Bounding the uncertainty in identification for control

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Contents

• Identification for control
• An engineer’s perspective to iteration
• The central issue: uncertainty bounding
• Achievements and challenges
“Here is a dynamical process with which you are allowed to experiment (preferably cheap).

*Design and implement a high-performance control system*."

Issues involved:

- Experiment design
- Modelling / identification
- Characterization of disturbances and uncertainties
- Choice of performance measure
- Control design and implementation

*experimental issues dependent on application area*
Classical experiments for finding control-relevant dynamics

- Ziegler/Nichols tuning rules for PID-controllers

- Relay feedback: amplitude and frequency at -180° phase
• Ad-hoc simple cases to be extended to general methodology for model-based control, including issues of robustness induced by model uncertainties

• Adaptive control so far only solves particular parts of the problem
Is it realistic to assume that this can be done in one shot?
Control bandwidth is based on model +...

If models are uncertain/approximate due to limited experiment, achievable performance needs to be discovered → modelling for control is learning (Schrama, 1992; Gevers, 1993)

Windsurfer approach

is learning

(Schrama, 1992; Gevers, 1993)
Uncertainty regions with probability $\alpha$
Classical reasoning for quantifying uncertainty

- Let identified models pass a validation test
- Assume that the real system belongs to the model set \((S \in M)\)
- Use analytical (variance) expressions for quantifying parameter variance and resulting model variance
Residual tests

When model passes test, there is no evidence in the data that the model is wrong
Classical reasoning for quantifying uncertainty

- Let identified models pass a validation test
- Assume that the real system belongs to the model set \((S \in M)\)
- Use analytical (variance) expressions for quantifying parameter variance and resulting model variance

However

- Validation does not guarantee \(S \in M\)
  validation is input-dependent
- Assumption preferably replaced by quantified condition
- Bias has to be taken into account when bounding uncertainty
Intriguing example – 4\textsuperscript{th} order process; 2\textsuperscript{nd} order model; white input

![Graphs showing amplitude, phase, and cross-correlation](Image)
Intriguing example – 4th order process; 2nd order model
Problems:

• Due to undermodelling, the noise is overestimated, leading to conservative 99% confidence bounds;

• Structure within $\hat{R}_{\epsilon u}(\tau)$ is not recognized

• “Solution”: improve noise modelling and apply vector valued test
Intriguing example – Vector-valued test over 128 lags

All models are validated

Exact noise model
Improved estimate
From residual
However

Even if we can improve validation test, it remains dependent on input-data (no expressions possible about frequency areas that are not excited)

Best case scenario (validation implies no-bias) is questionable

Argument for incorporating bias-term in quantifying model uncertainty
Dedicated procedures for uncertainty bounding

- Estimating carrier models with linearly parametrized models (linear regression)
- Taking account of bias by either bounding this term **deterministic** (Hakvoort, De Vries, Van den Hof, 1994, 1997), e.g.

\[ G_0(z) = \sum_{k=0}^{\infty} g_0(k) F_k(z) \quad |g_0(k)| \leq M \rho^k \]

or

**probabilistic** (stochastic embedding; Goodwin et al. 1992), or **high-order modelling** (model error modelling, Ljung, 1999)

- Leading to ellipsoidal uncertainty areas in parameter and \( f \)-domain, valid with a prechosen level of probability
Linear parametrizations with flexible structures

Modelling and Identification with Rational orthogonal basis functions
(Heuberger, Van den Hof, Wahlberg, Eds., 2005)

With several chapters of our Hungarian partners, József Bokor and Zóltan Szabó
Closed-loop identification

Advantage: loop signals $u$ and $y$ are shaped with sensitivity function $S = 1/(1+CG_0)$:

Identification of models, such that $\frac{CG_0}{1 + CG_0} - \frac{C\hat{G}}{1 + C\hat{G}}$ is small: models relevant for $C$.

Controlled experiments excite $u$ and $y$ in the right $f$-region, while keeping signal amplitudes bounded.
Achievements

• Insight into the structural relation between model construction and control
• Tools for closed-loop identification and uncertainty bound quantification
• Robustness analysis/synthesis tools for identified uncertainty models
• Iterative schemes for modelling and control tuning, renewing “classical” adaptive control
Challenges ahead

• Design of **cheap experiments**:
  • least disturbing,
  • satisfying process constraints
  • minimum length,
  providing sufficient information for performance improvement

  *Problem:* “all” theory is asymptotic in $N$
  Most important for process control applications

• **Performance monitoring**: when is re-identification economically viable?
Congratulations László!!