EINDHOVEN UNIVERSITY OF TECHNOLOGY

Identification of local models in interconnected systems – confounding variables, data-informativity and MATLAB toolbox

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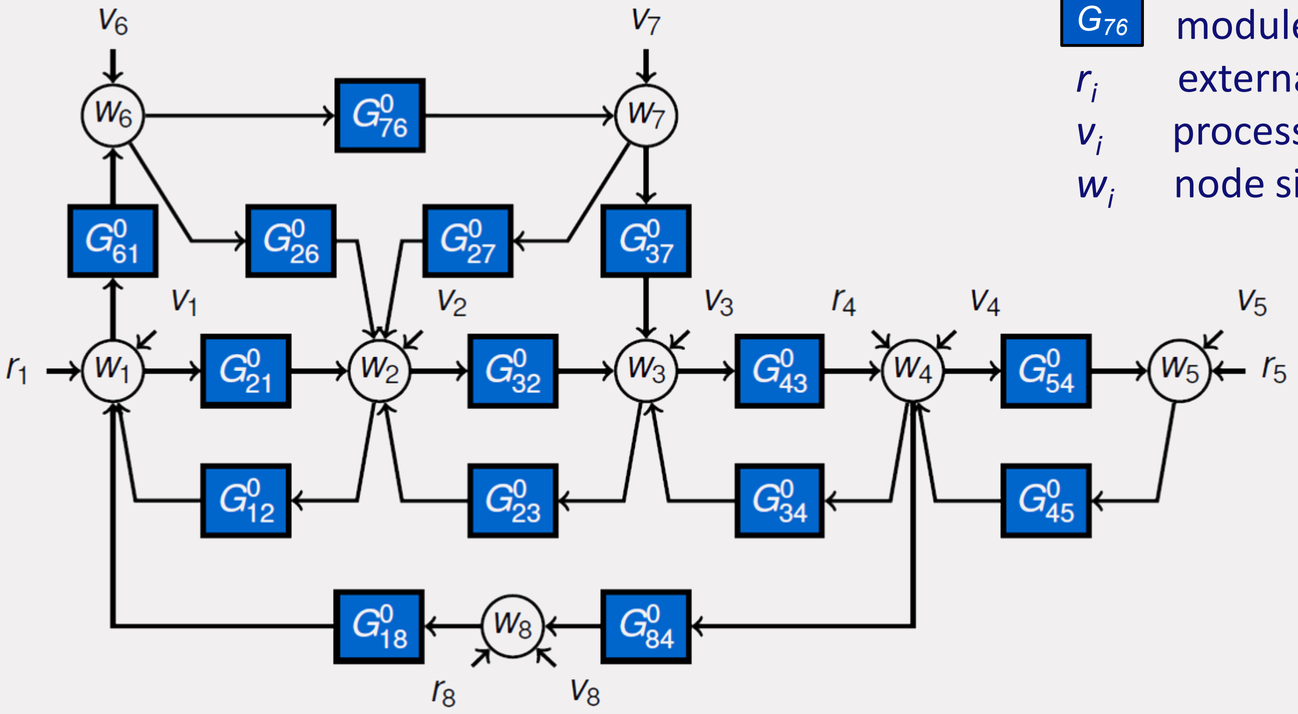
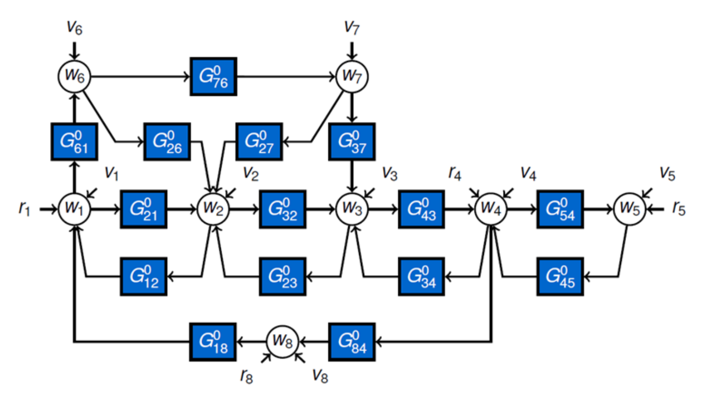
European Research Council

Dynamic network setup



G_{76} module
r_i external excitation
v_i process noise
w_i node signal

Dynamic network setup – Module framework



Many data-analytics and data-driven modeling challenges appear

Sensor locations: $\{w_k(t)\}_{k=\ldots};$

Actuator locations: $\{r_j(t)\}_{j=\ldots};$

- Estimate or validate a single module/subnetwork (known topology)
- Estimate or validate the full network
- Estimate or validate the topology
- Identifiability
- Detect a fault and diagnose its location
- Exploit active probing (experiment design)
- User prior knowledge of modules/topology
- Scalable algorithms

Dynamic network setup

Collecting all equations:

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & G^0_{12} & \cdots & G^0_{1L} \\ G^0_{21} & 0 & \ddots & G^0_{2L} \\ \vdots & \ddots & \ddots & \vdots \\ G^0_{L1} & G^0_{L2} & \cdots & 0 \end{bmatrix}}_{\text{Network matrix } G^0(q)} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + R^0 \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_K \end{bmatrix} + H^0 \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_p \end{bmatrix}$$

