Estimating Cutting Forces in Micromilling by Input Estimation from Closed-loop Data

Rogier S. Blom
Paul M.J. Van den Hof

Delft University of Technology

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Outline

1. Background and challenge
2. Problem statement
3. Approach to input estimation from closed-loop data
4. Simulation results
5. Conclusions
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1. Background and challenge
   - Micromilling with Active Magnetic Bearings spindles

2. Problem statement

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Micromilling with Active Magnetic Bearing spindles

**Micromilling** milling with tools with diameter $< 0.5\,mm$

**Applications** medical purposes, micro-electronics, etc.

**Challenge** Use active nature of AMB spindles for Process Monitoring and Control

Comparison of micro-endmill (0.2mm) and normal endmill (1.0mm)

Close-up of micromilling machine
Challenge

Control current signal

Displacement sensor

AMB Controller

Displacement signal

Magnetic coils

Micro-endmill
Challenge

Example of cutting forces in micromilling
Challenge

Cutting force estimation

Observe the cutting forces that arise during micromilling from the signals of the Magnetic Bearings.
Outline

1 Background and challenge

2 Problem statement
   - Design of cutting force estimator

3 Approach to input estimation from closed-loop data

4 Simulation results

5 Conclusions
Problem statement

- $G$: Model of the open-loop AMB spindle dynamics (MIMO, unstable)
- $K$: Controller and current amplifier
- $u_1$: Currents through the magnetic coils
- $u_2$: Cutting forces acting on tooltip
- $y_{1,2}$: Measurements of currents, displacements
- $v_{1,2}$: Measurement noise on currents, displacements (white)
Problem statement

Objective: Using
- model of plant $G$
- information on spectrum of the unknown input $u_2$
design linear filter $F$ on $y_{1,2}$ to create $\hat{u}_2(t)$ such that

$$\mathbb{E}|\hat{u}_2(t) - u_2(t - N)|^2$$

is minimized for fixed lag $N \geq 0$.

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3. Approach to input estimation from closed-loop data
   - Known $K(z)$
   - No explicit information on $K(z)$

4. Simulation results

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Input estimation from closed-loop data, known $K(z)$

If $K(z)$ is known, input $y_1$ is not needed.
Input estimation from closed-loop data, known $K(z)$

If $K(z)$ is known, input $y_1$ is not needed

Denote:
- $\Phi_{u_2}$: spectrum of $u_2$
- $\Phi_{y_2}$: the spectrum of $y_2$
- $\Phi_{u_2y_2}$: the cross spectrum between $u_2$ and $y_2$

The causal Wiener filter

Let the canonical spectral factorization of $\Phi_{y_2}$ be given by

$$\Phi_{y_2} = MRM^*$$

with $M$ minimum phase.

The filter causal $F$ that minimizes $\mathbb{E}|Fy_2 - u_2(t - N)|^2$ is given by

$$F = \{z^{-N}\Phi_{u_2y_2}M^{-*}\} + R^{-1}M^{-1}$$
Input estimation from closed-loop data, known $K(z)$

\[ y_2 = S(G_2 u_2 + v_2) \]
\[ S = (I + G_1 K)^{-1} \]

It is easily derived that:

1. $\Phi_{u_2 y_2} = \Phi_u G_2^* S^*$
2. $\Phi_{y_2} = S(G_2 \Phi_u G_2^* + R_{v_2}) S^*$
Input estimation from closed-loop data, known $K(z)$

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Causal Wiener filter for closed-loop data

Factorize $G_2 \Phi_u G_2^* + R_{v_2} = TRT^*$ such that $ST$ minimum phase. Then:

$$\Phi_{y_2} = (ST)R(ST)^*$$

With this it follows that

$$F = \left\{ z^{-N} \Phi_u G_2^* T^{-*}(z) \right\} + R^{-1} T^{-1}(z) S^{-1}(z)$$
Input estimation from closed-loop data, known $K(z)$

