

Data-driven model learning in linear dynamic networks

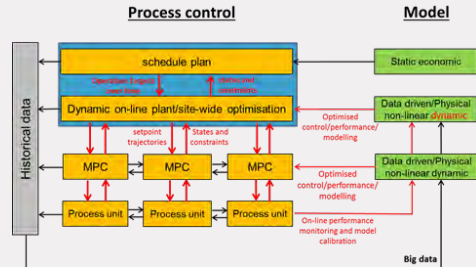
Paul Van den Hof

ICMS Symposium, “Getting a grip on complex systems”
4 April 2022, Eindhoven

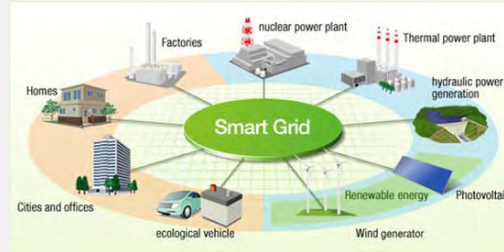
www.sysdynet.eu
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Introduction – dynamic networks

Decentralized process control



Smart power grid



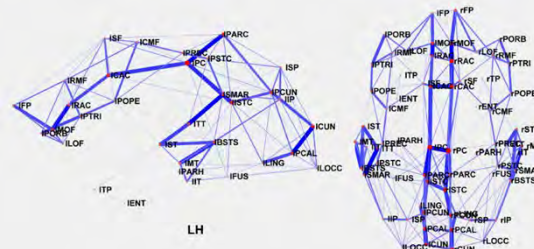
Pierre et al. (2012)



Complex machines

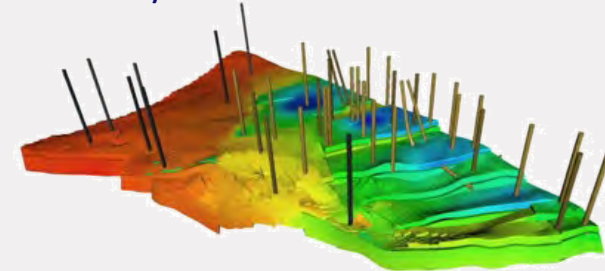


Brain network



P. Hagmann et al. (2008)

Hydrocarbon reservoirs



Mansoori (2014)

Introduction

Overall trend:

- (Large-scale) interconnected dynamic systems
- Distributed / multi-agent type monitoring, control and optimization problems
- Data is “everywhere”, big data era, AI/machine learning tools
- Model-based operations require accurate/relevant models
- → **Learning models from data** (including physical insights when available)

Introduction

Drivers for data-processing / data-analytics:

Providing the tools for **online**

- Model estimation / calibration / adaptation

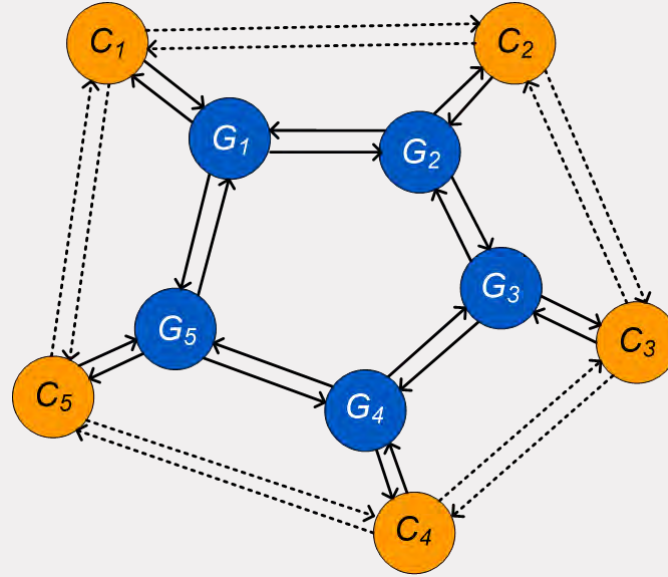
to accurately perform online model-based \mathbf{X} :

- Monitoring
- Diagnosis and fault detection
- Control and optimization
- Predictive maintenance
-



Introduction

Distributed / multi-agent control:

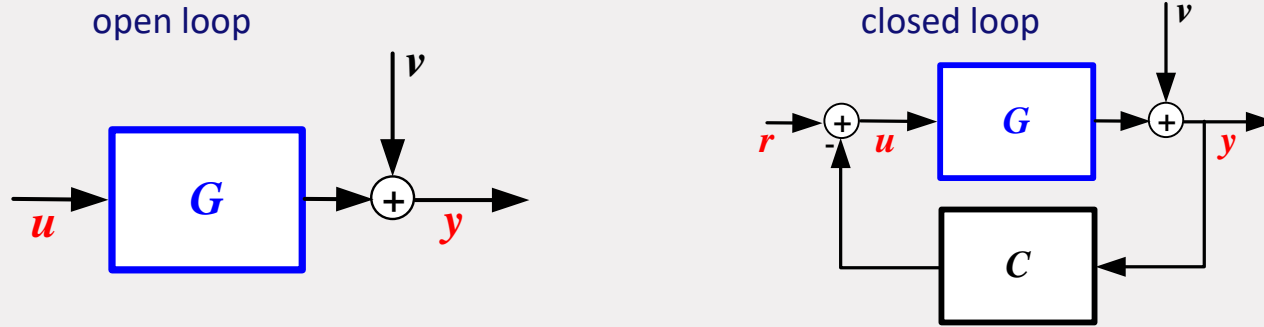


With both physical and communication links between systems G_i and controllers C_i

How to address data-driven modelling problems in such a setting?

Introduction

The classical (multivariable) data-driven modeling problems^[1]:



Identify a model of G on the basis of measured signals u, y (and possibly r), focusing on *continuous LTI dynamics*.

In interconnected systems (networks) the **structure / topology** becomes important to include

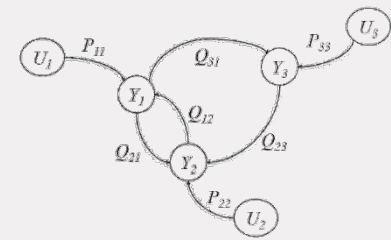
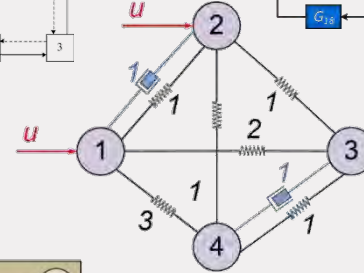
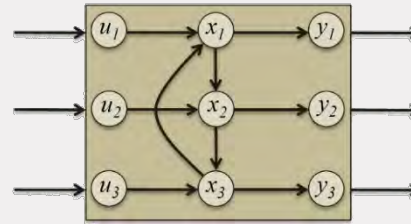
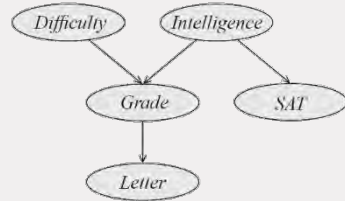
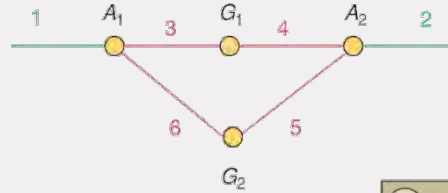
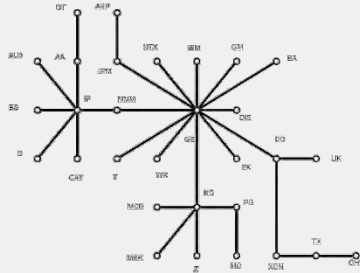
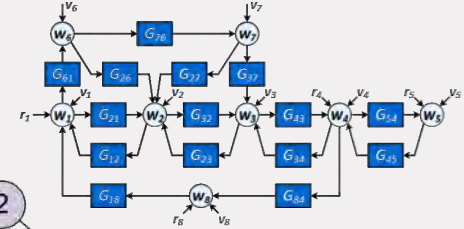
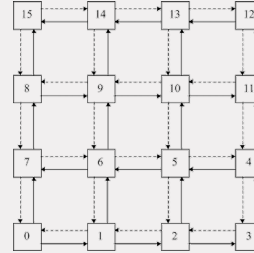
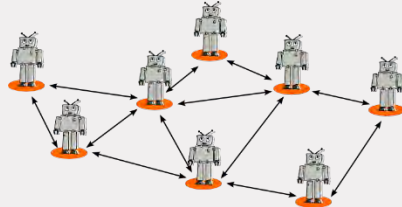
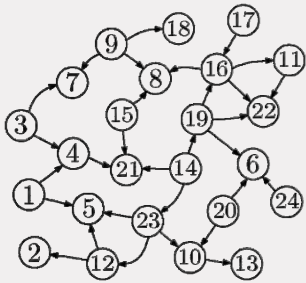
^[1] Ljung (1999), Söderström and Stoica (1989), Pintelon and Schoukens (2012)

Contents

- Introduction and motivation
- **How to model a dynamic network?**
- Single module identification
- Global network identification
- Diffusively coupled networks
- Extensions - Discussion

Dynamic networks for data-driven modeling

Network models



D. Materassi and M.V. Salapaka (2012)

R.N. Mantegna (1999)

www.momo.cs.okayama-u.ac.jp

J.C. Willems (2007)

D. Koller and N. Friedman (2009)

E.A. Carara and F.G. Moraes (2008)

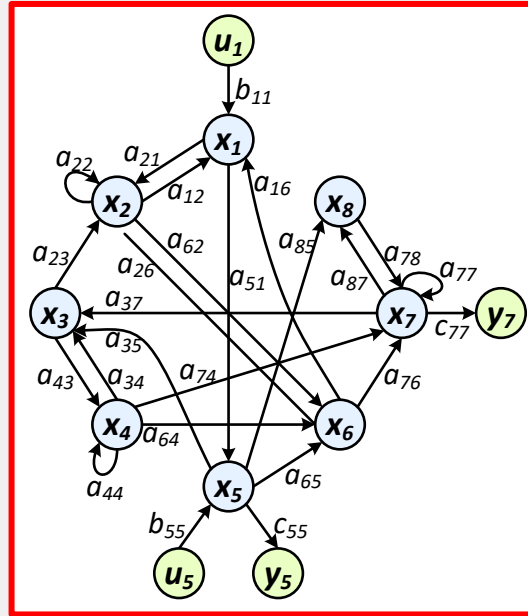
P.E. Paré et al (2013)

P.M.J. Van den Hof et al (2013)

X.Cheng (2019)

E. Yeung et al (2010)

Network models

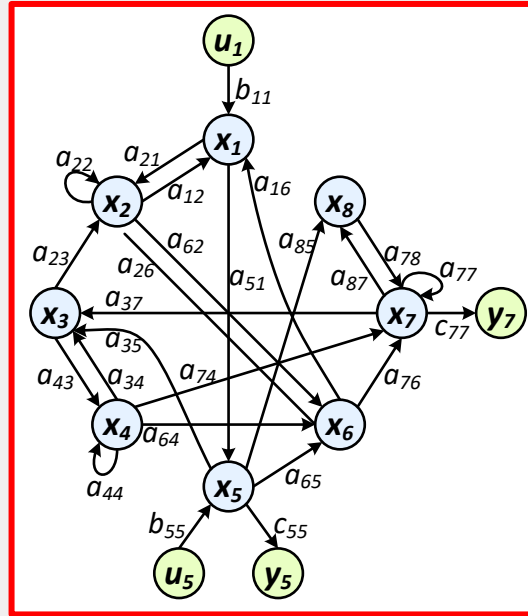


State space representation

$$\begin{aligned}x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k)\end{aligned}$$

- States as **nodes** in a (directed graph)
- State transitions (1 step in time) reflected by a_{ij}
- Transitions are encoded in **links**
- Effect of transitions are summed in the nodes
- Self loops are allowed
- Actuation (u) and sensing (y) reflected by separate links

Network models



State space representation

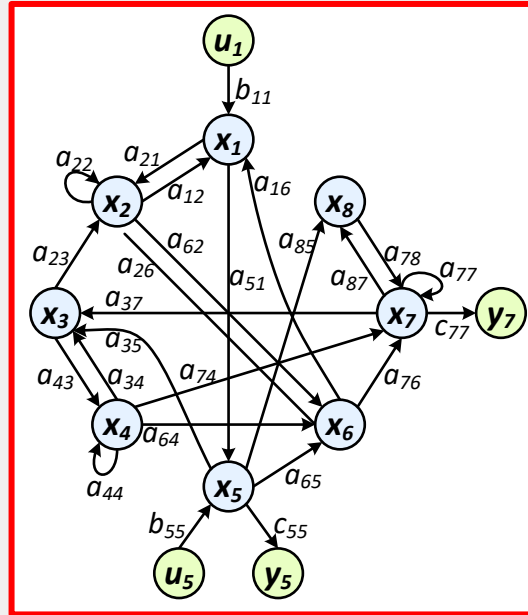
$$\begin{aligned}x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k)\end{aligned}$$

- Ultimate break-down of structure in the system
- to smallest possible level of detail

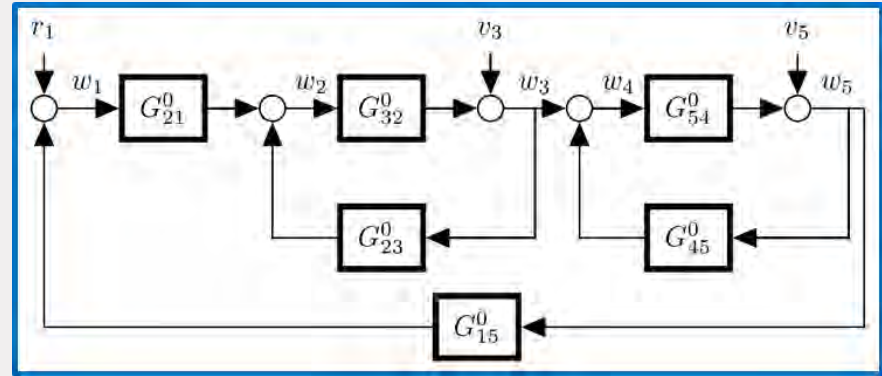
For data-driven modeling problems:

- Stronger role for measurable inputs and outputs
- i/o dynamics can be lumped in dynamic **modules**

Network models



State space representation [1]

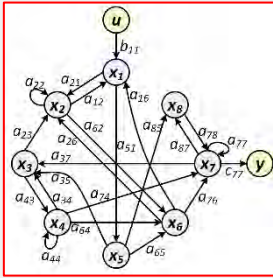


Module representation [2]

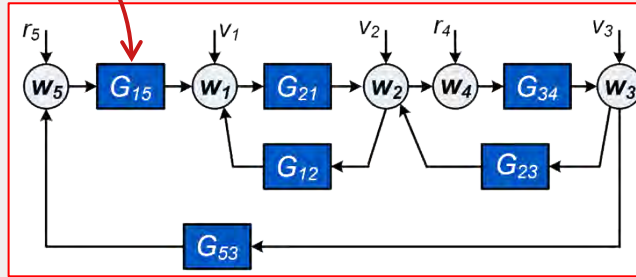
[1] Goncalves, Warnick, Sandberg, Yeung, Yuan, Scherpen,...

