Multi-step scalable least squares method for network identification with unknown disturbance topology

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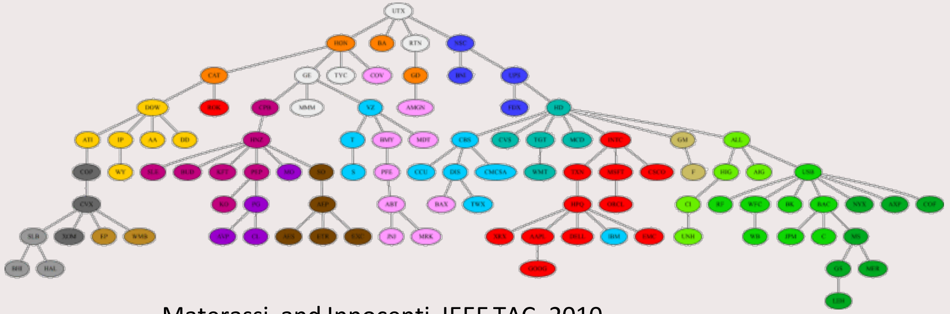
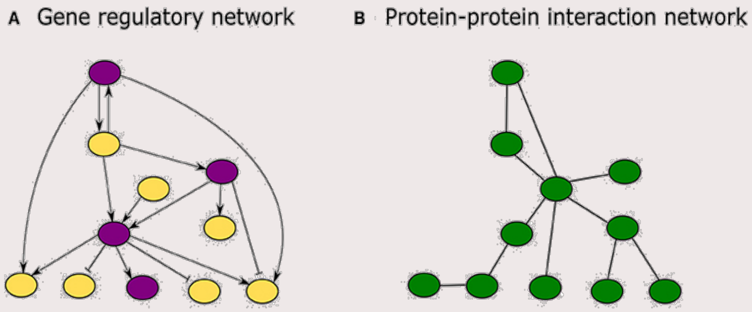
Contents

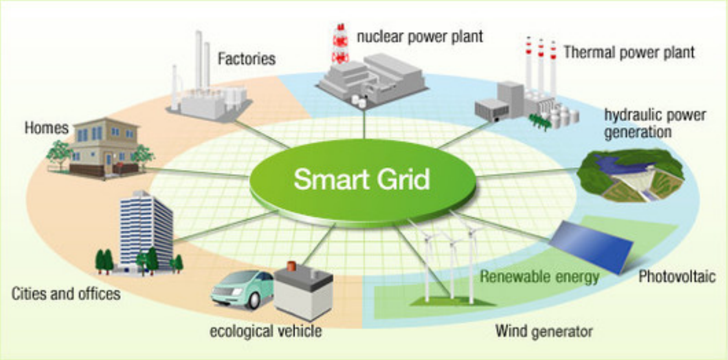
Introduction – Dynamic networks



Materassi and Innocenti, IEEE TAC, 2010



Vandereyken et al., Frontiers in plant science, 2018



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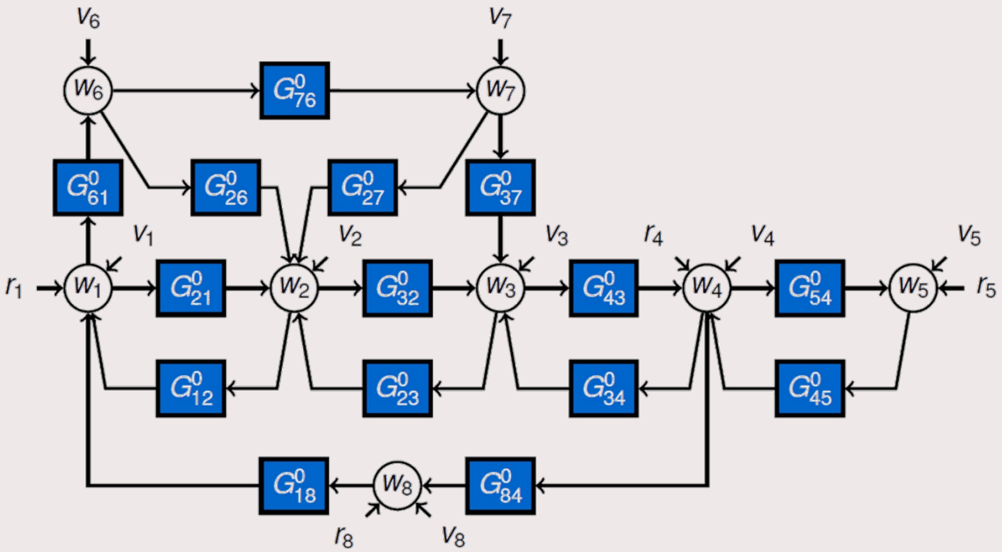
Introduction - Dynamic networks

