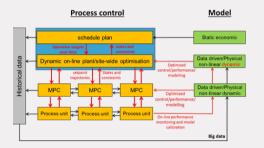






Introduction – dynamic networks

Decentralized process control



Thermal power plant hydraulic power generation **Smart Grid** Cities and offices

Smart power grid

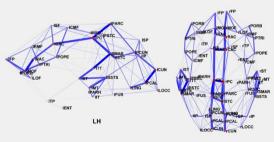




Complex machines

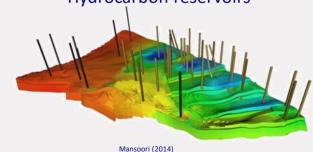


Brain network



P. Hagmann et al. (2008)

Hydrocarbon reservoirs





Overall trend:

- (Large-scale) interconnected dynamic systems
- Distributed / multi-agent type monitoring, control and optimization problems
- Data is "everywhere", big data era, Al/machine learning tools
- Model-based operations require accurate/relevant models
- > Learning models from data (including physical insights when available)



Drivers for data-processing / data-analytics:

Providing the tools for **online**

Model estimation / calibration / adaptation

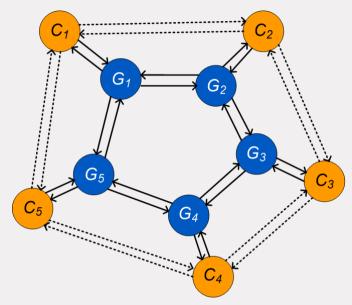
to accurately perform online model-based X:

- Monitoring
- Diagnosis and fault detection
- Control and optimization
- Predictive maintenance
- •





Distributed / multi-agent control:

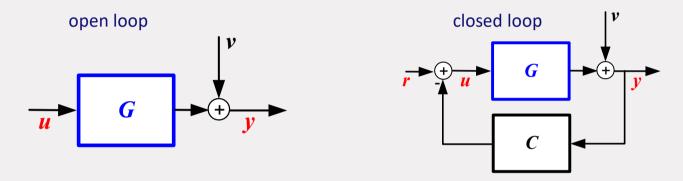


With both physical and communication links between systems G_i and controllers C_i

How to address data-driven modelling problems in such a setting?



The classical (multivariable) data-driven modeling problems [1]:



Identify a model of G on the basis of measured signals u, y (and possibly r), focusing on *continuous LTI dynamics*.

In interconnected systems (networks) the **structure / topology** becomes important to include



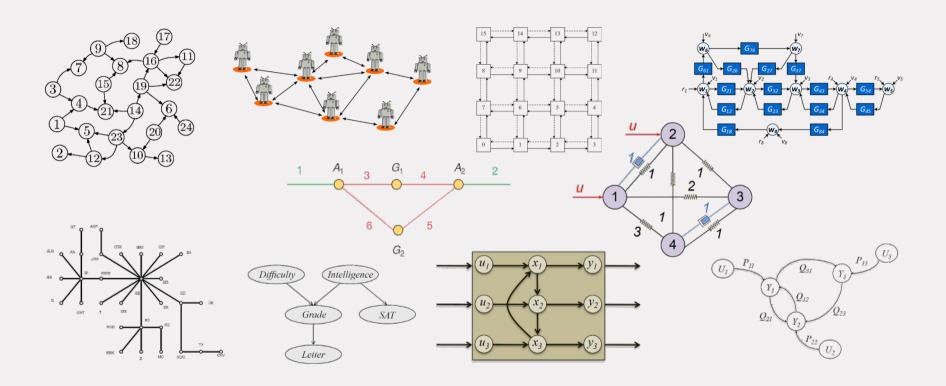


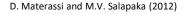
Contents

- Introduction and motivation
- How to model a dynamic network?
- Single module identification
- Global network identification
- Diffusively coupled networks
- Extensions Discussion

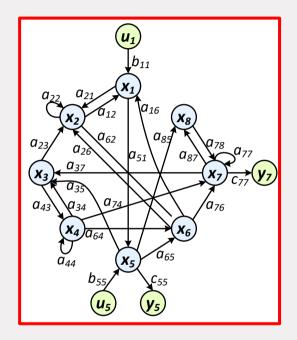


Dynamic networks for data-driven modeling







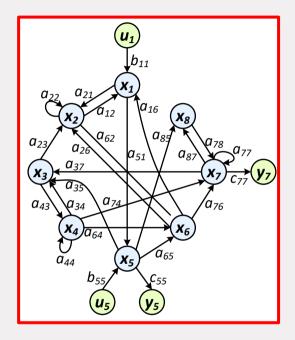


State space representation

$$x(k+1) = Ax(k) + Bu(k)$$
$$y(k) = Cx(k) + Du(k)$$

- States as **nodes** in a (directed graph)
- State transitions (1 step in time) reflected by a_{ij}
- Transitions are encoded in links
- Effect of transitions are summed in the nodes
- Self loops are allowed
- Actuation (u) and sensing (y) reflected by separate links





