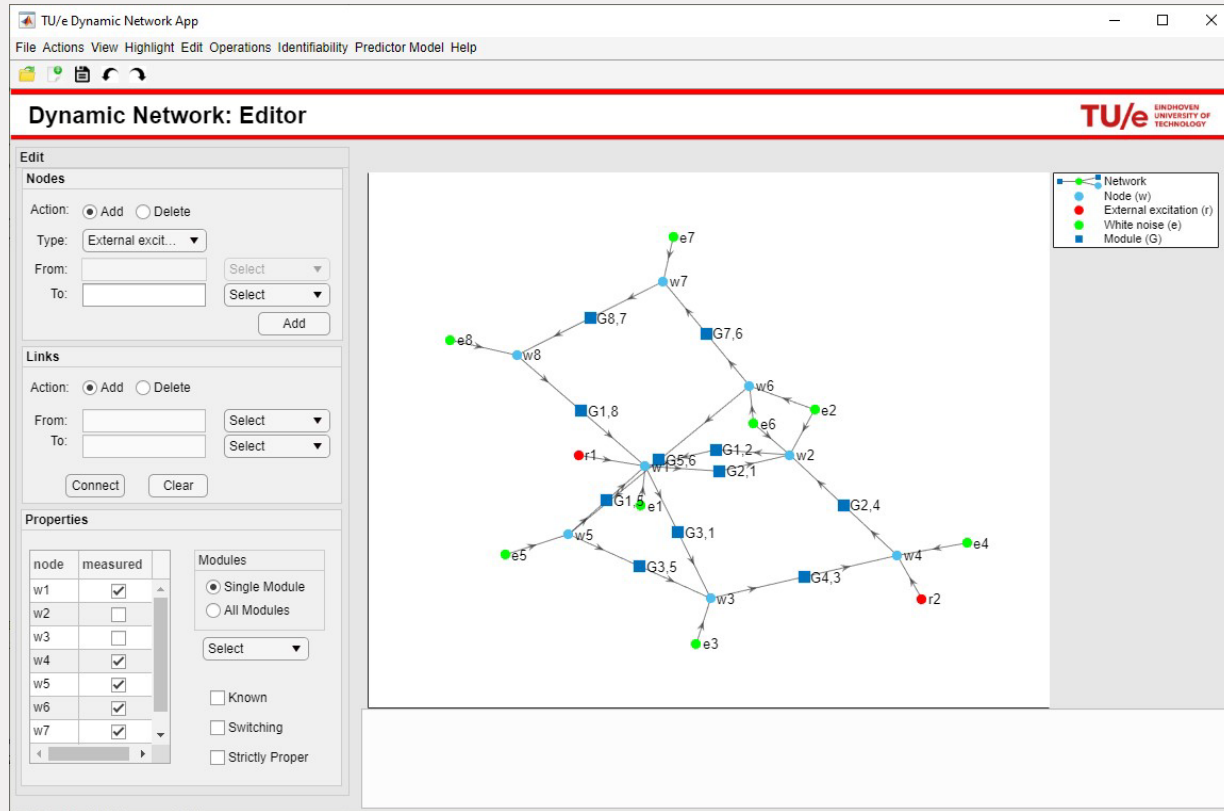
- And build the theory for related data-processing steps

# Conclusions

# Conclusions

- Rich framework
- Exploiting structure/topology
- Effective use of prior/physical information on model structures/ parameters
- So far, mainly developed for linear dynamics
- Free choice for actual estimation / machine learning algorithms
- Looking for attractive application opportunities in the biomed eng domain