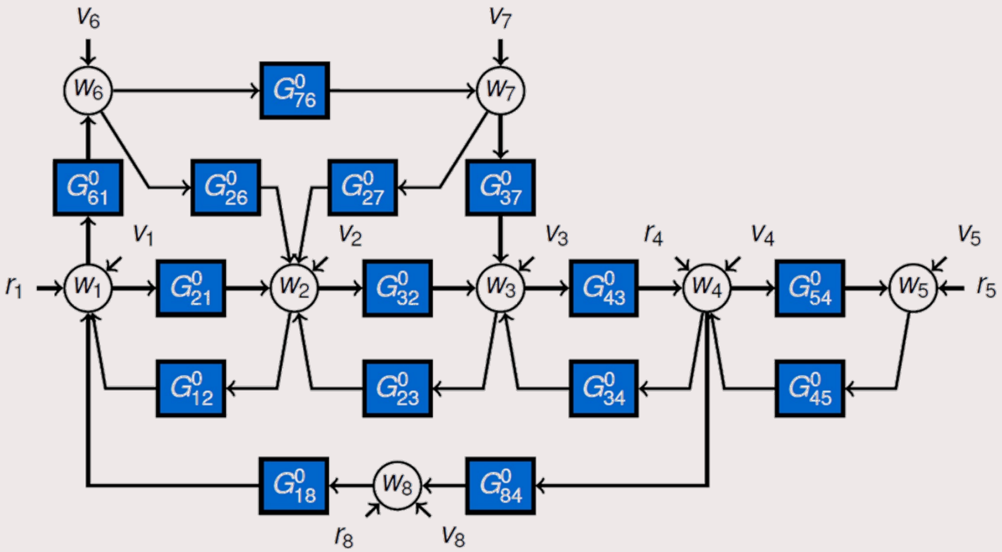
- G_{ij}^{0} : modules, Linear-time-invariant (LTI) transfer function
- $w(t)$: nodes
- r(t) : external excitation signals
- $v(t) = H^0 e(t)$: process noise



Dankers et al., Computers & Chemical Engineering, 2018.

Challenges in Dynamic networks

- Detection of network topology [1]
- Identification of modules in a dynamic network [2]
  - Local
  - **Full**



Networks are increasing in size and complexity

Thus there is a need for scalable and accurate identification methods

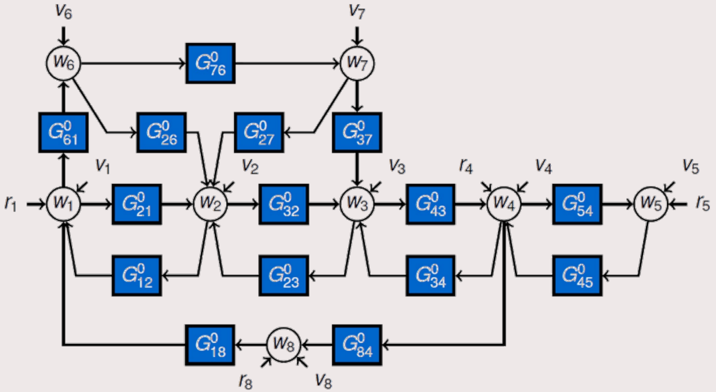
[1] Materassi et al., 2008, 2010, 2012, Chiuso et al., 2012. Shi et al., 2019. [2] Van den Hof et al.,2013. Dankers et al., 2015, 2016. Everitt et al., 2018. Galrinho et al. 2017, 2018, 2019. Weerts et al., 2017, 2018.

