System Identification in Dynamic Networks

Consistency Results for an Instrumental Variable Approach

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Relevance of identification in Dynamic Networks

Many Systems in engineering can be modelled as
dynamic networks

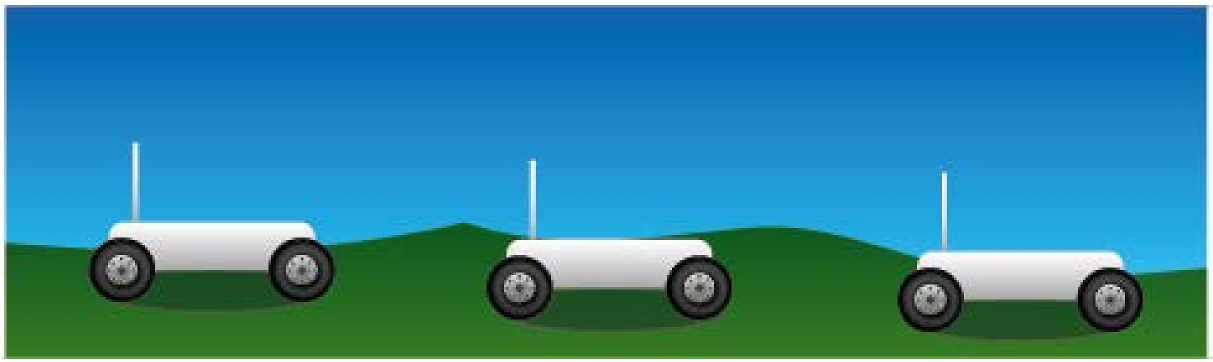
However

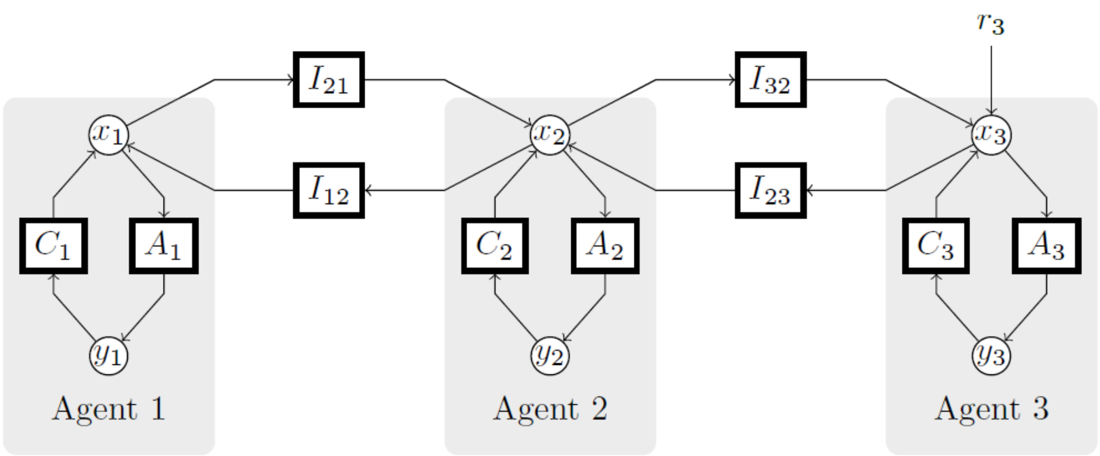
System Identification field used to thinking in terms of
Open-Loop, Closed-Loop, or MIMO

There are considerable advantages to bringing interconnection structure into the identification problem

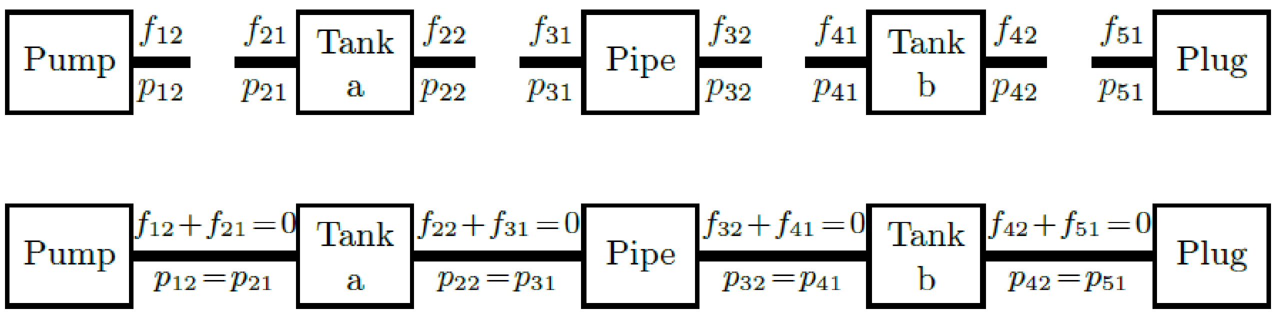
Many Systems Can be Modelled as Dynamic Networks

