Dynamic Network Structure Identification with Prediction-Error Methods:
Basic Examples

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Relevance of Presentation

Trend towards considering complex networks of interconnected systems (distributed control systems, biological systems, etc.).

Can standard identification tools be used to identify dynamics in complex networks?

> Used to identifying systems in well defined configurations (open-loop, closed-loop, etc.)

Question: What sort of problems could be encountered if the structure is not known a-priori?

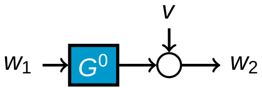
Outline of Presentation

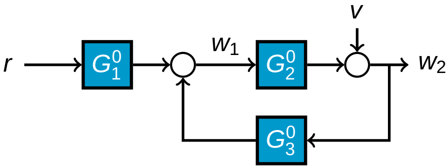
- **Introduction and Problem setting**
- Prediction-Error Method
- Simple Data Generating Systems to Illustrate Potential Problems
  - Cascaded System
  - Closed Loop System

Structure Identification - Early Contributions

Question whether an open loop or closed loop structure is present between two measured variables was addressed in late 70's, early 80's.[1][2].

Conclusion was: if each transfer is driven by an independent (noise) source, then distiction can be made.





[1] M.R. Gevers and B.D.O Anderson. "Representing of Jointly Stationary Stochastic Feedback Processes". In: *International Journal of Control* 33.5 (1981), pp. 777–809.

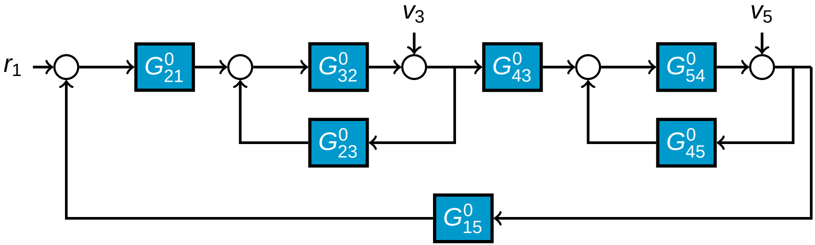
[2] P.E. Caines. "Weak and Strong Feedback Free Processes". In: *IEEE Transactions on Automatic Control* 21 (5 1976), pp. 737–739.

Identification of Networks

Renewed interest in structure detection in fields of biology, chemistry, economics, and machine learning, etc.[3][4].

Interested in more complex structures

Goal: detect interconnection architecture using only a given data set.



[3] D. Materassi, M.V. Salapaka, and L Giarrè. "Relations between structure and estimators in networks of dynamical systems". In: *Proceedings of 50th IEEE Conference on Decision and Control*. Orlando, USA, 2011.

[4] Y. Yuan et al. "Robust Dynamical Network Reconstruction". In: *Proceedings of 49th IEEE Conference on Decision and Control*. Atlanta, USA, 2010, pp. 810–815.

Focus of This Presentation

In literature:

- non-parametric (Wiener and Granger Filter) or FIR model structures have been used to detect interconnection architecture,
- and often the noise is not modelled,
- noise sources are assumed to be present at every measurement point.

In this paper:

- will use the Prediction-Error Method to detect the interconnection architecture,
- effect of modelling the noise will be investigated,
- attention will be given to the sensitivity of the approach to the presence of noise sources.