Dynamic network setup

$$\begin{bmatrix} w_1(t) \\ w_2(t) \\ \vdots \\ w_L(t) \end{bmatrix} = \begin{bmatrix} 0 & G^0_{12}(q) & \cdots & G^0_{1L}(q) \\ G^0_{21}(q) & 0 & \ddots & G^0_{2L,}(q) \\ \vdots & \ddots & \ddots & \vdots \\ G^0_{L1}(q) & G^0_{L2}(q) & \cdots & 0 \end{bmatrix} \begin{bmatrix} w_1(t) \\ w_2(t) \\ \vdots \\ w_L(t) \end{bmatrix} + R^0(q) \begin{bmatrix} r_1(t) \\ r_2(t) \\ \vdots \\ r_K(t) \end{bmatrix} + \begin{bmatrix} v_1(t) \\ v_2(t) \\ \vdots \\ v_L(t) \end{bmatrix}$$

$$w = G^0 w + R^0 r + v$$

$$w = G^0 w + R^0 r + H^0 e$$



Dynamic network setup

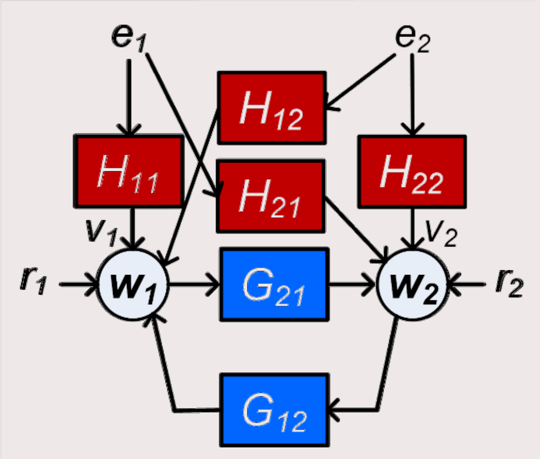
$$\begin{bmatrix} w_1(t) \\ w_2(t) \\ \vdots \\ w_L(t) \end{bmatrix} = \begin{bmatrix} 0 & G^0_{12}(q) & \cdots & G^0_{1L}(q) \\ G^0_{21}(q) & 0 & \ddots & G^0_{2L,}(q) \\ \vdots & \ddots & \ddots & \vdots \\ G^0_{L1}(q) & G^0_{L2}(q) & \cdots & 0 \end{bmatrix} \begin{bmatrix} w_1(t) \\ w_2(t) \\ \vdots \\ w_L(t) \end{bmatrix} + R^0(q) \begin{bmatrix} r_1(t) \\ r_2(t) \\ \vdots \\ r_K(t) \end{bmatrix} + \begin{bmatrix} v_1(t) \\ v_2(t) \\ \vdots \\ v_L(t) \end{bmatrix}$$

$$w = G^0 w + R^0 r + v$$

$$w = G^0 w + R^0 r + H^0 e$$

- Measurements of all $w(t)$ available
- $R^0 r$ is known
- G^0, H^0 rational transfer function matrices
- G^0 strictly proper
- H^0 monic
- Topology H^0 unknown

Disturbance $v(t) = He(t)$



Noise can be:

- Uncorrelated $$v(t) = \begin{bmatrix} H_{11} & 0 \\ 0 & H_{22} \end{bmatrix} \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix}$$
- Correlated $$v(t) = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix}$$
- Reduced rank $$v(t) = \begin{bmatrix} H_{11} \\ H_{21} \end{bmatrix} \begin{bmatrix} e_1(t) \end{bmatrix}$$

Available methods

	Variance
Indirect identification method [1]	Consistency
Methods that parametrize the disturbance model	Maximum Likelihood

[1] Ljung, Prentice-Hall, 1999.

Available methods

Method	Optimization problem	Parametrizes
Joint direct method[1]	Non-convex ✘	General, Box Jenkins model ✓

[1] Weerts et al., Automatica, 2018

Available methods

Method	Optimization problem		Parametrizes	
Joint direct method[1]	Non-convex	✗	General, Box Jenkins model	✓
Sequential Least Squares (SLS) [2]	Convex	✓	ARMAX model	✗
Sequential Linear Regressions (SLR) [3]	Convex	✓	FIR functions	✗
Weighted Null Space Fitting (WNSF) [4,5]	Convex	✓	ARMAX model, OE model	✗

However, these methods require known network and **disturbance** topology

[1] Weerts et al., Automatica, 2018 [2] Weerts et al., IFAC, 2018. [3] Dankers, technical note, 2019. [4] Galrinho et al., IEEE TAC 2019. [5] Fonken et al., IFAC 2020.