[2] VdH, Dankers, Goncalves, Warnick, Gevers, Bazanella, Hendrickx, Materassi, Weerts,...

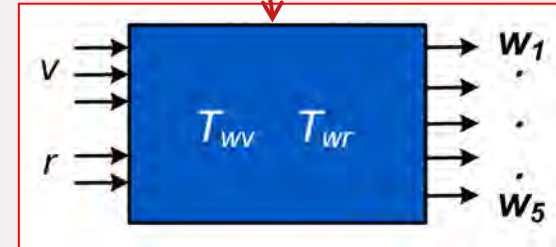
Dynamic network models - zooming



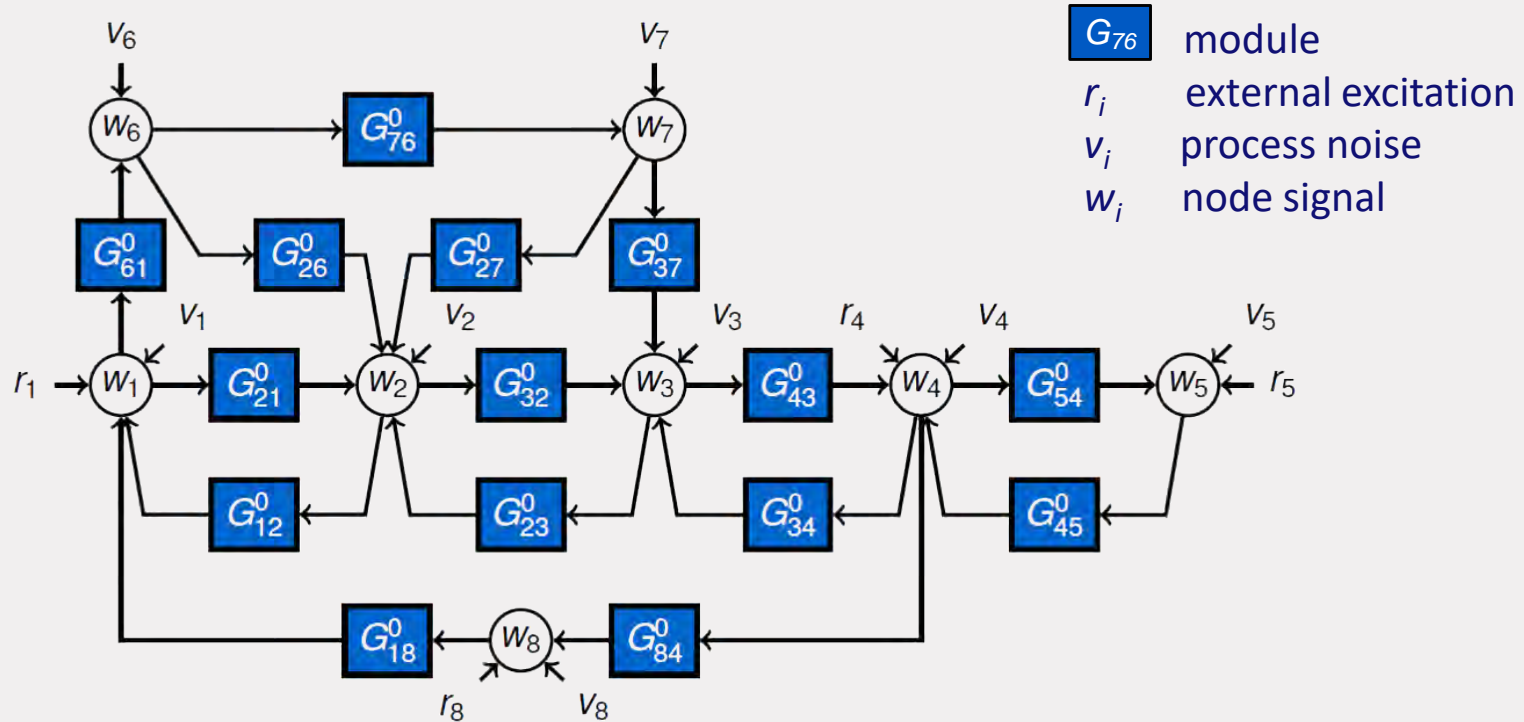
Increasing level of detail



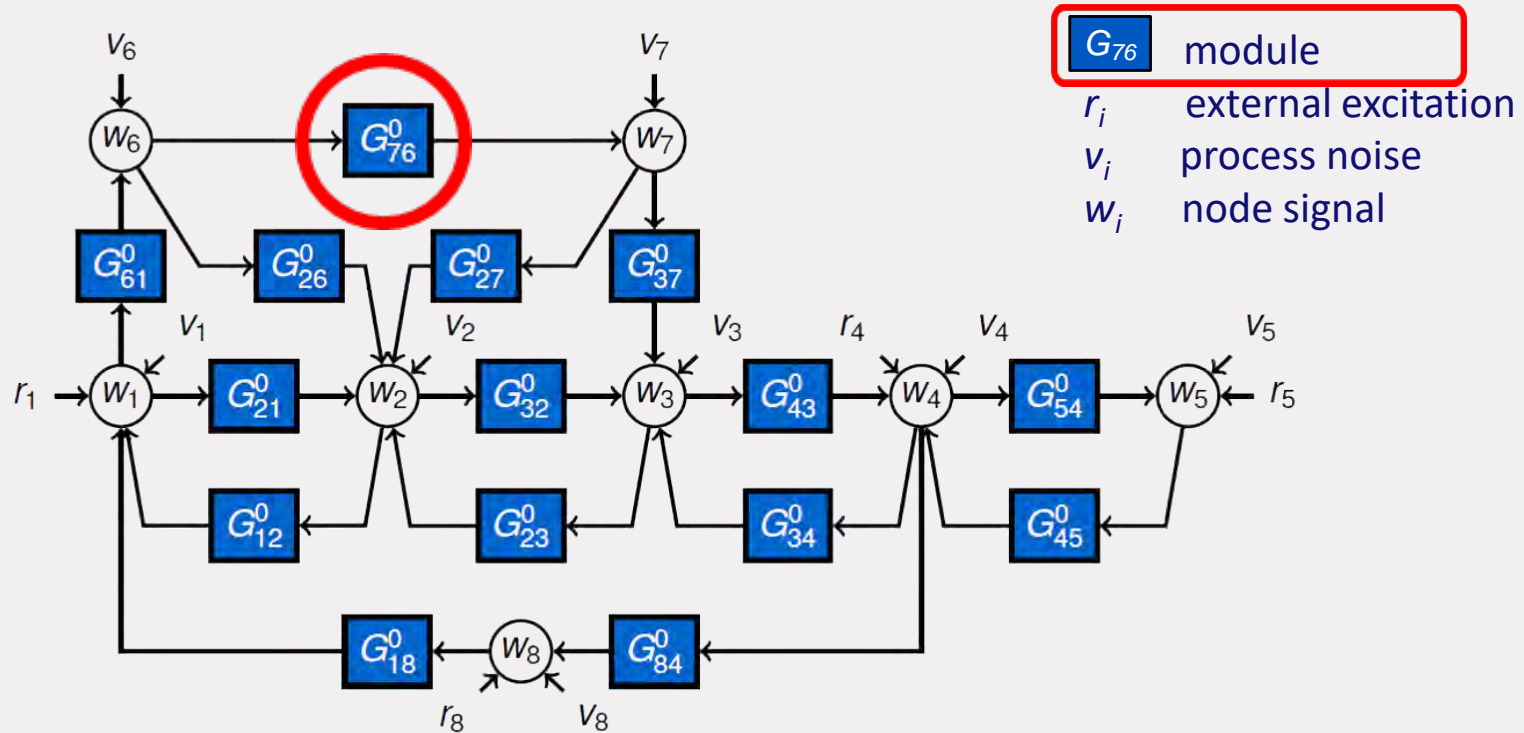
Decreasing structural information



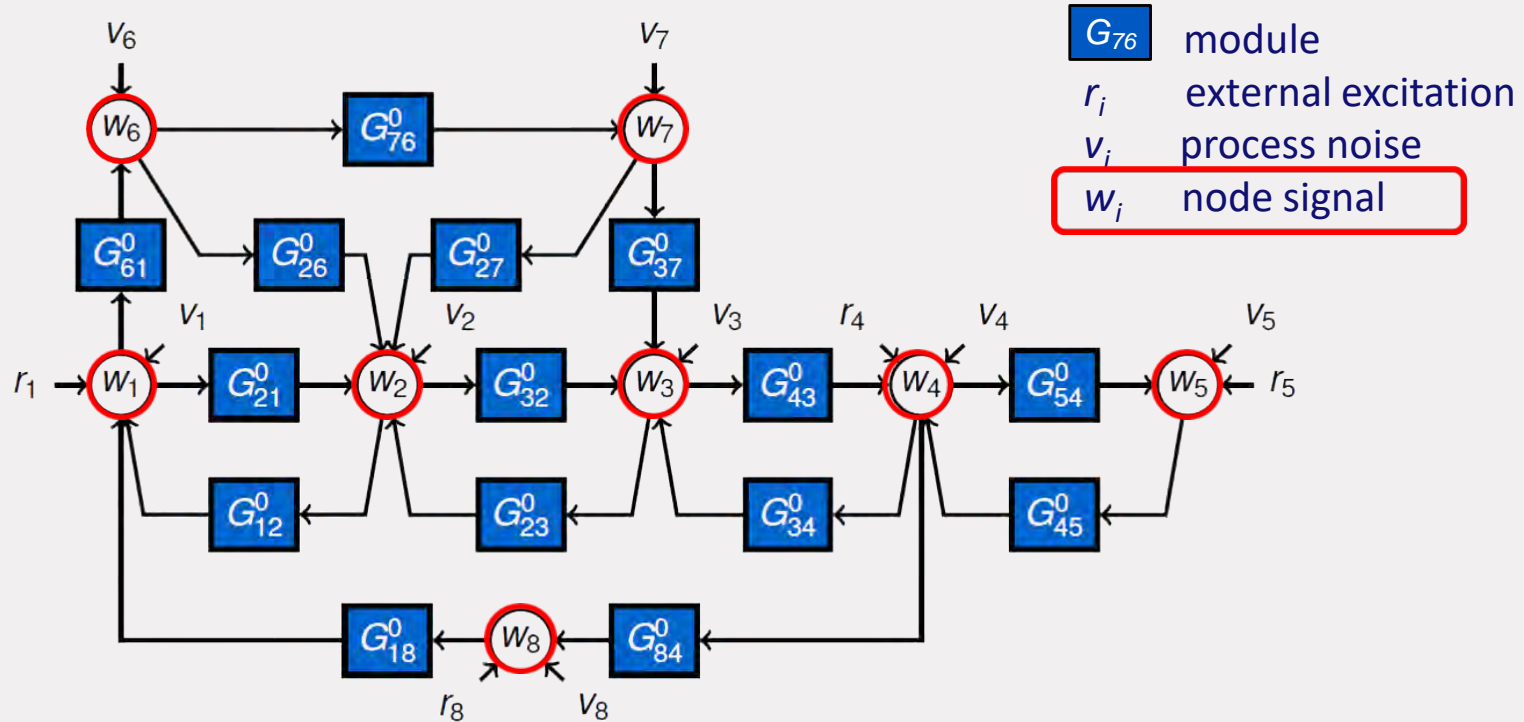
Dynamic network setup



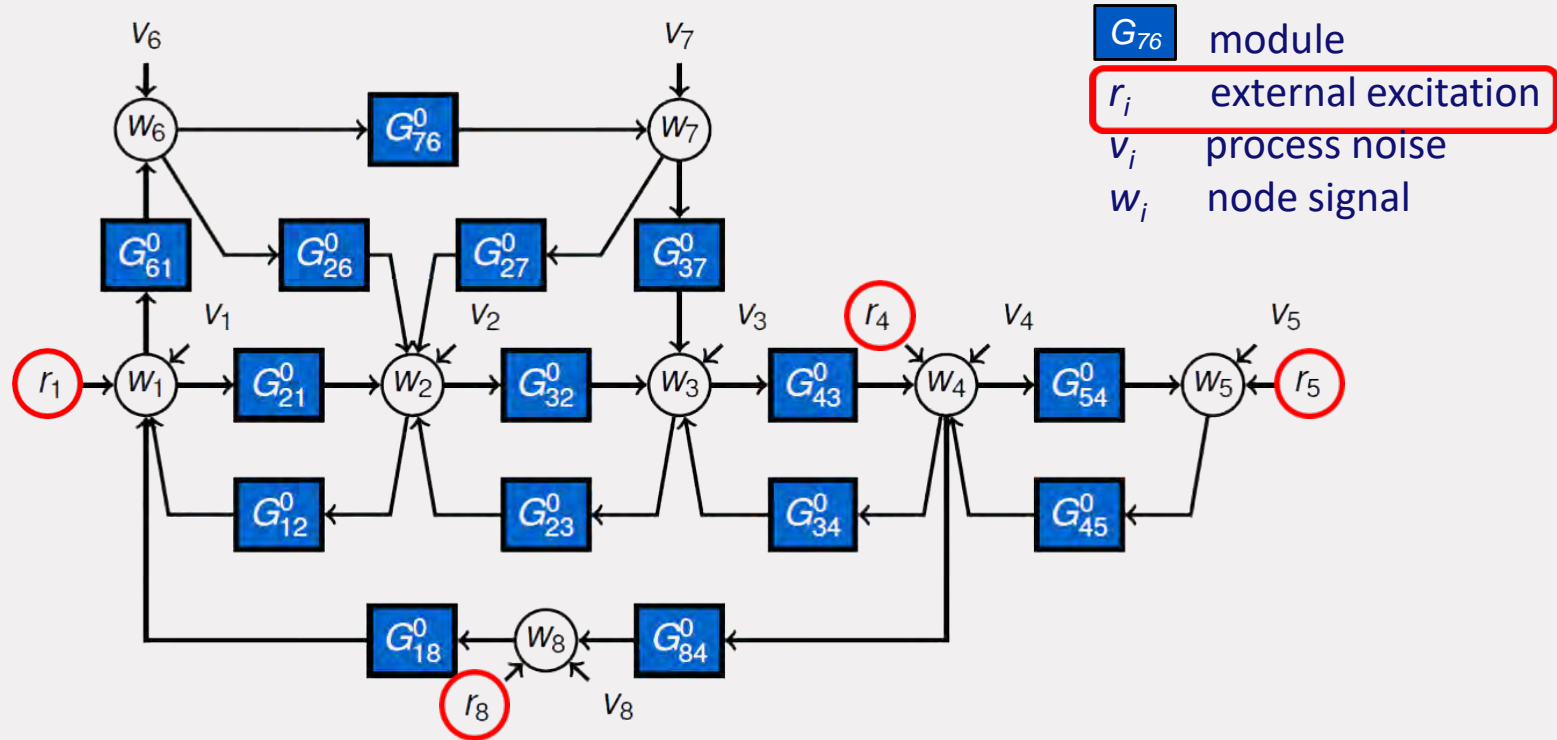
Dynamic network setup



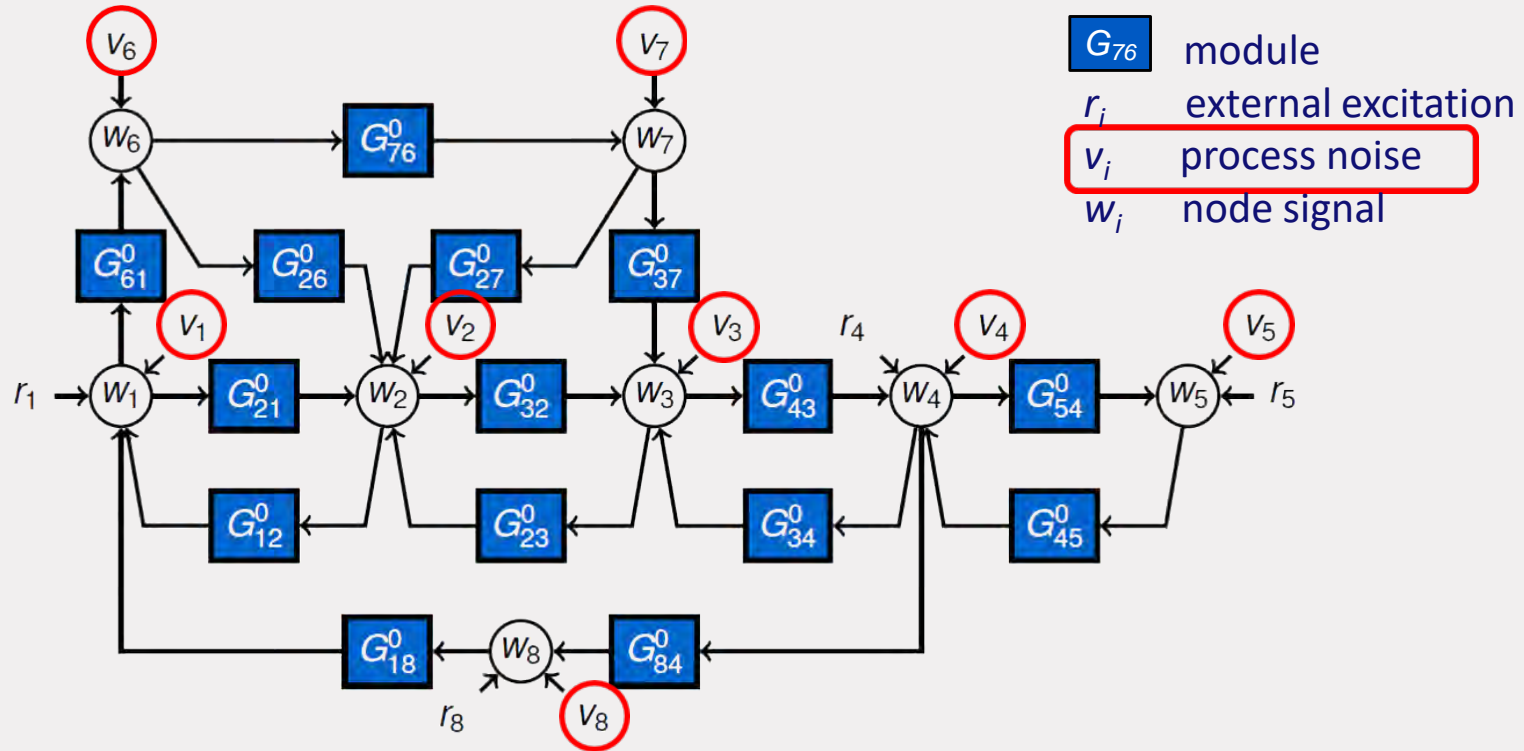
Dynamic network setup



Dynamic network setup



Dynamic network setup



Dynamic network setup

Basic building block:

$$w_j(t) = \sum_{k \in \mathcal{N}_j} G_{jk}^0(q) w_k(t) + r_j(t) + v_j(t)$$

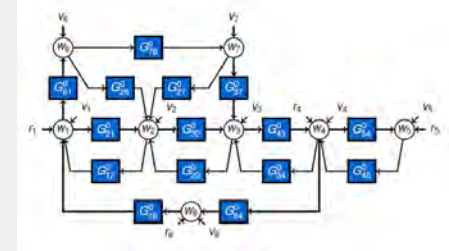
w_j : node signal

r_j : external excitation signal

v_j : (unmeasured) disturbance, stationary stochastic process

G_{jk}^0 : module, rational proper transfer function, $\mathcal{N}_j \subset \{\mathbb{Z} \cap [1, L] \setminus \{j\}\}$

q : shift operator, $q^{-1}w(t) = w(t-1)$



Node signals: w_1, \dots, w_L

Interconnection structure / topology of the network is encoded in \mathcal{N}_j , $j = 1, \dots, L$

Dynamic network setup

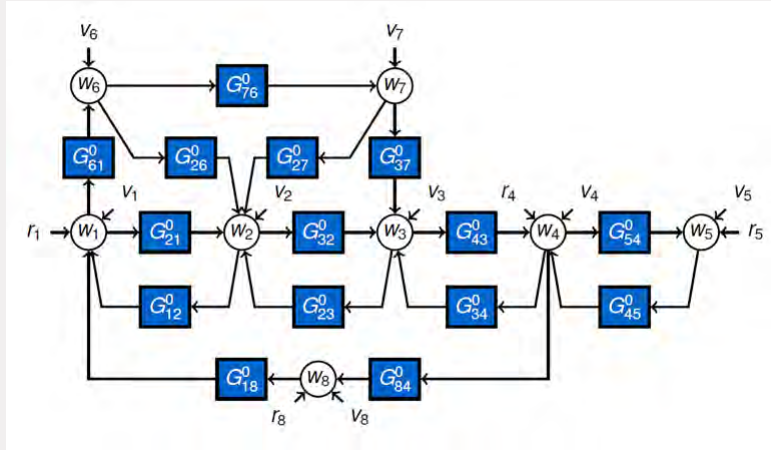
Collecting all equations:

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & G_{12}^0 & \cdots & G_{1L}^0 \\ G_{21}^0 & 0 & \cdots & G_{2L}^0 \\ \vdots & \cdots & \ddots & \vdots \\ G_{L1}^0 & G_{L2}^0 & \cdots & 0 \end{bmatrix}}_{\text{Network matrix } G^0(q)} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + R^0 \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_K \end{bmatrix} + H^0 \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_p \end{bmatrix}$$

$$w(t) = G^0(q)w(t) + R^0(q)r(t) + v(t); \quad v(t) = H^0(q)e(t); \quad \text{cov}(e) = \Lambda$$

- Typically R^0 is just a (static) selection matrix, indicating which nodes have an excitation signal.
- The topology of the network is encoded in the structure (non-zero entries) of G^0 .
- r and e are called **external signals**.

Dynamic network setup

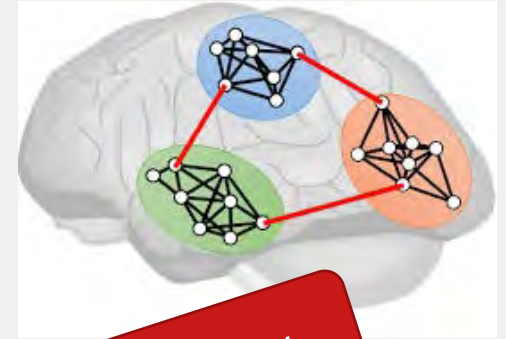
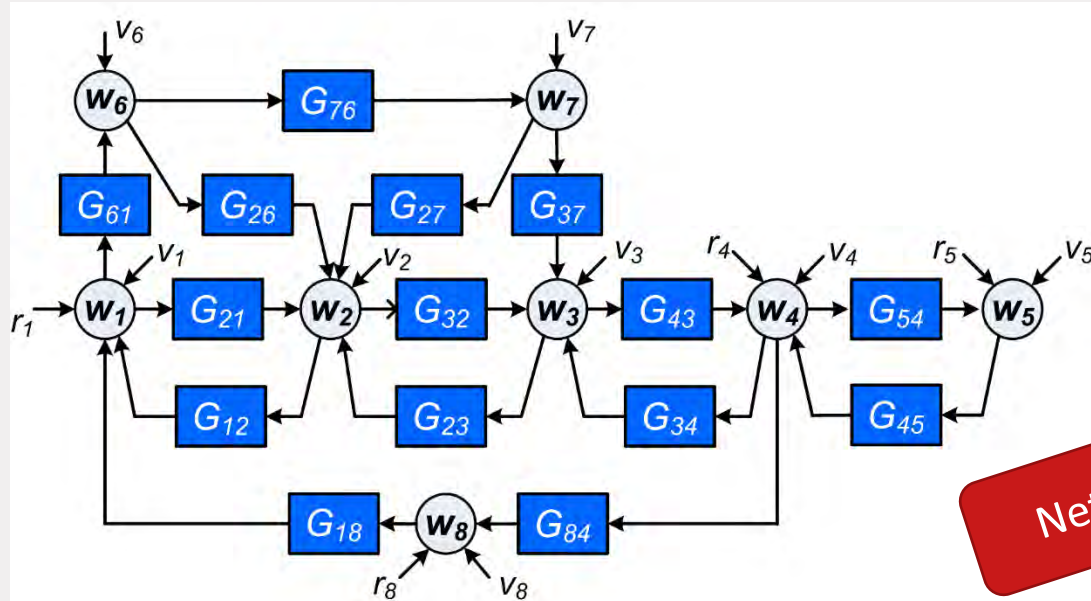


Many challenging data-driven modeling questions can be formulated

Measured time series:

$$\{w_i(t)\}_{i=1,\dots,L}; \quad \{r_j(t)\}_{j=1,\dots,K}$$

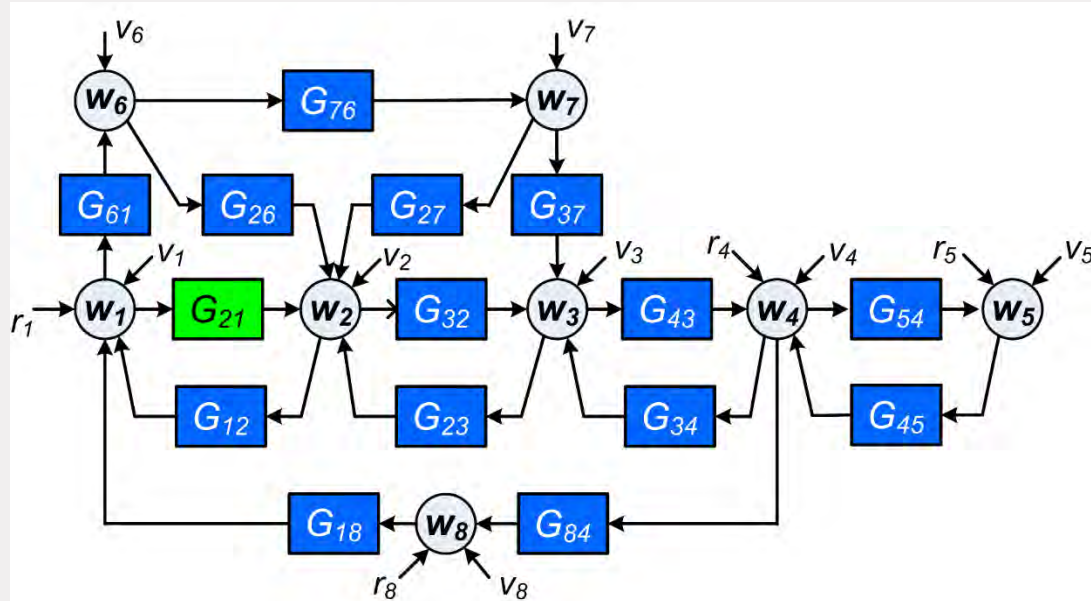
Model learning problems



Network identifiability

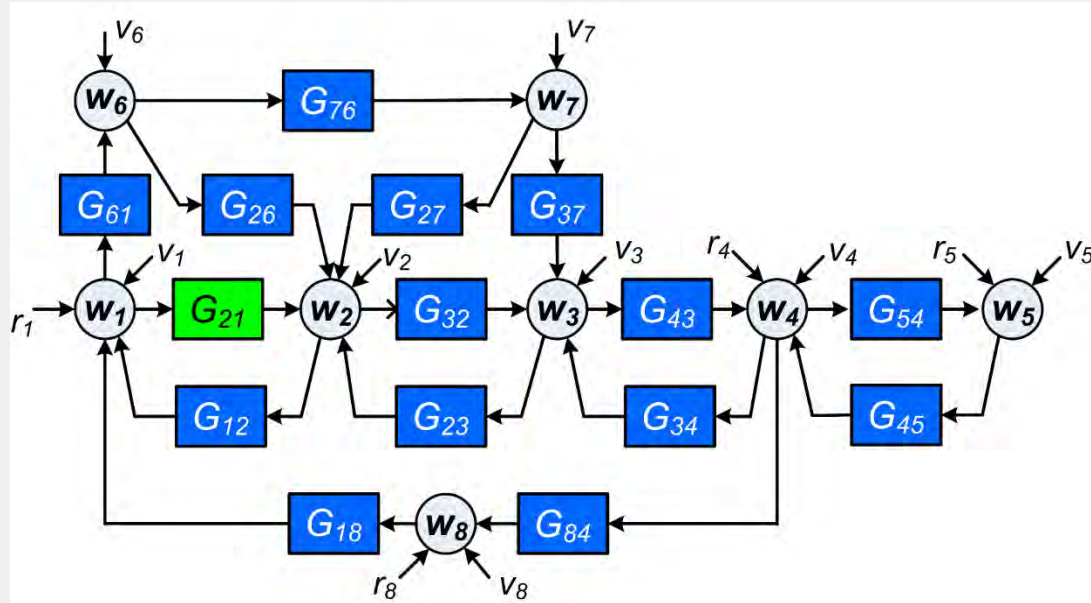
Under which conditions can we estimate the topology and/or dynamics of the full network?

Model learning problems



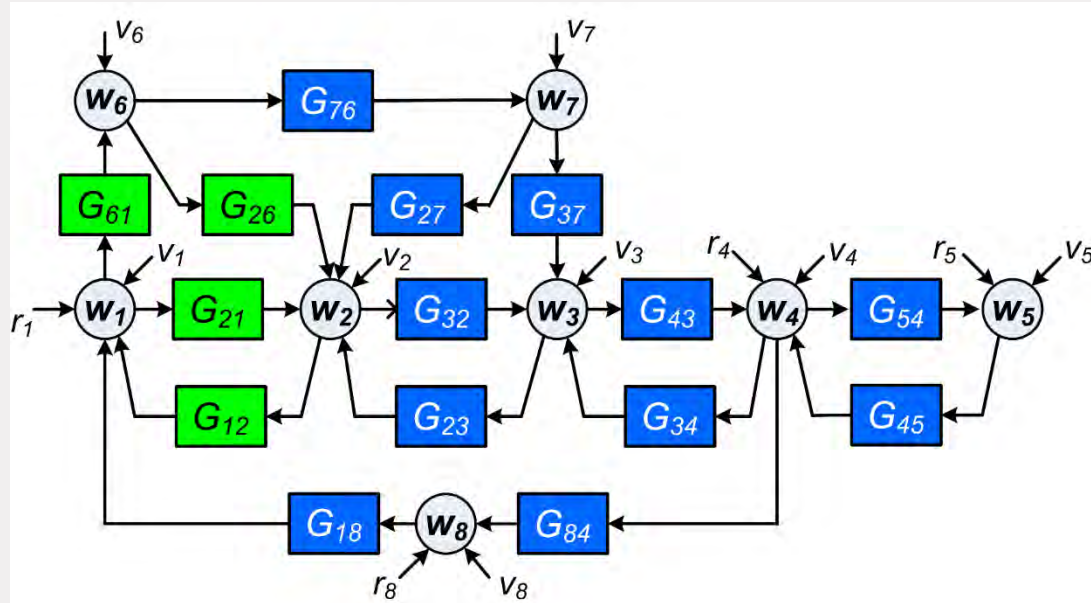
How/when can we learn a local module from data
(with known/unknown network topology)? Which signals to measure?