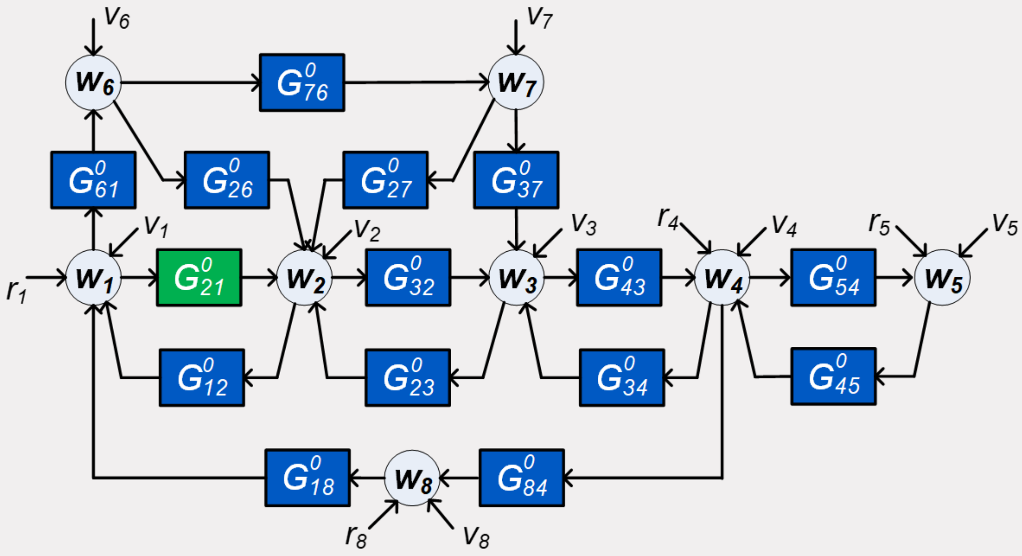
$$w(t) = G^0(q)w(t) + \underbrace{R^0(q)r(t)}_{u(t)} + v(t); \qquad v(t) = H^0(q)e(t); \quad cov(e) = \Lambda$$

- Typically R^0 is just a (static) selection matrix, indicating which nodes have an excitation signal.
- The topology of the network is encoded in the structure (non-zero entries) of G^0.
- r and e are called external signals.

[1] J. Gonçalves and S. Warnick, IEEE TAC, 2008.
[2] VdH et al., Automatica, 2013.

Single module identification

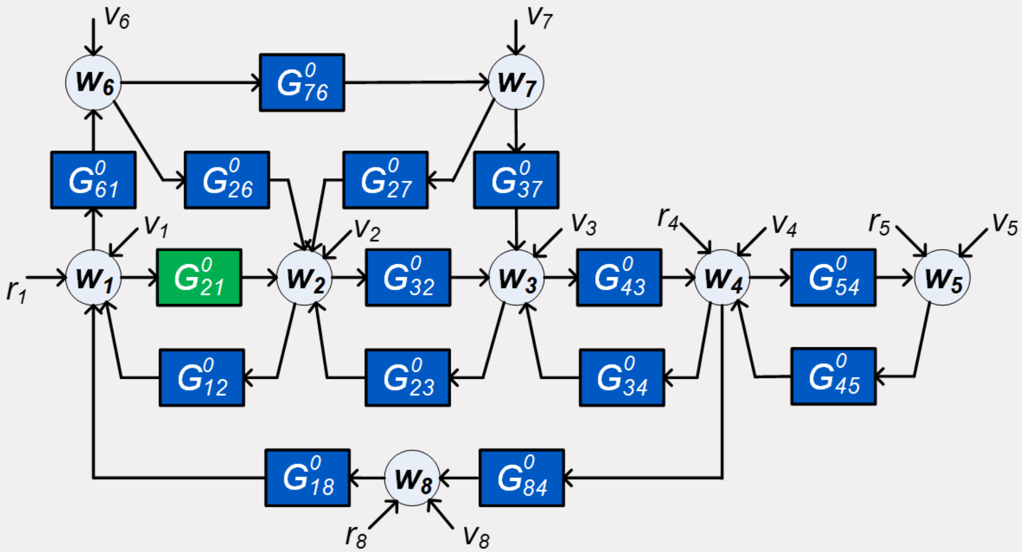
Single module identification



For a network with
known topology:

- Identify G^0_{21} on the basis of measured signals
- Which signals to measure? Preference for local measurements
- When is there enough excitation / data informativity?

Single module identification



Different types of methods:

Indirect methods [1,2,3]

- Rely on mappings $\boldsymbol{r} \rightarrow \boldsymbol{w}$ and on sufficient excitation signals $\boldsymbol{r}$

Direct methods [1,2,4]

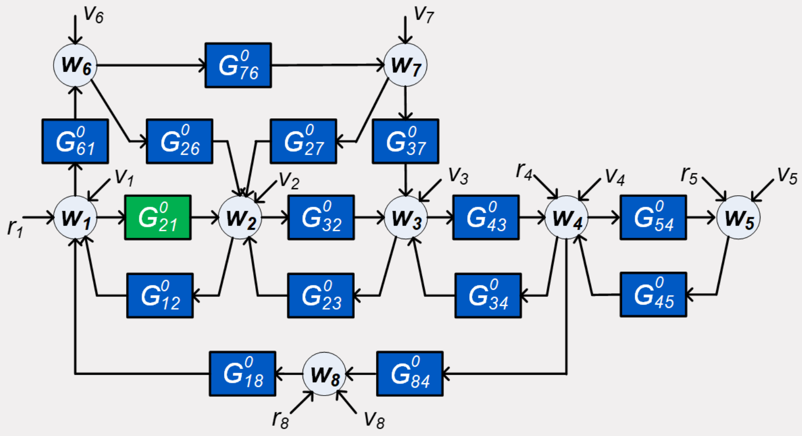
- Rely on mappings $\boldsymbol{w} \rightarrow \boldsymbol{w}$ and use excitation from both $\boldsymbol{r}$ and $\boldsymbol{v}$ signals

[1] PVdH et al., Automatica, 2013.
[2] A.G. Dankers et al., IEEE-TAC, 2016.
[3] M. Gevers et al., SYSID 2018.
[4] K.R. Ramaswamy et al., IEEE-TAC, 2021.