Problem statement

How can we effectively estimate dynamic networks with unknown disturbance topology, where the noise can be correlated and of reduced rank?

Effectiveness:

- Low computational burden - Scalable to large networks
  - Convex methods
  - MIMO → MISO
- Reduced variance
  - Parametrize disturbance model
  - Box Jenkins

Problem statement

How can we effectively estimate dynamic networks with unknown disturbance topology, where the noise can be correlated and of reduced rank?

- How do we obtain disturbance topology?
- How do we implement convex parametrization of a Box Jenkins model structure?

Disturbance topology detection

Why: Appropriate modeling of the disturbance

- Reduces variance
- Consistency

How: Stepwise procedure

Step 1: Noise rank p → **Step 2:** Noise correlation structure

$$v(t) = He(t) = \begin{bmatrix} H_{11} & \cdots & H_{1p} \\ \vdots & \ddots & \vdots \\ H_{L1} & \cdots & H_{Lp} \end{bmatrix} \begin{bmatrix} e_1(t) \\ \vdots \\ e_p(t) \end{bmatrix}$$

Disturbance topology detection

Step 1:
Noise rank
p

Step 1

- High order ARX model $\hat{w}(t|t-1) := \bar{\mathbb{E}}\{w(t)|w^{t-1}, r^t\}$
- Reconstruct innovation: $\hat{\varepsilon}_j = w_j - \hat{w}_j(t|t-1)$
- $\hat{\Lambda} = \frac{1}{N}\sum_{t=1}^{N} \hat{\varepsilon}\hat{\varepsilon}^\top$ Singular value decomposition: $\text{rank}(\hat{\Lambda}) = p$

Disturbance topology detection

Step 2:

Noise correlation structure

Step 2

- Use $\hat{\varepsilon}_j$ as additional measured input → Sequential Linear Regression [1]
- High order ARX model $\hat{w}(t|t-1) := \bar{\mathbb{E}}\{w(t)|w^{t-1}, r^t, e^{t-1}\}$
- Obtain estimates $G_{ji} = \sum_{k=1}^{n} g_k q^{-k}$ $H_{ji} = \sum_{k=1}^{n} h_k q^{-k}$
- Estimate noise correlation structure
  - Structure selection (AIC, BIC, Cross Validation)
  - Group Lasso

[1] Dankers, technical note, 2019.

Full network identification

Step 3:

- From nonparametric (high order) model to parametric model → WNSF

Nonparametric (high order) models

Step 1: Noise rank p → Step 2: Noise correlation structure → Step 3: Parametric BJ model

Weighted-Null Space Fitting [1] [2] [3] [4] [5]

Step 3:

Parametric BJ model

bias ↓

variance ↑

data

Step 1:

Estimate nonparametric (high order) FIR or ARX model

Choose model order n sufficiently large

$$G_{ji} = \sum_{k=1}^{n} g_k q^{-k}$$

variance ↓

Step 2:

Reduction to parametric model

$$\sum_{k=1}^{n} g_k q^{-k} = \frac{\sum_{k=1}^{m_l} l_k q^{-k}}{1 + \sum_{k=1}^{m_f} f_k q^{-k}}$$

variance ↓

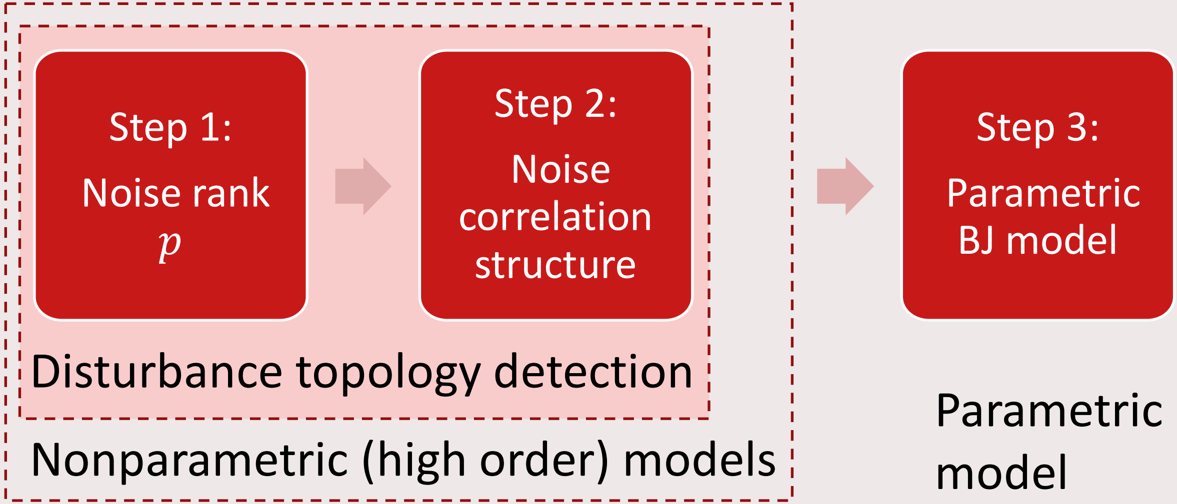
Step 3:

