Scenario-based robust optimization of water flooding in oil reservoirs enjoys probabilistic guarantees

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Closed-Loop Reservoir Management (CLRM)

Strategy that optimizes economic performance (life-cycle)

Closed-loop reservoir management

Jansen et al. (2005)
Challenges

- Large-scale, non-convex and non-linear optimization
- Uncertainty
  - Parametric uncertainty
- Economic uncertainty
  - Varying oil prices

Decision making (model-based economic optimization) under geological parametric and economic uncertainty
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• Scenario-based WCO under Geological uncertainty
Decision making under uncertainty
robust optimization

\[
\max (\text{Mean} - \gamma \text{Worst-case})
\]
Worst-case optimization (WCO)

Worst-case (robust) optimization:

$$\max_{u \in \mathcal{U}} \min_{\theta \in \Theta} J(u, \theta)$$

Reformulation

$$\text{WCO} : \left\{ \begin{array}{l}
\max_{u \in \mathcal{U}, z \in \mathbb{R}} \quad z \\
\text{s.t.} \quad z \leq \min_{\theta \in \Theta} J(u, \theta)
\end{array} \right.$$ 

$\theta \in \Theta$ is a representation of uncertainty space
Scenario-based worst-case optimization

Introduction

Scenario-based worst-case optimization

- sample the uncertainty space
- a finite number of realizations of the uncertain parameters

\[
\{\theta_1, \theta_2, \cdots, \theta_N\}
\]

\[
\max_{u \in \mathcal{U}} \min_{\theta_i \in \Theta} J(u, \theta_i) \quad i = 1, 2, \cdots, N
\]

\[
WCO_N : \begin{cases} 
\max_{u \in \mathcal{U}, z \in \mathbb{R}} & z \\
\text{s.t.} & z \leq \min_{i \in \{1, \ldots, N\}} J(u, \theta_i)
\end{cases}
\]
What can we claim about the achieved objective function value, when $u^*_N$ is applied in the presence of the unseen scenarios for $\theta$?

Given a number of samples $N$, can we quantify, e.g., in probabilistic terms, the robustness of the performance, i.e., the measure of the subset of $\Theta$ such that $J(u^*_N) \geq \max_{\theta \in \Theta} J(u^*_N, \theta)$?
Performance robustness probability

**Definition**: Let $u^*_N$ be an optimal solution to $WCO_N$. The performance robustness probability of $u^*_N$ in terms of NPV is defined as:

$$V(u^*_N) := \mathbb{P}_\Theta [J(u^*_N) \geq J(u^*_N, \theta \in \Theta)].$$

**$\epsilon$-level robust solution**: if $\epsilon \in (0,1)$, $V(u^*_N) \leq \epsilon$

- A-priori guarantees (convex optimization)
- A-posterior guarantees (wait-and-judge for non-convex optimization)
A sampled constraint is a support constraint for $WCO_N$ if its removal would alter the optimal solution of the problem.
Scenario-based WCO under economic uncertainty

Oil price variation $r_{k}^{oil}$ as source of uncertainty

$$J(u, \theta) = \sum_{k=1}^{K} \frac{\Delta t_k}{(1 + b)^{t_k/\tau}} (r_{k}^{oil} q_{k}^{oil} - r_{k}^{water} q_{k}^{water} - r_{k}^{inj} q_{k}^{inj})$$

$r_{k}^{oil}$ appears linearly in $J(u, \theta)$
Main result: Suppose that, for all $\theta = r_{k}^\text{oil} \in \Theta$, the function $u \rightarrow J(u, \theta)$ is concave. Let $u_N^*$ be the optimal solution to $WCO_N$ in,

$$N \geq \frac{2}{\epsilon} \left( K + \ln \left( \frac{1}{\beta} \right) \right).$$

Then, it holds

$$\mathbb{P}_\Theta^N \left[ V(u_N^*) > \epsilon \right] \leq \beta$$

An a-priori probabilistic bound
- Non-linear relation between NPV and uncertainty (permeability field)
- Use wait-and-judge theory to derive a-posterior bound

- Simulation example:
  - An ensemble of standard egg-model realizations (N = 100 members)
    - Economic parameters fixed
    - Optimize injection rates
    - Worst-case robust optimization
  
- Wait (find the solution) and judge (observe the number of support constraints)
upper-bound $\epsilon$ as a function of SC and the confidence level $(1 - \beta) = 0.99\%$
• Scenario-based optimal control solutions do enjoy probabilistic guarantees.

• In case of economic uncertainty, probabilistic statements hold depending on the number of scenarios used (a-priori bounds).

• In case of geological uncertainty, probabilistic statements are derived based on the computed optimal solution (a-posterior bounds).

• Extendable to other constrained problems in the water-flooding optimization.
Thank you!
Optimization solver

KNITRO:

A commercial solver for large-scale non-linear constraint optimization

Both interior-point (barrier) and active-set methods;

Programmatic interfaces: C/C++, Fortran, Java, Python;

Modeling language interfaces: AMPL ©, AIMMS ©, GAMS ©, MATLAB ©, MPL ©, Microsoft Excel Premium Solver ©;
Worst-case robust optimization

Handling geological uncertainty

\[
\max_u z \\
\text{s.t.} \quad z \leq J(u, \theta_i) \quad \forall i
\]

Worst-case increase: 3.60%
Average decrease: 1.54%
Worst-case robust optimization

Worst-case increase: 4.41%
Average decrease: 6.18%
Decision making under uncertainty

- A broad concept with ideas originating from various fields

Stochastic optimization

Robust optimization

Randomized methods/
Ensemble (scenario)-based approaches

Theory of risk