# Algorithms implemented in SYSDYNET MATLAB Toolbox



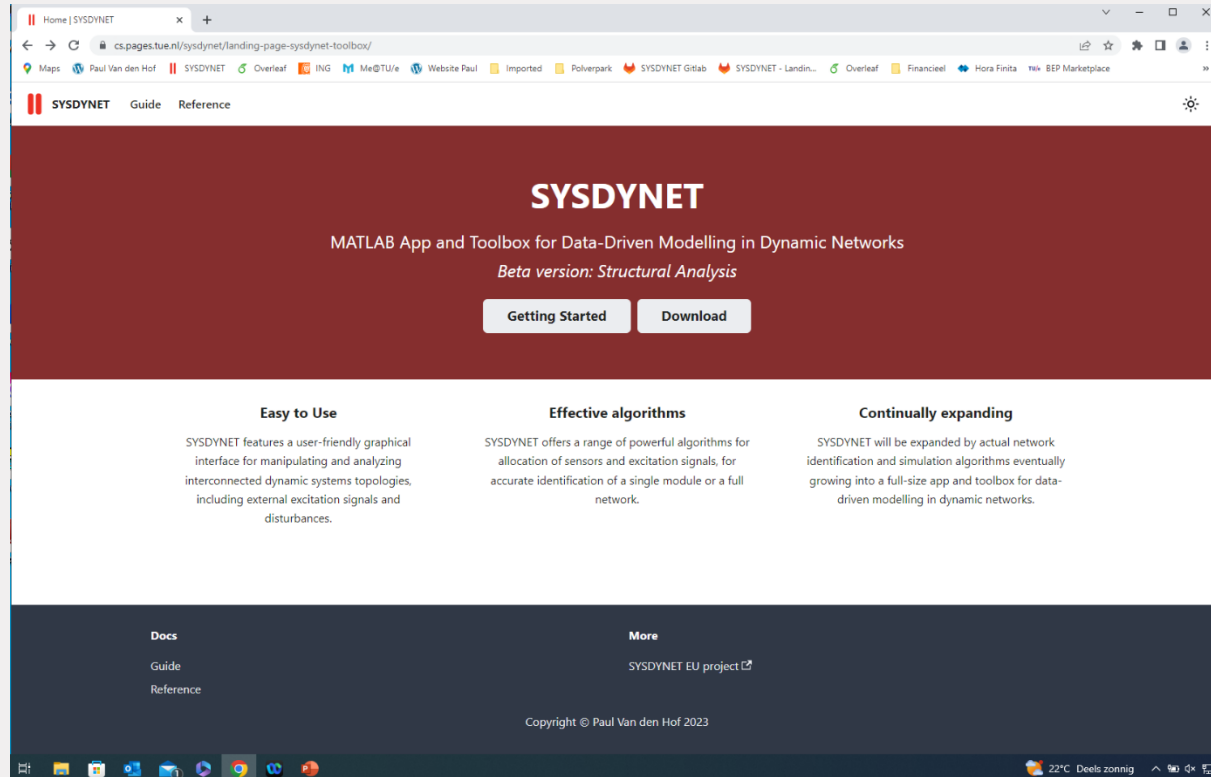
**Structural** analysis and operations on dynamic networks

- Edit and manipulate
- Assign properties to nodes and modules
- Immersion of nodes, PPL test
- Generic identifiability analysis and synthesis
- Predictor model selection for single module ID

to be complemented with

- estimation algorithms for single module and network ID;
- topology estimation
- Network simulation

# MATLAB Toolbox





# ERC SYSDYNET Team: data-driven modeling in dynamic networks

## Research team:



Arne Dankers

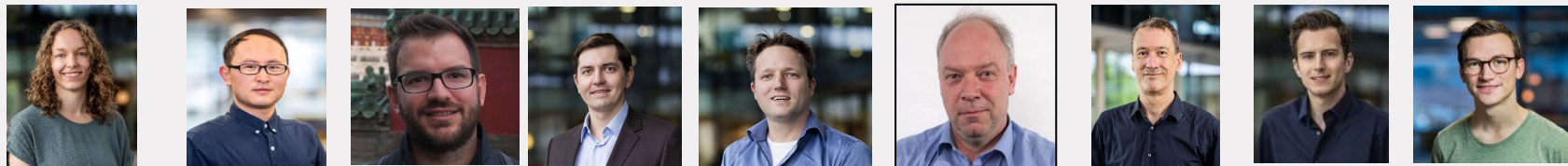
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Minneapolis, Vienna, Louvain-la-Neuve, Linköping, KTH Stockholm, Padova, Brussels, Salt Lake City, Lyon.

