Challenges in System Identification

From closed-loop to dynamic network identification

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Introduction

Model-based design and optimization is the dominant paradigm in the design – operation – maintenance of complex engineering systems

- Interacting robotics
- Smart (energy) grids
- Intelligent transportation systems
- Nano-positioning systems
- ……
## Future Requirements:

| Handling of          | highly complex interacting distributed systems that operate autonomously in a changing environment with changing objectives in a "learning" mode adapting to changing circumstances and maintain a verifiable high performance |
Introduction

Some required capabilities of models

1. **Accuracy assessment**
   on-line assessment of model validity

2. **Adaptability**
   flexible on-line updating of models (dynamics and interconnection structure)

3. **Active data-driven learning**
   demands on accuracy, autonomy, robustness
   → active probing for information

all relating to phenomena of data-driven modeling

Data-driven modeling becomes an integral part in virtually all complex engineering systems
Introduction

Example of current limitations:
Introduction

Example of current limitations:

- MPC projects in industry are highly dependent on accurate plant models and well-tuned controllers.
- Controllers and models are verified (identified) upon commissioning.
- When during operation circumstances change: MPC’s switched to “manual”.
- Loss of performance.
- Expensive experimental campaign to reidentify the models is the only way out.
Introduction

Next step in the development:

• Bring plant operation / automation to higher level of autonomy
• Monitor plant performance and detect changes on-line
• Generate probing signals when necessary and based on economic considerations (least costly experiments)
• Reidentify models and retune controllers on-line
• Keep high performance control
• Use economic performance criteria

Autoprofit

Autonomous economic model-based operation of industrial process systems
Introduction

Back to the core of the problem of data-driven modelling / identification of LTI models
Introduction

The classical identification problems:

- **Open loop**
  - Identify a plant model $\hat{G}$ on the basis of measured signals $u$, $y$ (and possibly $r$)

- **Closed loop**
  - Several classical methods available (PE, subspace, nonparam,..)
  - Well known results for identification *in known structure* (open loop, closed-loop, possibly known controller)
Introduction

Dynamical systems in emerging fields have a more complex structure:

- Distributed control system
- Dynamic network

Questions to be addressed:
- How to identify "single" transfers in a known (complex) structure?
- Can currently available tools from (closed-loop) identification be used for this purpose?
- How to identify the structure? (not focussed on here)
Contents

From open-loop and closed-loop identification to dynamic network identification

- Methods for (classical) closed-loop ID
- Dynamic network setup
- Network identification – Direct method
- Network identification – Two-stage method
- Illustrative example
- Two-stage method: generalizations
- Discussion
Closed-loop identification

Methods for closed-loop identification:

• Relying on full-order noise modelling
  (direct method)
  \((S \in \mathcal{M})\)

or:

• Relying on external excitation
  (indirect, two-stage, projection, IV)
  \((G_0 \in \mathcal{G})\)

Plant representation

\[ y(t) = G_0 u(t) + H_0 e(t) \]

- \(e\) white noise
- \(r\) and \(v\) uncorrelated
Closed-loop identification – Direct method

Model parametrization:
plant model $G(\theta)$, noise model $H(\theta)$

Prediction error:
$\varepsilon(t, \theta) = H(\theta)^{-1}[y(t) - G(\theta)u(t)]$

Parameter estimate:
$\hat{\theta}_N = \arg \min_{\theta} \frac{1}{N} \sum_{t=1}^{N} \varepsilon(t, \theta)^2$

Under weak regularity conditions:
$\hat{\theta}_N \rightarrow \theta^*$ for $N \rightarrow \infty$ w.p. 1

The plant model is consistently estimated ($G(\theta^*) = G_0$) if:

- Plant and noise model are correctly parametrized ($\mathcal{S} \in \mathcal{M}$)
- The feedback loop has at least one delay (no algebraic loop)
- For $z := \text{vec}(y, u)$ the power spectral density $\Phi_z(\omega) > 0 \quad \forall \omega$

[Ljung, 1987]
Closed-loop identification – Two-stage method

Decompose the input signal:

\[ u = u^r + u^v \]

such that \( u^r \) and \( v \) are uncorrelated

Prediction error:

\[ \varepsilon(t, \theta) = H(\rho)^{-1}[y(t) - G(\theta)u^r(t)] \]

A similar ``standard” (LS) identification approach then leads to:

The plant model is consistently estimated \((G(\theta^*) = G_0)\) if:

- Plant model is correctly parametrized \((G_0 \in \mathcal{G})\)
- Plant and noise model are independently parametrized
- \( u^r \) is persistently excitation of sufficient order, e.g.

\[ \Phi_{u^r}(\omega) > 0 \quad \forall \omega \]
Closed-loop identification – Two-stage method

Decompose the input signal:

\[ u = u^r + u^v \]

such that \( u^r \) and \( v \) are uncorrelated

How to do this decomposition?

