Identifiability in Dynamic Acyclic Networks with Partial Excitation and Measurement

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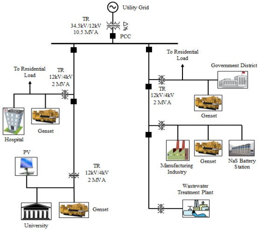
Outline

Introduction – Dynamic Networks

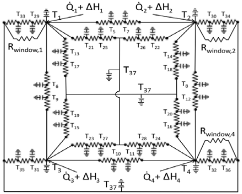
Appear in a wide range of applications

Power network



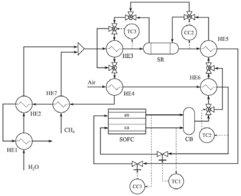
Chen, "Resilient Distribution Systems With Community Microgrids" 2016

RC circuit



K. Deng et al, Automatica 2014

Chemical process



Heo, S., et al. Chemical Engineering Science, 2015

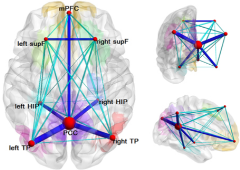
Cooperative robots

Robots in University of Groningen

Traffic network

N. Martin et al., IEEE NSE, 2019

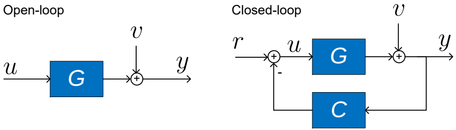
Brain networks



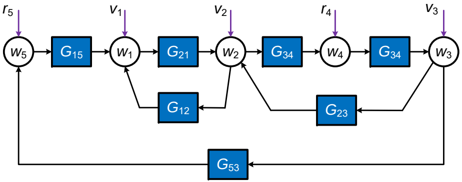
M. R. Knight IEEE Electrification Magazine 2016

Introduction

- Classical identification problems[1]:
  Identify a model of G based on measured signals u, y (and possibly r)

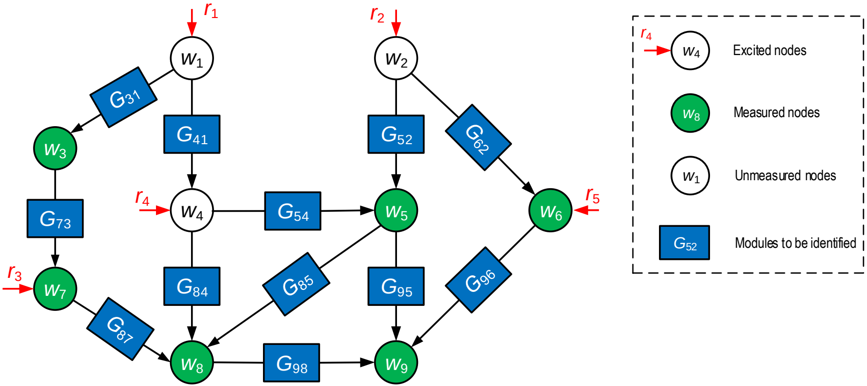


- How to address identification problems in a network setting?
  To analyze interconnection structure of the signals.



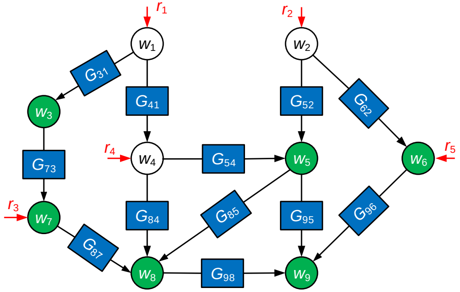
[1] Ljung (1999), Pintelon and Schoukens (2012)

Dynamic Acyclic Networks



We concentrate on acyclic networks, i.e. there does not exist a pair of vertices that can reach each other.

Network Model Setting



A compact form of the overall network:

$$
\begin{aligned}
w(t) &= G(q)w(t) + Rr(t) + v_{\mathrm{e}}(t),\\
y(t) &= Cw(t) + v_{\mathrm{m}}(t),
\end{aligned}
$$

- q^{-1}: delay operator
- $r(t)$, $y(t)$: measured external and internal signals
- R, C: indicate which nodes are excited and measured.

Assume:

- The network is *well-posed* and stable, i.e. $(I - G(q))^{-1}$ is proper and stable.
- All the entries of $G(q)$ are proper and stable transfer operators.

