Local identification in dynamic networks using a multi-step least squares method

42st Benelux Meeting on Systems and Control
Elspeet, Netherlands, March 23, 2023

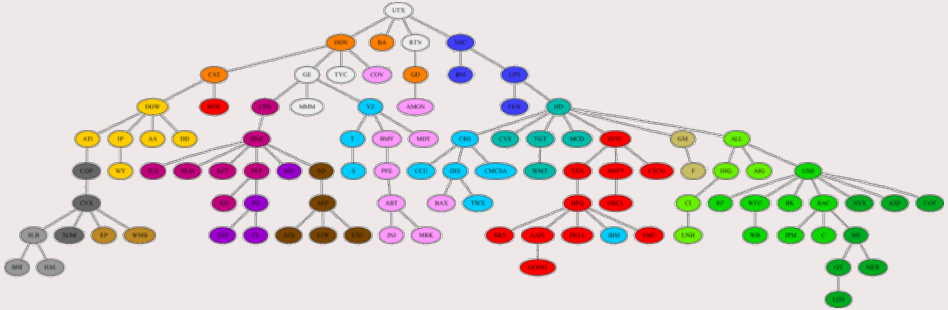
Stefanie Fonken, Karthik Ramaswamy, Paul Van den Hof

European Research Council

Electrical Engineering, CS group

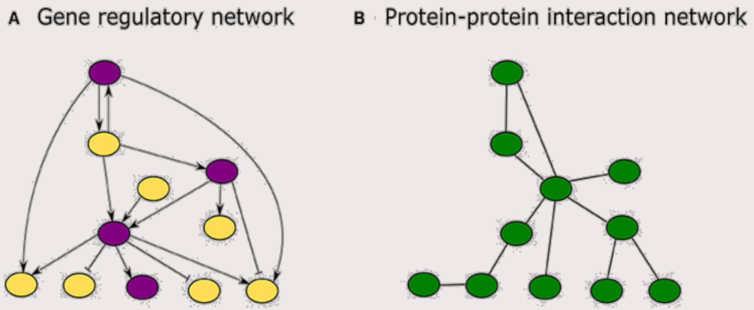
EINDHOVEN UNIVERSITY OF TECHNOLOGY

Networks

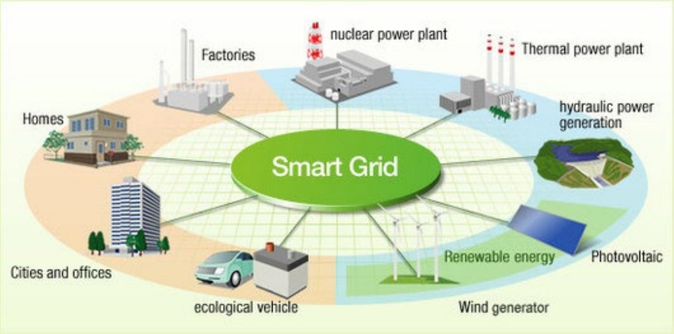


Materassi and Innocenti, IEEE TAC, 2010

www.envidia.com



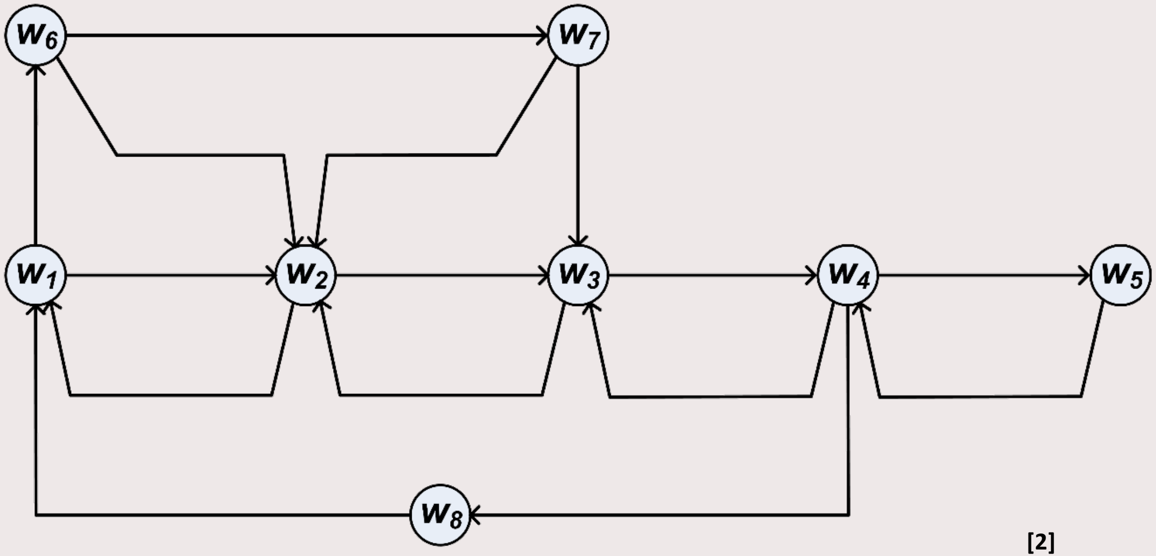
Vandereyken et al., Frontiers in plant science, 2018



www.betterworldsolutions.eu

Dynamic network framework [1] - Data-driven modeling of networks

- $w(t)$: Nodes

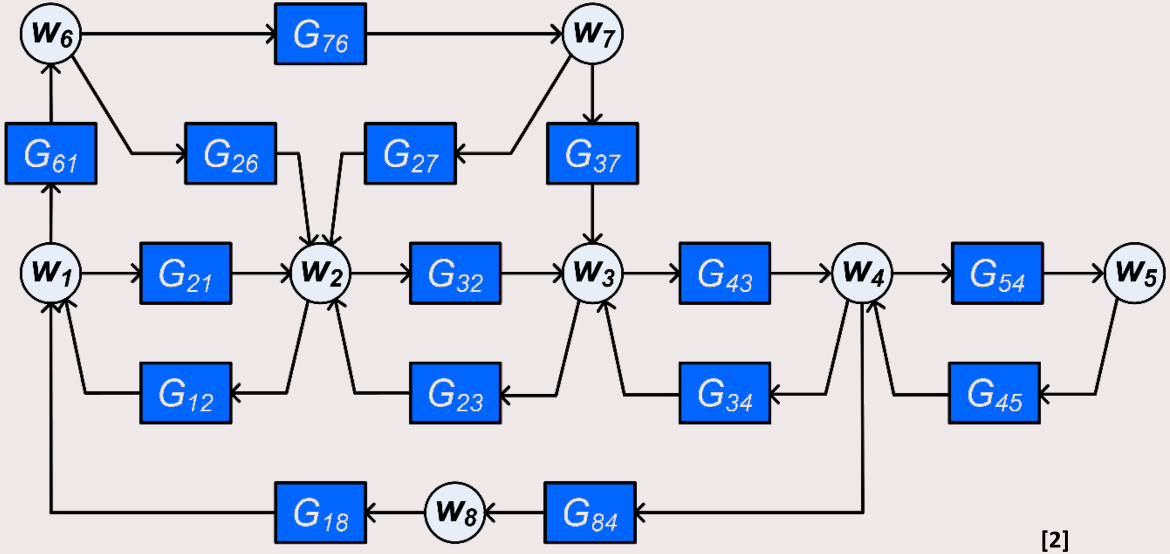


[2]