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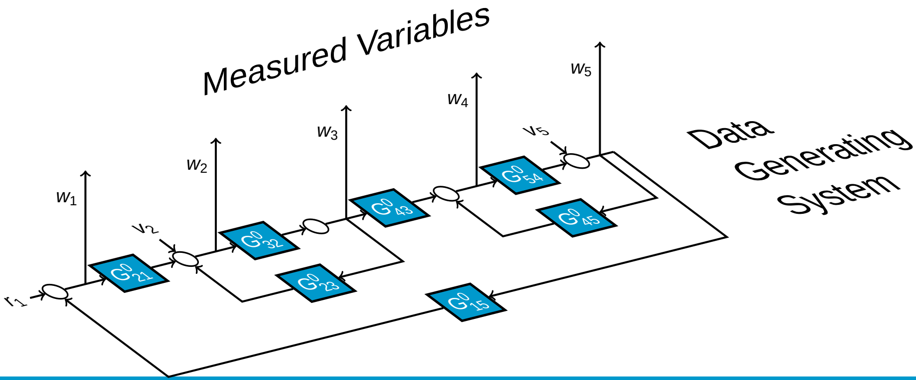
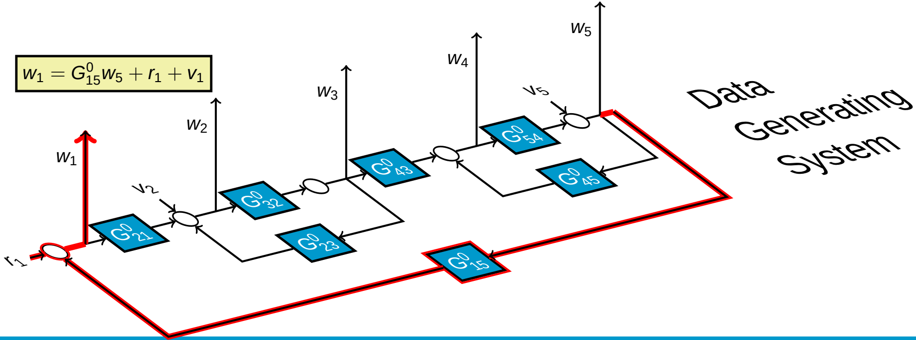
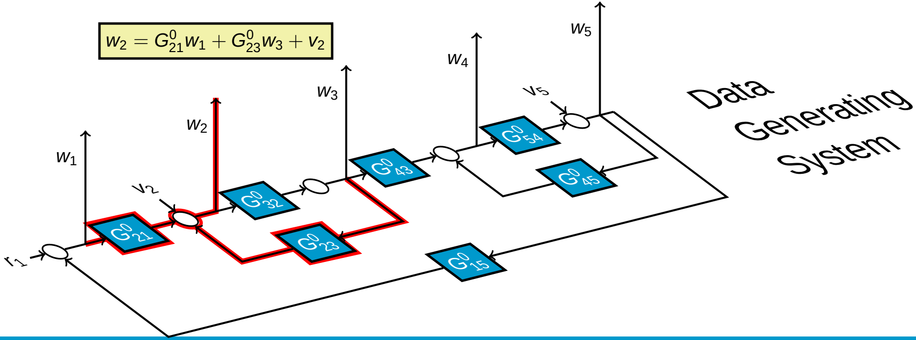
Measurements in an Interconnected System



Measurements in an Interconnected System



Measurements in an Interconnected System



Prediction Error Identification

The multi-input, single output one step ahead predictor model is:

$$\hat{w}_j(t|t-1,\theta) = H_j^{-1}(q,\theta)\Big(\sum_{k\in\mathcal{N}_j} G_{jk}(q,\theta)w_k(t)\Big) + \Big(1 - H_j^{-1}(q,\theta)\Big)y(t)$$

Some user choices:

- which measured variables w_k to incldue as inputs,
- include a noise model (OE vs. BJ).

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Prediction Error Identification

The prediction error is:

$$\varepsilon_j(t,\theta) = w_j - \hat{w}_j(t|t-1,\theta)$$

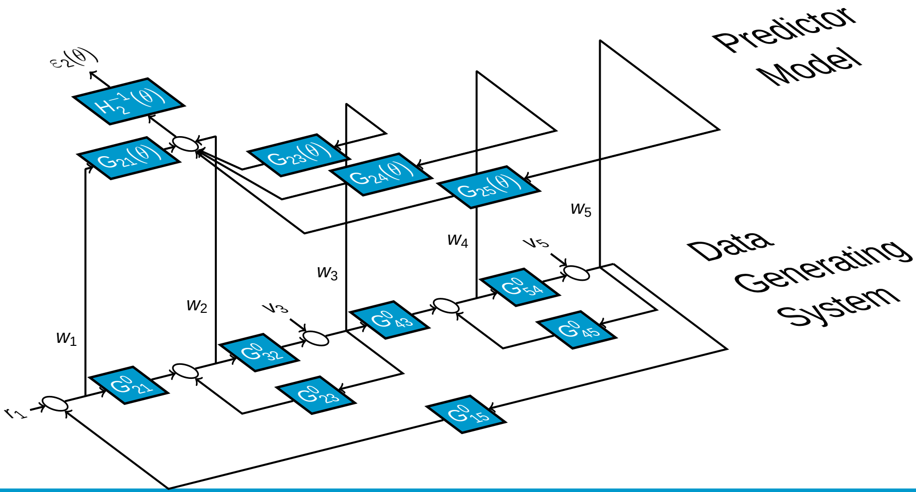
The prediction error estimate is defined by

$$\hat{\theta}_N = \arg\min_{\theta} \frac{1}{N} \sum_{t=0}^{N-1} \varepsilon_j(t,\theta)^2.$$