- Write: \( u = F_{ur}r + F_{uv}v \)
- Identify a model \( \hat{F}_{ur} \) on the basis of measured signals \( r \) and \( u \) (this is an open-loop problem)
- Construct \( \hat{u}^r = \hat{F}_{ur}r \)
- Use \( \hat{u}^r \) as an estimate of \( u^r \)

\( u^r \) is the projection of \( u \) onto the space of causally time-shifted versions of \( r \)

[Van den Hof & Schrama, 1993]
Closed-loop identification

Direct method
- Plant and noise model need to be identified simultaneously
- No algebraic loops

Two-stage/projection
- Plant model can be consistently identified without noise model
- External excitation is necessary

- Controller information is not required / utilized

Many more variants of closed-loop PE identification methods:
- **Indirect, Joint IO** [Ljung, 1999]
- **IV methods** [Gilson & Van den Hof, Automatica, 2005]
- **Dual-Youla** [Van den Hof, Annual Rev. Control, 1998]
- **Virtual Closed-Loop** [Aguero, Goodwin & Van den Hof, Automatica 2011]

see also [Forssel & Ljung, Automatica, 1999]
Question

• Can we utilize these tools for identification of transfer functions in a (complex) dynamic network?

Node and link structure:
• Nodes are signals
• Directed links (edges) are causal transfers
Network Setup

Formalizing one link (transfer between \( w_i \) and \( w_j \))

- On each node a disturbance \( v_j \) and a reference \( r_j \) might be present
- Reference signals are uncorrelated to noise signals
- \( N_j \): set of nodes that has a direct causal link with node \( j \)
Network Setup

Assumptions:
- Total of $L$ nodes
- Network is well-posed
  \[ I - G^0 \text{ invertible} \]
- Stable (all signals bounded)
- All $w_m, m = 1, \ldots, L$, measured, as well as all present $r_m$

\[
\begin{bmatrix}
  w_1 \\
  w_2 \\
  \vdots \\
  w_L
\end{bmatrix} = 
\begin{bmatrix}
  0 & G^0_{12} & \cdots & G^0_{1L} \\
  G^0_{21} & 0 & \cdots & G^0_{2L} \\
  \vdots & \vdots & \ddots & \vdots \\
  G^0_{L1} & G^0_{L2} & \cdots & 0
\end{bmatrix} 
\begin{bmatrix}
  w_1 \\
  w_2 \\
  \vdots \\
  w_L
\end{bmatrix} + 
\begin{bmatrix}
  r_1 \\
  r_2 \\
  \vdots \\
  r_L
\end{bmatrix} + 
\begin{bmatrix}
  v_1 \\
  v_2 \\
  \vdots \\
  v_L
\end{bmatrix}
\]
Network Setup

The set of nodes $\mathcal{N}_j$ is separated into

- $i$: our plant input
- $\mathcal{K}_j$: reflecting nodes $k$ with known transfers $G_{jk}^0$
- $\mathcal{U}_j$: reflecting nodes $k$ with unknown transfers $G_{jk}^0$
- $\mathcal{U}_j^i$: reflecting nodes $k \neq i$ with unknown transfers $G_{jk}^0$

Identification setting

- We focus on identifying a single transfer $G_{ji}^0$
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Applying direct method to input $w_i$ and output $w_j$ will lead to biased results
- if the prediction error can not be whitened, or equivalently
- If there are nodes in $\mathcal{U}_j$ that affect $w_j$

A MISO approach:

$$\varepsilon(t, \theta) = H_j(\theta)^{-1}[w_j - r_j - \sum_{k \in \mathcal{K}_j} G_{jk}^0 w_k - G_{ji}(\theta)w_i - \sum_{k \in \mathcal{U}_j^i} G_{jk}(\theta)w_k]$$

$\tilde{w}_j$ known

Simultaneous identification of transfers $G_{jk}^0, k \in \mathcal{U}_j$ and a noise model for $v_j$
Network Identification – Direct method

Result direct method

The plant model $G_{ji}(\theta)$ is consistently estimated ($G_{ji}(\theta^*) = G_{ji}^0$) if:

- All parametrized plant and noise models are correctly parametrized, $G_{jk}(\theta), \ k \in \mathcal{U}_j; \ H_j(\theta) \ (S \in \mathcal{M})$
- Every loop in the network that runs through node $j$ has at least one delay (no algebraic loop)
- $\Phi_z(\omega) > 0 \ \forall \omega, \ for \ z := vec\{w_j, \{w_k\}_k \in \mathcal{U}_j\}$ (excitation condition)
- Noise source $v_j$ is uncorrelated with all other noise terms in the network

[Dankers et al., CDC2012 submitted]
Network Identification – Direct method

Observation:

- Direct identification “works”
- Requires full noise models (whitened residual)
- Require a MISO approach (more transfers to be simultaneously identified)
- and a “strict” excitation condition

Next question:

- Can we solve the problem without the full noise models and without the MISO approach?
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Main approach:
• Look for an external reference signal that has a connection with $w_i$
• And that does not act as an unmodelled disturbance on $w_j$
Network Identification – Two-stage method

Algorithm:
• Determine whether there exists an $r_m$ such that $w_i^{r_m}$ is sufficiently exciting
• Construct:
  \[ \tilde{w}_j = w_j - r_j - \sum_{k \in \mathcal{K}_j} G^0_{j_k} w_k \]
  known terms

  \[ \varepsilon(t, \theta) = H_j(\rho)^{-1} [ \tilde{w}_j - G^0_{j_i}(\theta) w_i^{r_m}] \]
  with the noise model fixed or parametrized independently from $\theta$
• The unmodelled terms $G^0_{j_k} w_k$, $k \in \mathcal{U}_j^i$ appear as additional disturbance terms on the output $w_j$
Network Identification – Two-stage method

Result two-stage method

The plant model $G_{ji}(\theta)$ is consistently estimated ($G_{ji}(\theta^*) = G^0_{ji}$) if:

- The plant model $G_{ji}(\theta)$ is correctly parametrized, ($G^0_{ji} \in \mathcal{G}$)
- $w^{rm}_i$ is sufficiently exciting for identification of $G^0_{ji}$,
  e.g. $\Phi_{w^{rm}_i}(\omega) > 0 \ \forall \omega$
- All $w_k, k \in \mathcal{U}^i_j$ are uncorrelated to $r_m$

[Van den Hof et al., CDC2012 submitted]
Network Identification – Two-stage method

Observation:

- Consistent identification of single transfers is possible, dependent on network topology and reference excitation
- Full noise models are not necessary
- No conditions on uncorrelated noise sources, nor on absence of algebraic loops
- Excitation condition on single excitation signal
Illustrative example

Network with 5 nodes – Configurations for direct identification

\[ G_{21} \quad G_{32} \quad G_{45} \quad G_{54} \]

\[ G_{15} \]

Variables:
- \( v_1, v_2, v_3, v_4, v_5 \)
- \( w_1, w_2, w_3, w_4, w_5 \)
- \( r_1 \)
Illustrative example

Network with 5 nodes – Configurations for direct identification

\[ G_{21} \quad + \quad G_{32} \quad + \quad G_{23} \quad - \quad G_{54} \quad - \quad G_{15} \]

Inputs:
- \( v_1 \) connected to \( G_{21} \)
- \( r_1 \)

Outputs:
- \( w_1 \) from \( G_{21} \)
- \( w_2 \) from \( G_{32} \)
- \( w_3 \) from \( G_{23} \)
- \( w_4 \) from \( G_{54} \)
- \( w_5 \) from \( G_{15} \)

Connections: (wires not shown for clarity)
Illustrative example

Network with 5 nodes – Configurations for direct identification

\[ G_{21} + G_{32} + v_3 \]
\[ w_2, w_3 \]
\[ G_{23} - G_{54} \]
\[ v_4 \]
\[ v_5, w_4 \]
\[ G_{15} \]
Illustrative example

Network with 5 nodes – Configurations for direct identification

\[
\begin{align*}
G_{21} & \quad w_1 \quad + \quad v_1 \quad - \\
G_{32} & \quad w_2 \quad + \quad v_2 \quad - \\
G_{45} & \quad w_4 \quad - \\
G_{15} & \\
\end{align*}
\]
Illustrative example

Network with 5 nodes – Configurations for direct identification
Illustrative example

Network with 5 nodes – Configurations for direct identification
Illustrative example

Single transfers that can be identified by **two-stage identification**:

- Input signal correlated with $r_1$
- Output does not have unmodelled components correlated with $r_1$
Two-stage method – further results

Next questions:

- Can the conditions on $r_m$ be checked in a structured way?
- Generalization of the approach towards:
  - MISO transfers
  - Variance reduction through multiple external signals
Two-stage method – further results

Check on topological structure of the network for two-stage identification of \( G_{ji}^0 \)

Consider graph matrix: \( A \in \{0, 1\}^{L \times L} \) that represents the presence of causal transfers: \( A_{ji} = 1 \) if \( G_{ji}^0 \neq 0 \)

In our example: 
\[
A = \begin{bmatrix}
0 & 0 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]

Then \( [A^\ell]_{im} \) equals the number of different (causal) path connections from \( w_m \) to \( w_i \).
Two-stage method – further results

Let $\mathcal{R}$ be set of node numbers $m$ for which there is an $r_m$ present

Topological conditions on two-stage identification of $G_{j,i}^0$

- There exists an $m \in \mathcal{R}$ and $\ell \in 1, 2, \ldots, L$ such that $[A^\ell]_{im} \neq 0$
- For all $k \in U_j^i$ it holds that $[A^\ell]_{km} = 0$ for all $\ell \in 1, 2, \ldots, L$

Simple manipulations on the graph matrix.
Two-stage method – further results

Extension to multiple transfers (MISO)

\[ G_{21} + G_{32} + G_{23} - G_{15} + G_{45} \]

\[ v_1 \rightarrow w_1 \rightarrow G_{21} \rightarrow v_2 \rightarrow w_2 \rightarrow G_{32} \rightarrow v_3 \rightarrow w_3 \rightarrow G_{54} \rightarrow v_5 \rightarrow w_5 \]

\[ r_1 \rightarrow w_1 \rightarrow G_{23} \rightarrow w_2 \rightarrow G_{32} \rightarrow w_3 \rightarrow G_{54} \rightarrow w_4 \rightarrow G_{45} \]

\[ v_1, v_2, v_3, v_4, v_5; w_1, w_2, w_3, w_4, w_5 \]

\[ r_1, v_1, v_2, v_3, v_4, v_5; w_1, w_2, w_3, w_4, w_5 \]
Two-stage method – further results

Extension to multiple transfers (MISO)

Idea:
Combine $r_m$-correlated inputs to output $w_j$ into a MISO problem

- Determine for $r_m$ to which signals $w_k, k \in \mathcal{N}_j$ it is connected, $\rightarrow \mathcal{N}_j^m$
- Identify the transfers $G_{jk}, k \in \mathcal{N}_j^m$ in a MISO setting with the two-stage method
Two-stage method – further results

Extension to multiple transfers (MISO)

Identification of the transfers \( \begin{pmatrix} w_1 \\ w_3 \end{pmatrix} \rightarrow w_2 \) can be done consistently, provided the excitation conditions are satisfied.
Two-stage method – further results

Extension to multiple excitation signals

Contributes to worse signal-to-noise ratio, i.e. variance increase
Two-stage method – further results

Extension to multiple excitation signals

Remedy:

- Combine projections to as many external signals as possible
  \[ w_i^{r_{m_1}} + w_i^{r_{m_2}} + \cdots \]

- Even "reconstructable" noise signals can be used for this purpose
  \[ w_i^{r_{m_1}} + w_i^{v_{m_2}} + \cdots \]

A noise signal \( v_j \) is reconstructable if all transfers into node \( j \) are known; then \( v_j = w_j - r_j - \sum_{k \in N_j} G_{jk}^0 w_k \) is known
Two-stage method – further results

Extension to multiple excitation signals

- If $G_{32}$ is known
- noise signal $v_3$ can be reconstructed
- which excites $w_3$ to identify $G_{23}$
- and output $w_2$ is not disturbed by a $v_3$-dependent term
Identification of network structures

- So far considering the situation that the structure is known
- Identification of the structure (causality) is getting attention in the literature, in particular forms:
  - Tree-like structures (no loops)
  - Nonparametric methods (Wiener filter)
  - Mostly networks without external excitation
    
    see e.g. Materassi, Innocenti (TAC-2010)

- and in relation to compressive sensing methods for sparse identification

- Whenever loops are present and noises are non-white, the consistency results of closed-loop methods become important
  
  see e.g. Dankers et al., (IFAC SYSID 2012).
Summary

• Data-driven modelling will only grow in its importance for high-performing engineering systems
• Current framework for open/closed-loop identification has to be extended to dynamic networks
• Methods for closed-loop identification extend to this case with some new properties
• They are expected to provide the basic tools for dealing with the structure identification problem also
• Many new questions pop up……
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