[1] Van den Hof, et al. (2013) *Automatica*.
[2] Van den Hof, et al. (2018) *Computers & Chemical Engineering*

Dynamic network framework [1] - Data-driven modeling of networks

- $w(t)$: Nodes
- G_{ij}^{0} : Modules

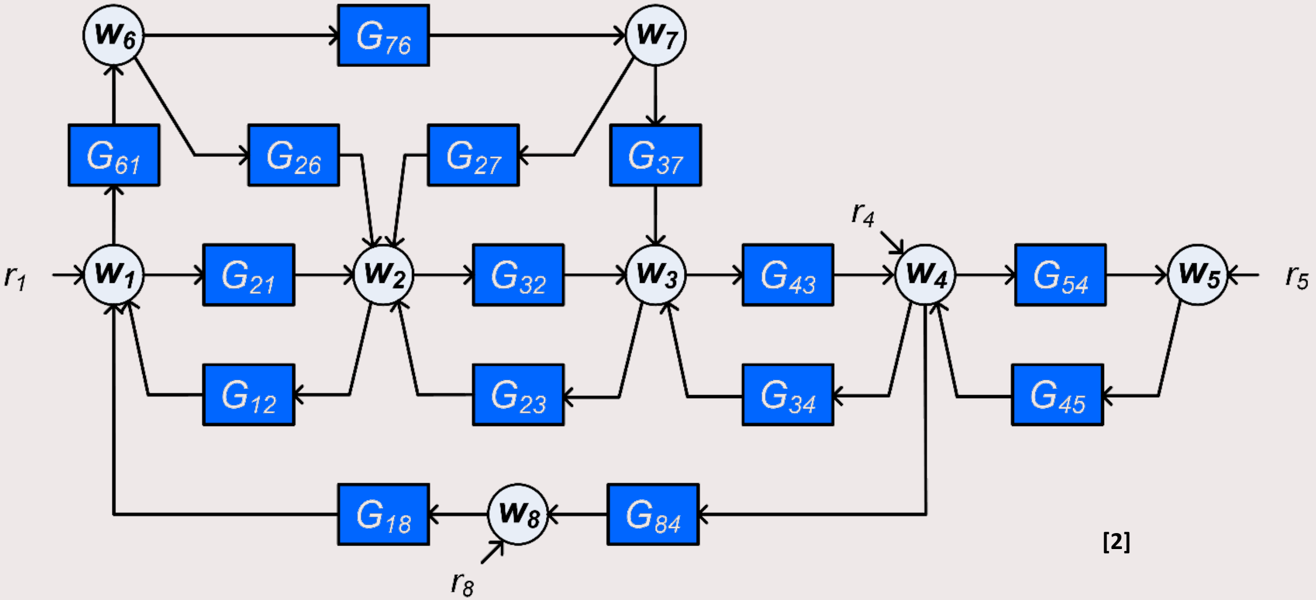


[2]

[1] Van den Hof, et al. (2013) *Automatica*.
[2] Van den Hof, et al. (2018) *Computers & Chemical Engineering*

Dynamic network framework [1] - Data-driven modeling of networks

- $w(t)$: Nodes
- G_{ij}^0 : Modules
- $r(t)$: Excitation

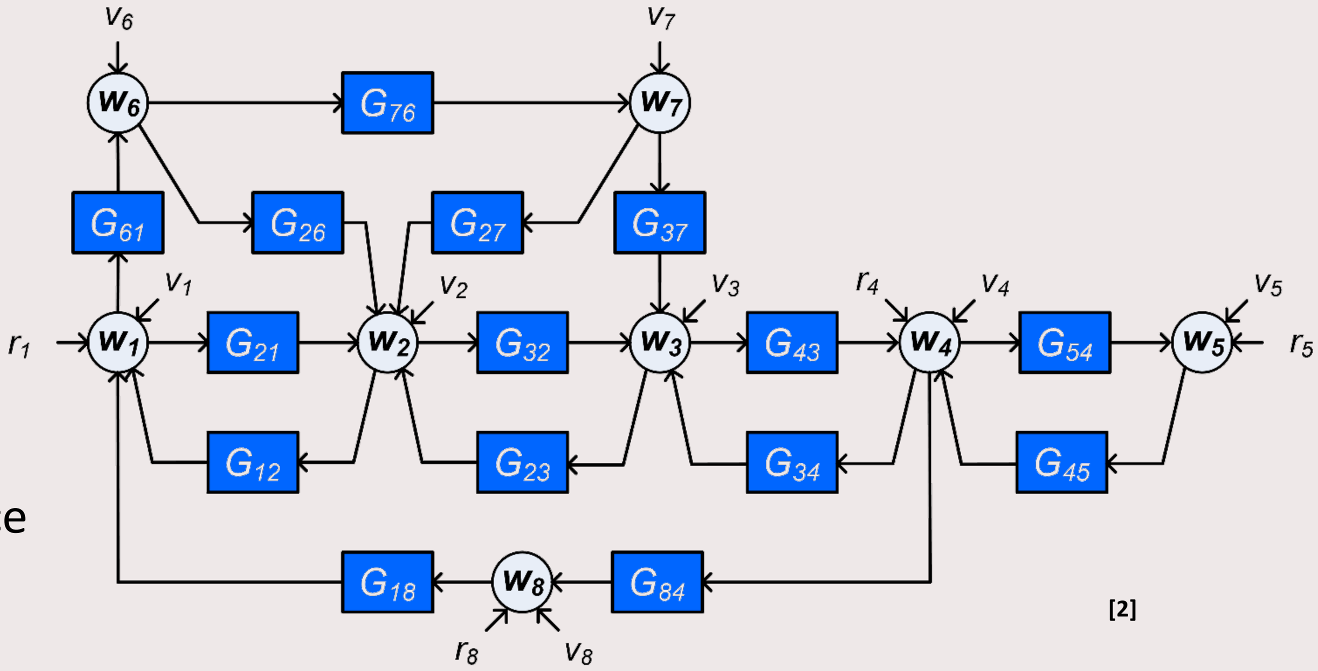


[2]

[1] Van den Hof, et al. (2013) *Automatica*.
[2] Van den Hof, et al. (2018) *Computers & Chemical Engineering*

Dynamic network framework [1] - Data-driven modeling of networks

- $w(t)$: Nodes
- G_{ij}^{0} : Modules
- $r(t)$: Excitation
- $v(t)$: Disturbance