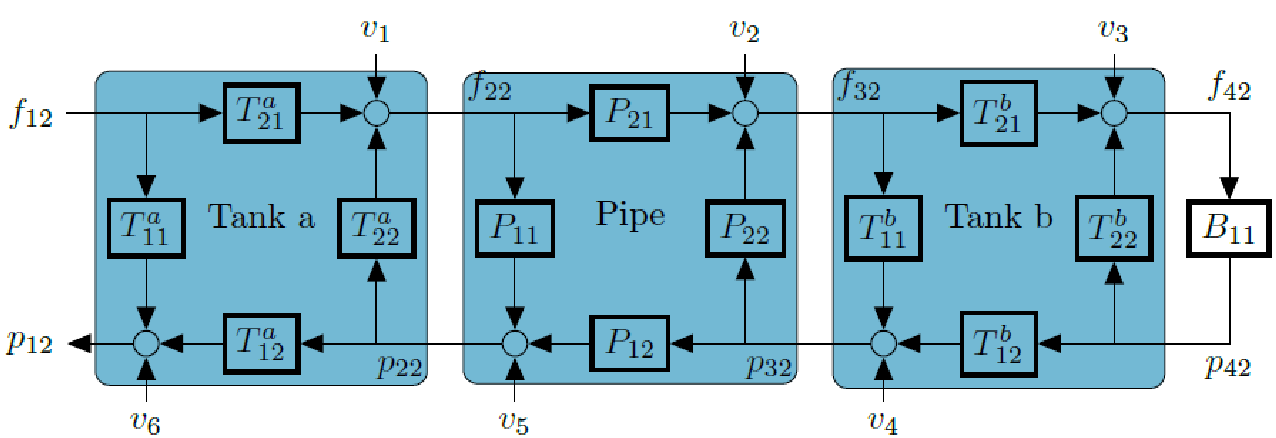
Distributed Control



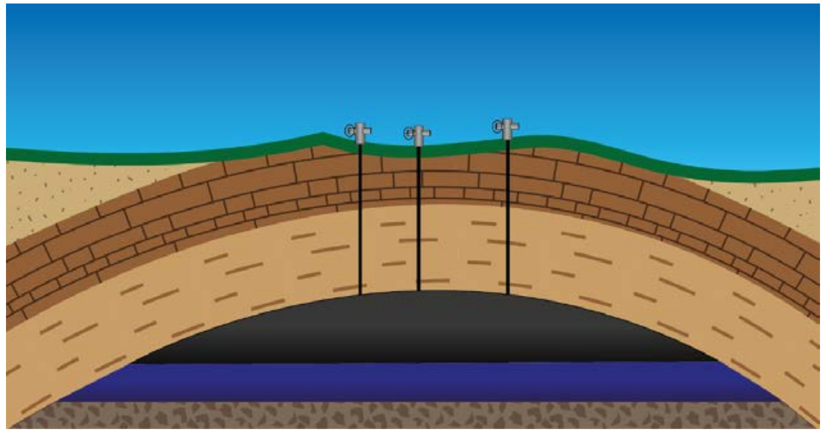


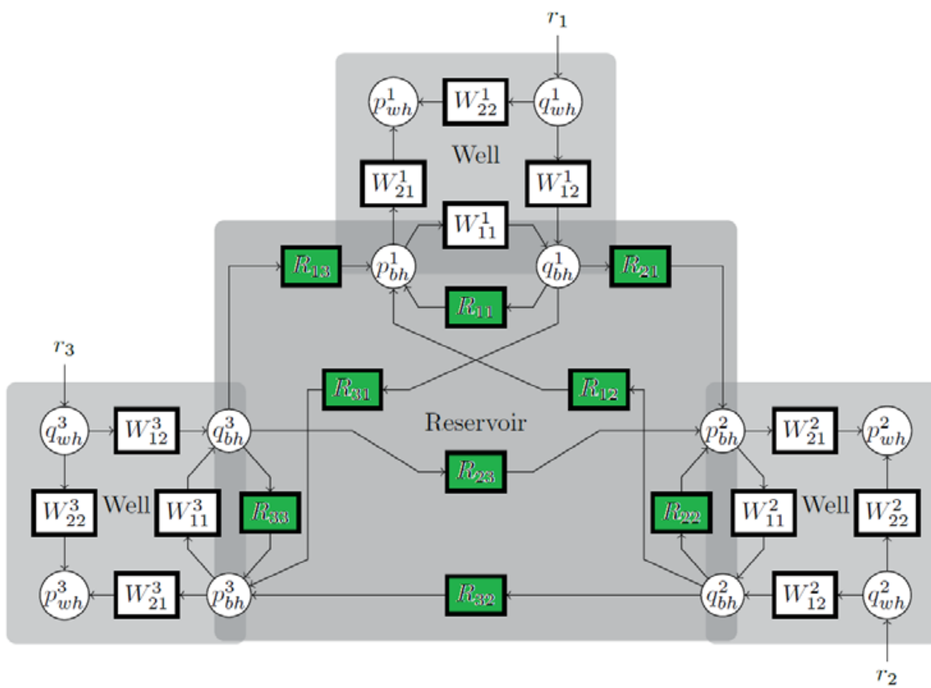
Many Systems Can be Modelled as Dynamic Networks





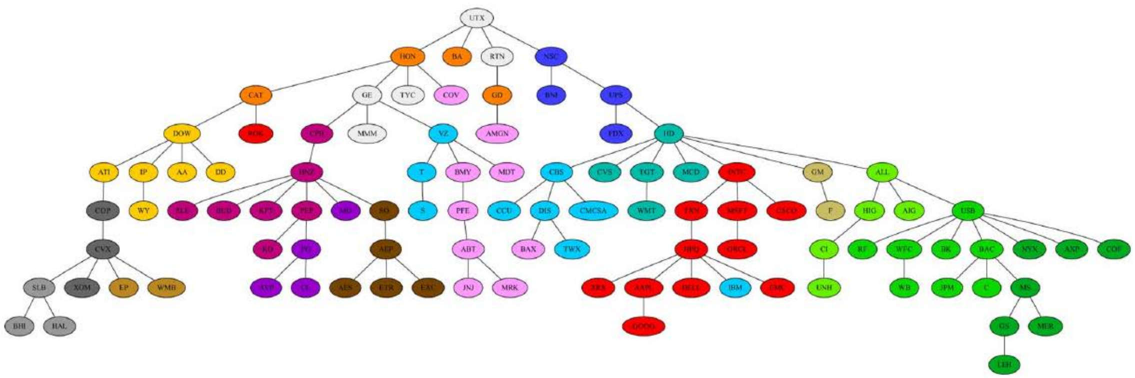
Many Systems Can be Modelled as Dynamic Networks