Model learning problems



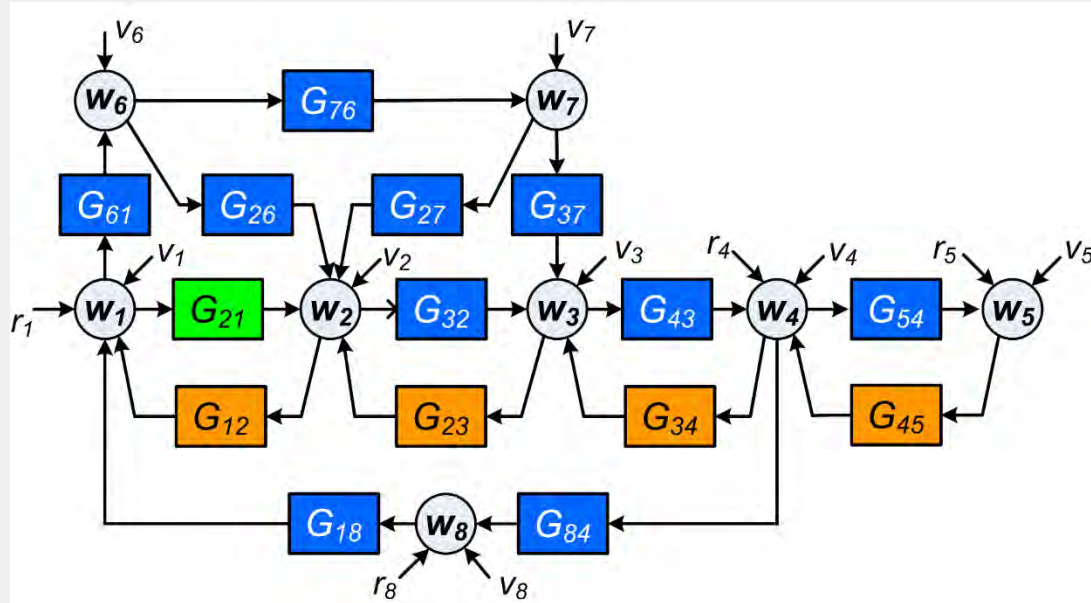
Where to optimally locate sensors and actuators?

Model learning problems



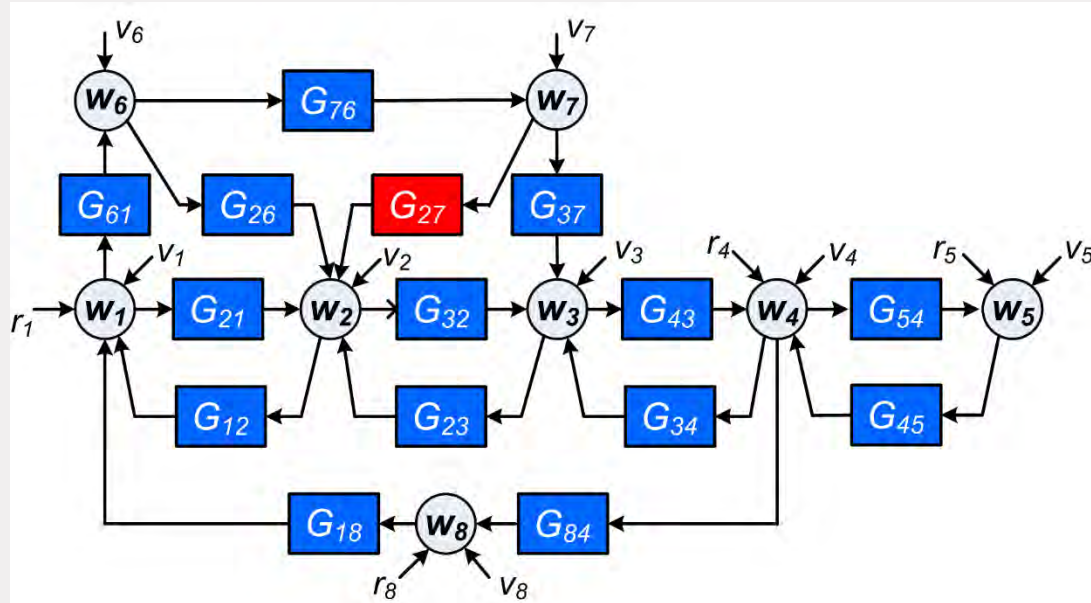
Same questions for a subnetwork

Model learning problems



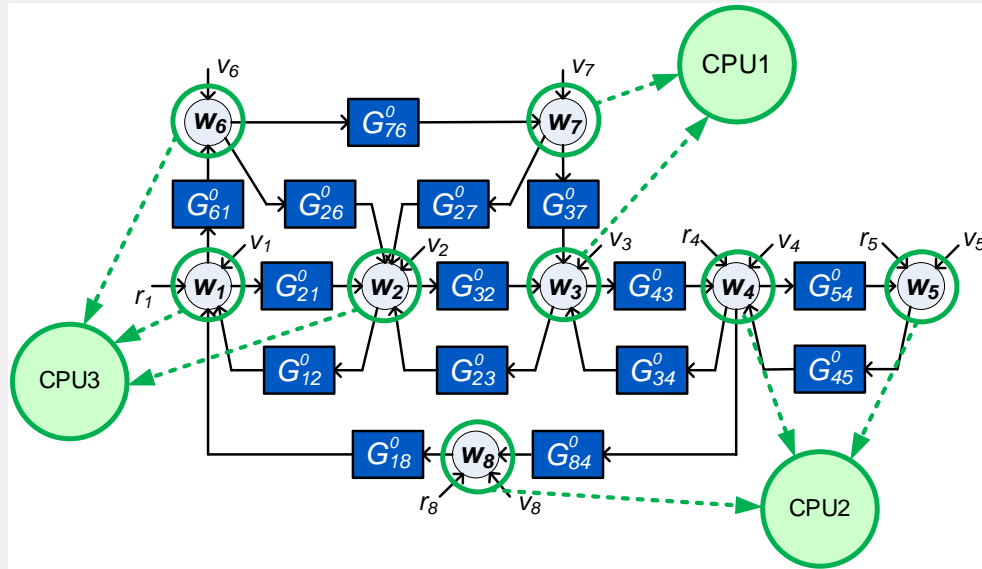
How can we benefit from known modules?

Model learning problems



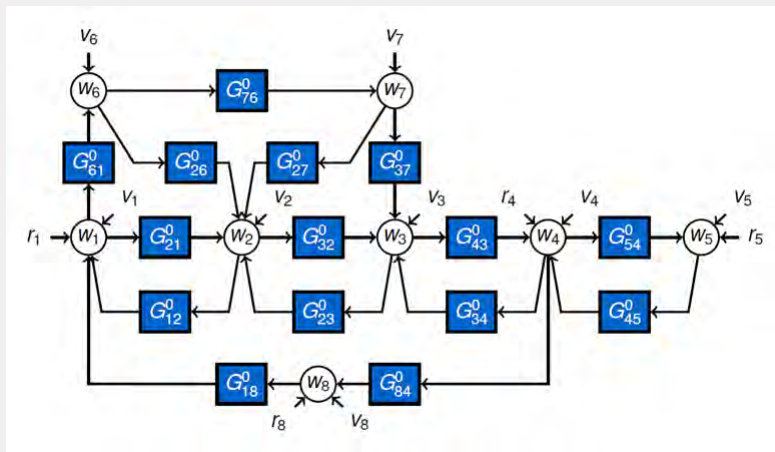
Fault detection and diagnosis; detect/handle nonlinear elements

Model learning problems



Can we distribute the computations?

Dynamic network setup



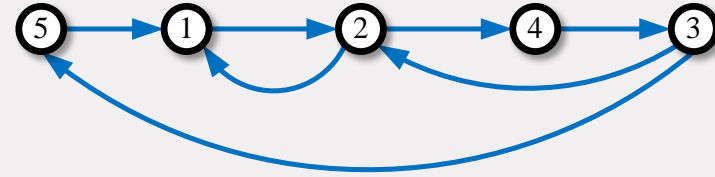
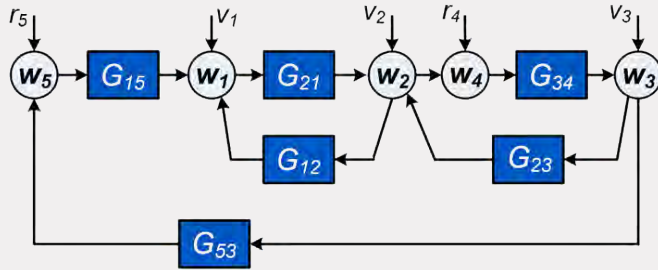
Measured time series:

$$\{w_i(t)\}_{i=1,\dots,L}; \{r_j(t)\}_{j=1,\dots,K}$$

Many challenging data-driven modeling questions can be formulated

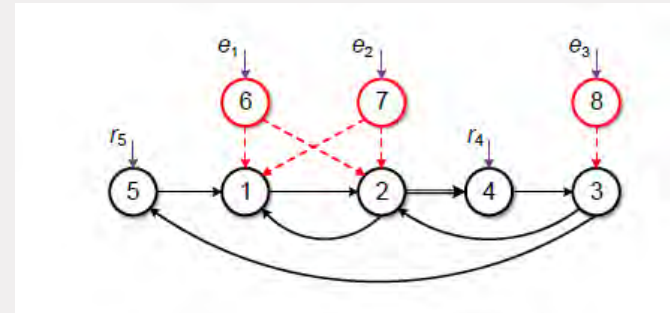
- Identification of a local module (known topology)
- Identification of the full network
- Topology estimation
- Identifiability
- Sensor and excitation allocation
- Fault detection
- User prior knowledge of modules
- Distributed identification
- **Scalable algorithms**

Dynamic network setup – directed graph

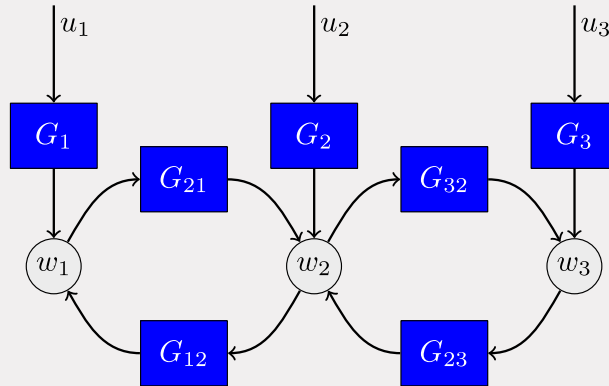
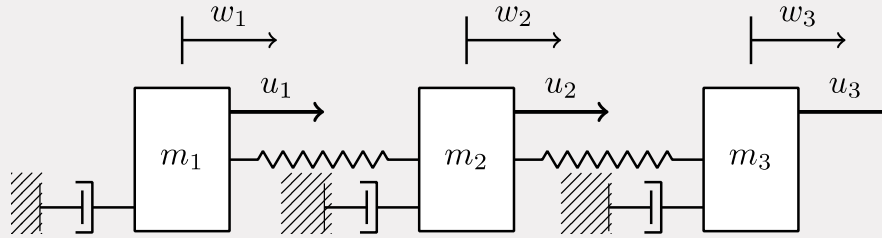


Nodes are vertices; modules/links are edges

Extended graph:
including the external signals
and disturbance correlations



Application: Networks of (damped) oscillators



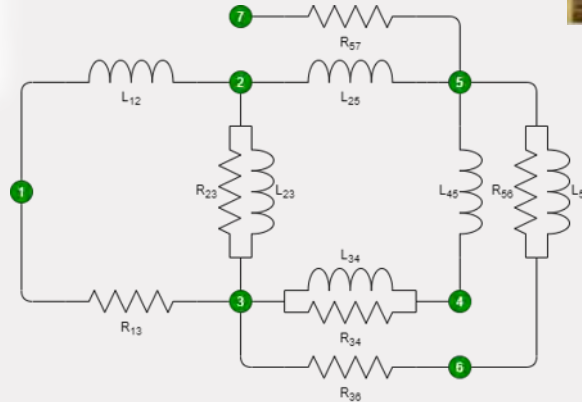
- Power systems, vehicle platoons, thermal building dynamics, ...
- Spatially distributed
- Bilaterally coupled

Application: Printed Circuit Board (PCB) Testing

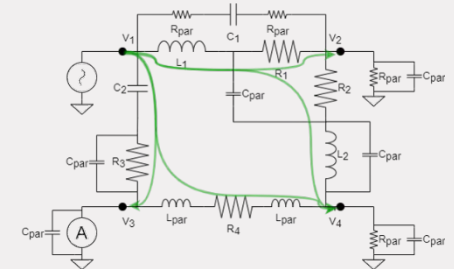


Detection of

- component failures
- parasitic effects



Source: Altium

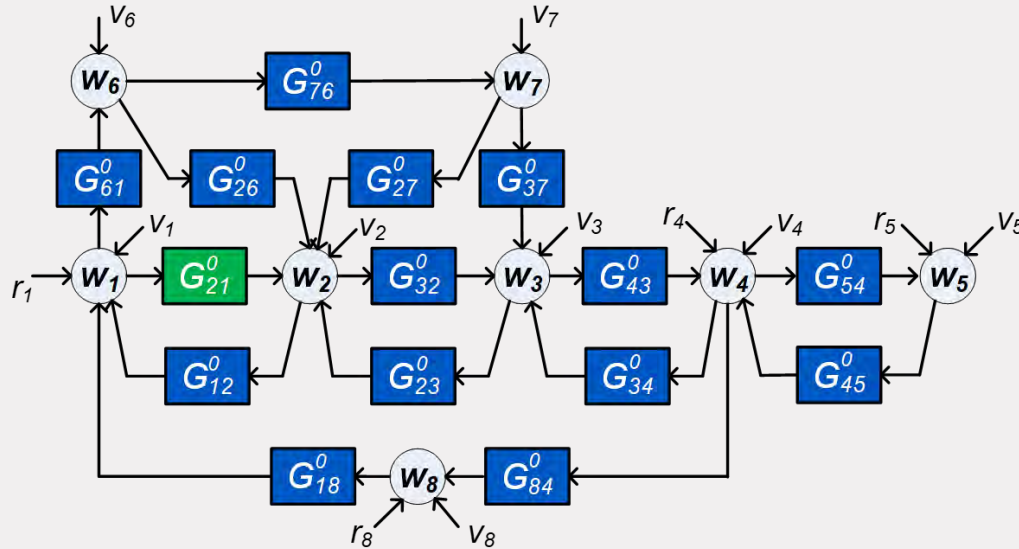


Contents

- Introduction and motivation
- How to model a dynamic network?
- **Single module identification**
- Global network identification
- Diffusively coupled networks
- Extensions - Discussion