Re-estimation of parametric model

[1] Galrinho, PhD thesis, 2018. [2] Galrinho et al., IEEE TAC 2019. [3] Galrinho et al., IFAC, 2018. [4] Fonken et al., IFAC 2020. [5] Ljung and Wahlberg, Advances in Applied Probability, 1992.

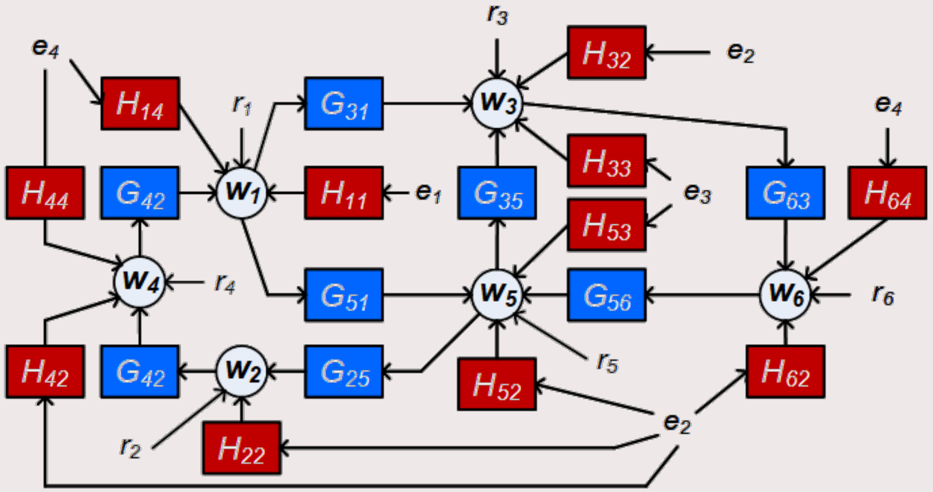
Method for full network identification [1]

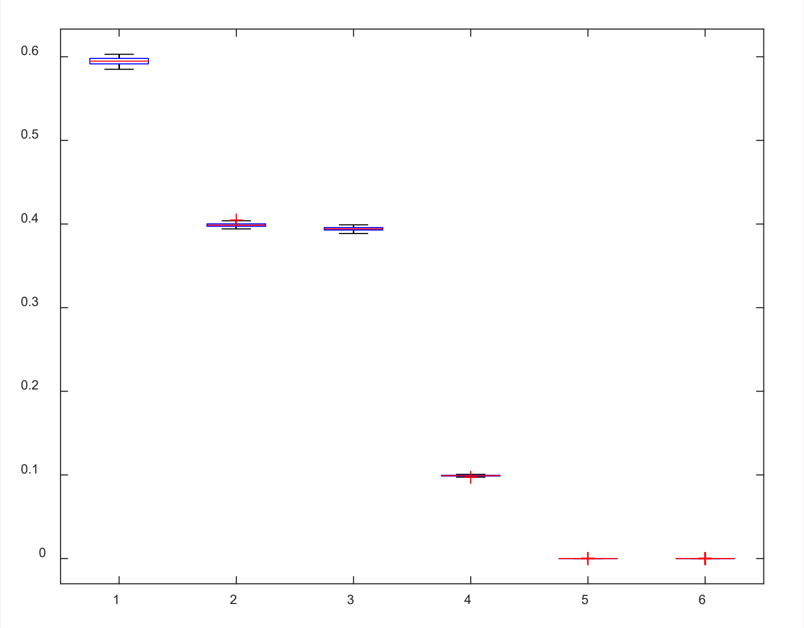
- Convex
- MISO predictors
- Consistency – path based data informativity conditions