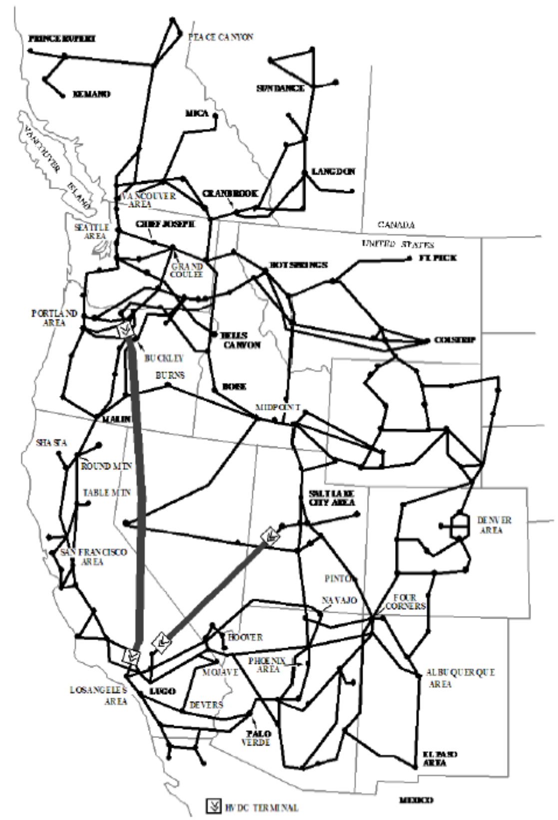


Many Systems Can be Modelled as Dynamic Networks

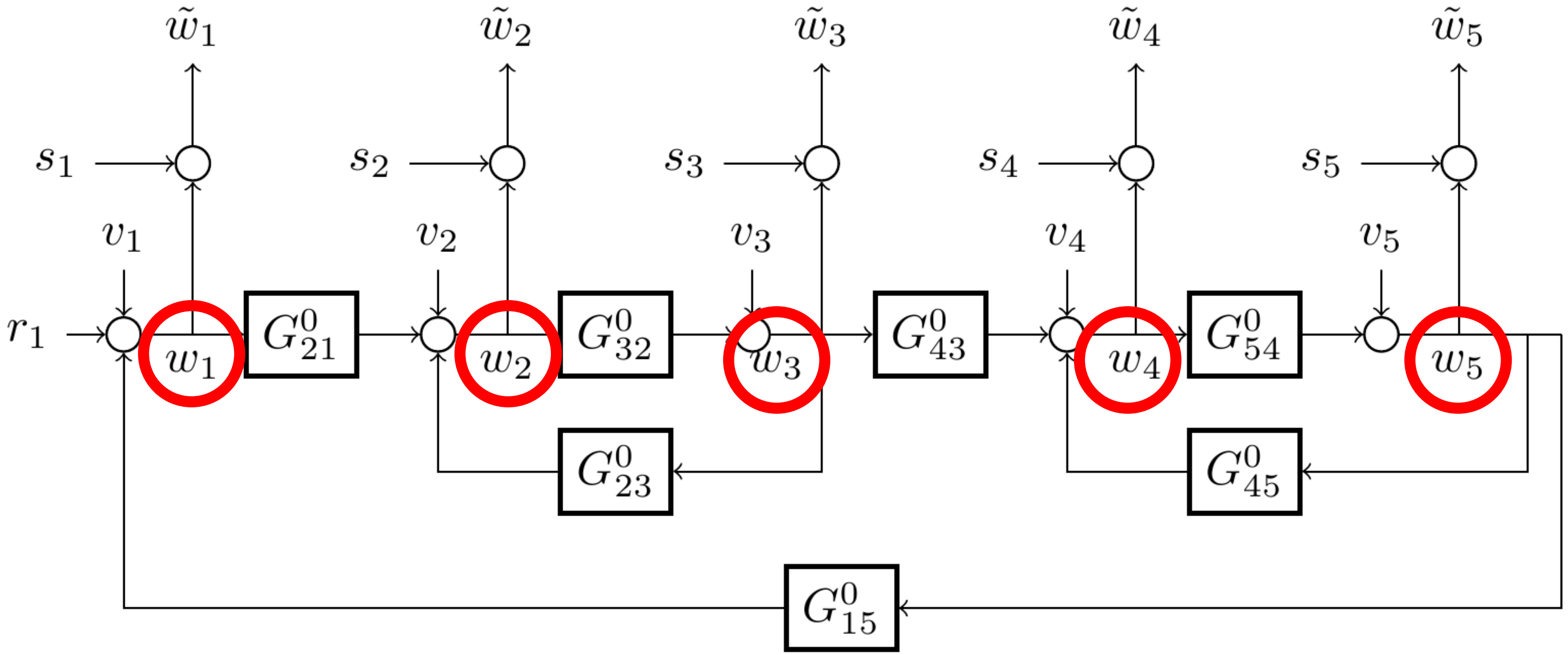
Power Systems



Financial Systems



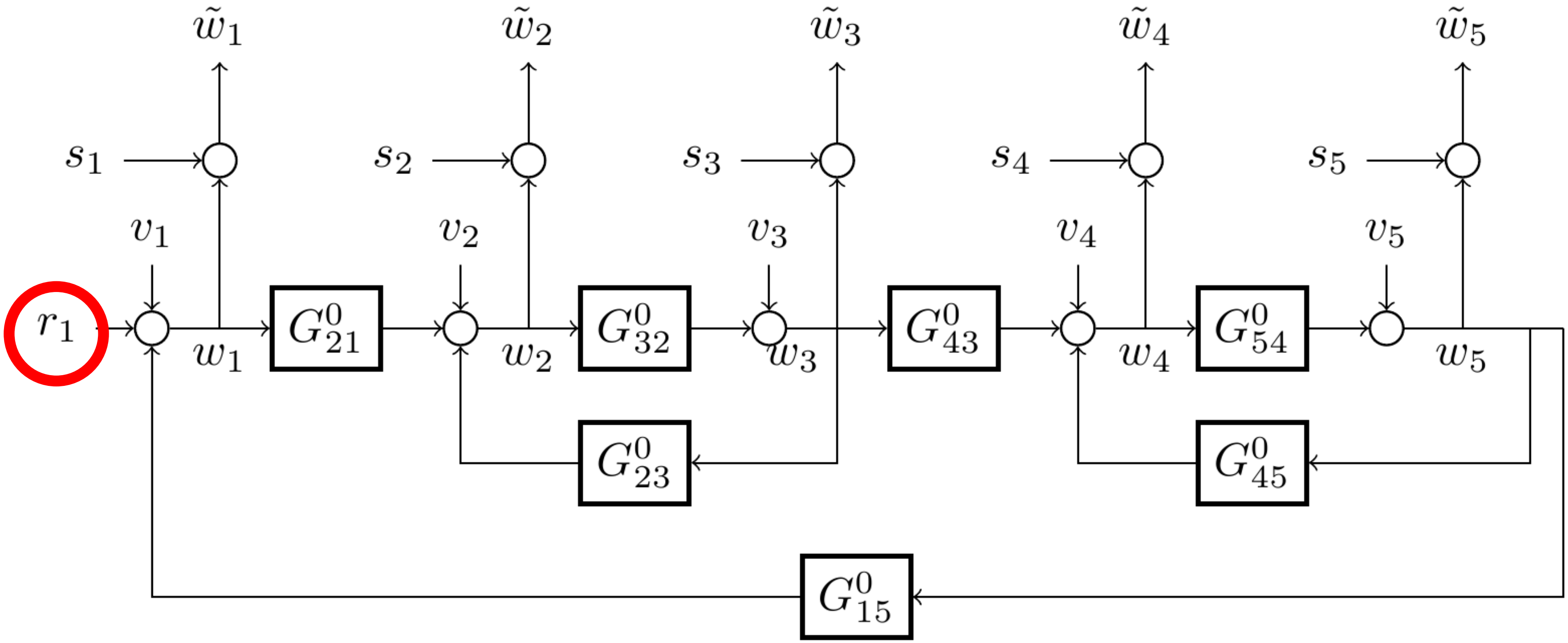
Dynamic Network Model



A Dynamic Network consists of:

- ***internal variables*** (**w_k**): measurable. Example: voltage, pressure, velocity, concentration, etc.

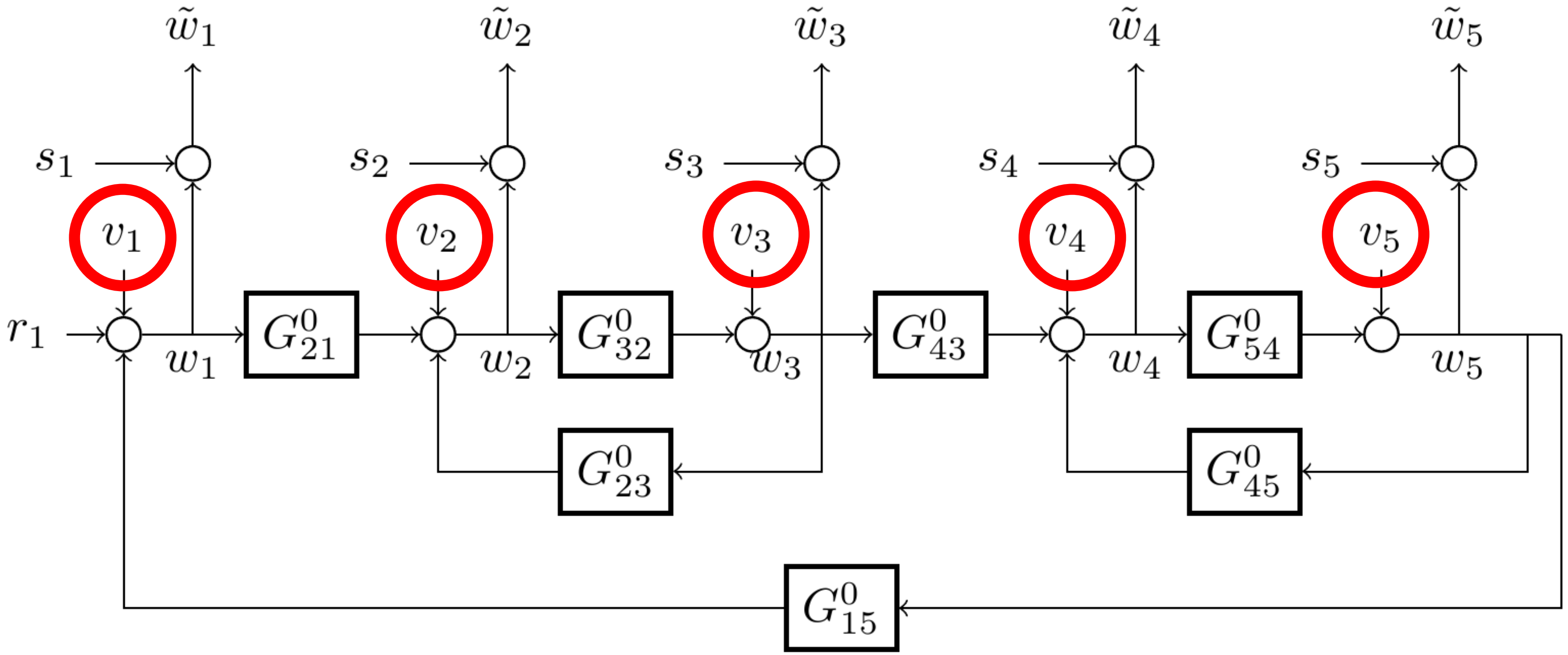
Dynamic Network Model



A Dynamic Network consists of:

- ***external variables*** (**r_k**): can be manipulated by user. Example: flow rate can be manipulated by a valve setting.