Single module identification

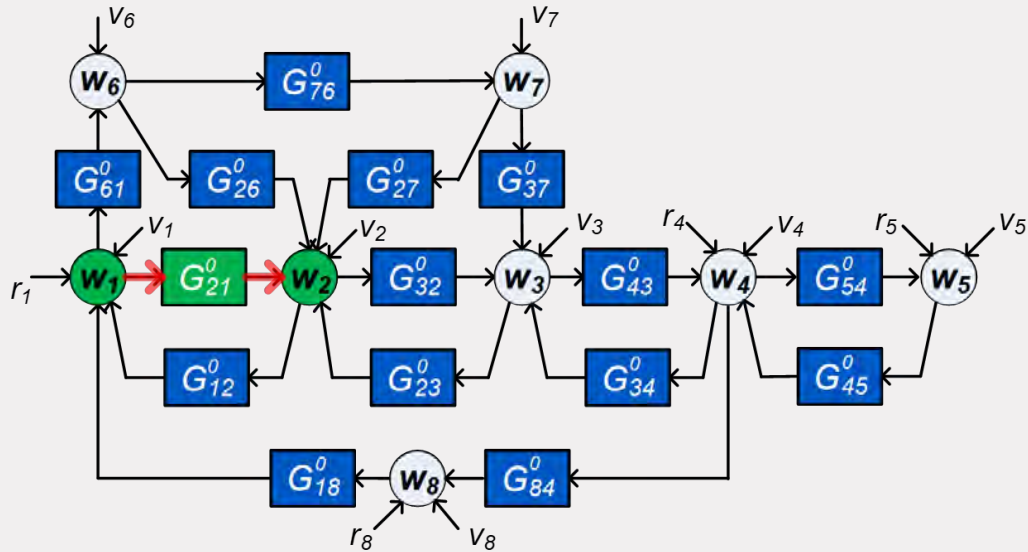
Single module identification



For a network with known topology:

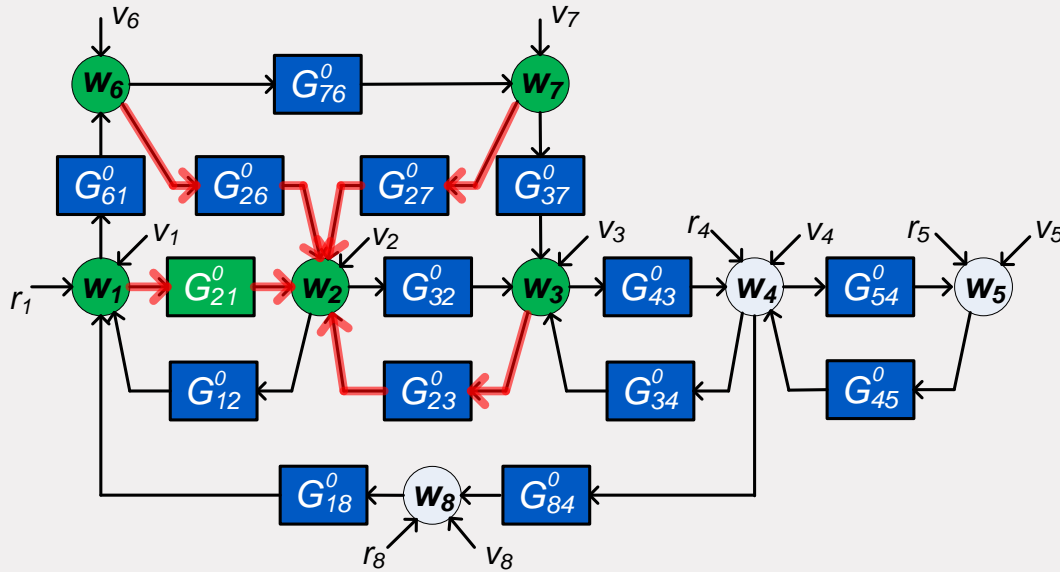
- Identify G_{21}^0 on the basis of measured signals
- Which signals to measure? Preference for local measurements
- When is there enough excitation / data informativity?

Single module identification



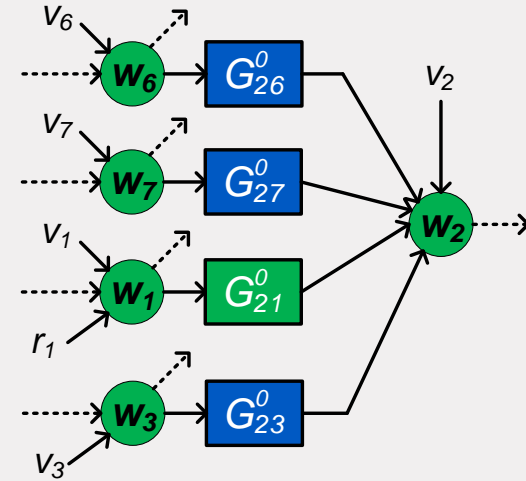
Naïve approach: identify based on w_1 and w_2 : in general does not work.

Single module identification



If noises v_k are correlated it may even be part of a MIMO problem

Identifying G_{21}^0 is part of a 4-input, 1-output problem



Single module identification

Identifying G_{21}^0 is part of a 4-input, 1-output problem

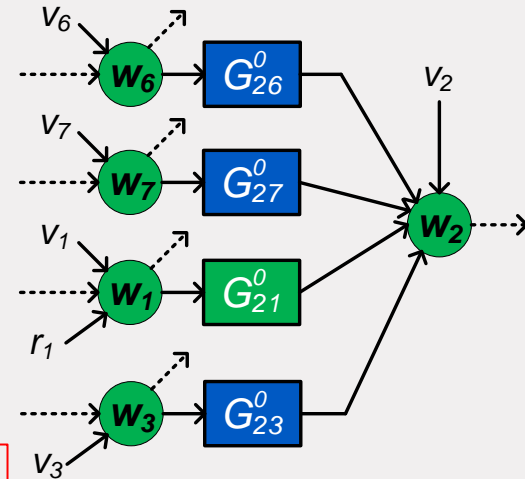
Input signals will be correlated:
similar as in a closed-loop situation

What is required for
identifiability / data informativity?

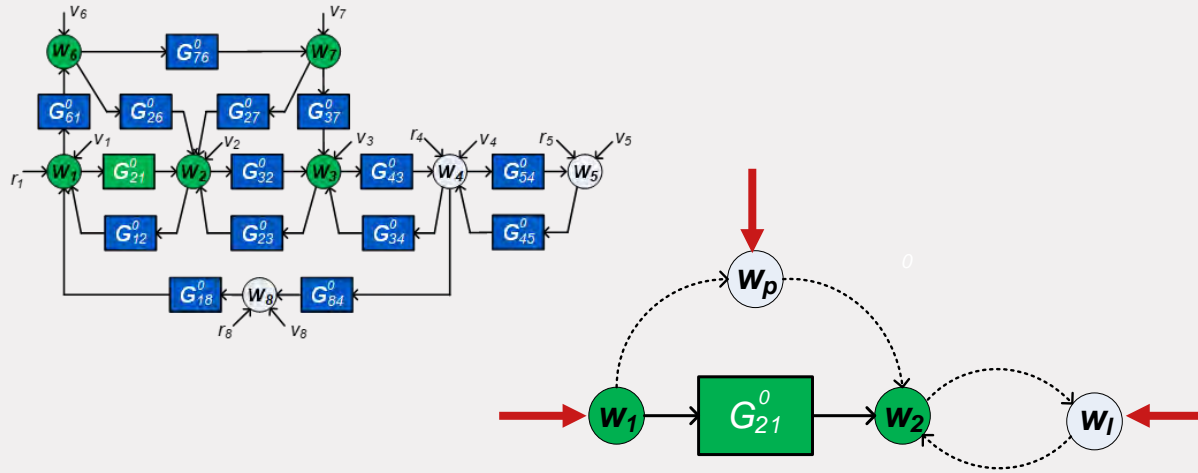


Ability to distinguish between models
independent of id-method

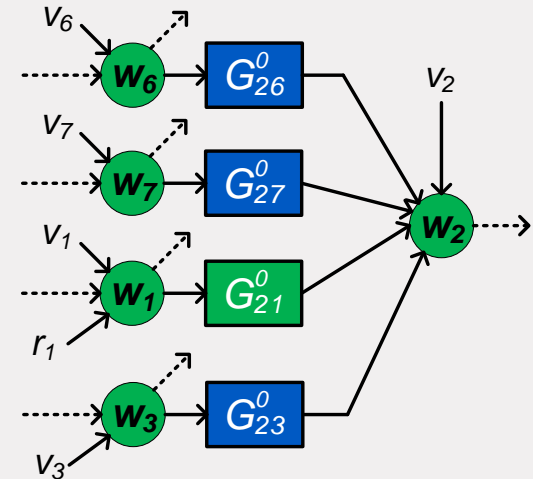
Information content of signals
dependent on id-method



Single module identification



Identifying G_{21}^0 is part of a 4-input, 1-output problem



All **parallel paths**, and **loops around the output**, plus input w_1 should have an independent external signal r or v and typically need to be **blocked** by a measured node

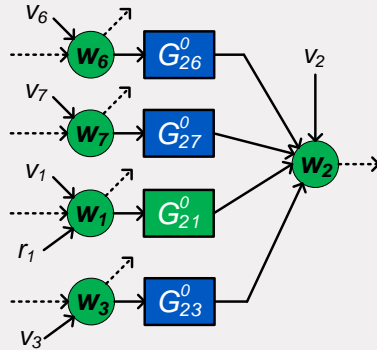
[1] Weerts et al., Automatica 2018, CDC 2018

[2] Bazanella et al. CDC2017; Hendrickx et al., IEEE-TAC, 2019.

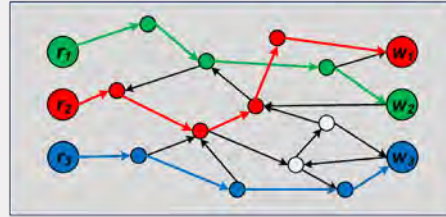
[3] Dankers et al., TAC 2016

[4] Shi et al., Automatica 2022

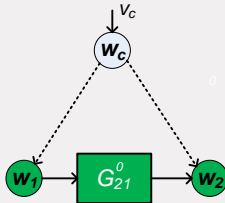
Single module identification



All inputs require an independent excitation (through vertex disjoint paths) from r, e



If excitation is relying on disturbances and correlated to v_2



Confounding variable [1][2]

To be handled by:

- Adding more input signals (blocking the cv)
- Including the input as output (MIMO) [3]

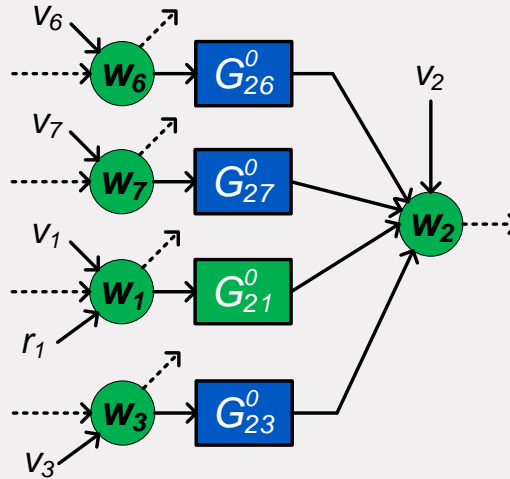
[1] J. Pearl, *Stat. Surveys*, 3, 96-146, 2009

[2] A.G. Dankers et al., *Proc. IFAC World Congress*, 2017.

[3] K.R. Ramaswamy et al., TAC November 2021

Single module identification

Typical solution:

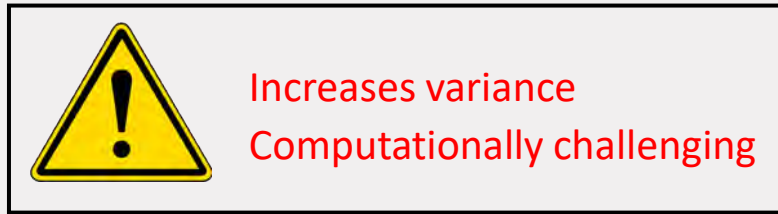


- MISO (sometimes MIMO) estimation problem
- to be solved by any (closed-loop) identification algorithm, e.g. direct/indirect method

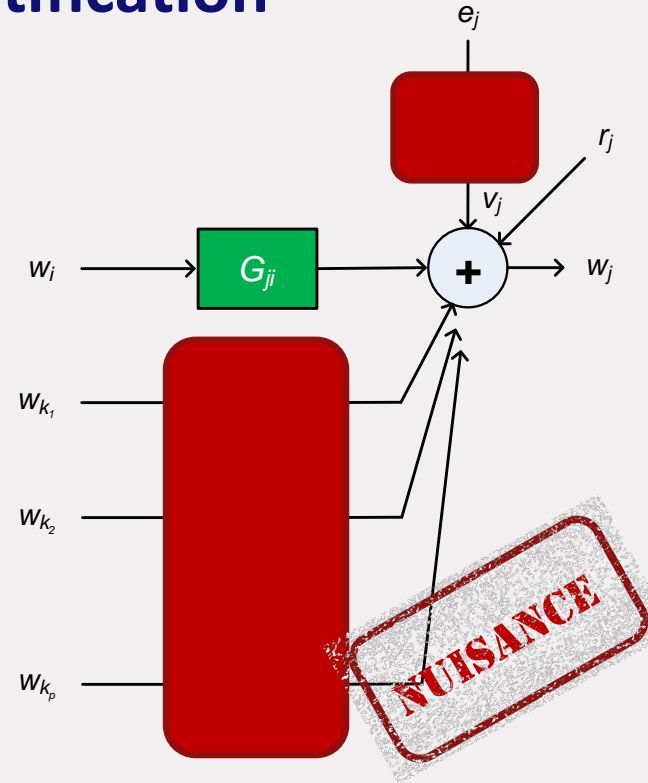
Machine learning in local module identification

- MISO identification with all modules parameterized
- Brings in two major problems :
 - ▶ Large number of parameters to estimate
 - ▶ Model order selection step for each module (CV, AIC, BIC)

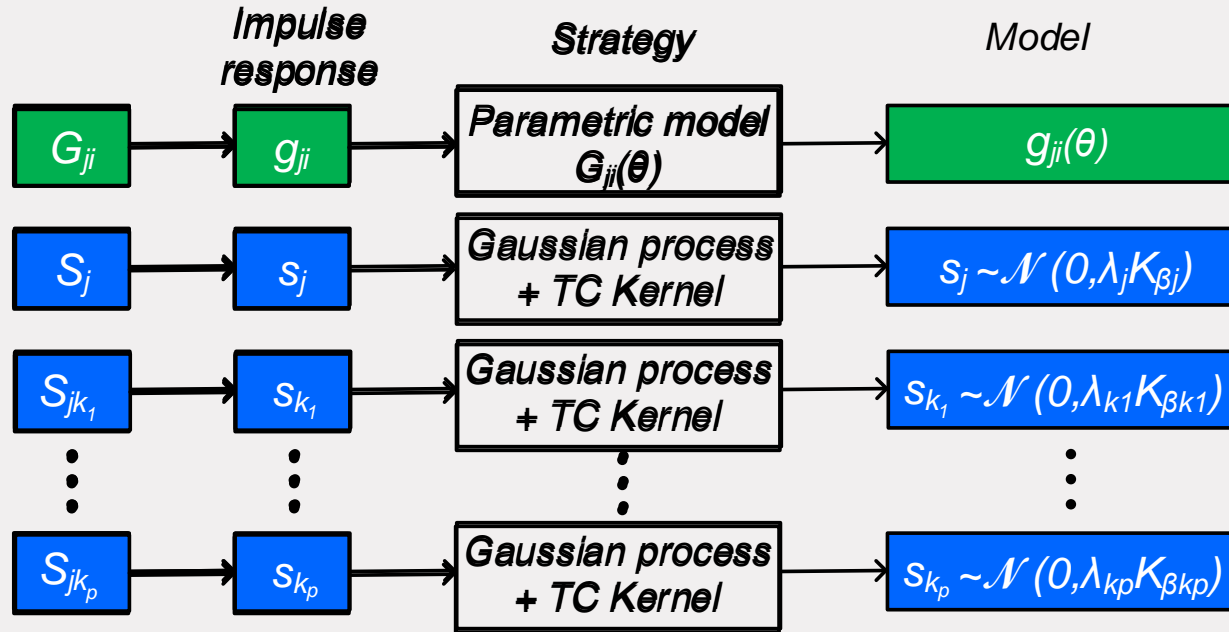
- For 5 modules, combinations = 244,140,625



- We need only the target module. No **NUISANCE**!