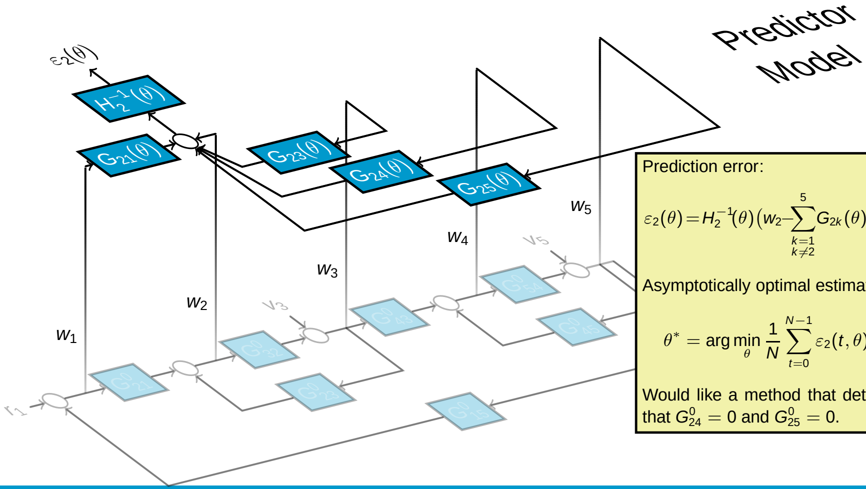
Assume that $\mathcal{S} \in \mathcal{M}$. Then the estimates are called consistent if

$$G_{jk}(q,\hat{\theta}_N) \rightarrow G_{jk}^0(q) \text{ and } H_j(q,\hat{\theta}_N) \rightarrow H_j^0(q) \text{ w.p. 1 as } N \rightarrow \infty$$

Predictor if Interconnection Architecture is Unknown



Predictor if Interconnection Architecture is Unknown



Prediction error:

$$\varepsilon_2(\theta) = H_2^{-1}(\theta)\Big(w_2 - \sum_{\substack{k=1\\k\neq 2}}^{5} G_{2k}(\theta) w_k\Big)$$

Asymptotically optimal estimate is

$$\theta^* = \arg\min_{\theta} \frac{1}{N}\sum_{t=0}^{N-1} \varepsilon_2(t,\theta)^2$$

Would like a method that detects that $G_{24}^0 = 0$ and $G_{25}^0 = 0$.

Connection to Wiener Filtering

In this paper it is shown that in fact an optimal Wiener Filter is equivalent to an asymptotically optimal OE model structure.

> Advantage: many tools available in the Prediction-Error Framework which allows to analyse the conditions required in order to obtain consistent estimates of the dynamics.

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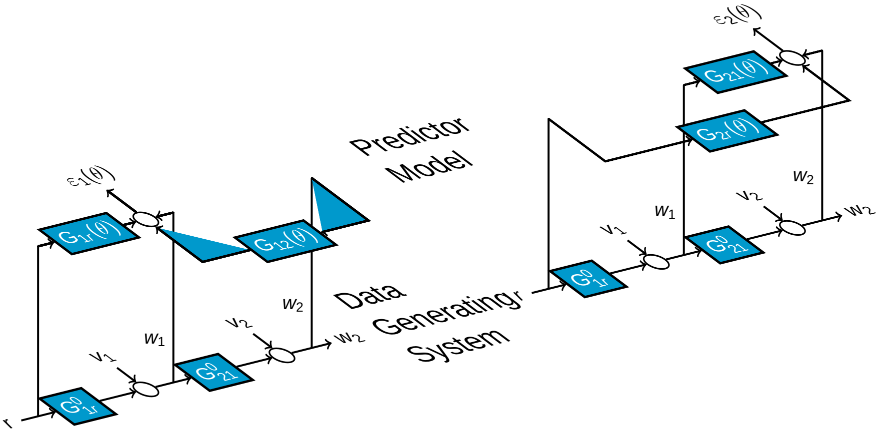
  - **Closed Loop System**

Cascade Data Generating System



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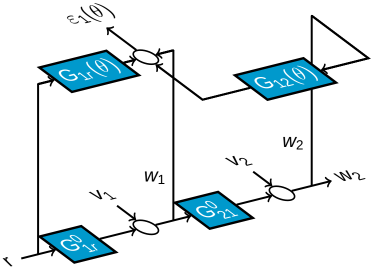
Cascade Data Generating System



- Assume noise is white.
- No noise model ($H_1(\theta) = 1$).
- Assume G^0_{1r} and G^0_{21} are strictly proper.