Network Identifiability

- From measurement data (r, y), we obtain transfer function:

$$T_{\mathcal{C},\mathcal{R}} = C \underbrace{(I - G)^{-1}}_{T} R$$

[2]Weerts et al., SYSID2015; Weerts et al., Automatica, 2018;
[3]Bazanella, CDC2017; Hendrickx et al., IEEE-TAC, 2019.

Network Identifiability

- From measurement data (r, y), we obtain transfer function:

$$T_{\mathcal{C},\mathcal{R}} = C \underbrace{(I - G)^{-1}}_{T} R$$

- Consider a network model set $\mathcal{M} := \{M(q,\theta) = (G(q,\theta), R, C), \theta \in \Theta\}$,
  Identifiability[2] of $\mathcal{M}$: all the models (i.e. the entries of G) in $\mathcal{M}$ can be distinguished from $T_{\mathcal{C},\mathcal{R}}$.

- **Generic identifiability**[3]: *almost all* models in $\mathcal{M}$ can be distinguished from $T_{\mathcal{C},\mathcal{R}}$ (excluding parameters that are in a subset of Θ with Lebesgue measure zero)

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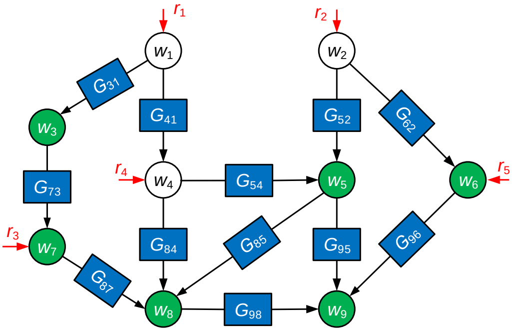
[2] Weerts et al., SYSID2015; Weerts et al., Automatica, 2018;
[3] Bazanella, CDC2017; Hendrickx et al., IEEE-TAC, 2019.

Problems

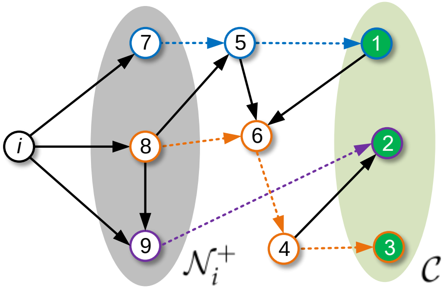


1. Under what conditions, the network model set is generically identifiable?
2. Which nodes are measured/excited to achieve generic identifiability.

Full Excitation - Path-Based Condition

Lemma (Path-based condition)

All transfer functions leaving node i is generically identifiable if and only if there are $|\mathcal{N}_i^+|$ vertex disjoint paths from $\mathcal{N}_i^+$ to $\mathcal{C}$.
The model set $\mathcal{M}$ is generically identifiable, if the condition holds for all $i \in \mathcal{V}$.



- all the nodes are excited
- $\mathcal{C}$: the set of measured nodes
- $\mathcal{N}_i^+$: the set of the out-neighbors of node i;

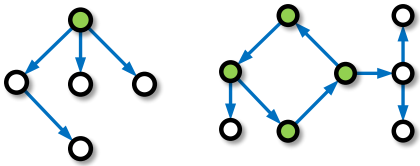
[4] Van der Woude, 1991, Hendrickx, Gevers & Bazanella, CDC 2017, Weerts et al., CDC 2018

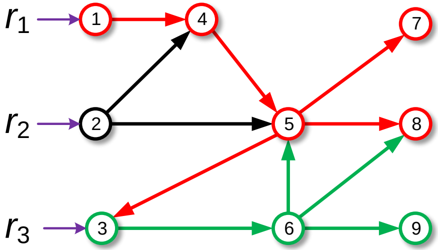
Full Measurement - Pseudotree-Covering Condition

Lemma (Pseudotree-Covering Condition[4])

The network model set $\mathcal{M}$ is generically identifiable if all the edges can be covered by a set of disjoint pseudotrees, and a root of each pseudotree is excited.

Pseudotrees: A connected directed graph with maximal indegree 1



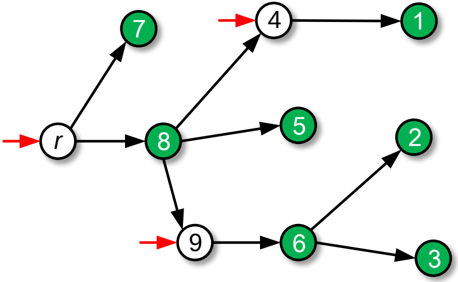


[5]Cheng et al., CDC2019; Cheng et al., IEEE-TAC2021

Partial Excitation/Measurement - Trees

Lemma (Trees[5])

A tree is generically identifiable if and only if its root is excited, all the leaves are measured, and the internal nodes are either excited or measured.