Single module identification -

local direct method

Single module identification



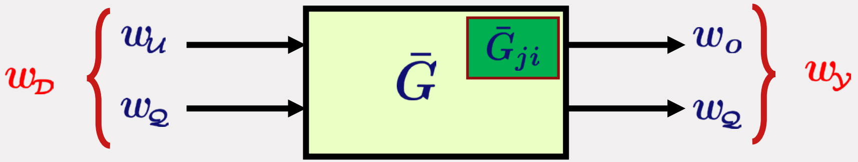
Local direct method:

(consistency and minimum variance properties)

Select a subnetwork:

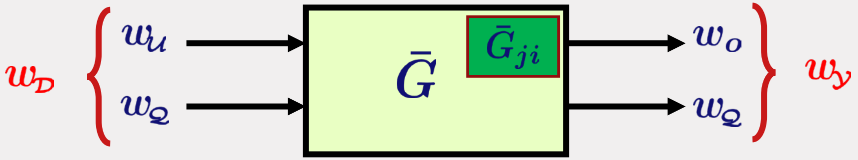
- Predicted outputs: $w_{\mathcal{Y}}$
- Predictor inputs: $w_{\mathcal{D}}$

such that prediction error minimization leads to an accurate estimate of G^0_{21}



Note: same node signals can appear in input and output

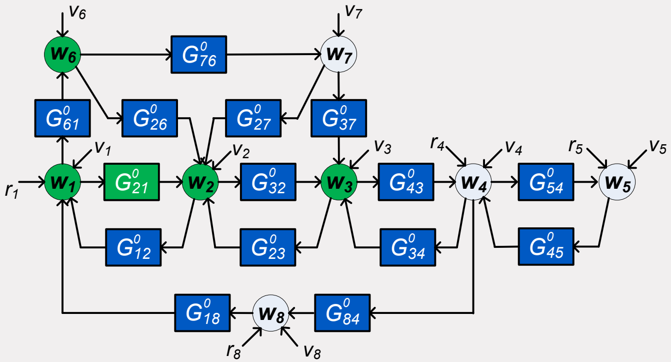
Single module identification

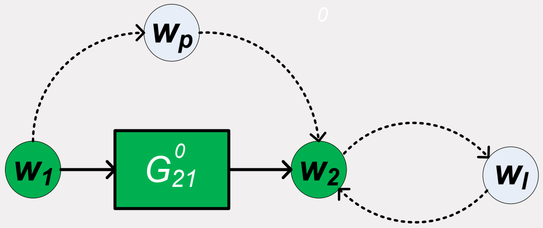


Conditions for arriving at an accurate (consistent) model estimate:

1. Module invariance: $\bar{G}_{ji} = G^0_{ji}$ when removing discarded nodes (immersion)
2. Handling of confounding variables
3. Data-informativity
4. *Technical condition on presence of delays (avoiding algebraic loops)*

Single module identification - module invariance





A sufficient condition for module invariance:

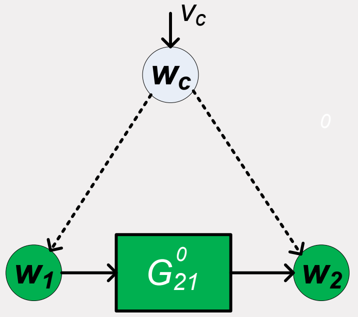
All parallel paths, and loops around the output,
should be "blocked" by a measured node that is present in $w_{\mathcal{D}}$

All other signals can be removed/immersed from the network[2]

Alternative graph-based formulation in terms of disconnecting sets in [3]

[1] Dankers et al., TAC 2016
[3] Shi et al., Automatica 2022
[2] Generalizations available in Linder&Enqvist (2017), Weerts et al, (2020)

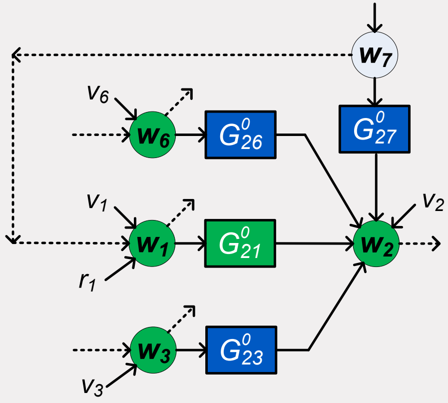
Single module identification - confounding variables



Confounding variables [1][2]**:**

Unmeasured signal that has (unmeasured) paths to both the input and output of an estimation problem.

In networks they can appear in two different ways:



Direct:

- If disturbances on inputs and outputs are correlated.