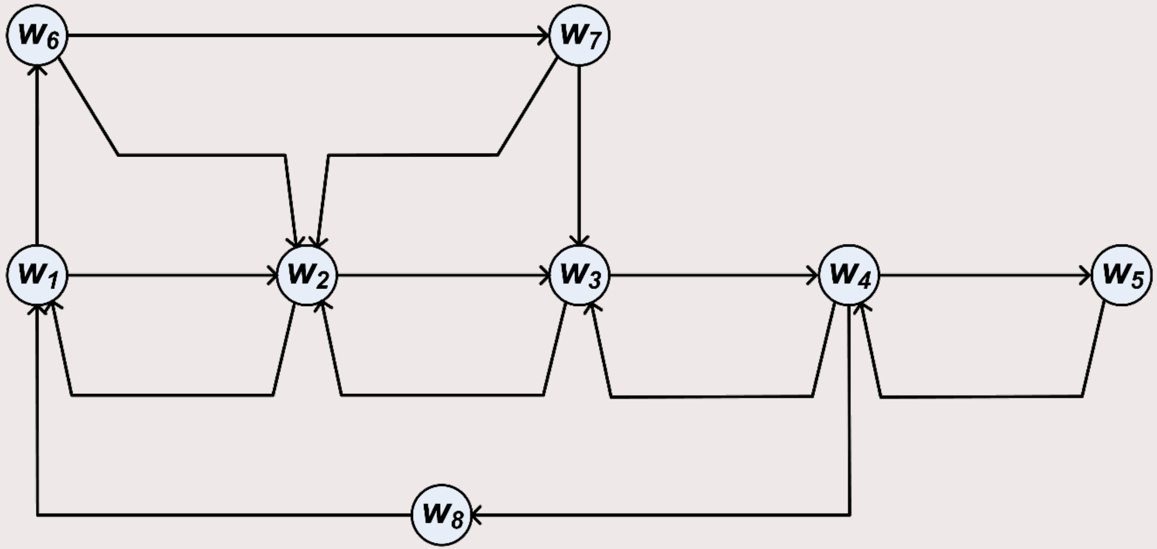


[2]

[1] Van den Hof, et al. (2013) *Automatica*.
[2] Van den Hof, et al. (2018) *Computers & Chemical Engineering*

Challenges in Dynamic networks

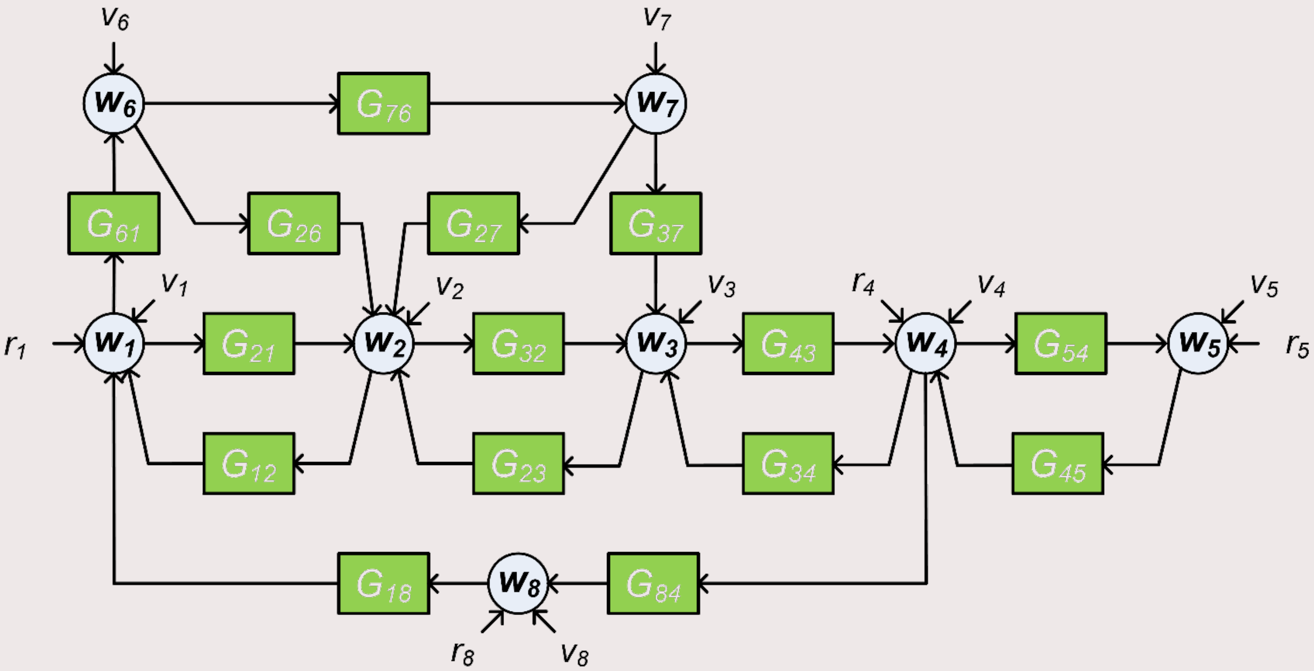
- Topology [1]



[1] Materassi, Innocenti, Salapaka, Shi,…
[2] Van den Hof, Dankers, Everitt, Galrinho, Weerts, Ramaswamy, Shi, …

Challenges in Dynamic networks

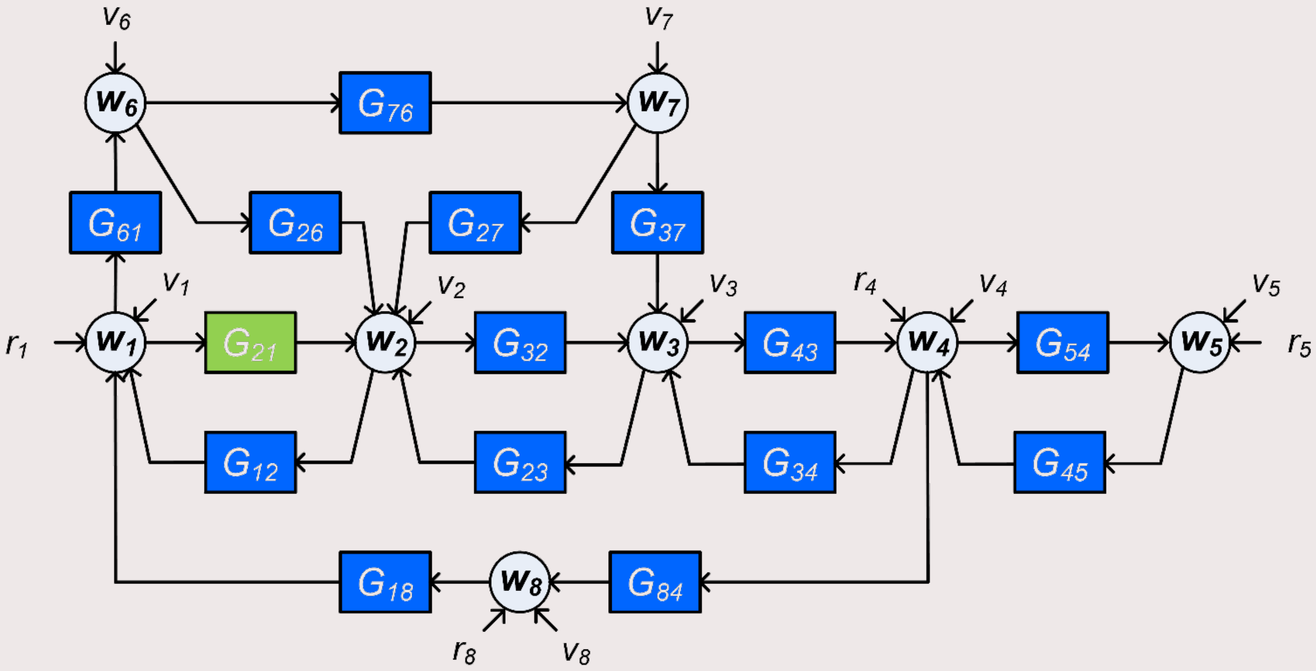
- Topology [1]
- Identification [2]
  - Full network