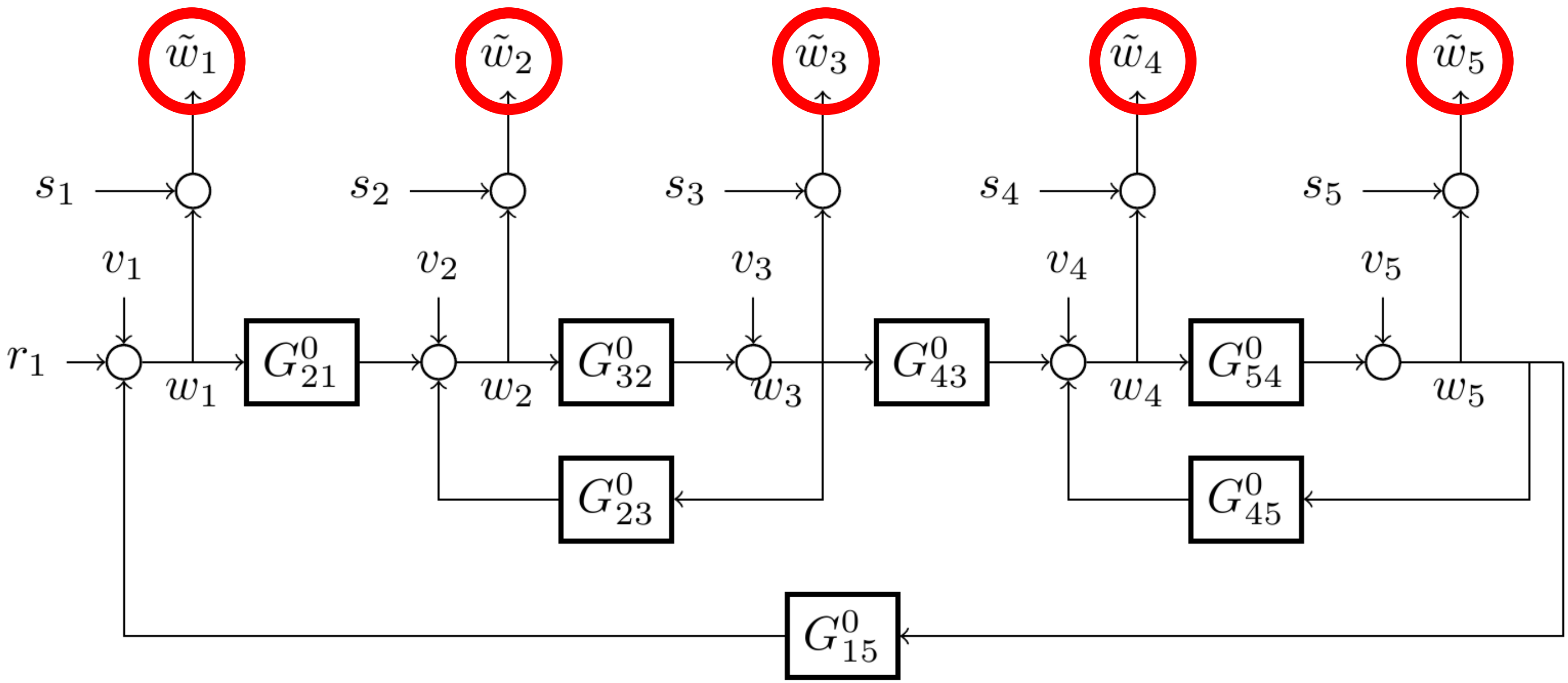
Dynamic Network Model



A Dynamic Network consists of:

- ***Process noise variables*** (v_k): unmeasurable disturbances. Example: thermal noise, electromagnetic radiation, wind.

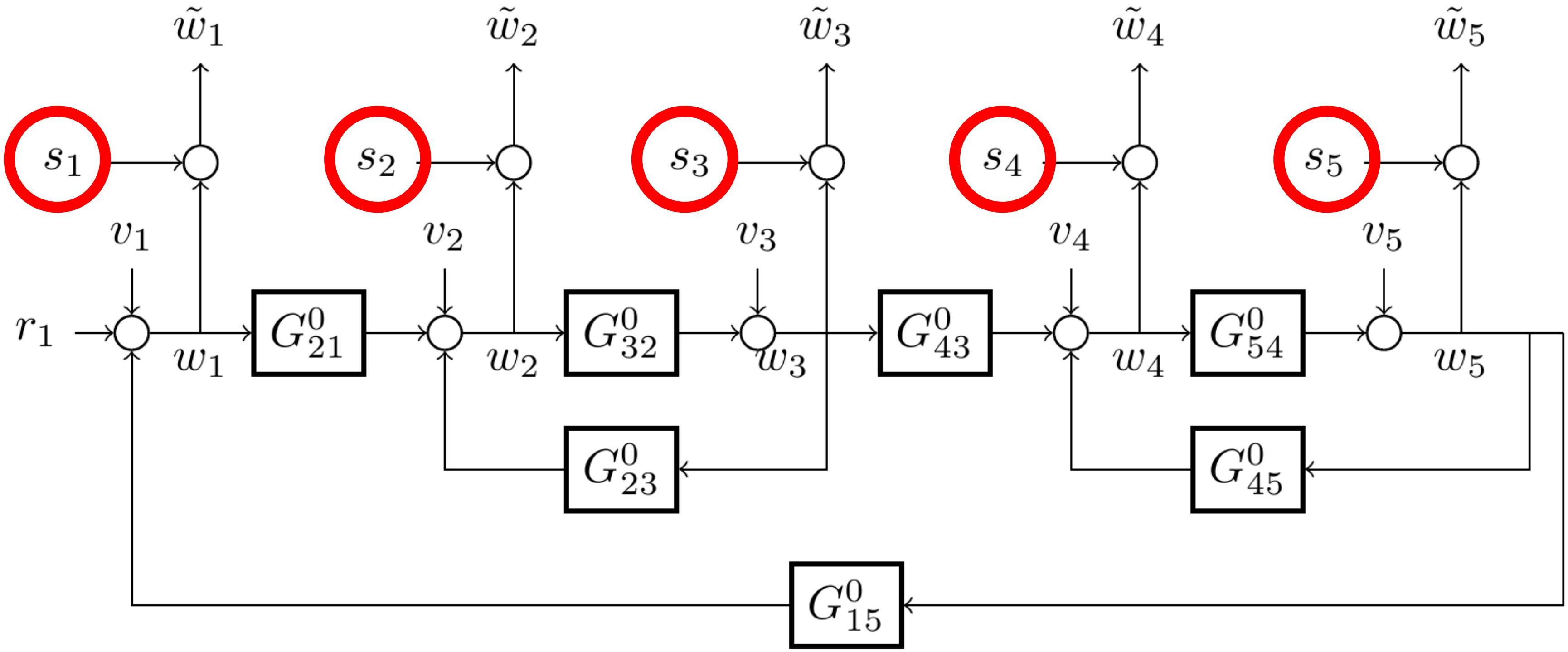
Dynamic Network Model



A Dynamic Network consists of:

- ***Measurements of internal variables*** ($\tilde{w}_k$): sampled, corrupted versions of the internal variables.

Dynamic Network Model



A Dynamic Network consists of:

- ***Sensor noise*** (s_k): error in recording the value of an internal variable.