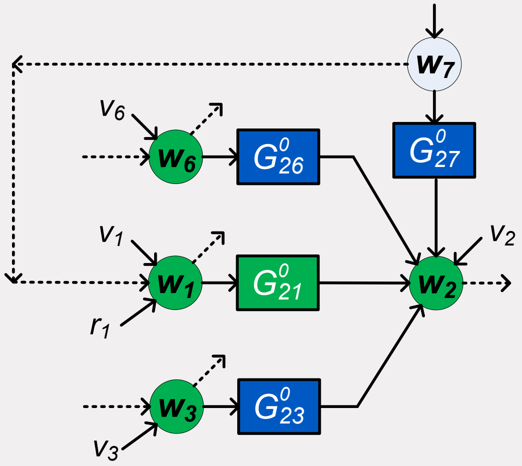
Indirect:

- If non-measured in-neighbors of an output affect signals in the inputs.

[1] J. Pearl, *Stat. Surveys, 3,* 96-146, 2009
[2] A.G. Dankers et al., *Proc. IFAC World Congress*, 2017.

Confounding variables

- **Direct** confounding variables



e.g., v_1 is correlated with v_2

In identification we know how to handle correlated disturbances: we model them!

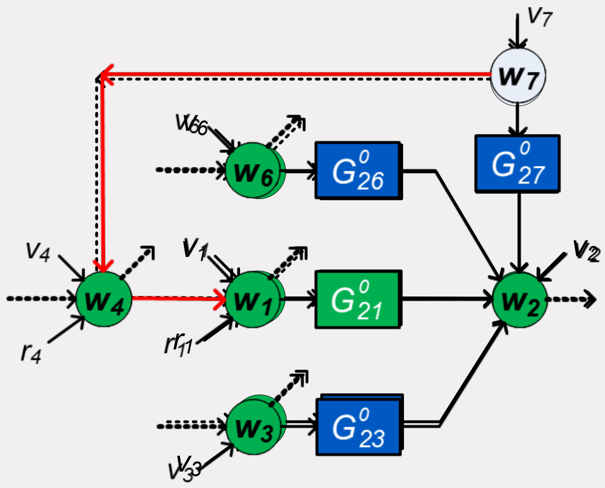
Solution:

Include w_1 as output and use a multivariate noise model

$$w_{\mathcal{D}} = \{w_1, w_3, w_6\} \quad w_{\mathcal{Y}} = \{w_1, w_2\}$$

Confounding variables

- **Indirect** confounding variable:



Non-measurable w_7 is a confounding variable

Two possible solutions:

1. Include w_4 ⟶ add predictor input

$$w_{\mathcal{D}} = \{w_1, w_3, w_4, w_6\} \quad w_{\mathcal{Y}} = \{w_2\}$$

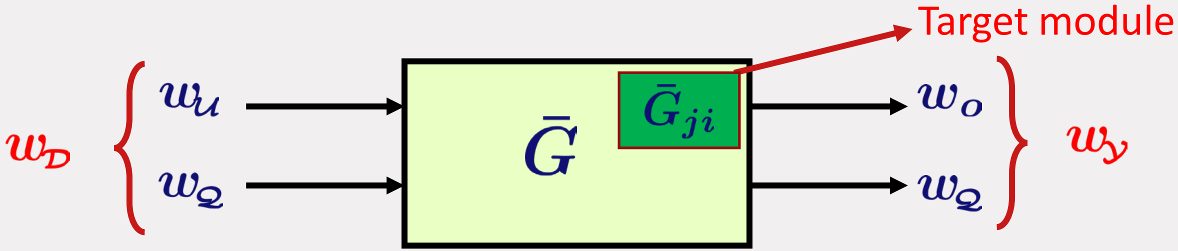
2. Predict w_1 too ⟶ add predictor output

$$w_{\mathcal{D}} = \{w_1, w_3, w_6\} \quad w_{\mathcal{Y}} = \{w_1, w_2\}$$

- There are degrees of freedom in choosing the predictor model

Local direct method

General setup:



Different algorithms for satisfying the 2 conditions (module invariance and conf. var.):

- Full input case: include all in-neighbors of $w_{\mathcal{Y}}$
- Minimum node signals case : maximize number of outputs
- User selection case (inputs first) : dedicated choice based on measurable nodes
- User selection case (outputs first) : dedicated choice based on measurable nodes

[1] A.G. Dankers et al., TAC 2016.
[2] K.R. Ramaswamy et al., TAC 2021.
[3] S. Shi et al., IFAC 2023.

Different strategies – direct method

Full input: $\begin{bmatrix} w_2 \\ w_3 \\ w_4 \\ w_6 \end{bmatrix} \rightarrow \begin{bmatrix} w_1 \\ w_3 \end{bmatrix}$

