IV methods for closed-loop system identification

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I. Closed-loop system identification

Solutions

- **Direct approaches: (y, u)**
  - direct identification of the process from the measured data $u$ and $y$

- **Indirect approaches: (y, u, r, C)**
  - identification of the CL transfer
  - deduction of the process model using the knowledge of $C$

- **Joint input/output approaches: (y, u, r)**
  - joint identification of the different transfers ($y/r$ and $u/r$)
  - deduction of the process model (two-stage method, coprime method e.g.)

Problem: correlation between $u$ and the noise
I. Closed-loop system identification

Goals of the study

- To find methods to **consistently identify** plant models of CL systems
- To use simple **linear regression** algorithms

Solutions

- To propose IV methods dedicated to the CL identification problem
- To answer the question: how to achieve the **smallest variance** of the estimate (and the consequence for the optimal choice of the design parameters: weights, filters, and instruments)?
I. Closed-loop system identification

II. Extended - IV methods

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IV. Approximate implementations of the optimal IV method

V. Example
II. Extended IV methods

System to be identified

$$S \begin{cases} y(t) = G_0(q)u(t) + H_0(q)e_0(t) \\ u(t) = r(t) - C(q)y(t) \end{cases}$$

with $G: G_0(q) = \frac{B_0(q)}{A_0(q)}$ of order $n_0$

controller $C$ of order $m$, $e_0(t)$ white noise with variance $\{\lambda_0\}$

ARX model set

$$M : y(t) = \psi^T(t) \theta + \varepsilon(t)$$

Extended IV (T. Söderström, P. Stoica)

$$\hat{\theta}_{xiv} = \arg \min_\theta \left[ \frac{1}{N} \sum_{t=1}^{N} z(t)L(q^{-1}) \left[ y(t) - \psi^T(t) \theta \right] \right]^2_Q$$

- $z(t) = \text{instruments such as } \begin{cases} \mathbf{E} \ z(t) \ \psi^T(t) \ \text{non singular} \\ 0 = \mathbf{E} \ z(t) \ e_0(t) \end{cases}$
- $L(q^{-1}) = \text{stable pre-filter}$
- $\| x \|^2_Q = x^T Q x$ with $Q$ a positive definite weighting matrix
II. Extended IV methods

Proposition: a closed-loop dedicated solution (clxiv)

- **Notations**
  - delayed extra-signal: \( \varphi_r(t) = [r(t - 1) \ldots r(t - n - m)]^T \)
  - OL regressor: \( \psi(t) = [y(t - 1) \ldots y(t - n) \ u(t - 1) \ldots u(t - n)]^T \)
  - CL regressor: \( \varphi(t) = [y(t - 1) \ldots y(t - n - m) \ r(t - 1) \ldots r(t - n - m)]^T \)

- **Tailor-made IV method** (M. Gilson, P. Van den Hof 2001) or BELS method
  - Goal: to estimate and to eliminate the bias induced by a LS estimation

- **Equivalent to an extended IV method with design parameters:**
  - instruments: \( z(t) = \varphi_r(t) \)
  - pre-filter: \( L(q^{-1}) = \) controller denominator
  - weighting matrix: \( Q = \left( \hat{R}_{\varphi_r \varphi} \hat{R}_{\varphi_r \varphi}^T \right)^{-1} \) with \( \hat{R}_{\varphi_r \varphi} = \frac{1}{N} \sum_{t=1}^{N} \varphi_r(t) \varphi(t) \)
  - regressor: \( \psi(\tau) \)
II. Extended IV methods

Proposition : a closed-loop dedicated solution (clxiv)

- **Covariance property \((G_0 \in G)\)**
  
  - \(\hat{\theta}\) asymptotically Gaussian distributed
    
    \[
    \sqrt{N}(\hat{\theta} - \theta) / \lambda \xrightarrow{\text{dist}} N(0, P_{xiv})
    \]

  - with
    
    \[
    P_{xiv} = \lambda_0 \left( R_{\psi}^T Q R_{\psi} \right)^{-1} R_{\psi}^T Q R_{zTzT} Q R_{\psi}^T \left( R_{\psi}^T Q R_{\psi} \right)^T
    \]

  - where

    - \(R_{\psi} = \overline{E}_{\psi}(t)L(q^{-1})\psi^T(t)\)
    
    - \(z_T(t) = \sum_{i=0}^{\infty} t_i \varphi(t - i)\)
    
    - \(T(q^{-1}) = L(q^{-1}) A_0(q^{-1}) H_0(q^{-1}) = \sum_{i=0}^{\infty} t_i q^{-i} \) (monic filter)
II. Extended IV methods

XIV = asymptotically unbiased results

Problem of the design parameter selection \((z(t), n_z, Q, L(q^{-1}))\)

considerable effects on the estimates and on \(P_{xiv}\)

Question: how to achieve the lowest value of \(P_{xiv}\)?
I. Closed-loop system identification

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III. Optimal CL IV method

- Lower bound of $P_{xiv}$ for any unbiased identification method (L. Ljung):
  Cramer-Rao bound

\[ P_{xiv}^{\text{opt}} = \lambda_0 \left[ \mathbb{E} \psi(t) \psi^T(t) \right]^{-1} \]

with

\[ \psi^T(t) = \left[ \frac{d}{dt} \hat{y}(t \mid \theta) \right]^T \bigg|_{\theta = \theta_0} = \left[ A_0(q^{-1})H_0(q^{-1}) \right]^{-1} \tilde{\psi}(t) \]

- Deduction: optimal IV estimator

\[ z(t) = \lambda_0^{-1} \left[ \left( A_0(q^{-1})H_0(q^{-1}) \right)^{-1} \tilde{\psi}(t) \right]^T \]

\[ n_z = 2n \]

\[ Q = I \]

\[ L(q^{-1}) = \left[ A_0(q^{-1})H_0(q^{-1}) \right]^{-1} \]