[1] submitted to Automatica, 2021

Results noise rank p estimation





Singular value decomposition of $\hat{\Lambda}$ for $N = 30000$

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Results disturbance topology detection

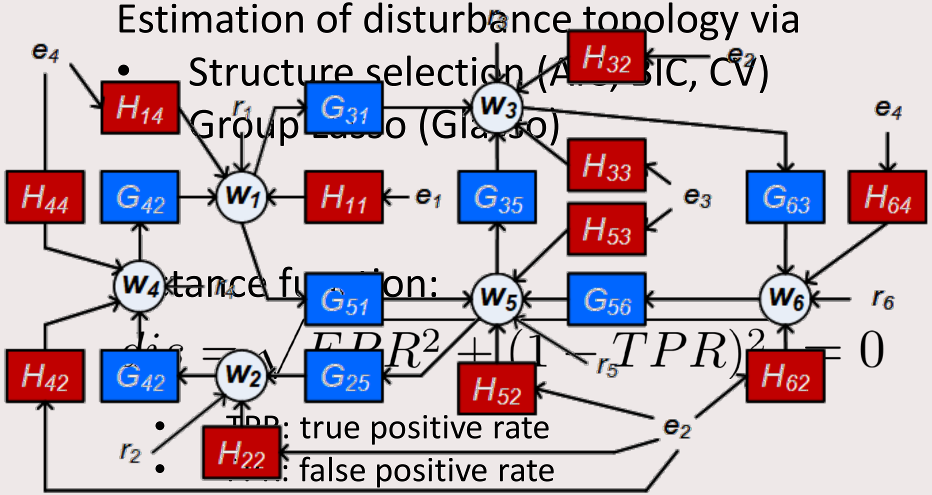
Estimation of disturbance topology via

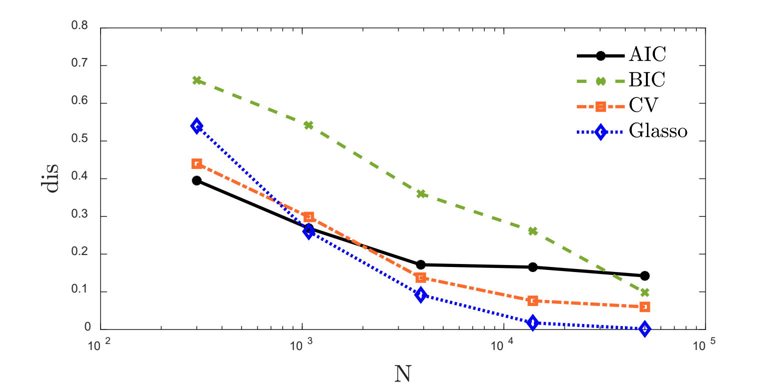
- Structure selection (AIC, BIC, CV)
- Group Lasso (Glasso)

Distance function:

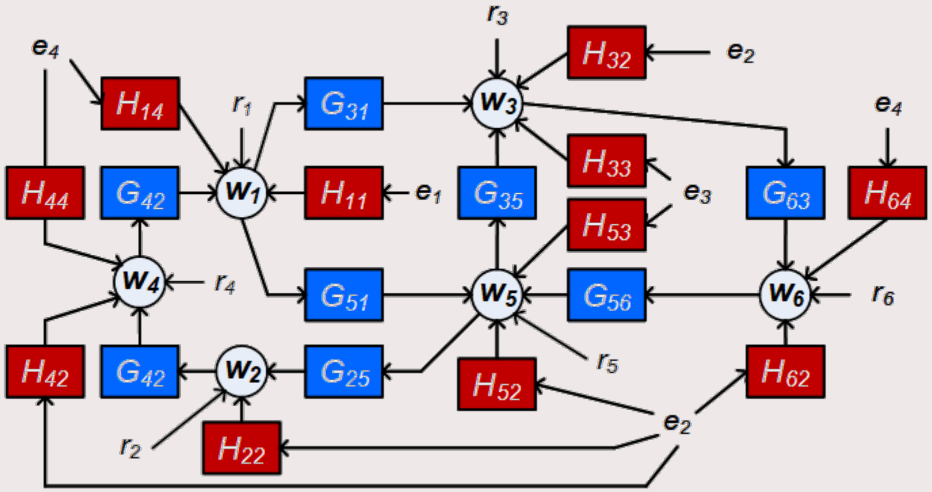
$$dis = \sqrt{FPR^2 + (1-TPR)^2} = 0$$

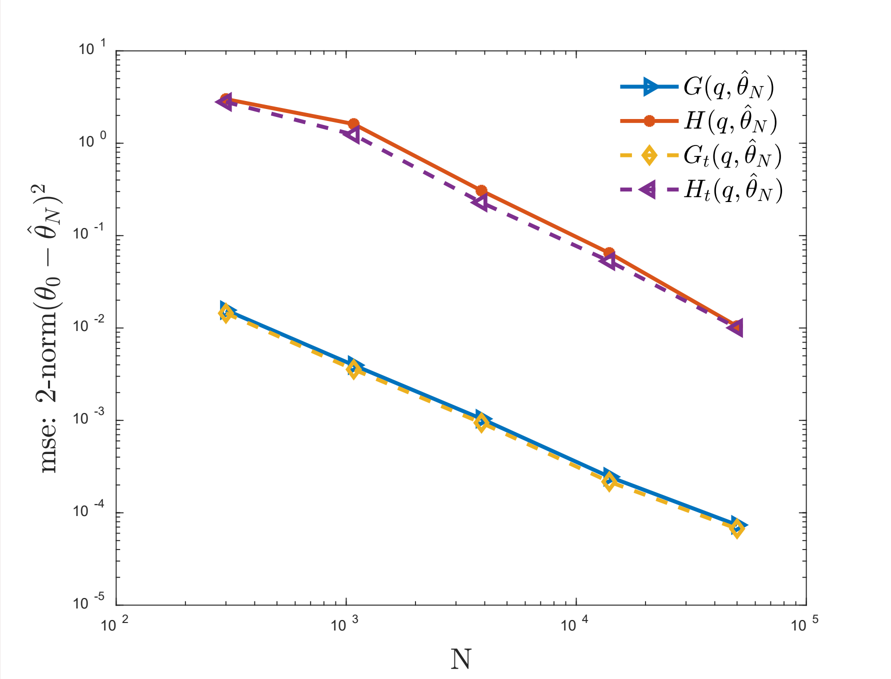
- TPR: true positive rate
- FPR: false positive rate