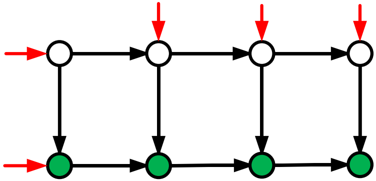


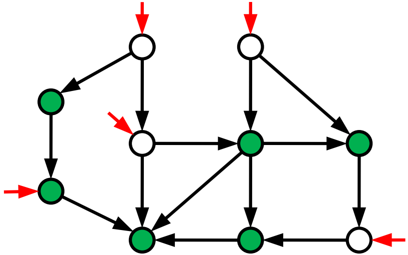
- root: the node has no in-neighbors
- leaf: the node has no out-neighbors
- internal nodes: the nodes that are neither root nor leaves

[6]Bazanella et al., CDC2019

General Acyclic Networks with Partial Excitation/Measurement

The above methods cannot be applied to acyclic networks:

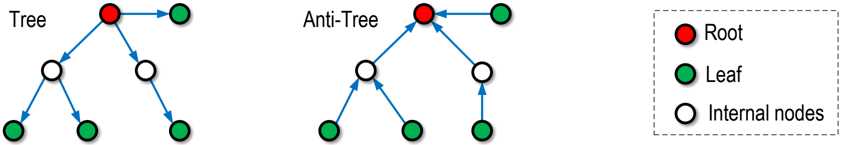




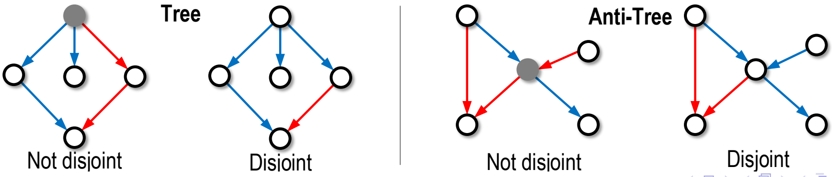
Problem

How to determine (generic) identifiability in acyclic networks?

Disjoint Trees and Disjoint Anti-Trees



- Two trees (anti-trees) are **disjoint** if they do not share common edges, and all the edges leaving from (pointing to) a node are in the same tree (anti-trees).
- Any acyclic network can be decomposed into a set of disjoint trees or anti-trees

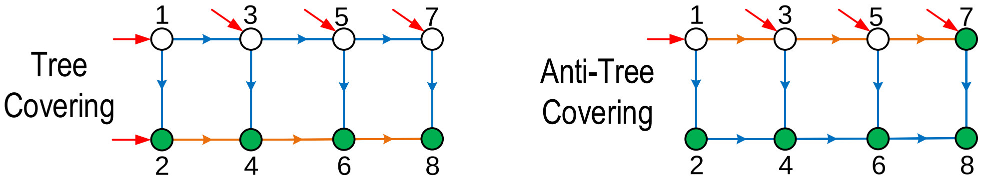


Generic Identifiability Condition

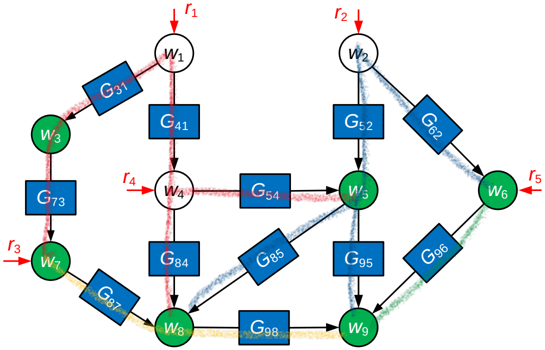
Theorem (Tree/Anti-tree covering)

Suppose that the underlying graph $\mathcal{G}$ is acyclic with $\mathcal{V} = \mathcal{R} \cup \mathcal{C}$. Then, $\mathcal{M}$ is generically identifiable if either of the following two conditions holds:

1. *$\mathcal{G}$ can be decomposed into a set of disjoint trees, and for each tree, its root is excited and all the leaves are measured.*
2. *$\mathcal{G}$ can be decomposed into a set of disjoint anti-trees, and for each anti-tree, its root is measured, and all the leaves are excited.*