[1] Materassi, Innocenti, Salapaka, Shi,…
[2] Van den Hof, Dankers, Everitt, Galrinho, Weerts, Ramaswamy, Shi, …

Challenges in Dynamic networks

- Topology [1]
- Identification [2]
  - Full network
  - **Local**



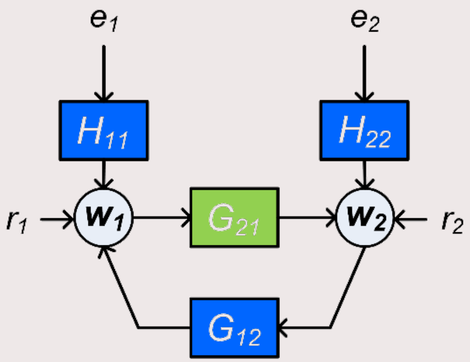
[1] Materassi, Innocenti, Salapaka, Shi,...
[2] Van den Hof, Dankers, Everitt, Galrinho, Weerts, Ramaswamy, Shi, ...

Local identification

Single module identifiability [1]

- Method independent
- Identifiability: $0\ r$-signal

Method	Transfer		# r-signals needed
Local Direct [2]	$w \rightarrow w$	Consistency & ML	0
Indirect[3]	$r \rightarrow w$	Consistency	1



[1] Shi, et al. (2021, 2022, 2023)
[2] Ramaswamy, et al. (2021)
[3] Gevers, et al. (2018), Hendrickx, et al. (2019), Bazanella, et al. (2019)

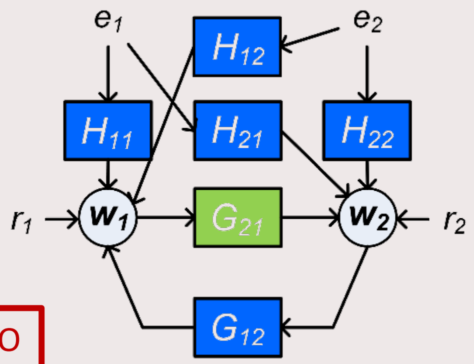
Local identification

Single module identifiability

- Method independent
- Identifiability: 1 r-signal

Method	Transfer		# r-signals needed
Local Direct [1]	$w \to w$	Consistency & ML	2 Conservative
Indirect	$r \to w$	Consistency	1

MIMO



 [1] Ramaswamy et al. (2021) *TAC*, van den Hof et al. (2017,2020,2023)

Research question

How can we obtain relaxed conservatism compared to current direct methods?

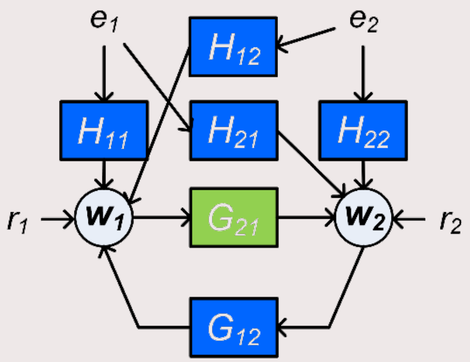
- Requires less r-signals
- Keep advantages of the current direct methods

Local identification

Single module identifiability

- Method independent
- Identifiability: 1 r-signal

Method	Transfer		# r-signals needed
Local Direct	$w \to w$	Consistency & ML	2
Indirect	$r \to w$	Consistency	1
Multi-step Least squares [1]			1



Full network identification

Multi-step least squares method [1,2]

Full network identification $w = Gw + He + r$

1. Estimate $r \rightarrow w$ with high-order ARX and reconstruct $\hat{e}$

2. Use $\hat{e}$ as additional measured input [2]

$$w = Gw + He + r$$

[1] Fonken, et al. (2022) *Automatica*
[2] Dankers, et al. (2019) *technical note*

Multi-step least squares method [1,2]

Full network identification $w = Gw + He + r$

1. Estimate $r \to w$ with high-order ARX and reconstruct $\hat{e}$

2. Use $\hat{e}$ as additional measured input [2]

$$w = Gw + (H - I)e + Ie + r$$

[1] Fonken, et al. (2022) *Automatica*
[2] Dankers, et al. (2019) *technical note*

Multi-step least squares method [1,2]

Full network identification $w = Gw + He + r$

1. Estimate $r \to w$ with high-order ARX and reconstruct $\hat{e}$

2. Use $\hat{e}$ as additional measured input [2]

$$w = Gw + (H - I)\hat{e} + Ie + r$$

[1] Fonken, et al. (2022) *Automatica*
[2] Dankers, et al. (2019) *technical note*

Multi-step least squares method [1,2]

Full network identification $$w = Gw + He + r$$

1. Estimate $\underline{r \rightarrow w}$ with high-order ARX and reconstruct $\hat{e}$ ← Indirect method

2. Use $\hat{e}$ as additional measured input [2]

$$w_j = \sum_k \underline{G_{jk} w_k} + \sum_\ell \underline{(H_{j\ell} - I_{jj})\hat{e}_\ell} + e_j + r_j$$ ← Direct method

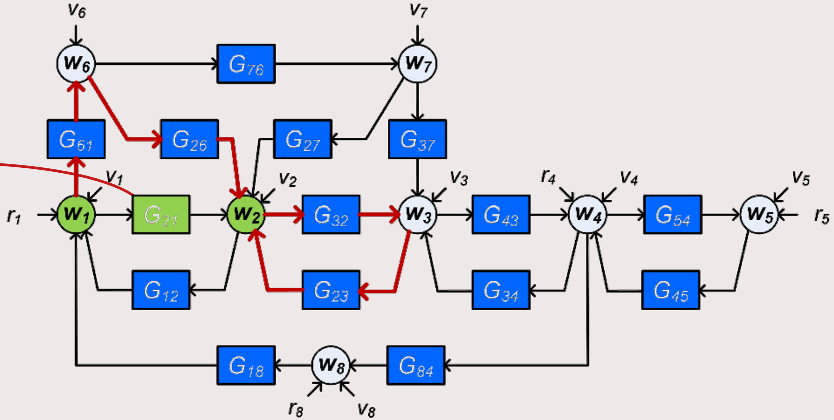
Estimate $\mathcal{L}$ MISO models $j = 1, \ldots, \mathcal{L}$

[1] Fonken, et al. (2022) *Automatica*
[2] Dankers, et al. (2019) *technical note*