Approach

Determine conditions under which it is possible to consistently estimate **one particular module** embedded in a dynamic network.

Conditions can then be extended to consistently identify **all or groups** of transfer functions in the network

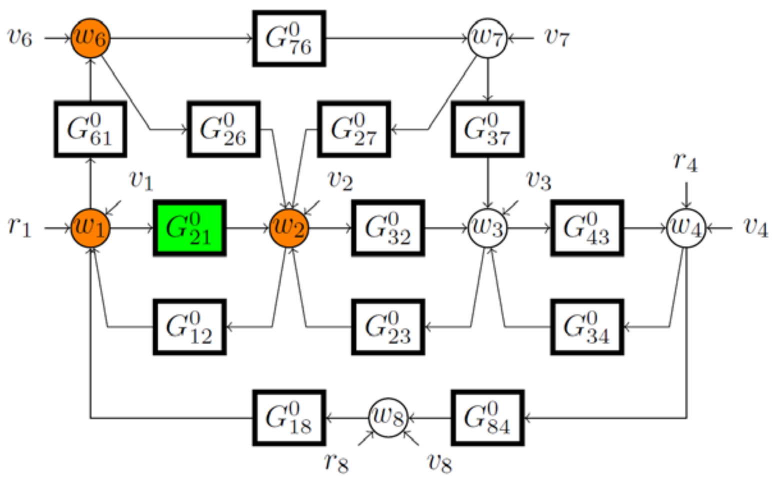
Conditions can also be extended to the case where **interconnection structure is unknown** (Chiuso, Goncalves, Materassi).

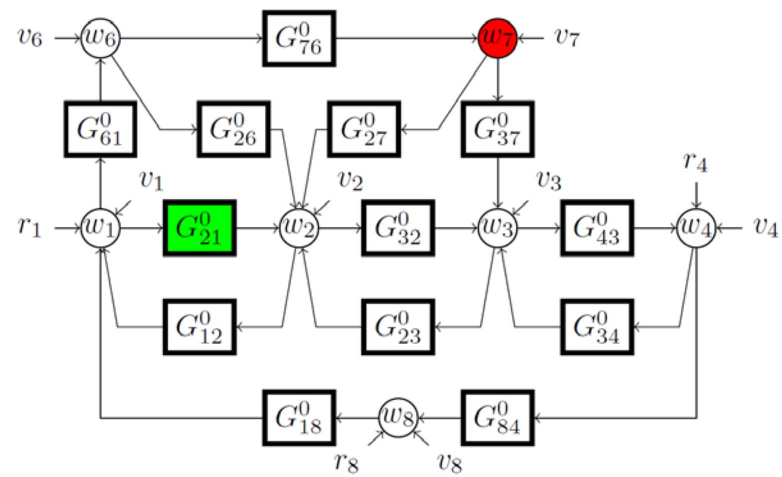
Advantages of bringing interconnection structure into the identification problem

- Local/distributed identification vs. global/centralized identification
- Requires weaker assumptions on noise
- Flexible choice of required measurements
- Number of transfer functions to identify is reduced $\Rightarrow$ less variance, less restrictive conditions on the informativity of the data
- Opportunities for variance reduction using extra measurements (sensor placement)
- Choice of cheapest actuation
- **Easily deal with sensor noise**

Identification Questions in Networks

- **Given a set of noise-free measurements**, is it possible to consistently **identify a module of interest**?



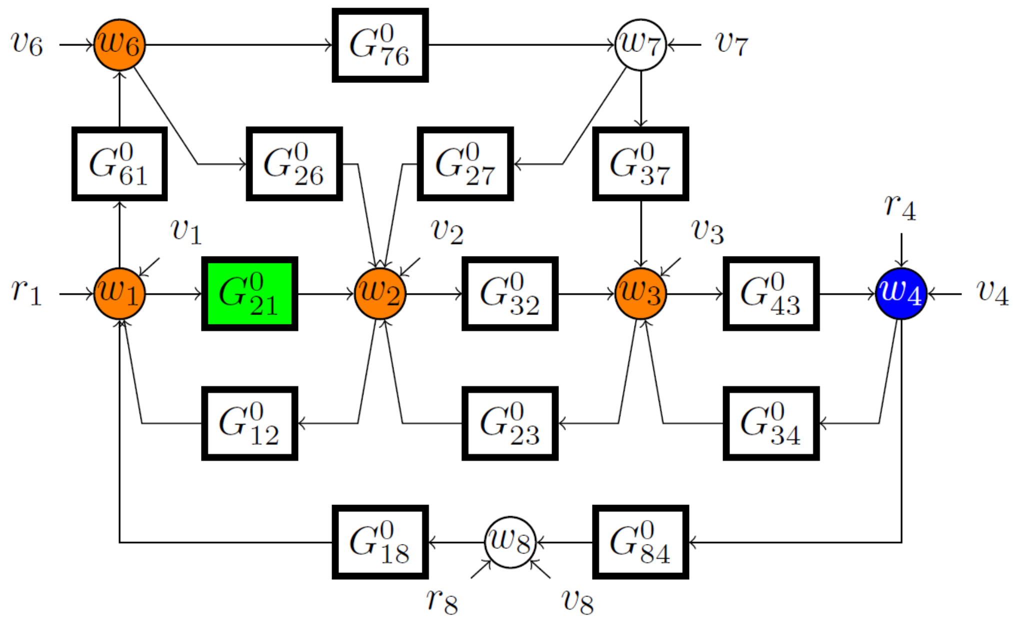


Sufficient conditions on the set of required measurements derived that ensure possibility of consistently identifying module of interest.

[Van den Hof, Dankers, Heuberger, Bombois, Automatica 2013; Dankers et al. ECC 2013, CDC 2013]

Identification Questions in Networks

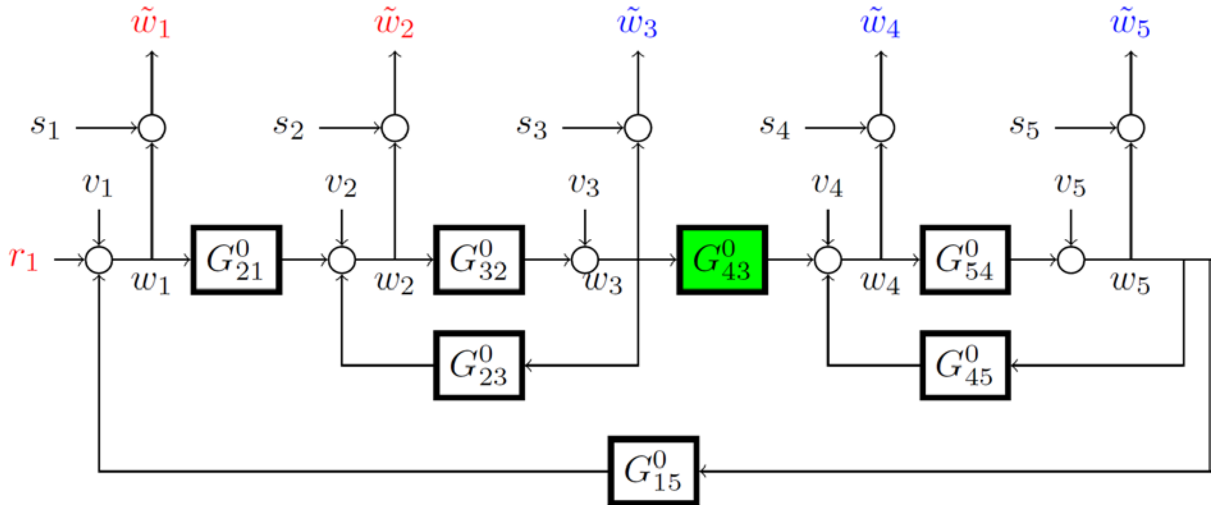
How can **"extra" measurements** be used to our benefit?