State space representation

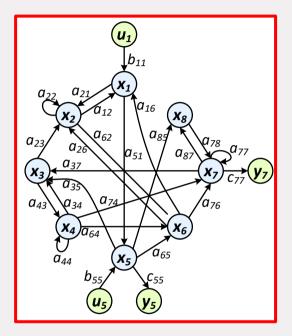
$$x(k+1) = Ax(k) + Bu(k)$$
$$y(k) = Cx(k) + Du(k)$$

- Ultimate break-down of structure in the system
- to smallest possible level of detail

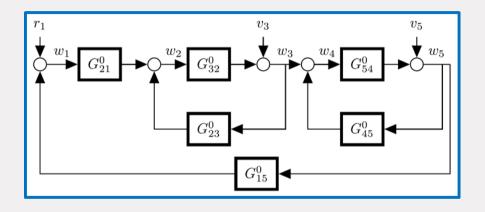
For data-driven modeling problems:

- Stronger role for measurable inputs and outputs
- i/o dynamics can be lumped in dynamic modules





State space representation [1]



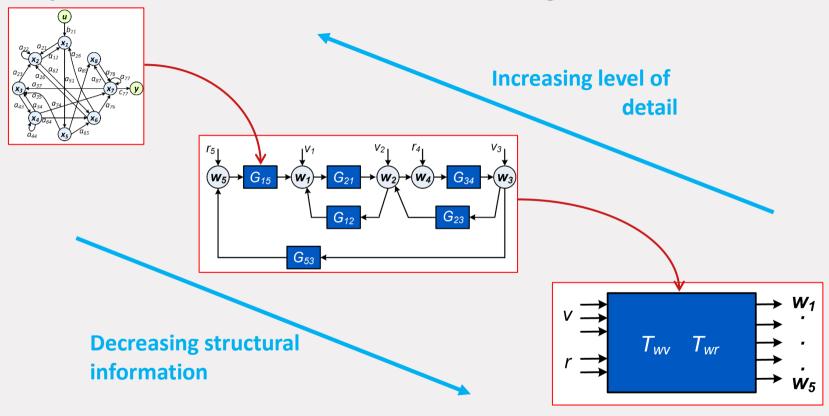
Module representation [2]