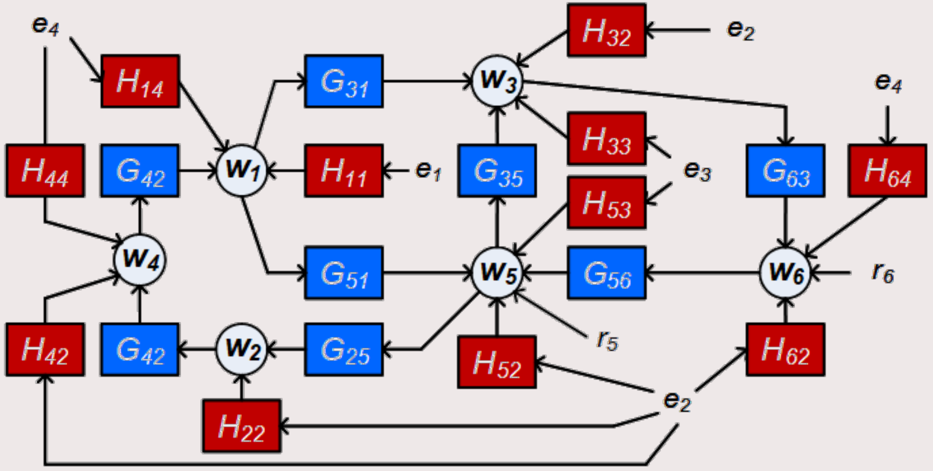
Results Identification

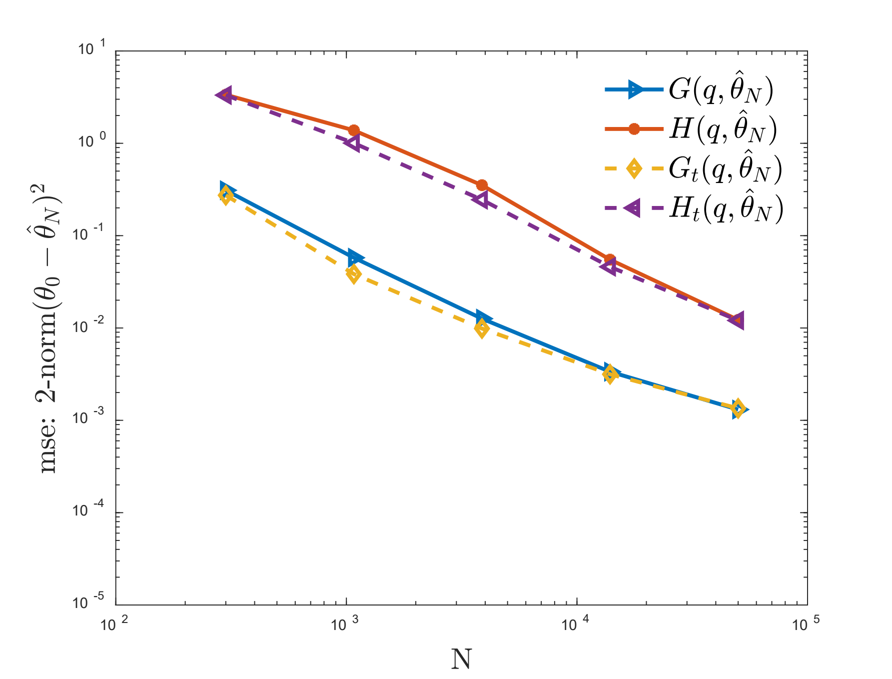




Average MSE(N): $\|\theta_0 - \hat{\theta}_N\|^2$

Results Identification





Average MSE(N): $\|\theta_0 - \hat\theta_N\|^2$

Conclusion

We effectively estimate dynamic networks with unknown disturbance topology by developing a multi-step least squares method

- Scalable due to low computational burden
  - Analytical solutions (least squares)
  - MISO optimization problems – parallel /sequential
- Reduced variance:
  - Estimation disturbance topology & include disturbance model in identification
  - For Box Jenkins model structures
- Consistent

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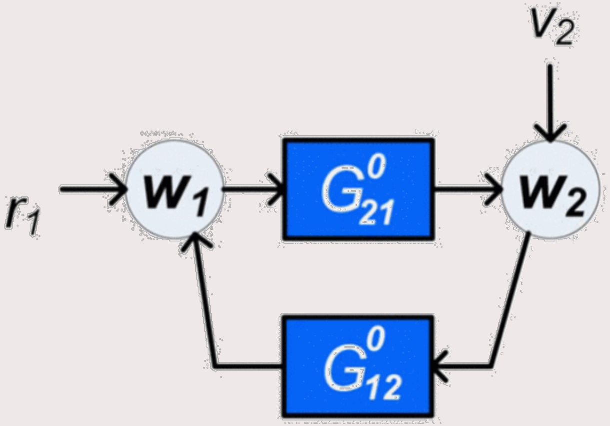
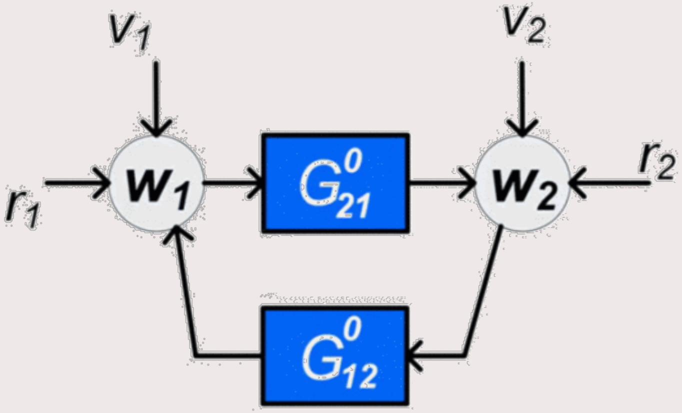
Classical identification VS Dynamic networks

Classic



Dynamic networks



 Van den Hof et al., CDC, 2017.