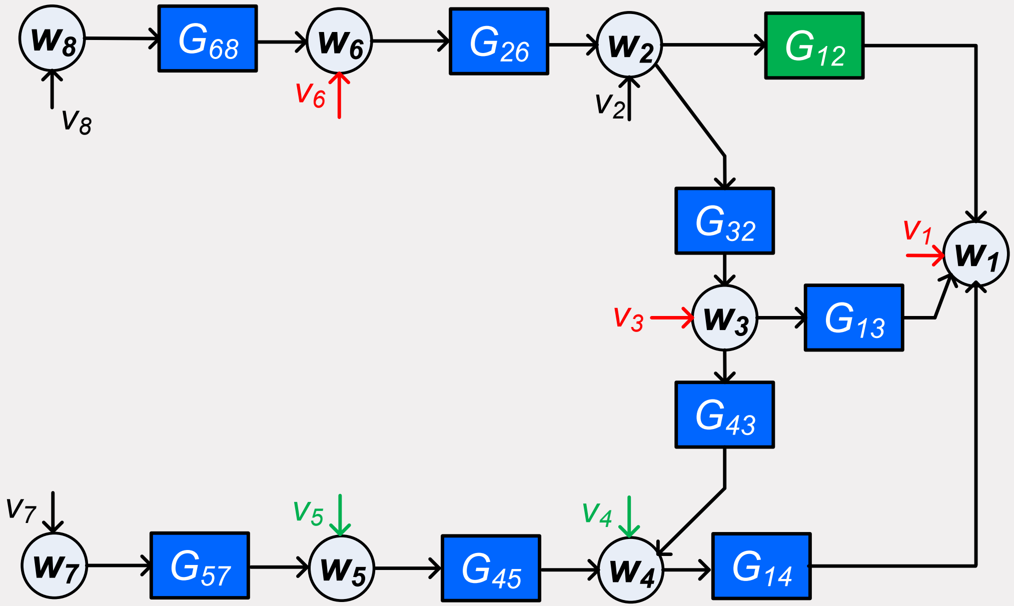
Minimum input: $\begin{bmatrix} w_2 \\ w_3 \end{bmatrix} \rightarrow \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$

User-selection: (based on w_1, w_2, w_3, w_5)

$$\begin{bmatrix} w_2 \\ w_3 \\ w_5 \end{bmatrix} \rightarrow \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_5 \end{bmatrix}$$

All achieving consistency / ML properties

Network with v_1 correlated with v_3 and v_6. v_4 correlated with v_5.



Single module identification

Serious degrees of freedom in selecting the predictor model to satisfy the first two conditions:

1. Module invariance – PPL test
2. Handling confounding variables
3. *Data-informativity*

While presuming that data-informativity can always be satisfied by adding sufficient # of r-signals.

WRONG!

Single module identification – data-informativity

Incorporating the role of external signals:

Original network model: $w(t) = G(q)w(t) + \underbrace{u(t)}_{R(q)r(t)} + H(q)e(t);$

Predictor model (subset of nodes):

$$w_{\mathcal{Y}}(t) = \bar{G}(q)w_{\mathcal{D}(t)} + \bar{J}(q)u_{\mathcal{K}}(t) + \bar{S}u_{\mathcal{P}}(t) + \bar{H}(q)\xi_{\mathcal{Y}}(t)$$

Effect of u on $w_{\mathcal{Y}}$ can appear in three different ways:

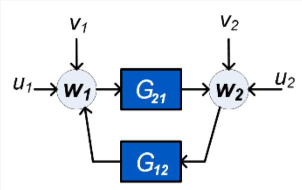
1. Incorporated in input $w_{\mathcal{D}}$
2. With a dynamic term $\bar{J}(q)$
3. With a constant unit-term in $\bar{S}$ (binary matrix)

[1] K.R. Ramaswamy et al., IEEE-TAC, 2021.

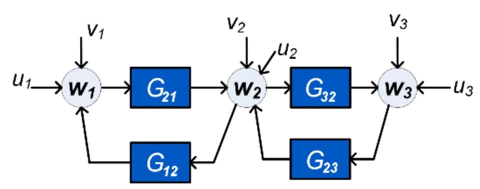
Single module identification – data-informativity

Examples for different roles of u:

$$w_{\mathcal{Y}}(t) = \bar{G}(q)w_{\mathcal{D}(t)} + \bar{J}(q)u_{\mathcal{K}}(t) + \bar{S}u_{\mathcal{P}}(t) + \bar{H}(q)\xi_{\mathcal{Y}}(t)$$



$$w_2 = G_{21}(q)w_1 + u_2 + v_2 \qquad u_{\mathcal{P}} = u_2 \qquad u_{\mathcal{K}} = \emptyset$$



$$w_2 = \frac{1}{1 + G_{32}G_{23}}[G_{21}w_1 + u_2 + v_2 + G_{23}(u_3 + v_3)]$$

$$u_{\mathcal{P}} = \emptyset \qquad u_{\mathcal{K}} = \{u_2, u_3\}$$

Dynamic term $\bar{J}(q)$ can be left unmodelled → higher level of ``disturbances''

Alternative: estimate the term with measured input $u_{\mathcal{K}}(t)$

Single module identification – data-informativity

Determining the different roles of excitation signals:

Given $\mathcal{Y}$ and $\mathcal{D}$, the sets $\mathcal{P}$ and $\mathcal{K}$ are determined through graphical conditions[1,2,3]:

- For $\ell \in \mathcal{Q}$, $u_\ell \in u_\mathcal{P}$ if all loops around w_ℓ pass through a node in $w_\mathcal{D}$
- $u_o \in u_\mathcal{P}$ if all loops around w_o pass through a node in $w_\mathcal{D}$ and all paths from w_o to $w_\mathcal{Q}$ pass through a node in $w_\mathcal{U}$.
- $u_\mathcal{Y} \in u_\mathcal{K}$ if $u_\mathcal{Y} \notin u_\mathcal{P}$
- For $\ell \notin \{\mathcal{Y} \cup \mathcal{D}\}$, $u_\ell \in u_\mathcal{K}$ if w_ℓ has a direct or unmeasured path to $w_\mathcal{Y}$

[1] Simple case where set $\mathcal{B} = \emptyset$
[2] Ramaswamy, PhD thesis 2022;
[3] VdH et al, IFAC 2023.