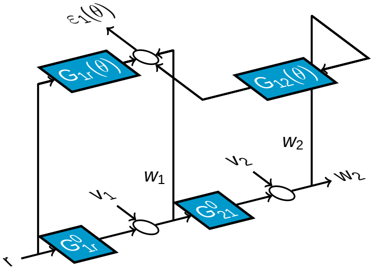
Prediction error:

$$
\begin{aligned}
\varepsilon_1(\theta) &= w_1(t) - \hat{w}_1(t|t-1,\theta) \\
&= w_1(t) - G_{12}(\theta)w_2(t) - G_{1r}(\theta)r(t) \\
&= \left(G^0_{1r} - G_{12}(\theta)G^0_{21}G^0_{1r} - G_{1r}(\theta)\right)r(t) \\
&\quad \left(1 - G_{12}(\theta)G^0_{21}\right)H^0_1 e_1(t) - G_{12}(\theta)H^0_2 e_2(t)
\end{aligned}
$$

σ_{v_1}	> 0	> 0	$= 0$	$= 0$
σ_{v_2}	> 0	$= 0$	> 0	$= 0$
$G_{12}(\theta^*)$	0	0	0	nonunique
$G_{1r}(\theta^*)$	G^0_{1r}	G^0_{1r}	G^0_{1r}	nonunique

Conclusion: As long as one of the noise sources is present, correct structure is identified.

Cascade Data Generating System

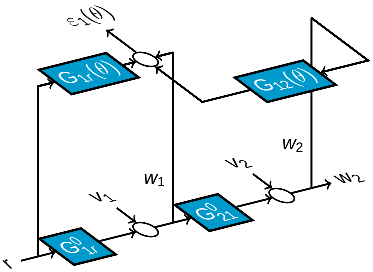


- Assume noise is not white.
- No noise model ($H_1(\theta) = 1$).
- Assume G^0_{1r} and G^0_{21} are strictly proper.
- Consider all combinations of presence of noise sources:

σ_{v_1}	> 0	> 0	$= 0$	$= 0$
σ_{v_2}	> 0	$= 0$	> 0	$= 0$
$G_{12}(\theta^*)$	biased	biased	0	nonunique
$G_{1r}(\theta^*)$	biased	biased	G^0_{1r}	nonunique

Conclusion: in general structure is not detected (only in the absence of v_1 is correct structure detected).

Cascade Data Generating System

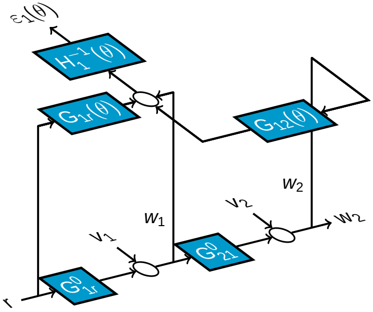


- Assume noise is not white.
- No noise model ($H_1(\theta) = 1$).
- Assume G^0_{1r} and G^0_{21} are not strictly proper.
- Consider all combinations of presence of noise sources:

σ_{v_1} σ_{v_2}	> 0 > 0	> 0 $= 0$	$= 0$ > 0	$= 0$ $= 0$
$G_{12}(\theta^*)$ $G_{1r}(\theta^*)$	biased biased	$(G^0_{21})^{-1}$ 0	0 G^0_{1r}	nonunique nonunique

Conclusion: again, in general structure is not detected. Only when v_1 is not present is structure correctly detected.

Cascade Data Generating System



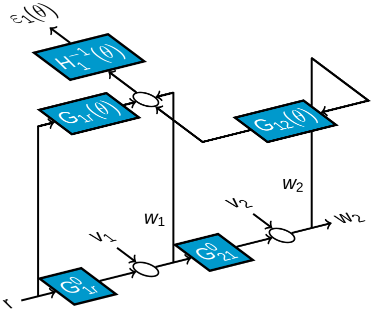
- Assume noise is not white.
- Noise is modelled.
- G^0_{1r} and G^0_{21} could be strictly proper or proper.
- Consider all combinations of presence of noise sources:

σ_{v_1} σ_{v_2}	>0 >0	>0 $=0$	$=0$ >0	$=0$ $=0$
$G_{12}(\theta^*)$ $G_{1r}(\theta^*)$	0 G^0_{1r}	nonunique nonunique	0 G^0_{1r}	nonunique nonunique

Conclusion: adding a noise model adds extra flexibility, which in some cases leads to non-unique estimates, which means strucuture is not detected.

Modelling the noise is only an advantage if both v_1 and v_2 are present.