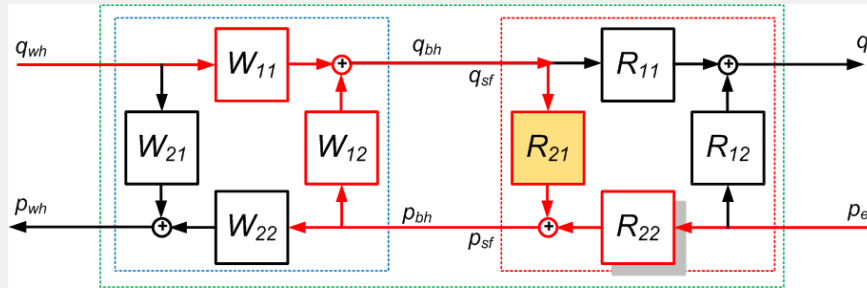
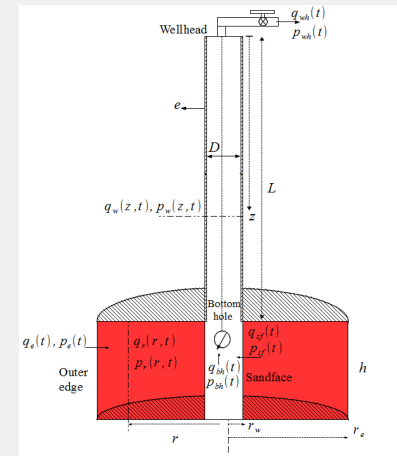
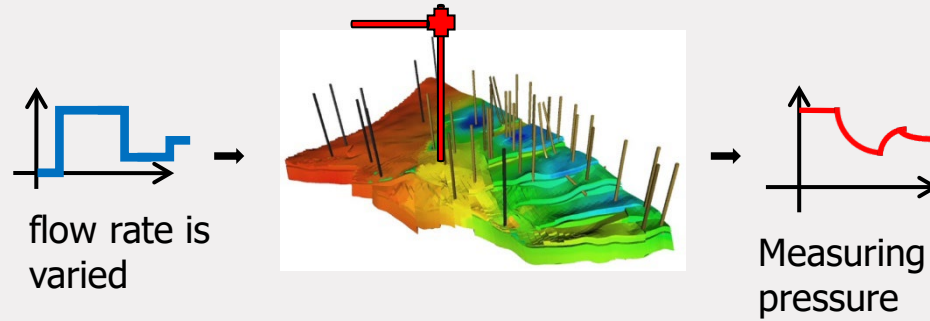
# Further reading

- P.M.J. Van den Hof, A. Dankers, P. Heuberger and X. Bombois (2013). Identification of dynamic models in complex networks with prediction error methods - basic methods for consistent module estimates. *Automatica*, Vol. 49, no. 10, pp. 2994-3006.
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- X. Cheng, S. Shi and P.M.J. Van den Hof (2022). Allocation of excitation signals for generic identifiability of linear dynamic networks. *IEEE Trans. Automatic Control*, Vol. 67, no. 2, pp. 692-705, February 2022.
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- S.J.M. Fonken, K.R. Ramaswamy and P.M.J. Van den Hof (2022). A scalable multi-step least squares method for network identification with unknown disturbance topology. *Automatica*, Vol. 141 (110295), July 2022.
- K.R. Ramaswamy, P.Z. Csurscia, J. Schoukens and P.M.J. Van den Hof (2022). A frequency domain approach for local module identification in dynamic networks. *Automatica*, Vol. 142 (110370), August 2022.
- S. Shi, X. Cheng and P.M.J. Van den Hof (2023). Single module identifiability in linear dynamic networks with partial excitation and measurement. *IEEE Trans. Automatic Control*, Vol. 68(1), pp. 285-300, January 2023.
- X. Bombois, K. Colin, P.M.J. Van den Hof and H. Hjalmarsson (2023). On the informativity of direct identification experiments in dynamical networks. *Automatica*, Vol. 148 (110742), February 2023.
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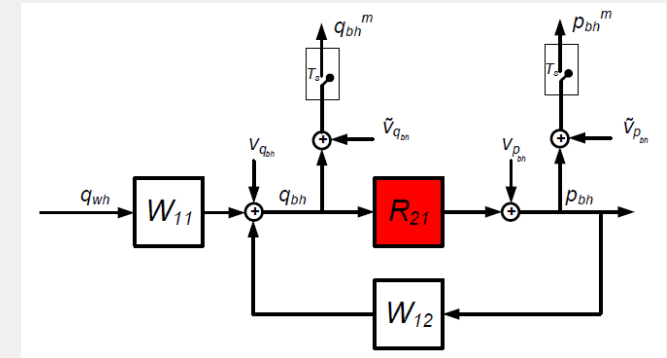
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# Use cases

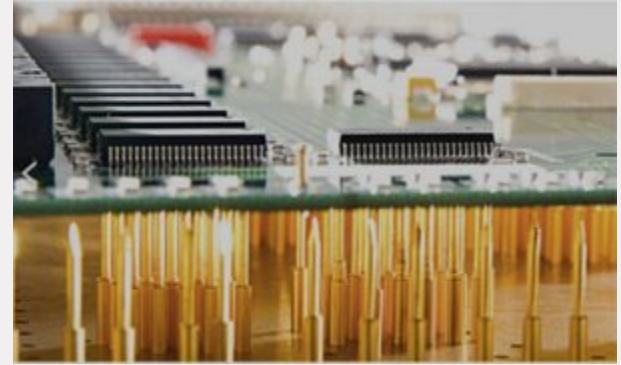
# Oil reservoir modeling through well testing