Machine learning in local module identification



- smaller no. of parameters
- simpler model order selection step
- scalable to large dynamic networks
- simpler optimization problems to estimate parameters



Maximize marginal likelihood of output data: $\hat{\eta} = \underset{\eta}{\operatorname{argmax}} p(w_j; \eta)$

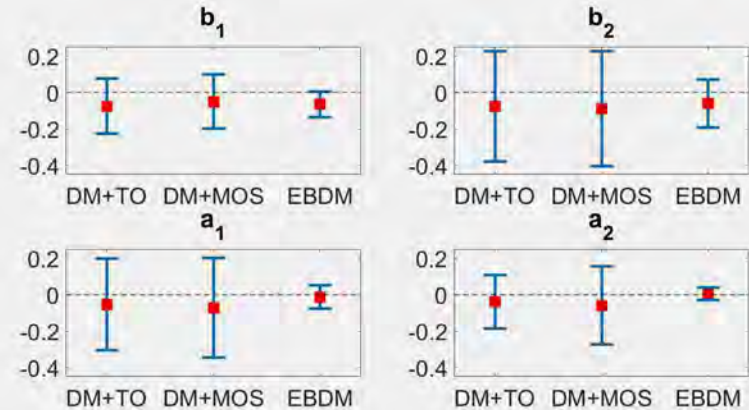
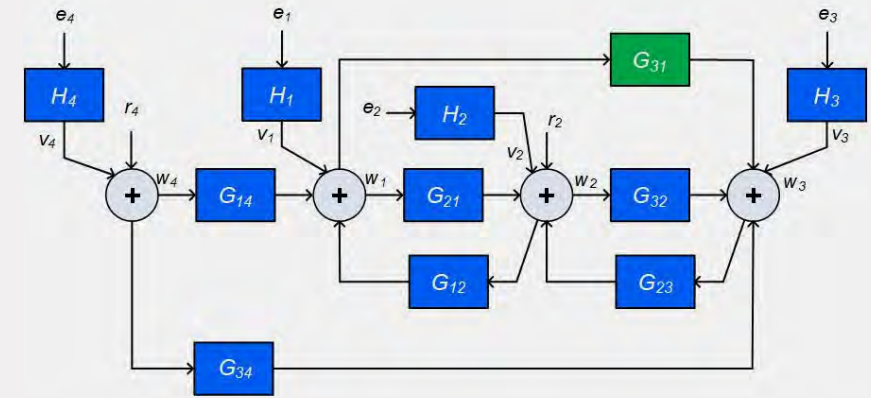
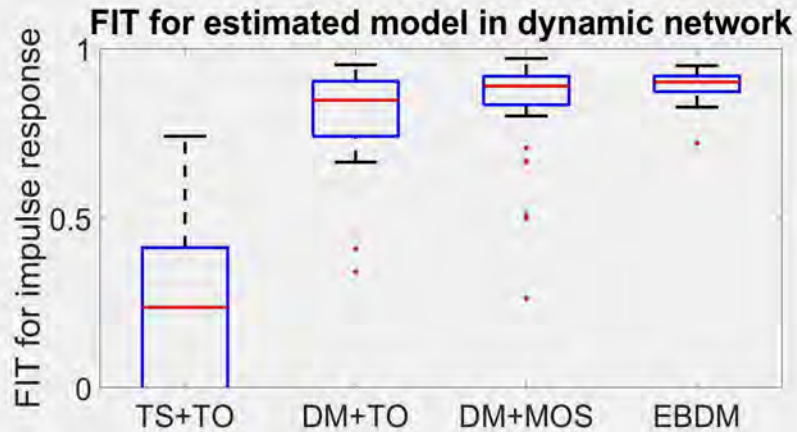
$$\eta := [\theta \quad \lambda_j \quad \lambda_{k_1} \quad \dots \quad \lambda_{k_p} \quad \beta_j \quad \beta_{k_1} \quad \dots \quad \beta_{k_p} \quad \sigma_j^2]^\top$$

[1] Everitt et al., Automatica 2017.

[2] K.R. Ramaswamy et al., Automatica, 2021.

Numerical simulation

- ▶ Identify G_{31} given data
- ▶ 50 independent MC simulation
- ▶ Data = 500



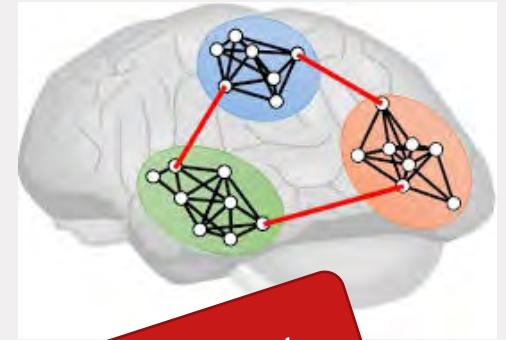
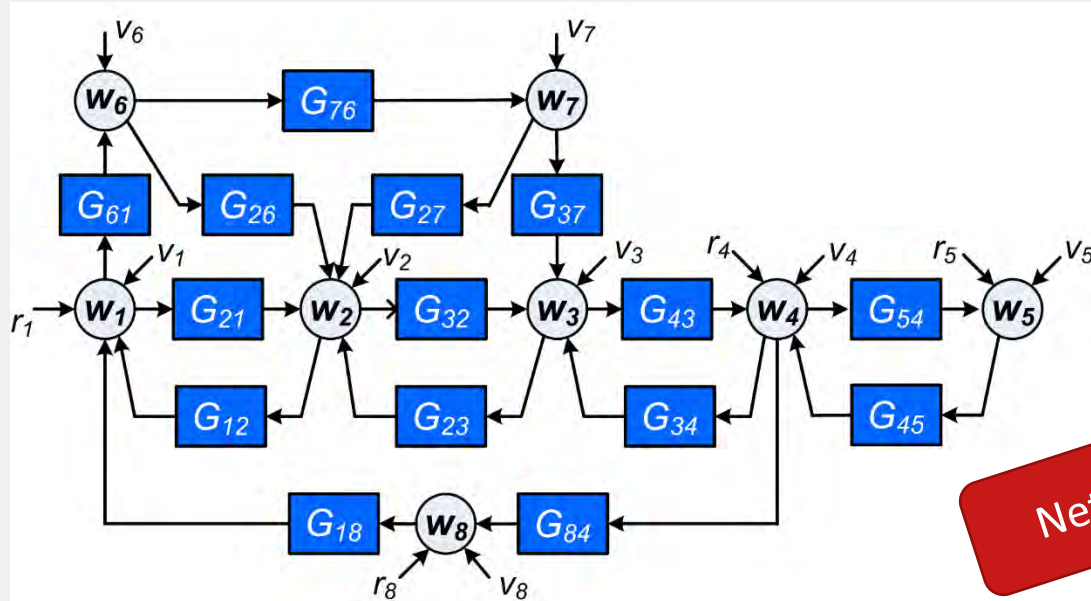
Summary single module identification

- Path-based conditions for **network identifiability** (where to excite?)
- Graph tools for checking conditions
- Degrees of freedom in selection of measured signals – sensor selection
- Methods for **consistent** and **minimum variance** module estimation, and effective (scalable) algorithms
- A priori known modules can be accounted for

Contents

- Introduction and motivation
- How to model a dynamic network?
- Single module identification
- **Global network identification**
- Diffusively coupled networks
- Extensions - Discussion

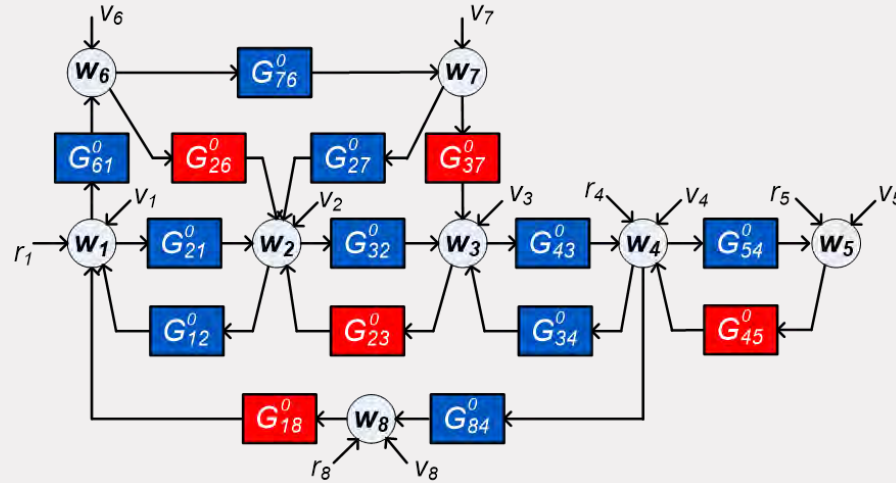
Full network identification



Network identifiability

Under which conditions can we estimate the topology and/or dynamics of the full network?

Network identifiability



blue = unknown
red = known

Question: Can different dynamic networks be *distinguished* from each other from measured signals w, r ?

Network identifiability

The identifiability problem:

The network **model**:

$$w(t) = G(q)w(t) + R(q)r(t) + \underbrace{H(q)e(t)}_{v(t)}$$

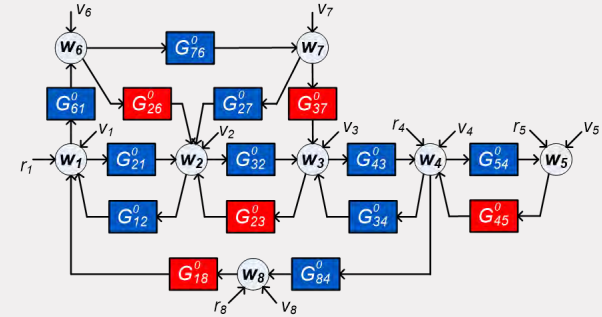
can be transformed with any rational $P(q)$:

$$P(q)w(t) = P(q)\{G(q)w(t) + R(q)r(t) + H(q)e(t)\}$$

to an **equivalent model**:

$$w(t) = \tilde{G}(q)w(t) + \tilde{R}(q)r(t) + \tilde{H}(q)e(t)$$

➡ **Nonuniqueness**, unless there are structural constraints on G, R, H .



[1] Weerts, Linder et al., Automatica, 2019.

[2] Bottegal et al., SYSID 2017

Network identifiability

Consider a **network model set**:

$$\mathcal{M} = \{(G(\theta), R(\theta), H(\theta))\}_{\theta \in \Theta}$$

representing structural constraints on the considered models:

- modules that are fixed and/or zero (topology)
- locations of excitation signals
- disturbance correlation

Generic identifiability of \mathcal{M} :

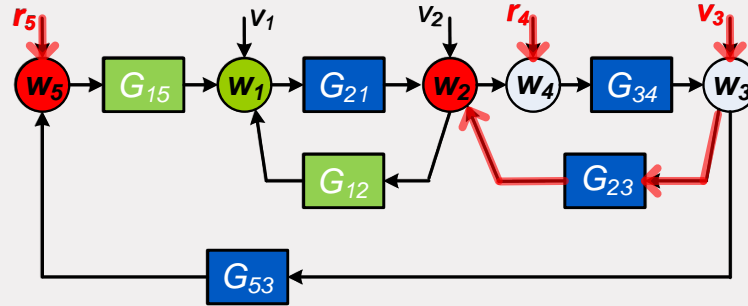
- There do not exist distinct equivalent models (generating the same data)
- for almost all models in the set.

[1] Weerts et al., SYSID2015; Weerts et al., Automatica, March 2018;

[2] Bazanella, CDC2017; Hendrickx et al., IEEE-TAC, 2019.

Example 5-node network

Conditions for identifiability \longrightarrow rank conditions on transfer function



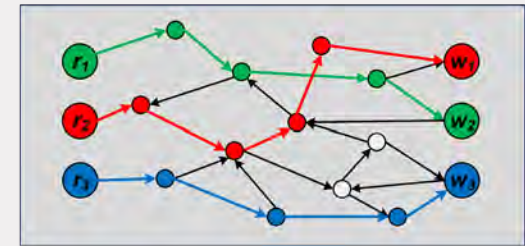
Full row rank of

$$\begin{bmatrix} v_3 \\ r_4 \\ r_5 \end{bmatrix} \rightarrow \begin{bmatrix} w_2 \\ w_5 \end{bmatrix}$$

For the **generic case**, the rank can be calculated by a graph-based condition^{[1],[2]}:

Generic rank = number of vertex-disjoint paths

2 vertex-disjoint paths \rightarrow full row rank 2



The rank condition has to be checked for all nodes.