Causal Wiener filter for closed-loop data

Factorize $G_2 \Phi_u G_2^* + R_{v_2} = T R T^*$ such that $S T$ minimum phase. Then:

$$\Phi_{y_2} = (S T) R (S T)^*$$

With this it follows that

$$F = \{ z^{-N} \Phi_u G_2^* T^{-*}(z) \} + R^{-1} T^{-1}(z) S^{-1}(z)$$

How to proceed:

- step 1. Find factorization $G_2 \Phi_u G_2^* + R_{v_2} = T R T^*$
- step 2. Find causal part of $z^{-N} \Phi_u G_2^* T^{-*}(z)$
Step 1. Factorization of $G_2 \Phi_u G_2^* + R_{v_2}$ using state space realization

- Let a realization of $G$ be given by \( \begin{pmatrix} A & B_1 & B_2 \\ C & 0 & 0 \end{pmatrix} \)

**Conditions**

- $A$ not necessarily Hurwitz
- $(A, B_1)$ and $(A, B_2)$ stabilizable, $(A, C)$ detectable
- \( \begin{bmatrix} A - \lambda I & B_2 \\ C & 0 \end{bmatrix} \) has full column rank for all $\lambda \in \mathbb{C}$, $|\lambda| \geq 1$
Step 1. Factorization of $G_2\Phi_u G_2^* + R_{v_2}$ using state space realization

- Let a realization of $G$ be given by
  \[
  \begin{pmatrix}
  A & B_1 & B_2 \\
  C & 0 & 0
  \end{pmatrix}
  \]

- Let $\Phi_{u_2} = G_u R_u G_u^*$ be the canonical spectral factorization with minimal realization of $G_u$ given by
  \[
  \begin{pmatrix}
  A_u & B_u \\
  C_u & 0
  \end{pmatrix}
  \]

- Define cascaded system $G_2 G_u$, which has realization
  \[
  \begin{pmatrix}
  A_c & B_c \\
  C_c & 0
  \end{pmatrix} = \begin{pmatrix}
  A_u & 0 & B_u \\
  B_2 C_u & A & 0 \\
  0 & C & 0
  \end{pmatrix}
  \]
Step 1. Factorization of $G_2 \Phi_u G_2^* + R_{v_2}$ using state space realization

With this, we can obtain the desired factorization

$$G_2 \Phi_u G_2^* + R_{v_2} = (G_2 G_u) R_u (G_u^* G_2^*) + R_{v_2}$$

$$= [C_c (zI - A_c)^{-1} L + I] R[*]^*$$

$$= TRT^*$$

with

- $L = A_c P C_c R^{-1}$,
- $R = R_{v_2} + C_c P C_c^T$, and
- $P$ the unique p.d. solution of the DARE

$$P = A_c P A_c^T + B_c R_u B_c^T - L R L^T.$$
Step 2. Deriving the causal filter

\[ F = \{ z^{-N} G_u(z) R_u G_c(z)^* T^*(z) \} + R^{-1} T^{-1}(z) S^{-1}(z) \]

- Split \( W(z) \) in a causal and anti-causal part: \( W(z) = W_1(z) + W_2(z) \)
- Then \( \{ z^{-N} W(z) \}_+ = z^{-N} W_1(z) + \{ z^{-N} W_2(z) \}_+ \)
- \( \{ z^{-N} W_2(z) \}_+ \) can be found by truncating the Laurent expansion of \( W_2(z) \)
No explicit information on $K(z)$

- $S$ is the only factor in $F$ that depends on $K(z)$

$$\hat{u}_2 = (z^{-N} W_1(z) + \{z^{-N} W_2\}_+) R^{-1} T^{-1}(z) S^{-1} y_2$$
No explicit information on $K(z)$