Errors-in-variables problem in closed-loop setting

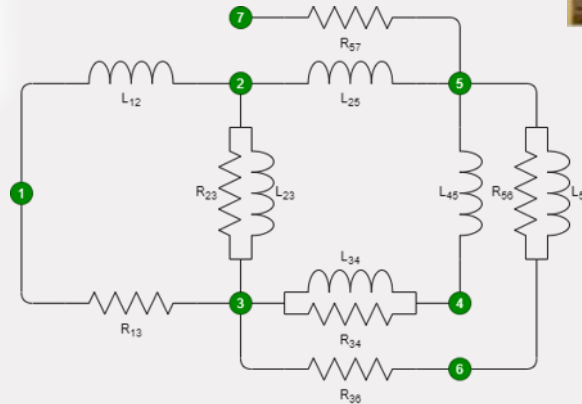


# Application: Printed Circuit Board (PCB) Testing

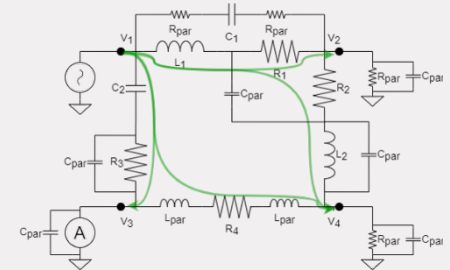


Detection of

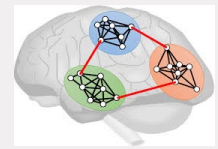
- component failures
- parasitic effects



Source: Altium



# Neurodynamic effect of listening to Mozart music



Identifying changes in network connections in the brain, after intensely listening for one week (Sonate K448), based on fMRI data

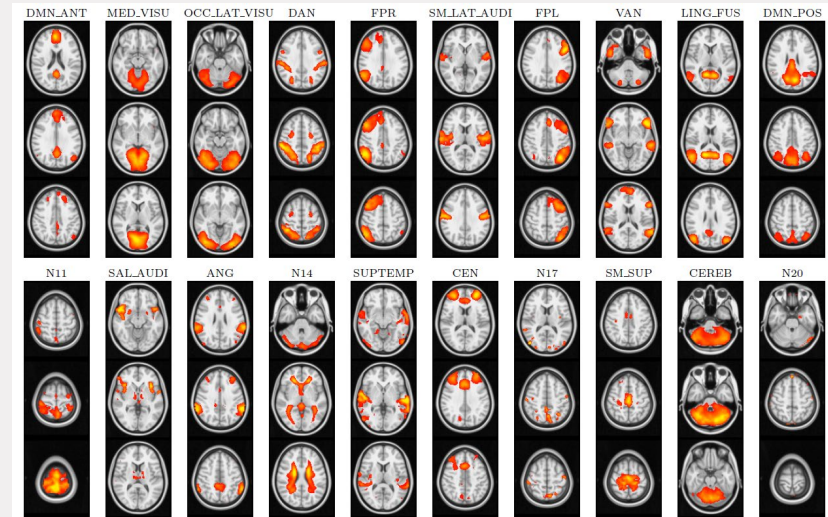
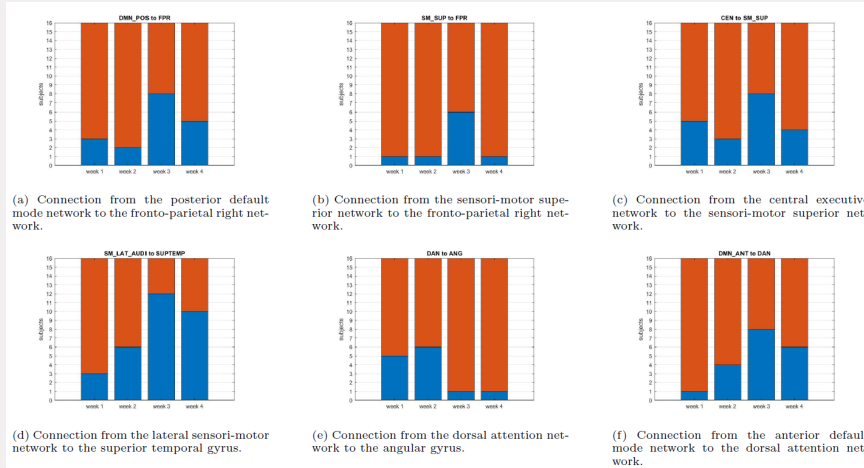
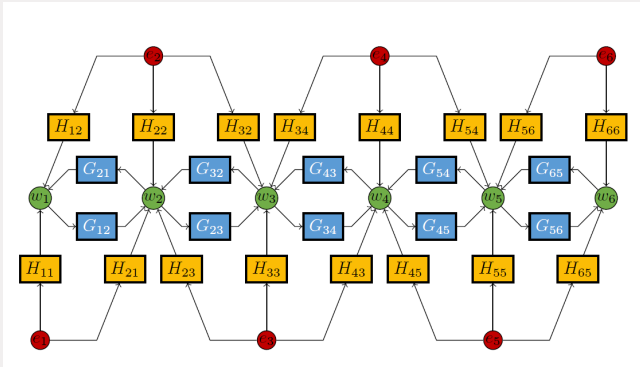
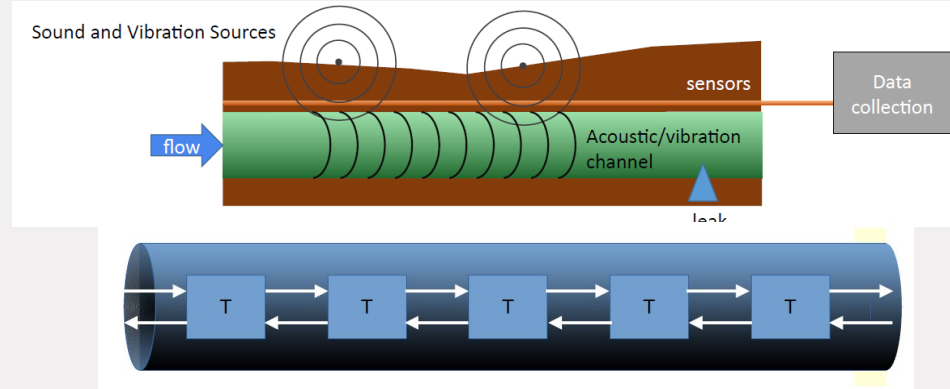