[1] Van der Woude, 1991

[2] Hendrickx, Gevers & Bazanella, CDC 2017, TAC 2019

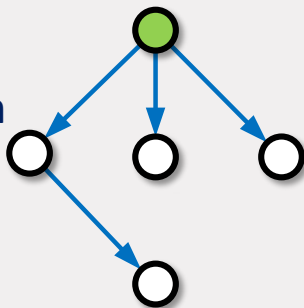
Synthesis solution for network identifiability

Allocating external signals for **generic identifiability**:

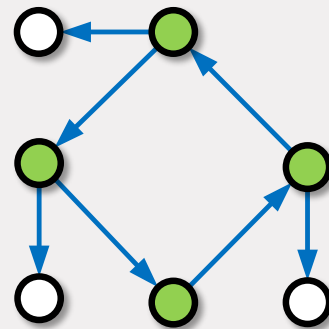
1. Cover the graph of the network model set by a set of **disjoint pseudo-trees**

Pseudo-trees:

Tree with root in green



Cycle with outgoing trees;
Any node in cycle is root

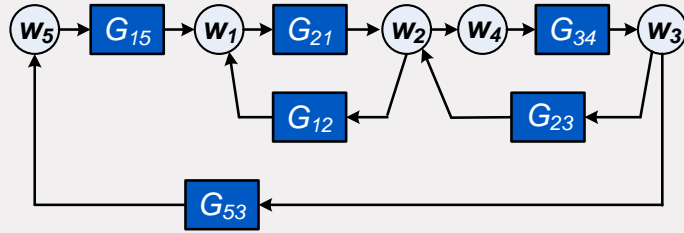


Edges are **disjoint** and all out-neighbours of a node are in the same pseudo-tree

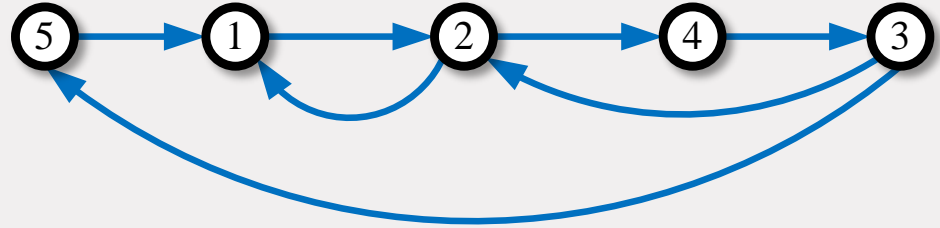
2. Assign an independent external signal (r or e) at a root of each pseudo-tree.

This guarantees **generic identifiability** of the model set.

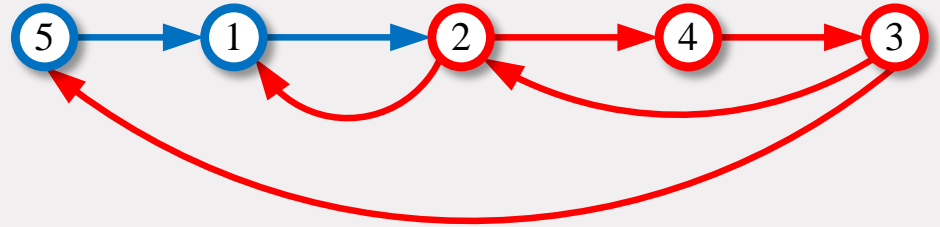
Where to allocate external excitations for network identifiability?



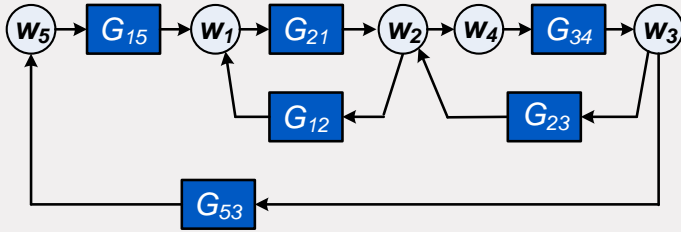
All indicated modules are parametrized



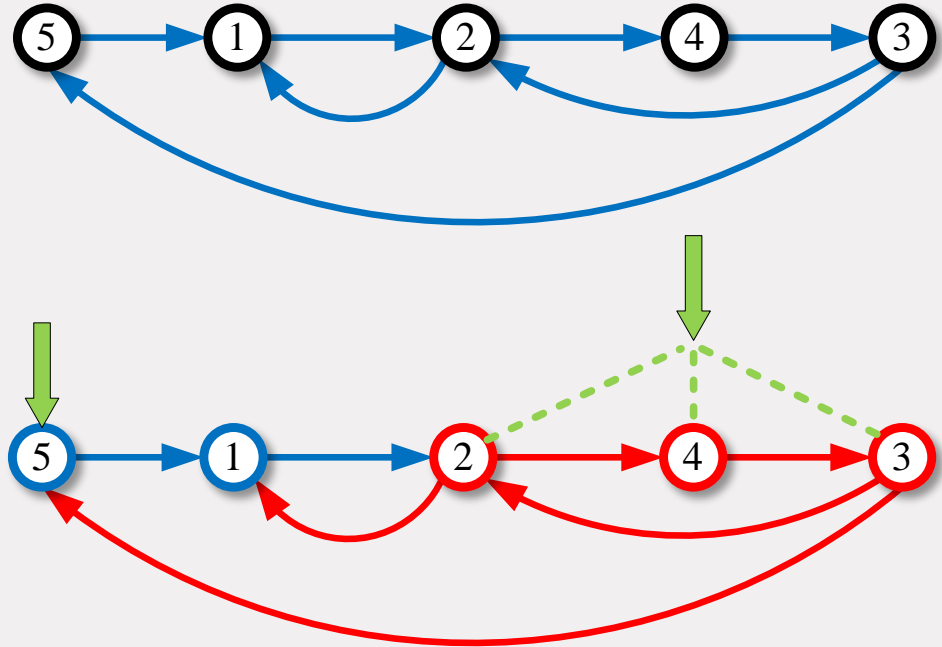
Two disjoint pseudo-trees



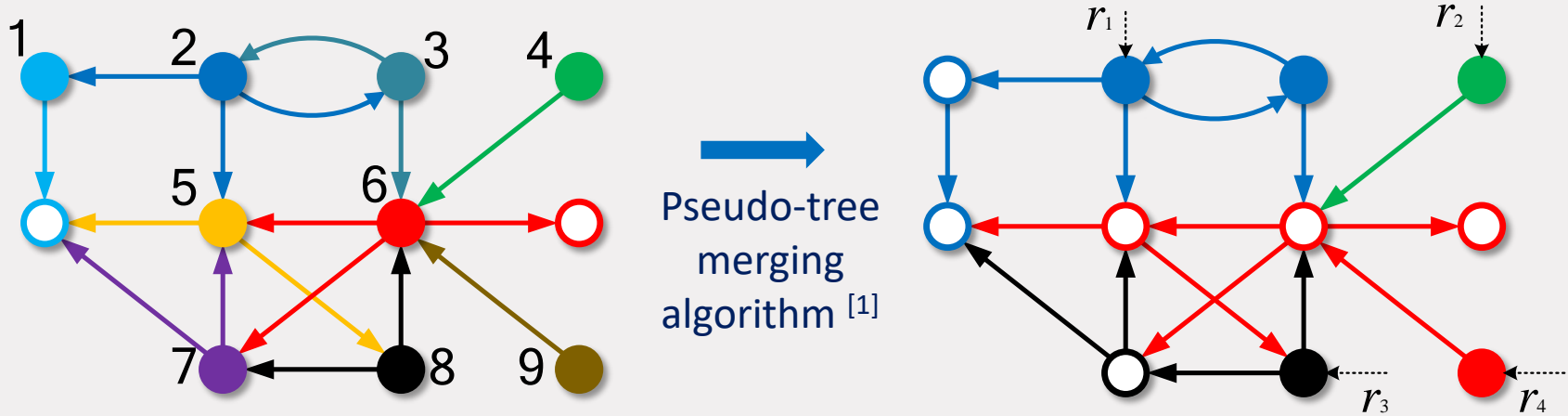
Where to allocate external excitations for network identifiability?



Two independent excitations
guarantee
generic network identifiability



Where to allocate external excitations for network identifiability?



- Nodes are signals w and external signals (r, e) that are input to parametrized link
- Known (nonparametrized) links do not need to be covered

Summary identifiability of full network

Identifiability of network model sets is determined by

- Presence and location of external signals, and
 - Correlation of disturbances
 - Topology of parametrized modules
- Graphic-based tool for synthesizing allocation of external signals

Extensions:

- Situations where not all node signals are measured ^[1]

[1] Bazanella, CDC 2019.

Algorithms for identification of full network

(Prediction error) identification methods will typically lead to large-scale **non-convex** optimization problems

Convex relaxation algorithms are being developed^[1,2] as well as machine learning tools

[1] Weerts, Galrinho et al., SYSID 2018

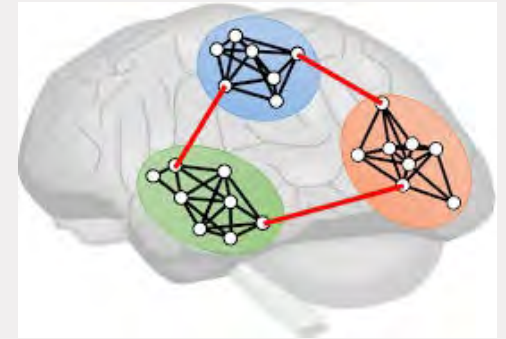
[2] Fonken et al., Automatica 2022, to appear.

Topology identification

- Topology resulting from full dynamic model
- Alternative: non-parametric models (Wiener filters ^[1]) or kernel-based approaches ^{[2][3]}
- modeling module dynamics by Gaussian processes, kernel with 2 parameters for each dynamic module
- Optimizing likelihood of the data as function of parameters and topology:

$$p(\{w(t)\}_{t=1}^N | \theta, \mathcal{G})$$

- Forward-backward search over topologies + empirical Bayes (EM) for parameters

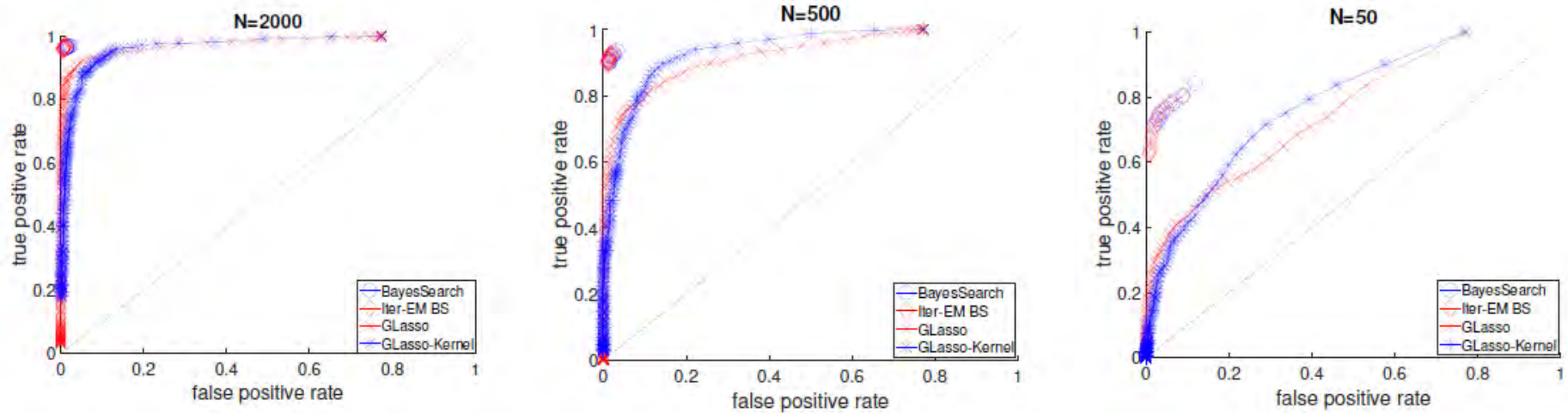


[1] Materassi & Innocenti, TAC 2010.

[2] Chiuso & Pillonetto, Automatica, 2012.

[3] Shi, Bottegal, PVdH, ECC 2019

Topology identification



50 MC realizations of network with 6 nodes.

Neurodynamic effect of listening to Mozart music

Identifying changes in network connections in the brain, after intensely listening for one week

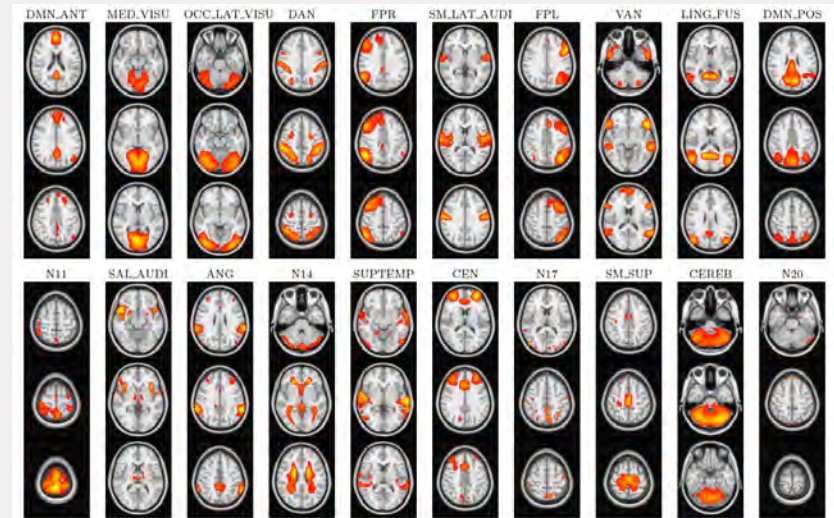
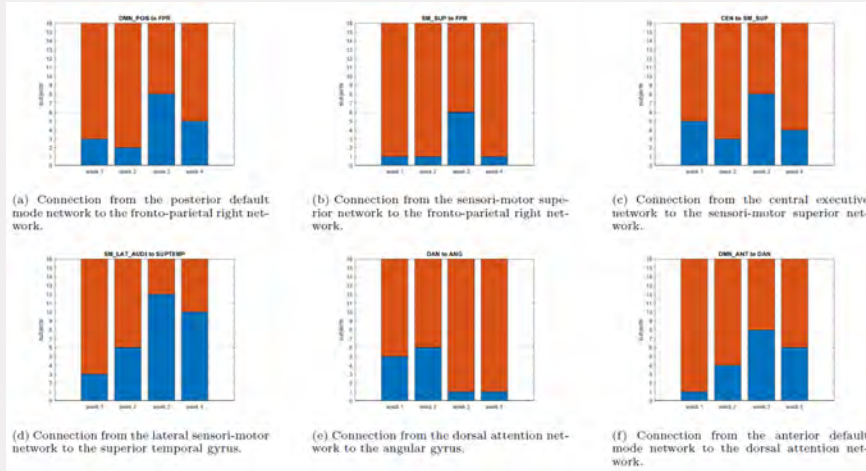


Figure 3: Spatial maps of the 20 active brain networks found through the ICA decomposition. Each image consists of 3 relevant horizontal slices of the brain, where the spatial map is indicated by the red color scale.