Single module identification – data-informativity

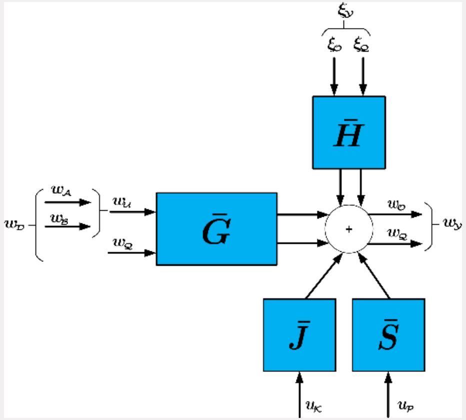
Predictor model equation:

$$w_{\mathcal{Y}}(t) = \bar{G}(q,\theta)w_{\mathcal{D}}(t) + \bar{H}(q,\theta)\xi_{\mathcal{Y}}(t) + \bar{J}(q,\theta)u_{\mathcal{K}}(t) + \bar{S}u_{\mathcal{P}}(t)$$

Typical data-informativity condition:

κ persistently exciting $\quad \Phi_{\kappa}(\omega) > 0$ for almost all ω

$$\kappa(t) := \begin{bmatrix} w_{\mathcal{D}}(t) \\ \xi_{\mathcal{Y}}(t) \\ u_{\mathcal{K}}(t) \end{bmatrix}$$ inputs of the predictor model



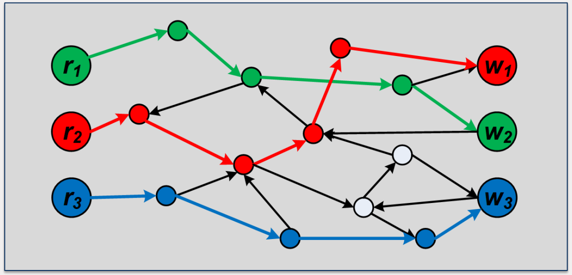
Rank-based condition can generically be satisfied based on a graph-based condition

[1] L. Ljung, 1989.
[2] K.R. Ramaswamy et al., IEEE-TAC, 2021.

Data informativity (path-based condition)

A signal $y(t) = F(q)x(t)$ with x persistently exciting, is persistently exciting iff F has **full row rank**.

This condition can be verified in a generic sense, by considering the **generic rank** of F [1],[2]



$b_{\mathcal{R}\to\mathcal{W}} = 3$

linking to the maximum number of **vertex disjoint paths** between inputs and outputs

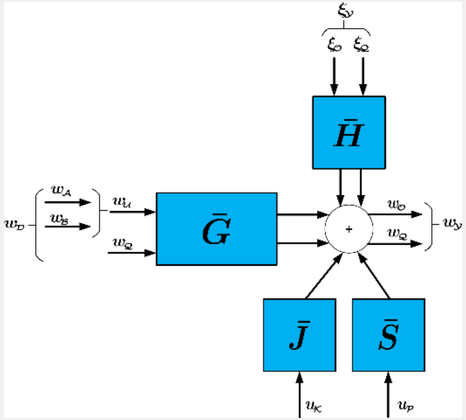
κ persistently exciting holds **generically** if there are $dim(\kappa)$ **vertex disjoint paths** between external signals $\{u, e\}$and $\kappa = \begin{bmatrix} w_{\mathcal{D}} \\ \xi_{\mathcal{Y}} \\ u_{\mathcal{K}} \end{bmatrix}$

Equivalently:

$dim(w_{\mathcal{D}})$ vertex disjoint paths between $\{u, e\} \backslash \{\xi_{\mathcal{Y}}, u_{\mathcal{K}}\}$ and $w_{\mathcal{D}}$

[1] Van der Woude, 1991
[2] Hendrickx, Gevers & Bazanella, CDC 2017, TAC 2019.
[3] VdH et al., CDC 2020.

Data informativity (path-based condition)



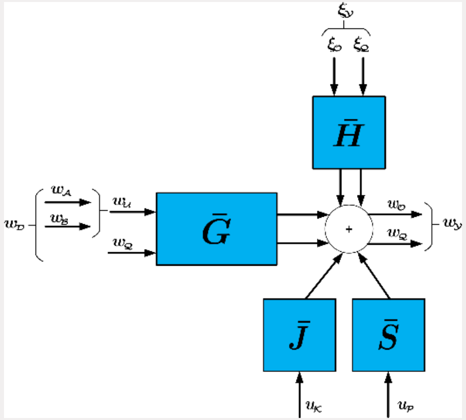
Specific result for networks with **full rank disturbances**:

Every node signal in $w_{\mathcal{Q}}$ requires an excitation in $u_{\mathcal{P}}$ having a 1-transfer to $w_{\mathcal{Y}}$

$$w_{\mathcal{Y}}(t) = \bar{G}(q,\theta)w_{\mathcal{D}}(t) + \bar{H}(q,\theta)\xi_{\mathcal{Y}}(t) + \bar{J}(q,\theta)u_{\mathcal{K}}(t) + \bar{S}u_{\mathcal{P}}(t)$$

- For every node in $w_{\mathcal{Q}}$ we need a *u*-excitation
- More expensive experiments with growing # outputs
- A node $w_{\mathcal{Q}}$ whose excitation appears in $u_{\mathcal{K}}$ can never be sufficiently excited

Data informativity (path-based condition)



Specific result for networks with **full rank disturbances**:

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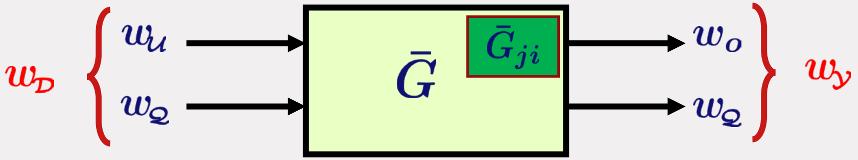
Additional condition for a node $w_\mathcal{Q}$ to be effectively ``excitable'':

Every loop around a node in $w_\mathcal{Q}$ should be blocked by a node in $w_\mathcal{D}$.