Multi-step least squares method

Full network identification → local identification

1. Select subnetwork w_k
   Immerse [1] w_k

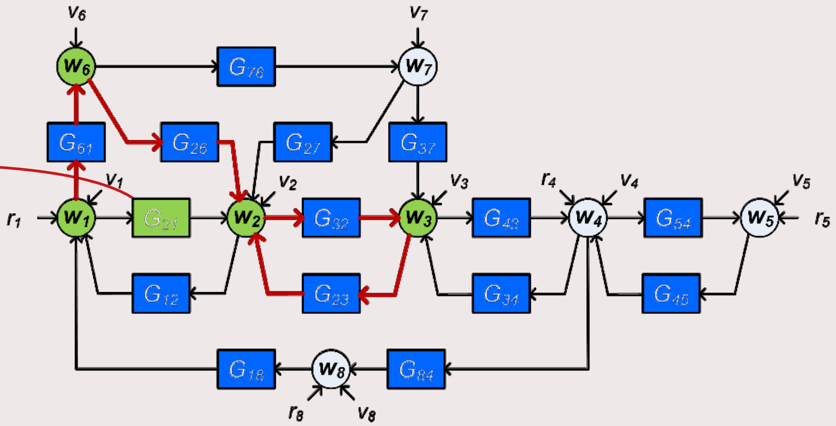


 [1] Danker, et al. (2016) *Automatica*

Multi-step least squares method

Full network identification → local identification

1. Select subnetwork w_k
   Immerse [1] w_k



[1] Danker, et al. (2016) *Automatica*

Local identification

Full network → Immersed network

$$w = G^0 w + He + r$$

$$w_{\mathcal{S}} = \bar{G}_{\mathcal{S}} w_{\mathcal{S}} + \bar{H}_{\mathcal{S}} \xi_{\mathcal{S}} + u_{\mathcal{S}}$$

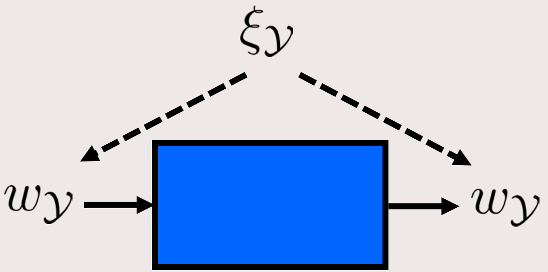
Local identification [1,2]

- Decompose $w_{\mathcal{S}}$ [1]

  $j \in \mathcal{Y}$

$$\begin{bmatrix} w_{\mathcal{Y}} \\ \hline w_{\mathcal{U}} \end{bmatrix} = \begin{bmatrix} \bar{G} \\ \bar{G}_{\mathcal{U}} \end{bmatrix} \begin{bmatrix} w_{\mathcal{Y}} \\ \hline w_{\mathcal{U}} \end{bmatrix} + \left[\begin{array}{c|c} \bar{H} & 0 \\ \hline 0 & \bar{H}_{\mathcal{U}} \end{array}\right] \begin{bmatrix} \xi_{\mathcal{Y}} \\ \hline \xi_{\mathcal{U}} \end{bmatrix} + \begin{bmatrix} u_{\mathcal{Y}} \\ \hline u_{\mathcal{U}} \end{bmatrix}$$ [2]

- Confounding variable → Correlated noise



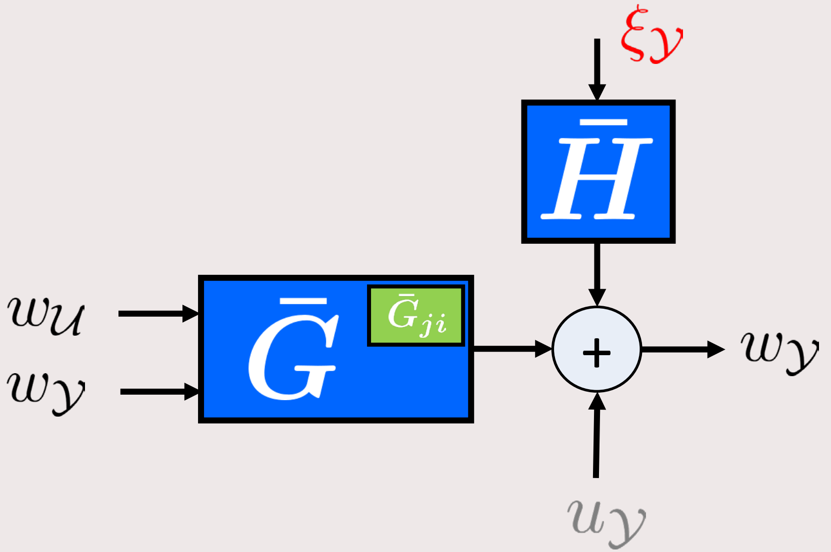
$$w_{\mathcal{Y}} = \bar{G} \begin{bmatrix} w_{\mathcal{Y}} \\ \hline w_{\mathcal{U}} \end{bmatrix} + \bar{H}\xi_{\mathcal{Y}} + u_{\mathcal{Y}}$$

[1] Shi, et al. (2023) *IFAC*
[2] Ramaswamy, et al. (2021) *Automatica*

Local identification

- No confounding variable for
  $w_{\mathcal{U}} \rightarrow w_{\mathcal{Y}}$

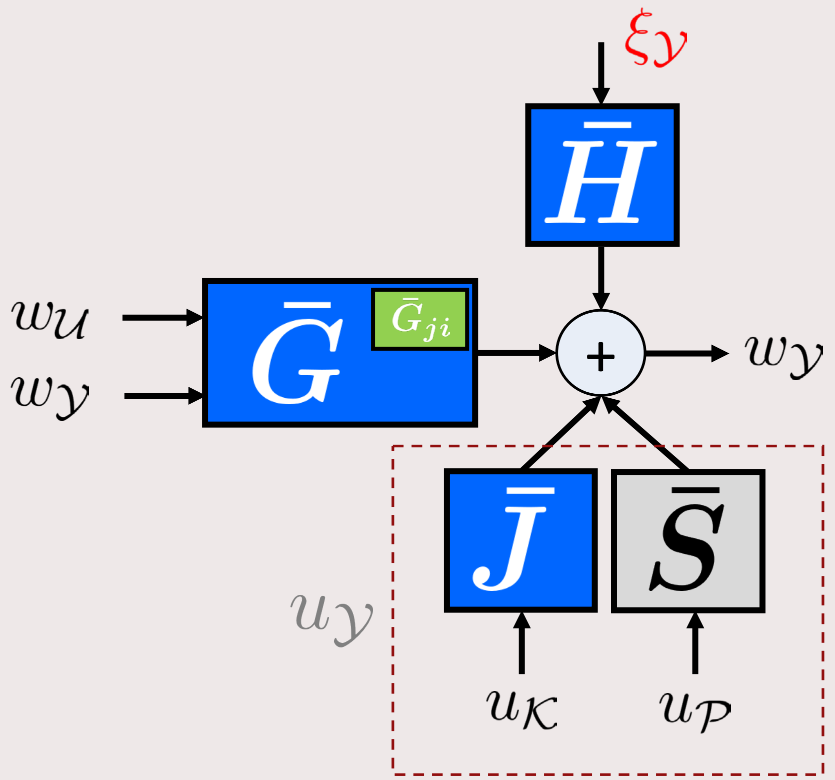
$$w_{\mathcal{Y}} = \bar{G}\begin{bmatrix} w_{\mathcal{Y}} \\ w_{\mathcal{U}} \end{bmatrix} + \bar{H}\xi_{\mathcal{Y}} + u_{\mathcal{Y}}$$