Cascade Data Generating System



Discussion:

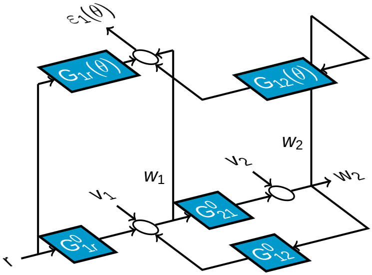
In open-loop identification

- it is possible to consistently identify transfers without noise models;
- the added flexibility of a noise model will not be a hinderance.

Both properties disappear in the structure detection problem.

Location and presence of noise sources is important when attempting to detect the structure.

Single Closed Loop Data Generating System

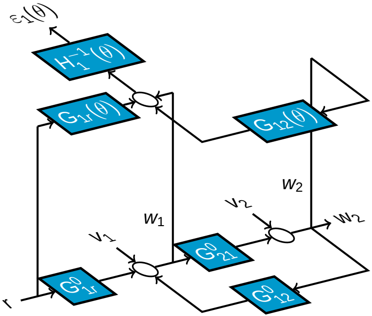


- Assume noise is colored.
- No noise model ($H_1(\theta) = 1$).
- Assume G^0_{1r} and G^0_{21} are strictly proper.
- Consider all combinations of presence of noise sources:

σ_{v_1} σ_{v_2}	> 0 > 0	> 0 $= 0$	$= 0$ > 0	$= 0$ $= 0$
$G_{12}(\theta^*)$ $G_{1r}(\theta^*)$	biased biased	nonunique nonunique	G^0_{12} G^0_{1r}	nonunique nonunique

Conclusion: in general structure is not detected (only in the absence of v_1 is correct structure detected).

Single Closed Loop Data Generating System



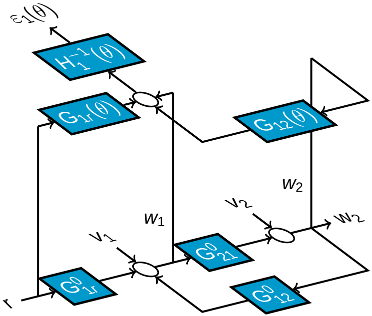
- Assume noise is colored.
- Noise is modelled.
- Assume G^0_{1r} and G^0_{21} are strictly proper.
- Consider all combinations of presence of noise sources:

σ_{v_1} σ_{v_2}	>0 >0	>0 $=0$	$=0$ >0	$=0$ $=0$
$G_{12}(\theta^*)$ $G_{1r}(\theta^*)$	G^0_{12} G^0_{1r}	nonunique nonunique	G^0_{12} G^0_{1r}	nonunique nonunique

Conclusion: adding a noise model adds extra flexibility, which in some cases leads to non-unique estimates, which means strucuture is not detected.

Modelling the noise is only an advantage if both v_1 and v_2 are present.

Single Closed Loop Data Generating System



Conclusions, similar to the cascaded data generating system,

- All predictors obtained consistent estimates of G_{12}^0 and G_{1r}^0 if noise source v_2 is present.
- Even modelling the noise could not obtain consistent estimates if noise source v_2 not present.

Location and presence of noise sources is important when attempting to detect the structure.

Advantage of External Excitation

The structure detection problem seems to be sensitive to the presence/absence of noise sources.

In the closed loop identification literature, Instrumental Variable (IV) and Two-Stage Methods are able to obtain consistent estimates without noise modelling.

This suggests that perhaps the IV and Two-Stage Methods could be applied to the structure identification problem, and be less sensitive to the presence/absence of noise sources.

Conclusion

Recap:

Tools from the Prediction-Error Framework were used to analyse the possibilty of detecting the structure of a network using only measurements.

Results:

- By showing the (asymptotical) equivalence of Wiener Filtering Framework to the Prediction Error framework, open up the structure detection field to a wide range of analysis tools.
- Based on simple test cases, it is shown that common knowledge in the open and closed loop system identification literature does not always hold for the structure detection problem.

Future Work:

- More complex cases have since been considered[5]
- The IV and Two-Stage Methods should be investigated for use in the structure detection problem[6].

[5] A. Dankers et al. "Dynamic Network Identification Using the Direct Prediction Error Method". In: *Proceedings of 51 IEEE Conference on Decision and Control*. submitted. 2012.

[6] P.M.J. Van den Hof et al. "Identification in Dynamic Networks with Known Interconnection Topology". In: *Proceedings of 51 IEEE Conference on Decision and Control*. submitted. 2012.

Questions???