- The true noise model has to be known
- Optimal accuracy cannot be achieved in practice!
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IV. Approximate implementation

To achieve optimal IV method, we need:

- A noise model to construct \( L(q^{-1}) \) and \( z(t) \)
- A first model of \( G_0(q) \) to compute \( \hat{\psi}(t) \)

Proposed solution:

- To estimate first **consistent** (and not necessarily efficient) models of \( G_0(q) \) and \( H_0(q) \)
- Only **second-order statistical property** effects in the resulting accuracy

3 bootstrap techniques
IV. Approximate implementation

- Preliminary remark:

⚠️ Major difference between open-loop and closed-loop cases

- In closed-loop, **output AND input** are correlated with the noise

→ **Consequences on the construction of the instrument** $z(t)$

- uncorrelated to the noise part of $u(t)$
- correlated to the noise-free part of $u(t)$
IV. Approximate implementation

1st solution: extension of the IV4 method (L. Ljung) to the closed-loop case (CLIV4)

- **Step 1:** LS estimation of $G_0 \Rightarrow \hat{G}_1(q)$
- **Step 2:**
  - Generation of the instrument $z_1(t) = \text{estimation of } \tilde{y}(t)$
  - $z_1(t)$ composed of $\tilde{y}_1(t) = \frac{C(q) \hat{G}_1(q)}{1 + C(q) \hat{G}_1(q)} r(t)$ and $\tilde{u}_l(t) = \frac{1}{1 + C(q) \hat{G}_1(q)} r(t)$
  - IV estimation of $G_0 \Rightarrow \hat{G}_2(q)$
- **Step 3:** noise model estimation, AR structure $\Rightarrow \hat{L}(q^{-1})$
- **Step 4:**
  - generation of the instrument $z_2(t)$ on the basis of $C(q), \hat{G}_2(q)$ and $r(t)$
  - IV estimation using $z_2(t), \hat{L}(q^{-1})$

2nd solution: improvement of the 1st solution by using a more sophisticated noise modeling procedure (armacel) - (CLIV4-armacel)
IV. Approximate implementation

3rd solution: (clivh)

- **Step 1:** high order LS estimate of $G_0 \Rightarrow \hat{G}_1(q)$ and $\hat{H}_1(q)$

- **Step 2:**
  - **Generation of the instrument** $z(t)$ on the basis of $\hat{G}_1(q)$, $C(q)$ and $r(t)$
    - $z(t) = $ estimation of $\hat{\psi}(t)$
  - **Computation of the pre-filter**
    \[
    \hat{L}(q^{-1}) = \hat{A}_1(q^{-1}) \hat{H}_1(q) = 1 \quad (\text{arx model})
    \]

- **Step 3:** IV estimation using $z(t)$ and $\hat{L}(q^{-1})$
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V. Example

Process to be identified

\[
\begin{align*}
S & \quad \begin{cases} 
y(t) = G_0(q)u(t) + H_0(q)e_0(t) \\
u(t) = r(t) - C(q)y(t)
\end{cases}
\end{align*}
\]

with

\[
G_0(q) = \frac{0.5 q^{-1}}{1 - 0.8 q^{-1}} \quad (n = 1), \quad C(q) = \frac{0.0012 + 0.0002 q^{-1} - 0.001 q^{-2}}{0.5 - 0.9656 q^{-1} + 0.4656 q^{-2}} \quad (m = 2)
\]

\[
H_0(q) = \frac{1 - 1.5 q^{-1} + 1.045 q^{-2} - 0.3338 q^{-3}}{1 - 2.35 q^{-1} + 2.09 q^{-2} - 0.6675 q^{-3}}
\]

Characteristics

- \( r(t) = \) deterministic sequence (realization of a random binary signal)
- \( e_0(t) = \) white noise
- Monte Carlo simulation with \( \text{snr}=15 \, \text{dB} \)
- Application of 5 methods
V. Example

Methods

- **clxiv**: closed-loop extended method (= tailor-made IV method)
- **cliv4**: extension of the IV4 method to the closed-loop case
- **cliv4_arma**: improvement of the cliv4 method
- **clivh**: bootstrap method using a high LS technique in the first step
- **cliv**: method proposed by Söderström, Stoica, Trullson (1987):
  - \( z(t) = [r(t) \ r(t - 1) \ \cdots \ r(t - 2n)]^T \) (extra-signal)
  - \( \psi(t) = \text{regressor} \)
- **Benchmark**: generation of \( L(q) \) and \( z(t) \) using the true noise and process models (for illustration purpose)
V. Example

Results: Bode diagrams of the models identified by the six methods

- clxiv
- cliv4
- cliv4_arma
- clivh
- cliv
- benchmark

Unbiased results
Bootstrap and cliv methods better than clxiv (or tailor-made iv)
V. Example

Results: representation of the distance between the real and the estimated TF processes

\[ g(\omega) = \frac{1}{MC} \sum_{k=1}^{MC} \left| G_0(e^{i\omega}) - \hat{G}_k(e^{i\omega}) \right| \]

- best results obtained with cliv4_arma method
- the use of a more sophisticated noise model improves the result
- clivh method more appropriate than cliv4
Conclusion

- Presentation of several IV methods
- Proposition of an extended-IV method dedicated to the closed-loop case
- Formulation of the optimal closed-loop IV estimator
- Discussion on the choice of the design parameters to approximate the optimal closed-loop IV estimator

Tailor-made IV method = unbiased

but

To obtain attractive variance properties

apply bootstrap methods
IV methods for closed-loop system identification

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