Local identification

Immersed network: $u_{\mathcal{Y}}$ [1]

$$\underbrace{u_{\mathcal{Y}} = \bar{J}(q)u_{\mathcal{K}}}_{\text{Dynamic due to immersion}} + \underbrace{\bar{S}u_{\mathcal{P}}}_{\text{Known}}$$



[1] Ramaswamy, et al. (2021) *Automatica*

Multi-step least squares method [1,2]

Local identification

$$w_{\mathcal{Y}} = \bar{G}(q)\begin{bmatrix} w_{\mathcal{Y}} \\ w_{\mathcal{U}} \end{bmatrix} + \bar{H}(q)\xi_{\mathcal{Y}} + \bar{J}(q)u_{\mathcal{K}} + \bar{S}u_{\mathcal{P}}$$

1. Estimate $\underline{u_{\mathcal{K}\cup\mathcal{P}} \rightarrow w}$ with high-order ARX and reconstruct $\hat{\xi}_{\mathcal{Y}}$

Indirect method

2. Use $\hat{\xi}_{\mathcal{Y}}$ as additional measured input [2]

$$w_j = \sum_{k\in\mathcal{N}_j^-} \underline{\bar{G}_{jk}w_k} + \sum_{\ell\in\mathcal{Y}} \underline{(\bar{H}_{j\ell} - I_{jj})\hat{\xi}_\ell} + \xi_j + \sum_{m\in\mathcal{K}_j} \bar{J}_{jm}u_m + u_{\mathcal{P}_j}$$

Direct method

Estimate the j-th MISO model $j \in \mathcal{Y}$

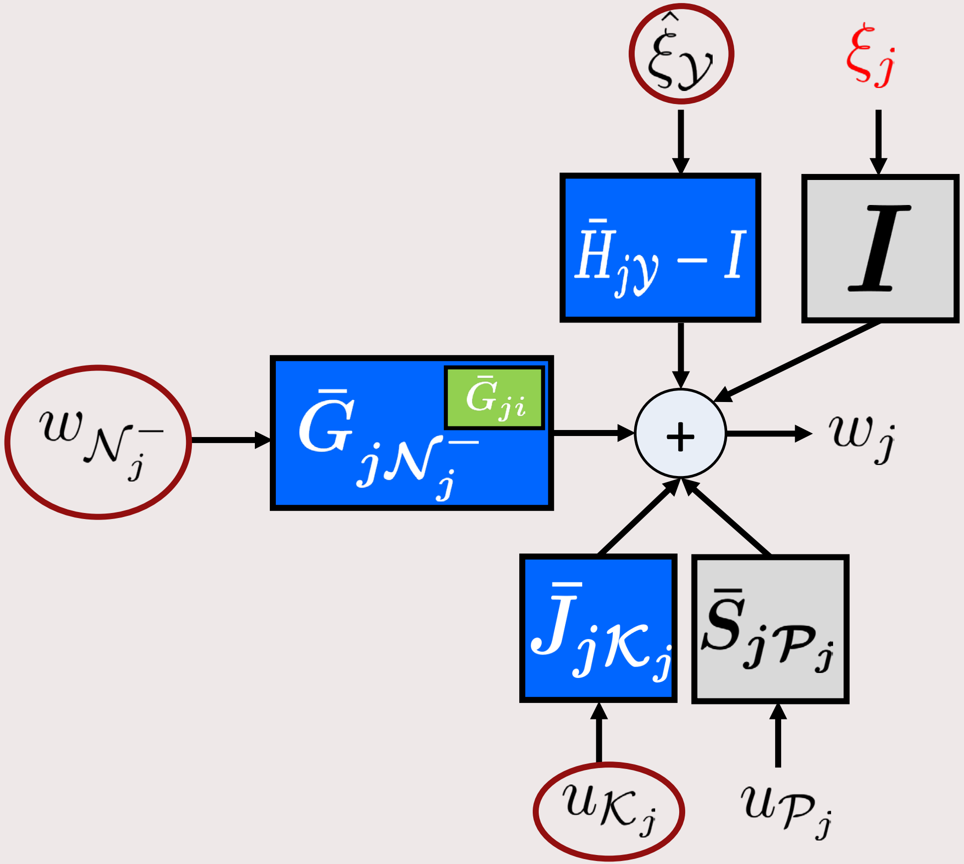
[1] Fonken, et al. (2022) *Automatica*
[2] Dankers, et al. (2019) *technical note*

Local identification

Data informativity [1]

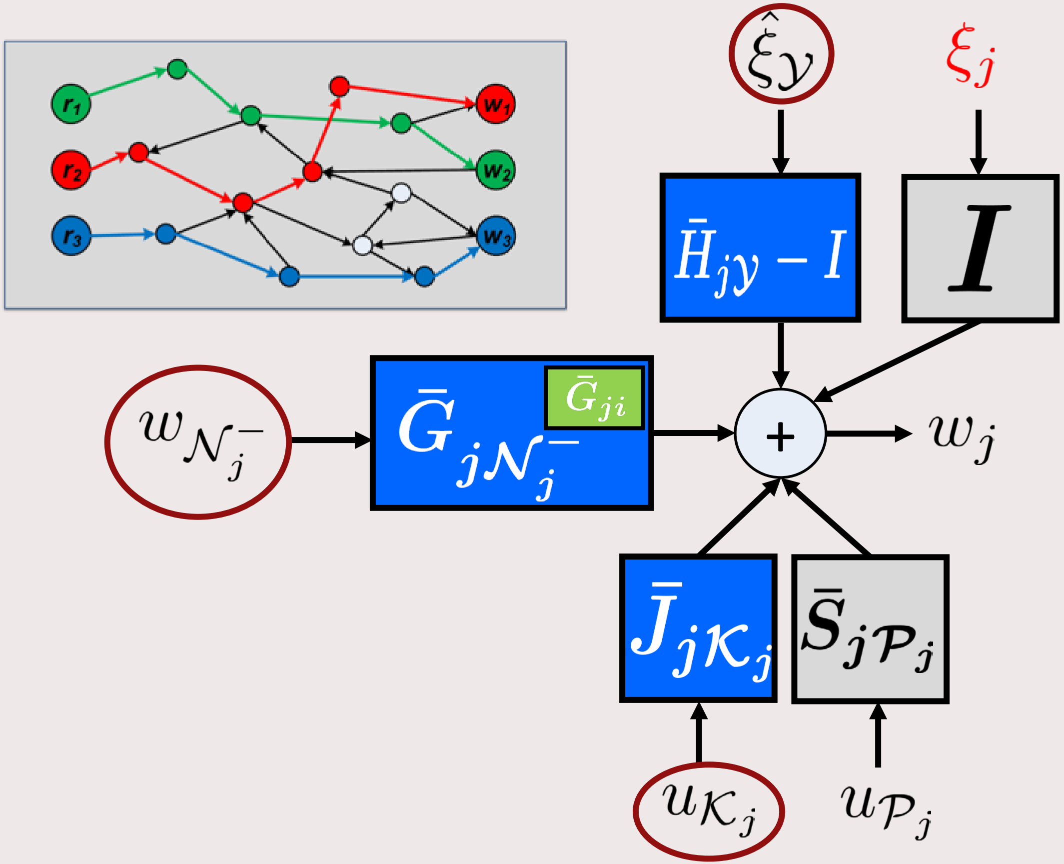
$$\Phi_\kappa \succ 0$$

$$\kappa = \begin{bmatrix} w_{\mathcal{N}_j^-} \\ \xi_{\mathcal{Y}} \\ u_{\mathcal{K}_j} \end{bmatrix}$$