This additional graph-based condition needs to be integrated in the predictor model algorithms

[1] VdH et al., IFAC 2023

Single module identification – direct method

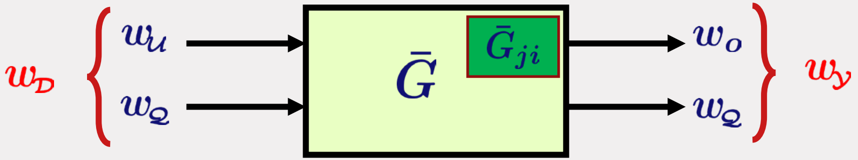


Conditions for arriving at an accurate model:

1. Module invariance: $\bar{G}_{ji} = G^0_{ji}$
2. Handling of confounding variables
3. Data-informativity
4. *Technical conditions on presence of delays*

Path-based conditions on the network graph

Single module identification – direct method



Are we done……??

Are there alternatives that can do the job with smaller # excitation signals?

Single module identification -

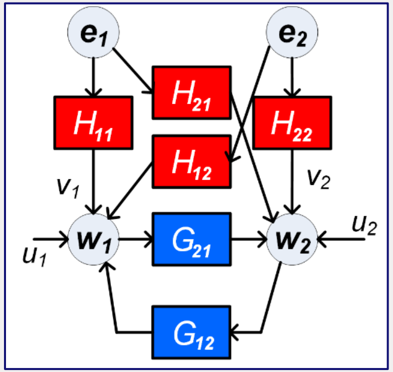
multi-step method

2-node example

Target: identify G_{21} with direct method

Predictor model: $\underbrace{\{w_1\}}_{w_\mathcal{D}} \rightarrow \underbrace{\{w_1, w_2\}}_{w_\mathcal{Y}}$

$w_\mathcal{Q} = \{w_1\} \quad w_\mathcal{U} = \emptyset$



Step 1:

Neither u_1 nor u_2 can contribute to $u_\mathcal{P}$ ⟶ data informativity condition is not satisfied

Step 2: Change predictor model to: $\underbrace{\{w_1, w_2\}}_{w_\mathcal{D}} \rightarrow \underbrace{\{w_1, w_2\}}_{w_\mathcal{Y}}$

Both u_1 and u_2 contribute to $u_\mathcal{P}$ ⟶ data informativity condition is satisfied

Both u_1 and u_2 need to be present, while an indirect method requires only u_1!

Single module identification – multi-step method

Confounding variable handling leads to multi-output models, that appear to be costly in terms of required excitation

Alternative multi-step method -
originally developed for full network identification[1] as computationally attractive.

Principle steps[2]:

1. Choose a selection of nodes w_s such that the PPL condition is satisfied
2. Immerse all other nodes
3. Estimate a high order ARX model for the mapping $u \to w_s$ in the immersed network
4. Use the estimated model to reconstruct the innovation signal
5. Use the reconstructed innovation as a measured input in a parametric estimation
6. Always ending up with a MISO predictor model

[1] Fonken et al., Automatica 2022

[2] Fonken et al., CDC 2023; Poster I-5 ERNSI 2023.

Single module identification – multi-step method

Alternative way of confounding variable handling

Relaxed data-informativity conditions:

$\Phi_\kappa(\omega) > 0$ for almost all ω

Multi-step method

$$\kappa(t) := \begin{bmatrix} w_{\mathcal{N}_j^-}(t) \\ \xi_{\mathcal{Y}}(t) \\ u_{\mathcal{K}_j}(t) \end{bmatrix}$$

Local direct method

$$\kappa(t) := \begin{bmatrix} w_{\mathcal{D}}(t) \\ \xi_{\mathcal{Y}}(t) \\ u_{\mathcal{K}}(t) \end{bmatrix}$$

$w_{\mathcal{N}_j^-}$: in-neighbors of w_j in immersed model

$u_{\mathcal{K}_j}$: subset of $u_{\mathcal{K}}$

Smaller dimension of κ leads to less excitation signals required !

[1] Fonken et al., Automatica 2022

[2] Fonken et al., CDC 2023; Poster I-5 ERNSI 2023.

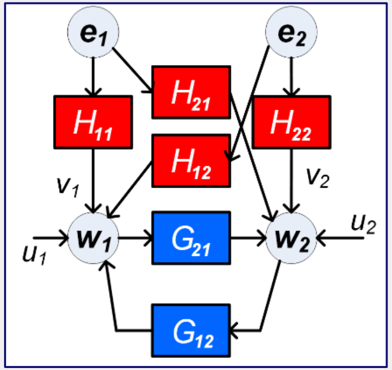
2-node example – multi-step method

Target: identify G_{21}

Predictor model in final step:

$$\underbrace{\{w_1\}}_{w_{\mathcal{D}}} \rightarrow \underbrace{\{w_2\}}_{w_{\mathcal{Y}}}$$

with reconstructed $\hat{e}_1, \hat{e}_2$
as additional predictor model inputs.