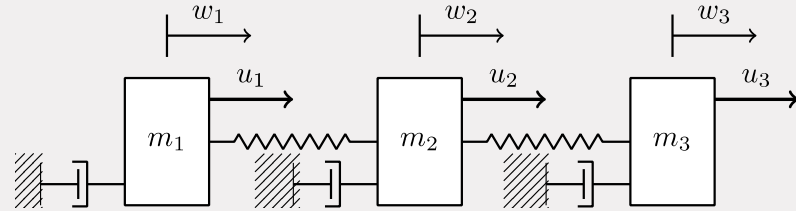
Contents

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- **Diffusively coupled networks**
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Diffusively coupled networks

Back to the basics of physical interconnections

In connecting physical systems, there is often no predetermined direction of information ^[1]



Example: resistor / spring connection in electrical / mechanical system:



Resistor

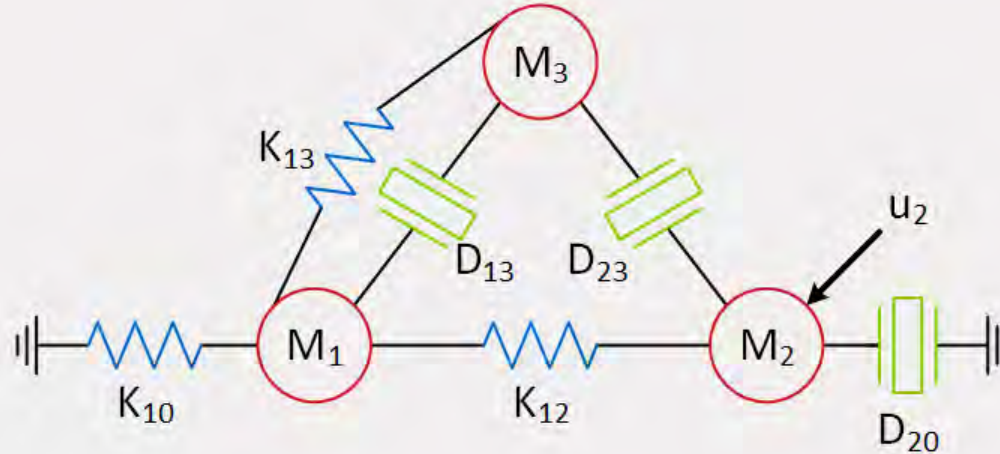
$$I = \frac{1}{R} (V_1 - V_2)$$

Spring

$$F = K(x_1 - x_2)$$

Difference of node signals drives the interaction: **diffusive coupling**

Diffusively coupled physical network

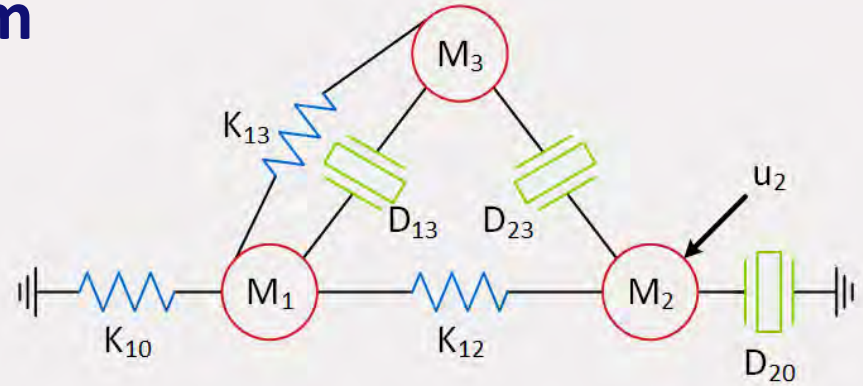


Equation for node j :

$$M_j \ddot{w}_j(t) + D_{j0} \dot{w}_j(t) + \sum_{k \neq j} D_{jk} (\dot{w}_j(t) - \dot{w}_k(t)) + K_{j0} w_j(t) + \sum_{k \neq j} K_{jk} (w_j(t) - w_k(t)) = u_j(t),$$

Mass-spring-damper system

- Masses M_j
- Springs K_{jk}
- Dampers D_{jk}
- Input u_j



$$\begin{aligned}
 & \begin{bmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{bmatrix} \begin{bmatrix} \ddot{w}_1 \\ \ddot{w}_2 \\ \ddot{w}_3 \end{bmatrix} + \begin{bmatrix} 0 & & \\ & D_{20} & \\ & & 0 \end{bmatrix} \begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \end{bmatrix} + \begin{bmatrix} K_{10} & & \\ & 0 & \\ & & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \\
 & + \begin{bmatrix} D_{13} & 0 & -D_{13} \\ 0 & D_{23} & -D_{23} \\ -D_{13} & -D_{23} & D_{13} + D_{23} \end{bmatrix} \begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \end{bmatrix} + \begin{bmatrix} K_{12} + K_{13} & -K_{12} & -K_{13} \\ -K_{12} & K_{12} & 0 \\ -K_{13} & 0 & K_{13} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 0 \\ u_2 \\ 0 \end{bmatrix}
 \end{aligned}$$

$$\left[\underbrace{A(p)}_{\text{diagonal}} + \underbrace{B(p)}_{\text{Laplacian}} \right] w(t) = u(t) \quad A(p), B(p) \text{ polynomial} \quad p = \frac{d}{dt}$$

Mass-spring-damper system

$$\left[\underbrace{A(p)}_{\text{diagonal}} + \underbrace{B(p)}_{\text{Laplacian}} \right] w(t) = u(t) \quad A(p), B(p) \text{ polynomial}$$

$$\left[\underbrace{Q(p)}_{\text{diagonal}} - \underbrace{P(p)}_{\text{hollow \& symmetric}} \right] w(t) = u(t)$$

This fully fits in the earlier **module** representation:

$$w(t) = Gw(t) + \underbrace{Rr(t) + He(t)}_{Q^{-1}(p)u(t)}$$

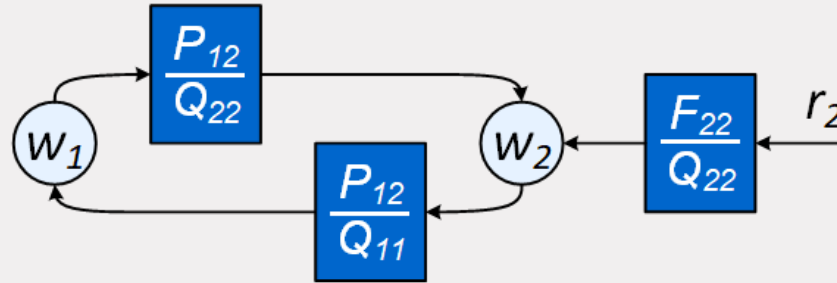
with the additional condition that:

$$G(p) = Q(p)^{-1}P(p) \quad Q(p), P(p) \text{ polynomial}$$

$P(p)$ symmetric, $Q(p)$ diagonal

Module representation

Consequences for node interactions:



- Node interactions come in pairs of modules
- Where numerators are the same

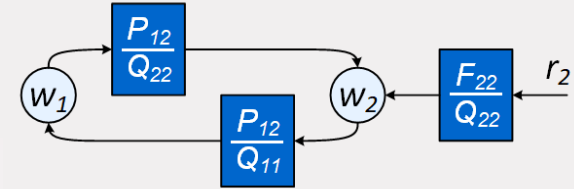
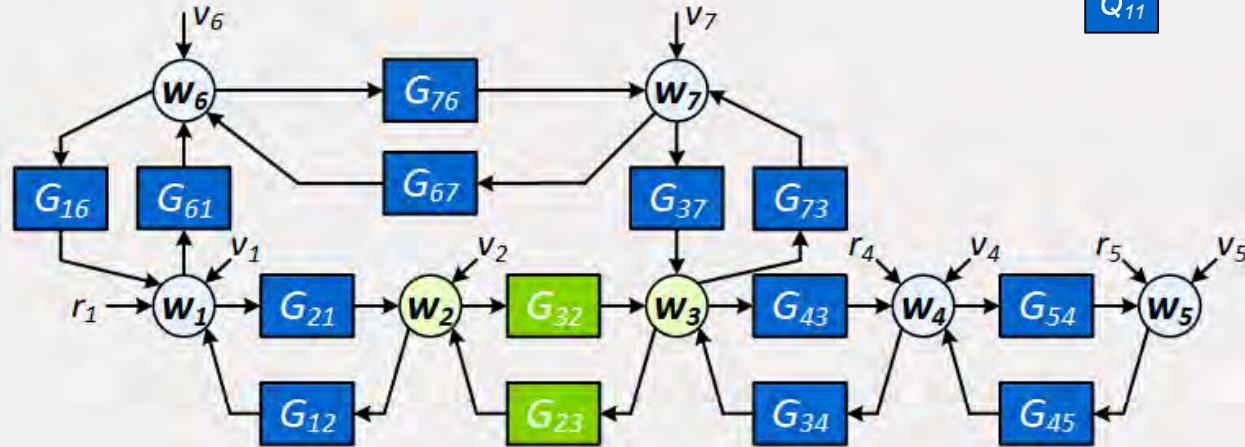
Framework for network identification remains the same

- Symmetry can simply be incorporated in identification

Local network identification

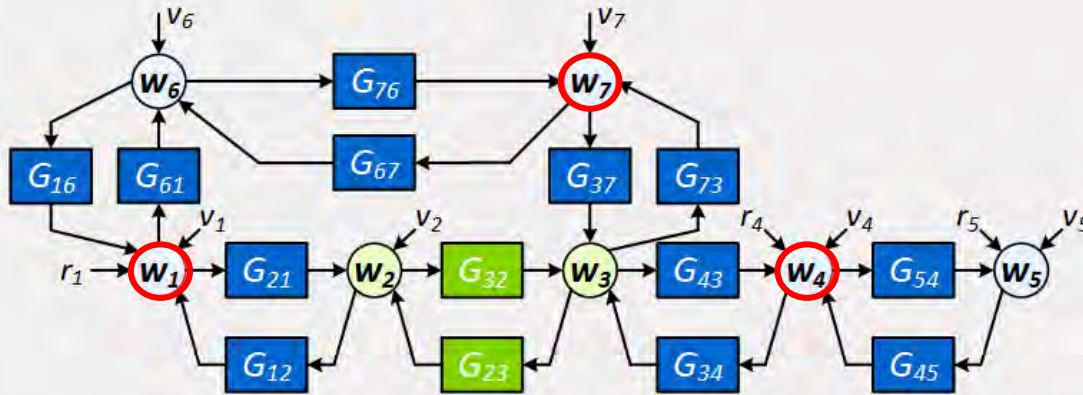
Identification of **one** (physical) interconnection

Identification of **two** modules G_{jk} and G_{kj}



Immersion conditions

For simultaneously identifying two modules in one interconnection:



The parallel path and loops-around-the-output condition, now simplifies to:

Measuring/exciting all neighbouring nodes of w_2 and w_3 leads to a solution

Summary diffusively coupled networks

- Diffusively coupled networks fit within the module framework (special case)
 - no restriction to second order equations
- Earlier identification framework can be utilized
- Local identification is well-addressed (and stays really local)
- Framework is fit for representing **cyber-physical systems**
(combining physical bi-directional links, and cyber uni-directional links).

Extensions - Discussion

Extensions - Discussion

- **Including sensor noise** ^[1]
 - Errors-in-variables problems can be more easily handled in a network setting
- **Distributed estimation (MISO models)** ^[2]
 - Communication constraints between different agents
 - Recursive (distributed) estimator converges to global optimizer (more slowly)
- **Experiment design** ^{[3],[4]}
 - design of least costly experiments

[1] Dankers et al., Automatica, 2015.

[2] Steentjes et al., IFAC-NECSYS, 2018.

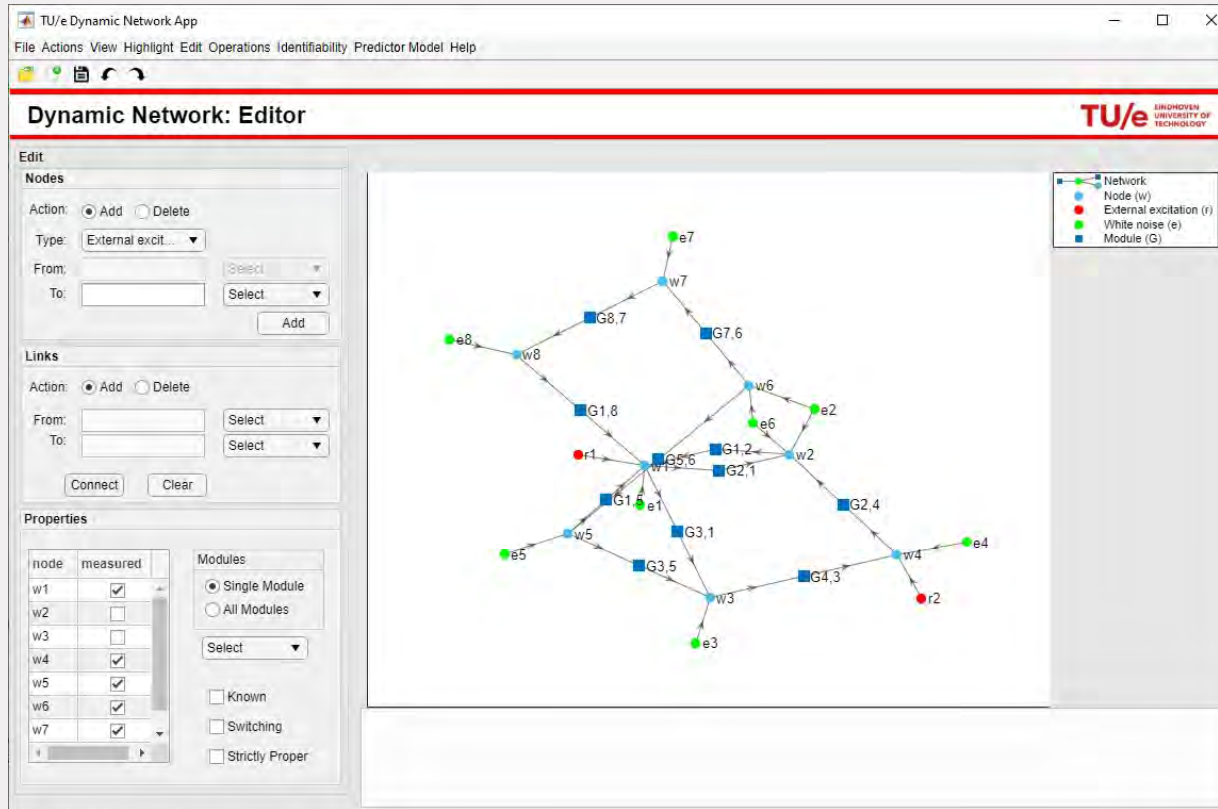
[3] Gevers and Bazanella, CDC 2015.

[4] Morelli, Bombois et al., ECC 2019;

Summary

- **Dynamic network modeling:**
intriguing research topic with many open questions
- The (centralized) LTI framework is only just the beginning
- Further move towards data-aspects related to distributed control
- and large-scale aspects
- and more real-life applications

Matlab Toolbox



ERC SYSDYNET Team: data-driven modeling in dynamic networks

Research team:



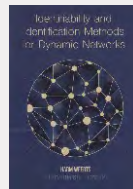
Karthik
Ramaswamy



Arne Dankers



Harm Weerts



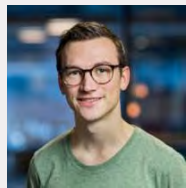
Shengling Shi



Giulio Bottegal



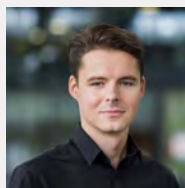
Xiaodong Cheng



Mannes Dreef



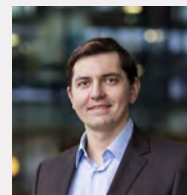
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Further reading

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- S. Shi, X. Cheng and P.M.J. Van den Hof (2023). Single module identifiability in linear dynamic networks with partial excitation and measurement. To appear in *IEEE Trans. Automatic Control*, January 2023.

The end