[1] van den Hof et al. (2020, 2023), Hendrickx, et al. (2019)

Local identification



Data informativity [1]

$$\Phi_\kappa \succ 0$$

κ is persistently exciting holds generically if there are $dim(\kappa)$ vertex disjoint paths from

$$\begin{bmatrix} \xi_{\mathcal{S}} \\ u_{\mathcal{L}} \end{bmatrix} \rightarrow \begin{bmatrix} w_{\mathcal{N}_j^-} \\ \xi_{\mathcal{Y}} \\ u_{\mathcal{K}_j} \end{bmatrix}$$

[1] van den Hof et al. (2020, 2023), Hendrickx, et al. (2019)

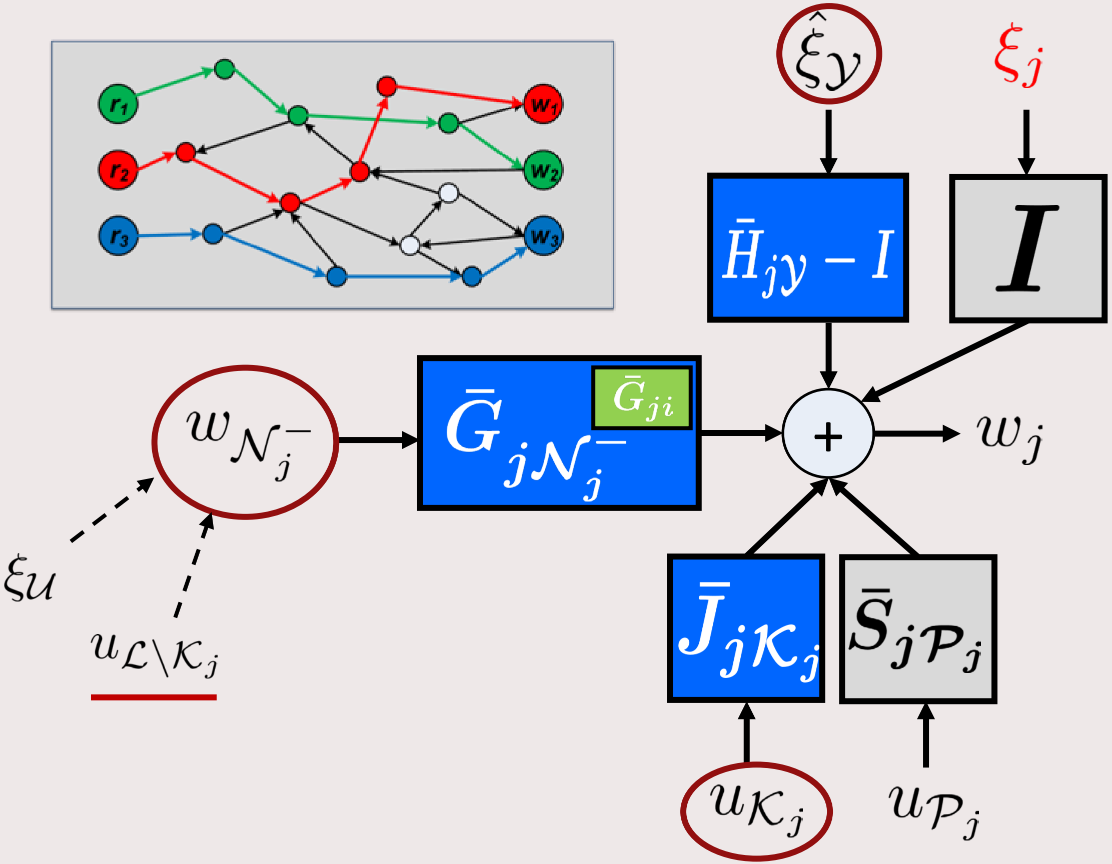
Local identification

Data informativity [1]

$$\Phi_\kappa \succ 0$$

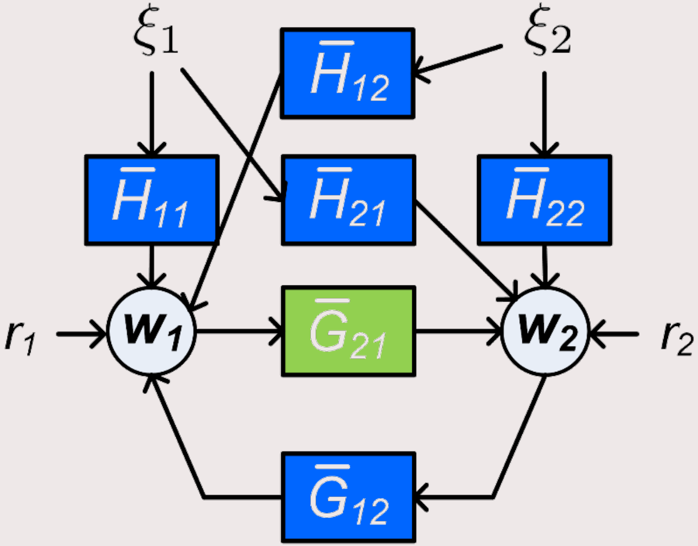
$w_{\mathcal{N}_j^-}$ is persistently exciting holds generically if there are $dim(w_{\mathcal{N}_j^-})$ vertex disjoint paths from

$$\begin{bmatrix} \xi_{\mathcal{U}} \\ u_{\mathcal{L}\setminus\mathcal{K}_j} \end{bmatrix} \rightarrow w_{\mathcal{N}_j^-}$$



 [1] van den Hof et al. (2020, 2023), Hendrickx, et al. (2019)