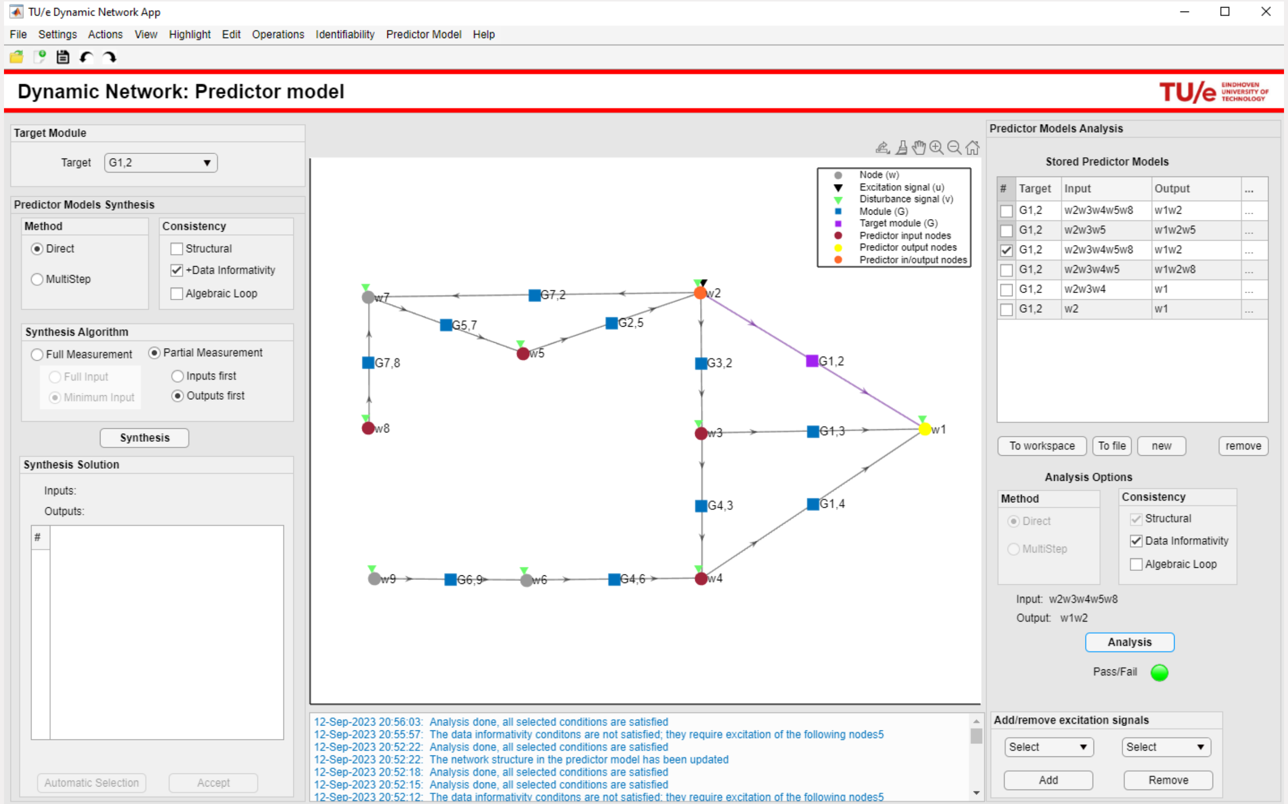


Exciting w_1 through either u_1 or u_2 is sufficient for data informativity

SYSDYNET

MATLAB App and Toolbox

Algorithms implemented in SYSDYNET App and Toolbox



Structural analysis and operations on dynamic networks

- Edit and manipulate
- Assign properties to nodes and modules
- Immersion of nodes, PPL test
- Generic identifiability analysis and synthesis
- Predictor model selection for single module ID

to be complemented with

- estimation algorithms for single module and full network ID;
- topology estimation

Version beta-0.2.0, September 2023, to be downloaded from www.sysdynet.net

TU/e

Summary

- All conditions for a consistent module estimate can be formulated (in the generic case) in terms of **path-based conditions** that the predictor model should satisfy.

- **Data-informativity** has to be taken into account and requires its own set of conditions.

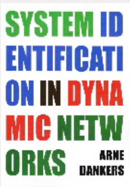
- There is room for **flexibility** in terms of sensor/actuator placement

- The **multi-step method** provides remarkable advantages in terms of excitation requirements

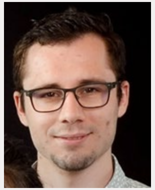
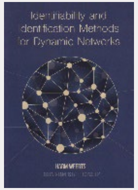
- All algorithms implemented in the **Matlab App/Toolbox**

ERC SYSDYNET Team: data-driven modeling in dynamic networks

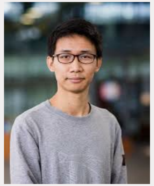
Research team:

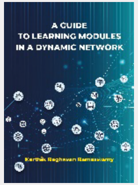
Arne Dankers

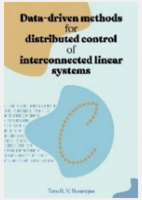
Harm Weerts




Shengling Shi

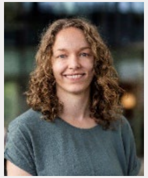



Karthik Ramaswamy

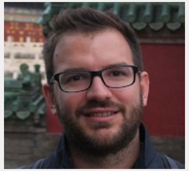
Tom Steentjes

Lizan Kivits



Stefanie Fonken

Xiaodong Cheng



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