Figure 3: Spatial maps of the 20 active brain networks found through the ICA decomposition. Each image consists of 3 relevant horizontal slices of the brain, where the spatial map is indicated by the red color scale.



# Leak detection in gas pipelines with acoustic sensors

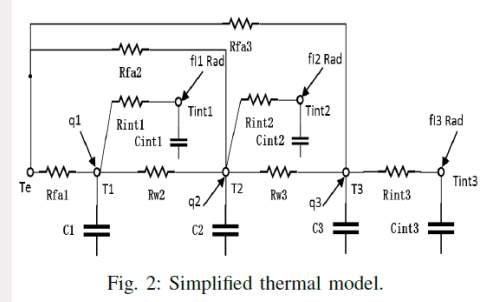
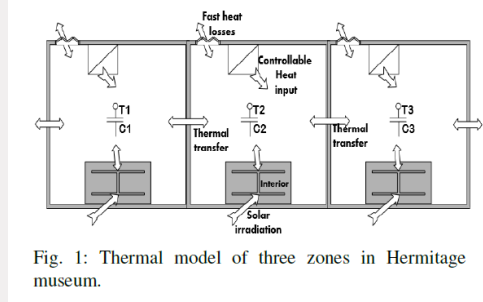


- Use operational data to detect changes in network model dynamics
- Map model changes to physical causes

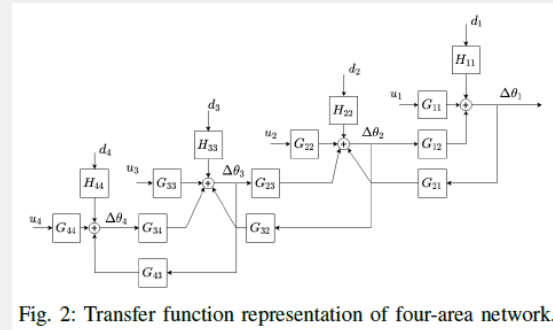


# Additional projects

- Distributed identified model-based predictive control of a building climate system (Museum Hermitage Amsterdam)<sup>[1]</sup>



- Data-driven modeling and control of a four-area power network<sup>[2]</sup>



[1] X. Chen, J.H.A. Ludlage, M. Lazar (2019)

[2] A. Anupama, T.R.V. Steentjes, M. Lazar (2021)