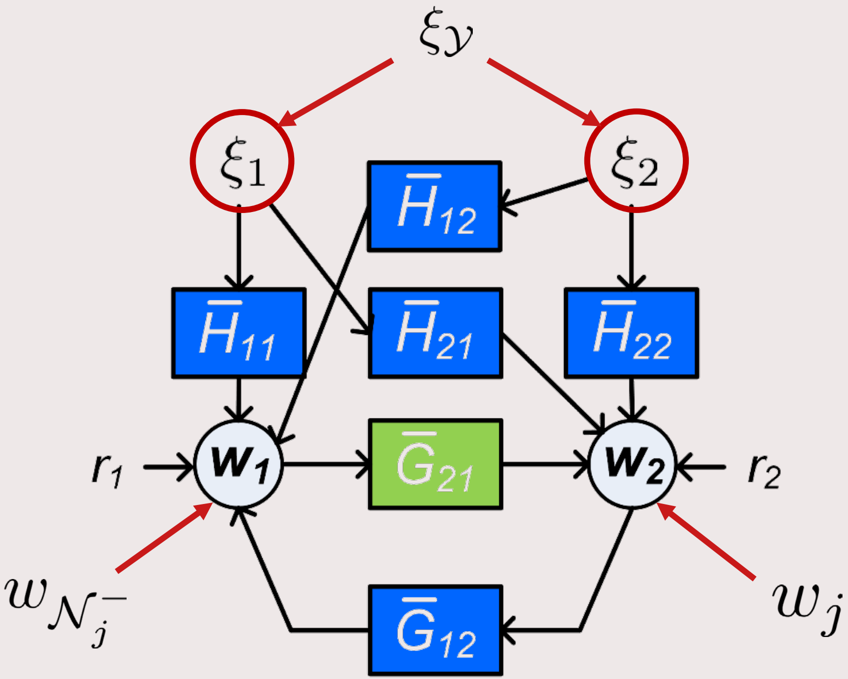
Local identification



$$\Phi_\kappa \succ 0$$

$$\begin{bmatrix} \xi_{\mathcal{U}} \\ u_{\mathcal{L}\setminus\mathcal{K}_j} \end{bmatrix} \rightarrow w_{\mathcal{N}_j^-}$$

Local identification



$$\xi_{\mathcal{U}} = \emptyset$$

$$u_{\mathcal{L}\setminus\mathcal{K}_j} = r_1, r_2 \qquad \notin \mathcal{K}_j = \emptyset$$

$$\Phi_\kappa \succ 0$$

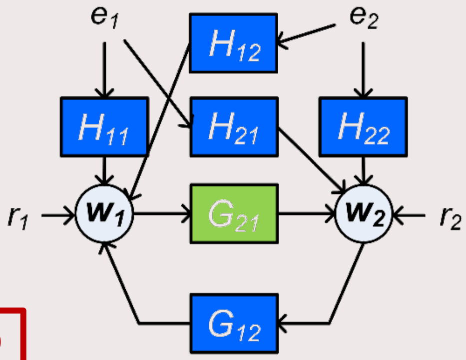
$$\left.\begin{bmatrix} \xi_{\mathcal{U}} \\ u_{\mathcal{L}\setminus\mathcal{K}_j} \end{bmatrix} \to w_{\mathcal{N}_j^-} \right\} \quad \begin{bmatrix} \emptyset \\ r_1, r_2 \end{bmatrix} \to w_1$$

w_1 is persistently exciting holds generically if there is 1 vertex disjoint paths from r_1 or $r_2 \to w_1$

Local identification

Single module identifiability

- Method independent
- Identifiability: $1\ r$-signal



Method	Transfer		# r-signals needed
Local Direct	$w \rightarrow w$	Consistency & ML	2
Indirect	$r \rightarrow w$	Consistency	1
Multi-step Least squares	1. Indirect 2. Direct	Consistency & ML?	1

MIMO

MISO

Conclusion

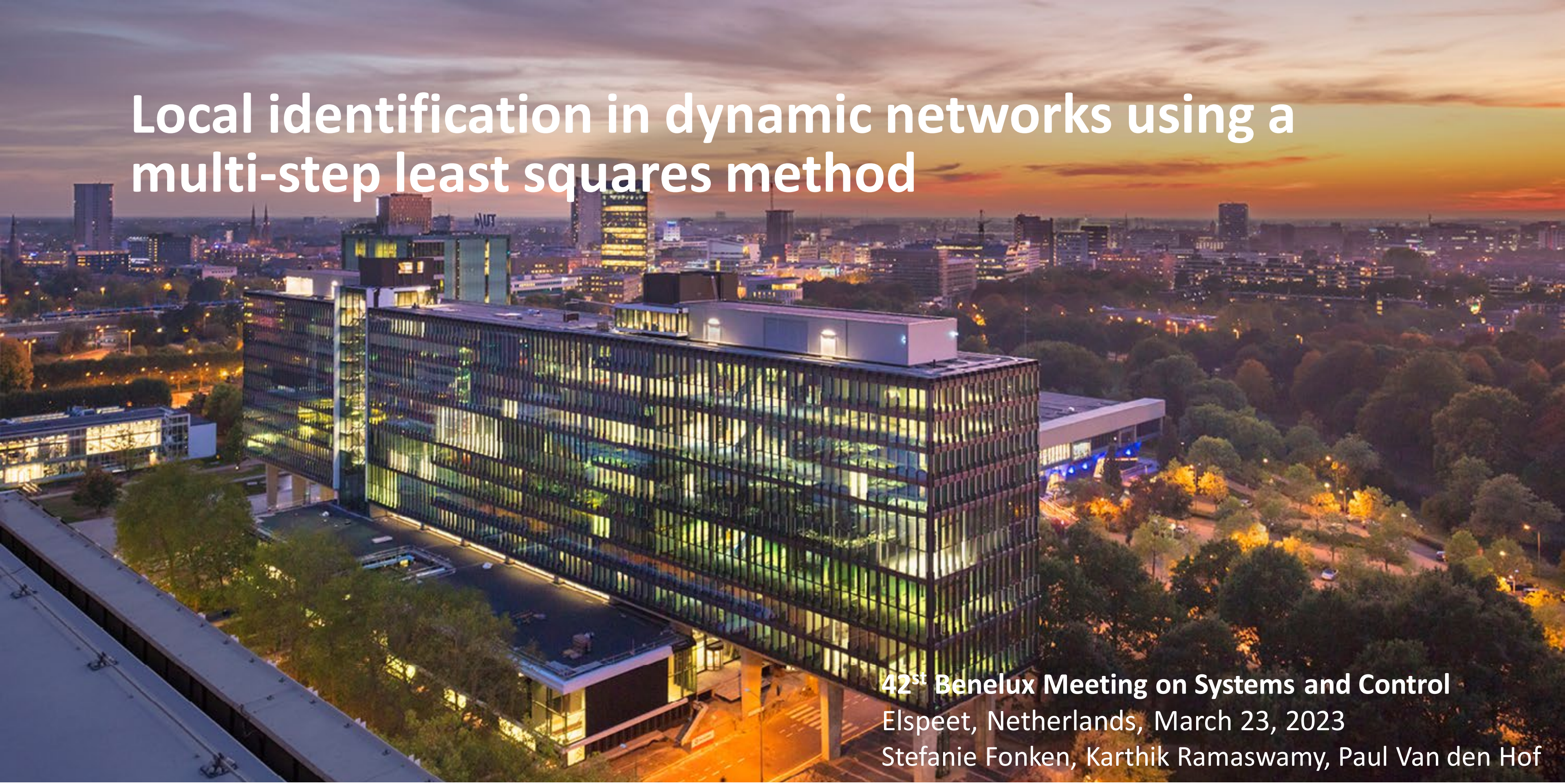
Combining indirect and direct methods:

- Requires less r-signals than current direct method

- Keeps advantages of the current direct methods

&

- Parametrize with Weighted Null Space Fitting → Convex



Local identification in dynamic networks using a multi-step least squares method

42st Benelux Meeting on Systems and Control
Elspeet, Netherlands, March 23, 2023
Stefanie Fonken, Karthik Ramaswamy, Paul Van den Hof