- $S$ is the only factor in $F$ that depends on $K(z)$

$$\hat{u}_2 = (z^{-N} W_1(z) + \{z^{-N} W_2\}_+) R^{-1} T^{-1}(z)(I + G_1 K) y_2$$
No explicit information on \( K(z) \)

\[ S \text{ is the only factor in } F \text{ that depends on } K(z) \]

\[ \hat{u}_2 = (z^{-N} W_1(z) + \{z^{-N} W_2\}_+) R^{-1} T^{-1}(z) (I + G_1 K) y_2 \]

If \( v_1 \) negligible, \( y_1 = -K(z)y_2 \). With this:

\[ \hat{u}_2 = (z^{-N} W_1(z) + \{z^{-N} W_2\}_+) R^{-1} T^{-1}(z) \begin{bmatrix} -G_1 & I \end{bmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \]
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  \]

- This filter is optimal for any $K(z)$
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Results: Simulation with Simulink

- Simulation performed based on parameters of micromilling setup in laboratory
- Open loop AMB system modeled using first principles
- Cutting forces are simulated using model from the micromilling literature
Results: Simulation with Simulink

First test:
- Rotational speed is 10,000 rpm, $T = 25\mu s$
- Random walk model for input spectrum
- Input estimator constructed for $N = 0$ and $N = 40$

Results:
- Filter for $N = 40$ outperforms filter for $N = 0$
- Estimator for $N = 0$ appears to yield delayed estimation results
Error analysis

Observe that the estimation error for $N = 0$ consists of two terms:

$$e = \hat{u}_2 - u_2 = (F_2 G_2 - I)u_2 + F_2 v_2$$
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$$e = \hat{u}_2 - u_2 = (F_2 G_2 - I)u_2 + F_2 v_2$$

- $|F_2 G_2| \approx I$ for lower frequencies
- Then $F_2 G_2$ acts as a pure delay for those frequencies if the group delay

$$\tau_g = -\frac{d}{d\omega} \arg F_2 G_2(e^{j\omega})$$

is constant in that frequency range.
Error analysis

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e = \hat{u}_2 - u_2 = (F_2 G_2 - I)u_2 + F_2 v_2
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- \(|F_2 G_2| \approx 1\) for lower frequencies
- Then \( F_2 G_2 \) acts as a pure delay for those frequencies if the group delay

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\tau_g = -\frac{d}{d\omega} \arg F_2 G_2(e^{j\omega})
\]

is constant in that frequency range.
Error analysis

Estimation error for $N > 0$:

$$e = \hat{u}_2 - z^{-N}u_2 = (F_2G_2 - z^{-N}I)u_2 + F_2v_2$$

- Estimation error decreases for increasing $N$
- Key question: do both terms of $e$ decrease for increasing $N$?

Plot of $\sqrt{\frac{1}{k} \sum_k e^2}$ for increasing $N$
Error analysis

Estimation error for $N > 0$:

$$ e = \hat{u}_2 - z^{-N}u_2 = (F_2 G_2 - z^{-N}I)u_2 + F_2 v_2 $$

Plot of $\sqrt{\frac{1}{k} \sum_k e^2}$ for increasing $N$

Plot of $\|\sigma_v F_2\|$ for increasing $N$
The effect of delay

Observations

- Specifying a filter without lag \((N = 0)\) results in a filter with a lag.
- Specifying a filter with this lag, results in a filter with better performance.
- Specifying a filter with a larger lag, improves the results even further.
Results: Improved spectral model of input

Second test:
- Rotational speed 50,000 rpm;
- $\Phi_u$ has high density at frequencies related to the rotational speed;
- Filter derived for $N = 0$ and $N = 40$.

Results:
- Filter with lag again performs better
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Conclusions

- An optimal input estimator was developed to estimate the cutting forces in micromilling from AMB signals
- No additional sensors are needed
- No knowledge on the AMB controller is needed, if exact measurements of the control currents are available
- The estimator has an adjustable delay allowing to trade off the estimation error against the lag
- There exists a minimum delay that can be attained
- Estimation results can be improved by using a priori information on the spectral content of the cutting forces.