Example



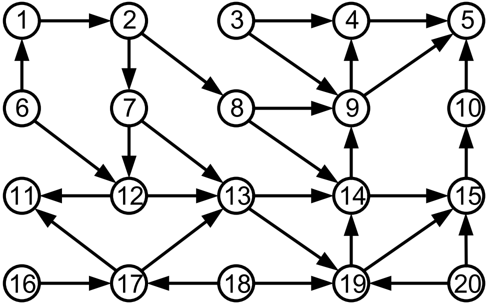
all the edges are covered by four trees ✓   roots are excited ✓   leaves are measured ✓

$\Rightarrow$ **identifiability**

Actuator/Sensor Allocation (Application of Condition 1)

Initialization:

- All the sources are excited, and all the sinks are measured;
- Nodes having more than one in-neighbors are measured (implied by Condition 1).



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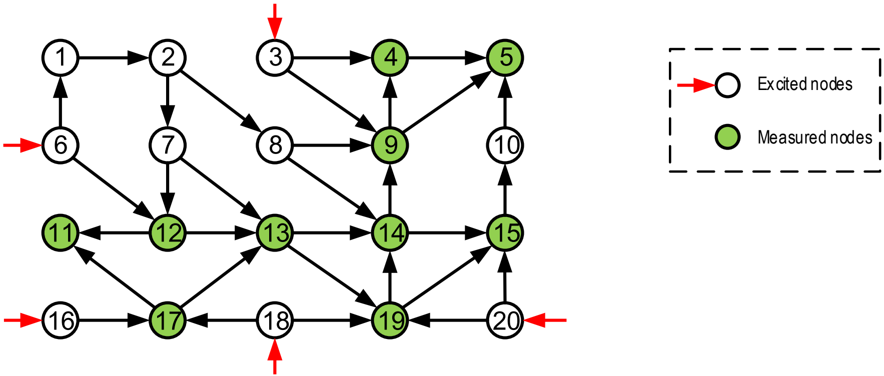
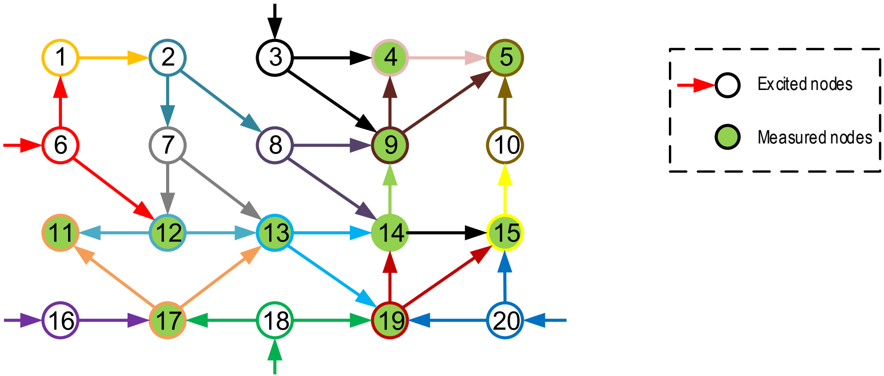


Actuator/Sensor Allocation (Application of Condition 1)

A greedy tree merging procedure:

- Partition the graph into a set of smallest trees (i.e. a node with its out-going edges)
- Merge two trees if their union is still a tree



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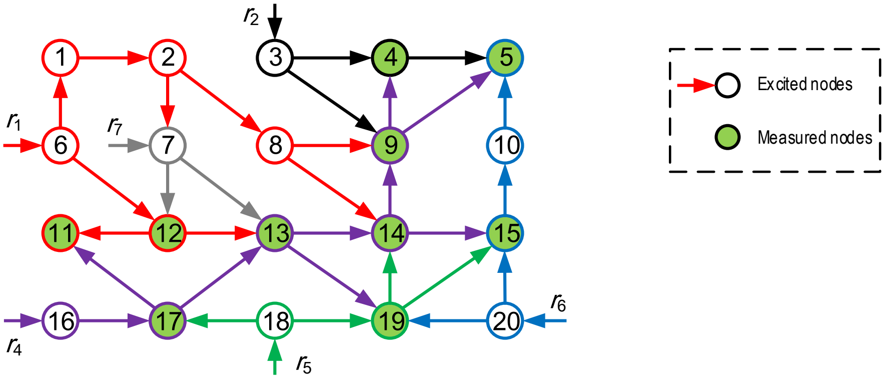

- A sufficient condition to characterize the generic identifiability in acyclic networks with partial excitation/measurement
- Extensions: cyclic networks contain known modules and correlated noise signals

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Thank you for your attention!