1. "Extra" measurements can be used to reduce the variance of the estimate in the presence of sensor noise (Hjalmarsson, Wahlberg, Gunes)

2. "Extra" measurements can be used to eliminate bias due to sensor noise.

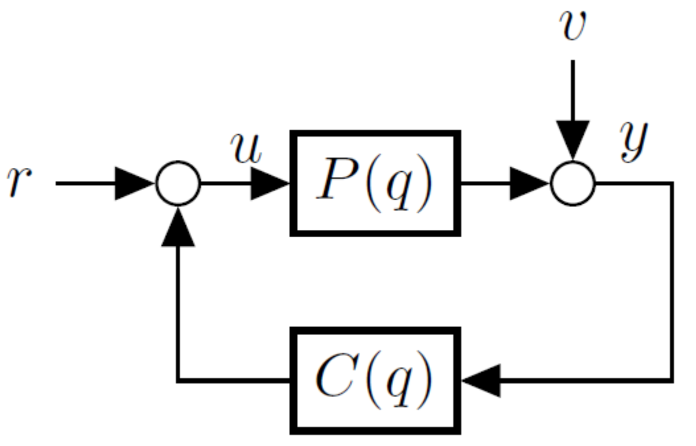
Errors-in-Variables Identification



Three cases of increasing generality:

1. **Noise free External variables present**
   (can use Two Stage or IV method)
2. **Extra measurements available where there is no path from output to extra measurement**
   (can use standard IV method)
3. **Extra measurements available**
   (use generalized IV method)

Basic Closed-Loop Instrumental Variable Method



The IV solution is defined as:

$$\boldsymbol{\theta_{IV} = sol\{\bar{E}[(y(t) - \phi(t)\theta)Z(t)] = 0\}}$$

Where

$$\phi(t) = [-y(t-1) \cdots y(t-n_a)\, u(t) \cdots u(t-n_b)]$$

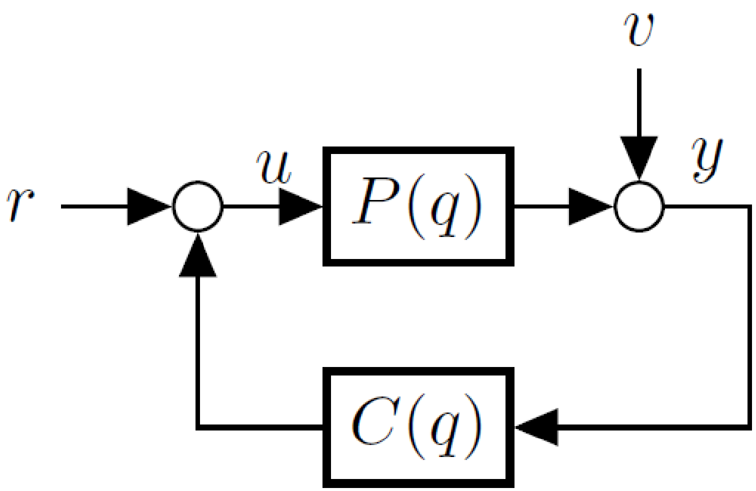
$$Z(t) = [r(t) \cdots r(t-n_a-n_b)]$$

Basic Closed-Loop Instrumental Variable Method

This is the prediction error with ARX model structure

$$G(q,\theta) = \frac{B(q,\theta)}{A(q,\theta)}$$

$$H(q,\theta) = \frac{1}{A(q,\theta)}$$



$$\bar{E}[(y(t) - \phi(t)\theta)Z(t)] = 0$$

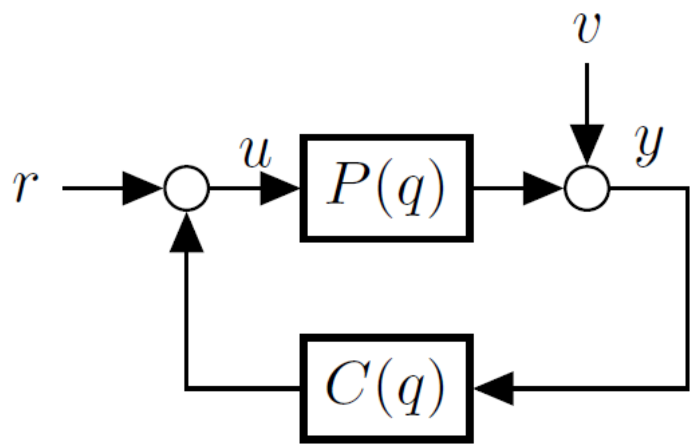
$$\bar{E}[(A(q,\theta)y(t) - B(q,\theta)u(t))r(t-\tau)] = 0, \qquad \tau = 0, \dots, n_a + n_b$$

$$\bar{E}[\epsilon(t,\theta)r(t-\tau)] = 0, \qquad \tau = 0, \dots, n_a + n_b$$

Compact description of BCLIV method →

$$R_{\epsilon r}(\tau) = 0, \qquad \tau = 0, \dots, n_a + n_b$$

Basic Closed-Loop Instrumental Variable Method



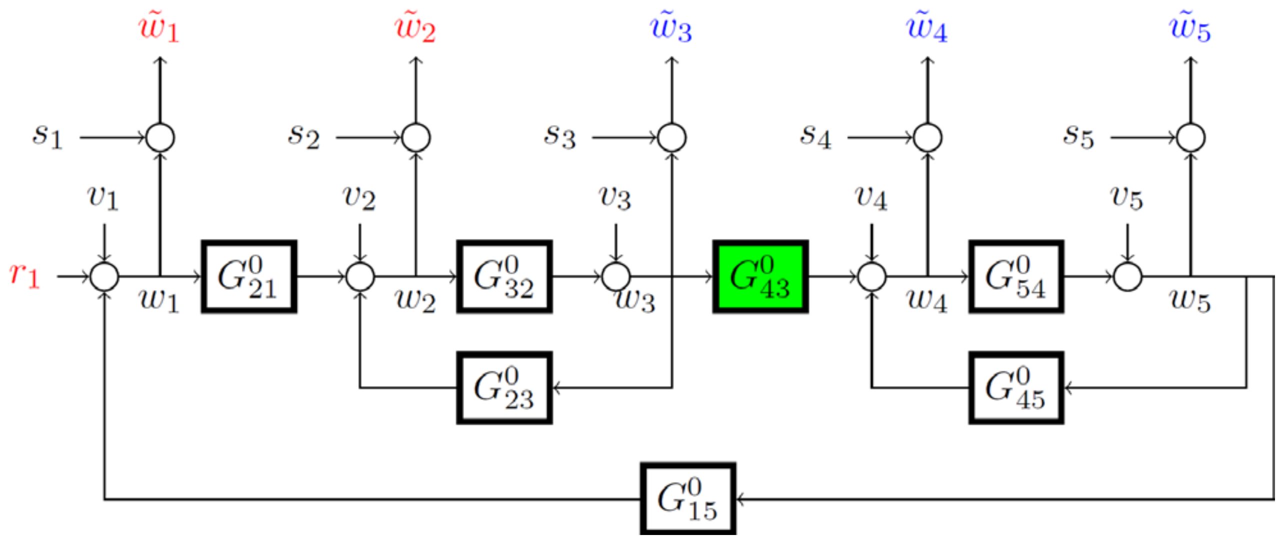
The Equivalence relation

$$R_{\epsilon r}(\tau) = 0, \qquad \tau = 0, \ldots, n_a + n_b \Leftrightarrow G(q, \theta) = P(q)$$

Holds if the following conditions are satisfied:

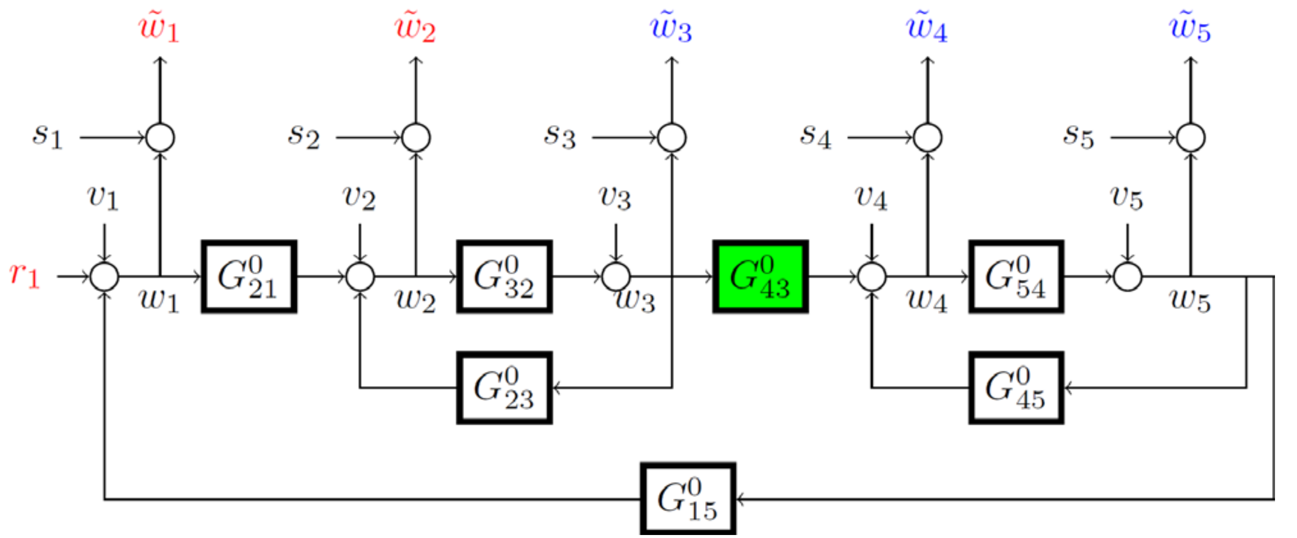
- *The data is informative*
- *Process noise v is uncorrelated to r*
- *There exists a θ^0 such that $G(q, \theta^0) = P(q)$*

Reasoning is extendable to networks and sensor noise



- Any external variable or measured variable that is not a predictor input is a candidate instrumental variable
- Sensor noise does not affect the equivalence relation!

Reasoning is extendable to networks and sensor noise



- Collect all variables chosen as instruments in z:

$$z(t) = [r_{k_1}(t) \cdots r_{k_n}(t)\ \tilde{w}_{l_1}(t) \cdots \tilde{w}_{l_m}(t)]$$

- Choose internal variables with direct connection to output (w_j) as predictor inputs

Reasoning is extendable to networks and sensor noise

The equivalence relation

$$\boldsymbol{R_{\epsilon z}(\tau) = 0, \qquad \tau = 0, \ldots, n_z \Leftrightarrow G_{jk}(q,\theta) = G_{jk}^{0}(q), \forall k \in N_j}$$

Holds if the following conditions are satisfied:

- *There is no path from w_j to any of the instrumental variables.*
- *The data is informative.*
- *Sensor noise of predictor inputs is uncorrelated to sensor noise of instrumental variables*
- *Process noise on output is uncorrelated to all v_k with paths to w_j*
- *There exists a θ^0 such that $G_{jk}(q,\theta^0) = G_{jk}^0(q), \forall k \in N_j$*

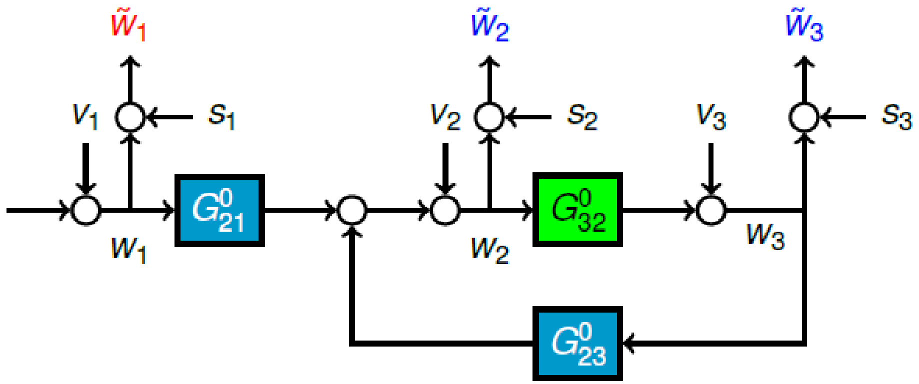
Restrictive Condition

There is no path from w_j to any of the instrumental variables.

Places a restriction on candidate instrumental variables

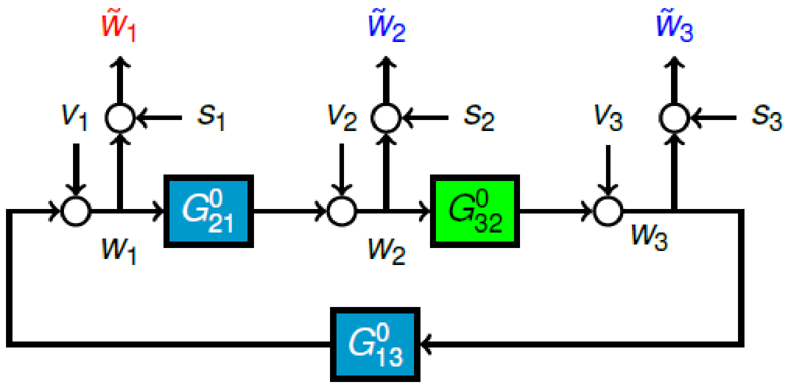
Required because instruments need to be uncorrelated to process noise on output (in any identification method the noise affecting the output must somehow be made uncorrelated to the predictor inputs)

Restrictive Condition



Objective: identify G^0_{32}.
Choose $\tilde{w}_2$ and $\tilde{w}_3$ as predictor inputs

$\tilde{w}_1$ can be used as instrumental variable



$\tilde{w}_1$ can **not** be used as instrumental variable

From the Direct Closed-Loop Method we know that exact noise modelling can be used to deal with the problem that the predictor inputs are correlated to v_j.

Extend IV Method

Apply same reasoning to IV method:

Switch from an ARX model structure to a Box-Jenkins model structure so that exact noise modelling is possible.

$$G_{jk}(q,\theta) = \frac{B_{jk}(q,\theta)}{F_{jk}(q,\theta)}, k \in N_j$$

$$H_j(q,\theta) = \frac{C_j(q,\theta)}{D_j(q,\theta)}$$

IV Method with Flexible Model Structure

The equivalence relation

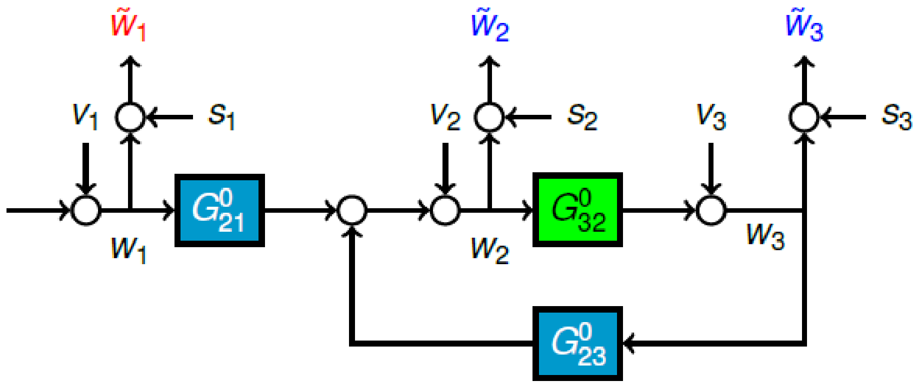
$$\{ \boldsymbol{R_{\epsilon z}(\tau) = 0, \tau = 1, \dots, n_z} \} \Leftrightarrow \begin{Bmatrix} \boldsymbol{G_{jk}(q,\theta) = G^0_{jk}(q), \forall k \in N_j} \\ \boldsymbol{H_j(q,\theta) = H^0_j(q)} \end{Bmatrix}$$

Holds if the following conditions are satisfied:

- *The data is informative.*
- *Sensor noise of predictor inputs is uncorrelated to sensor noise of instrumental variables*
- *Process noise on output is uncorrelated to all* v_k *with paths to* w_j
- *There exists a* θ^0 *such that* $G_{jk}(q,\theta^0) = G^0_{jk}(q), \forall k \in N_j$

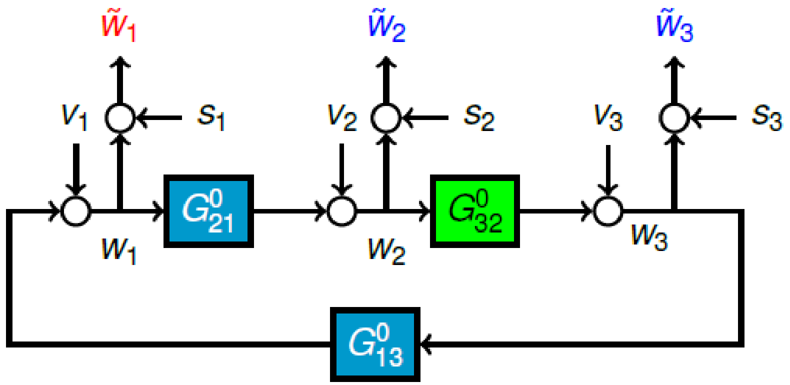
No more condition on the allowable set of candidate instrumental variables!

Extended IV Method



Objective: identify G_{32}^0.
Choose $\tilde{w}_2$ and $\tilde{w}_3$ as predictor inputs

$\tilde{w}_1$ can be used as instrumental variable



$\tilde{w}_1$ **can** be used as instrumental variable

Standard IV vs. Extended IV

Advantage of standard IV is that θ_{IV} can be obtained by linear regression

This is no longer the case for the Extended IV.

Implementation of Extended IV

Solving the set of equations:

$$R_{\varepsilon z}(\tau, \theta) = 0 \text{ for } \tau = 1, \dots, n_z$$

Is equivalent to finding θ that minimizes

$$V_{n_z}(\theta) = \sum_{\tau=1}^{n_z} R_{\varepsilon z}^2(\tau, \theta)$$

← **A sum of squared error objective function**

Implementation of Extended IV

$$R_{\varepsilon z}(\tau) = \bar{E}\left[\left(H_j^{-1}(q,\theta)\left(w_j(t) - \sum_{k \in N_j} G_{jk}(q,\theta) w_k(t)\right)\right) z(t-\tau)\right]$$

$$R_{\varepsilon z}(\tau) = H_j^{-1}(q,\theta)\left(\underbrace{\boldsymbol{R_{w_j z}(\tau)}}_{\text{“output”}} - \sum_{k \in N_j} G_{jk}(q,\theta)\underbrace{\boldsymbol{R_{w_k z}(\tau)}}_{\text{“inputs”}}\right)$$

Which has the same form as the prediction error using a BJ model structure!

Implementation of Extended IV

Minimize

$$V_{n_z}(\theta) = \sum_{\tau=1}^{n_z} R^2_{\varepsilon z}(\tau, \theta)$$

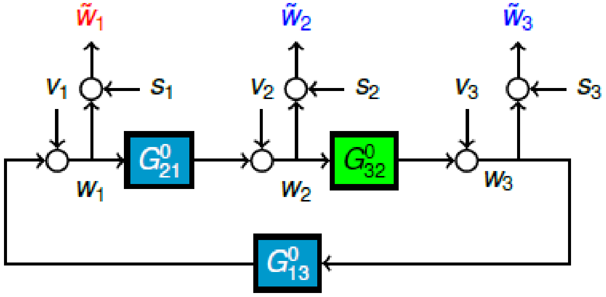
where

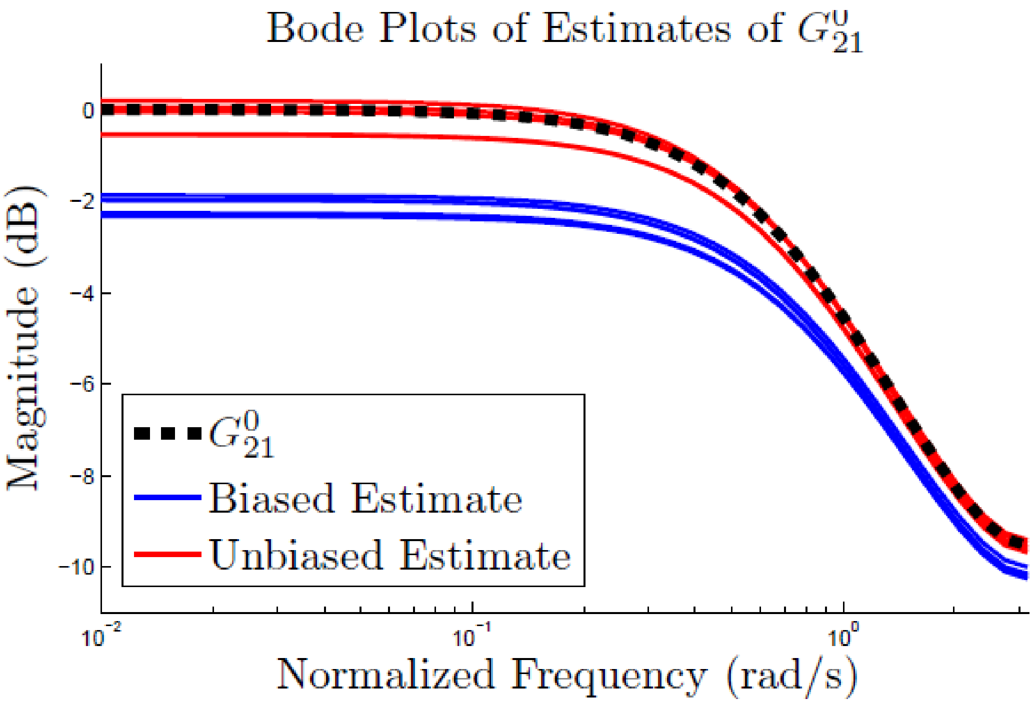
$$R_{\varepsilon z}(\tau) = H_j^{-1}(q,\theta)\left(\boldsymbol{R_{w_j z}(\tau)} - \sum_{k \in N_j} G_{jk}(q,\theta)\boldsymbol{R_{w_k z}(\tau)}\right)$$

Which is a standard prediction error optimization problem: can use MATLAB system identification toolbox!

Question: what is best choice for n_z?

Simulation Results





Blue: Direct Closed Loop Method (bias due to sensor noise)

Red: Extended IV Method with BJ model structure (no bias)

Concluding Remarks

Consistent estimation of G_{ji}^{0} is possible based on only noisy measurements

- Errors in variables problems become way much simpler in a dynamic network setting

Questions?