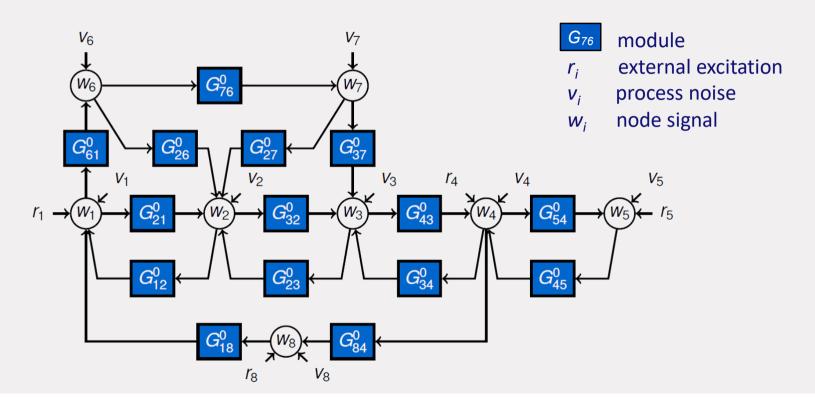




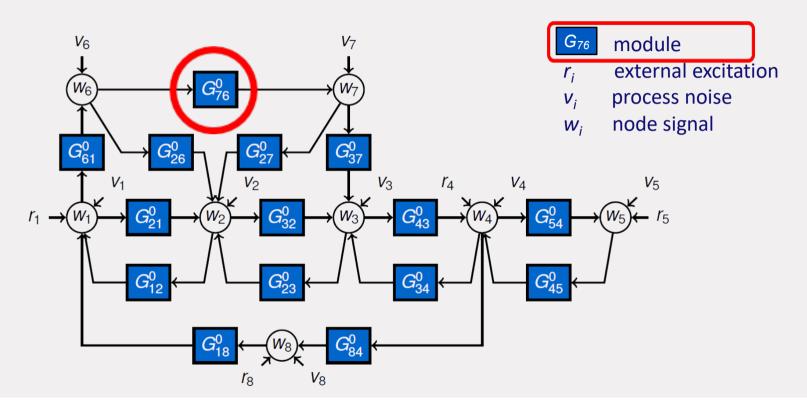
Dynamic network models - zooming



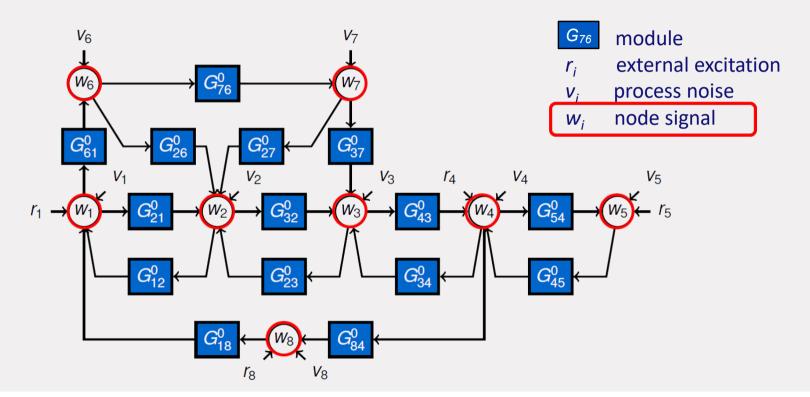




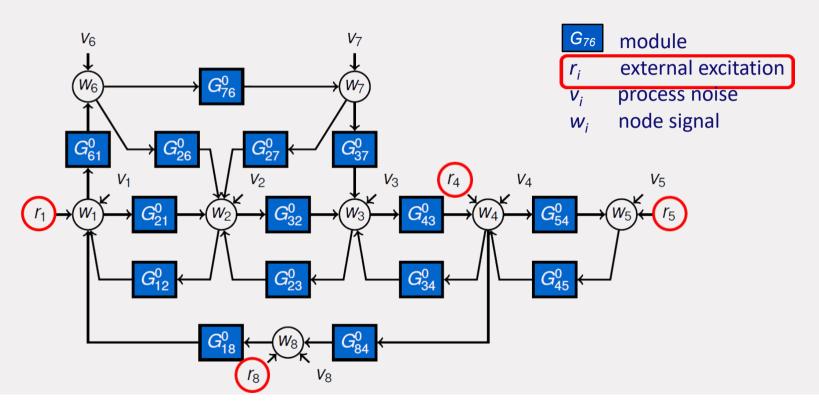




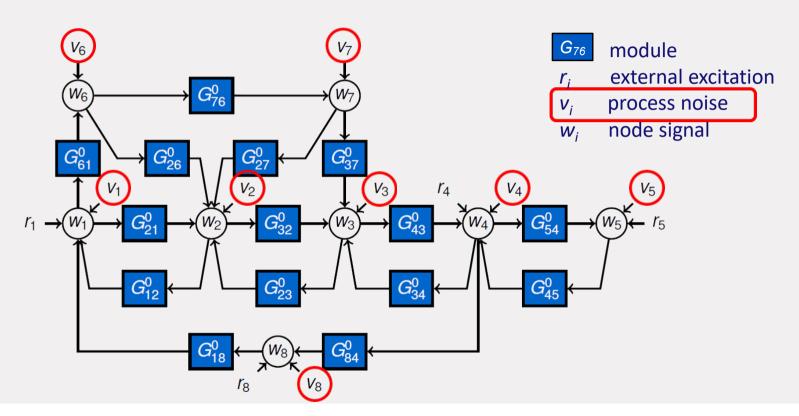














Basic building block:

$$w_j(t) = \sum_{k \in \mathcal{N}_j} G^0_{jk}(q) w_k(t) + r_j(t) + v_j(t)$$

 w_j : node signal

 r_j : external excitation signal

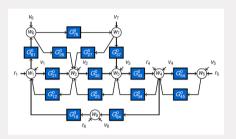
 v_j : (unmeasured) disturbance, stationary stochastic process

 G^0_{jk} : module, rational proper transfer function, $\mathcal{N}_j \subset \{\mathbb{Z} \cap [1,L] ackslash \{j\}\}$

q: shift operator, $q^{-1}w(t)=w(t-1)$

Node signals: $w_1, \cdots w_L$

Interconnection structure / topology of the network is encoded in $\mathcal{N}_j,\ j=1,\cdots L$





Collecting all equations:

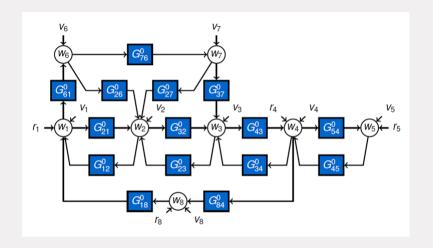
$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} = \begin{bmatrix} 0 & G_{12}^0 & \cdots & G_{1L}^0 \\ G_{21}^0 & 0 & \cdots & G_{2L}^0 \\ \vdots & \ddots & \ddots & \vdots \\ G_{L1}^0 & G_{L2}^0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + R^0 \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_K \end{bmatrix} + H^0 \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_p \end{bmatrix}$$

Network matrix $G^0(q)$

$$w(t)=G^0(q)w(t)+R^0(q)r(t)+v(t); \hspace{0.5cm} v(t)=H^0(q)e(t); \hspace{0.5cm} cov(e)=\Lambda$$

- Typically ${m R}^{m 0}$ is just a (static) selection matrix, indicating which nodes have an excitation signal.
- The topology of the network is encoded in the structure (non-zero entries) of G^0 .
- r and e are called external signals.



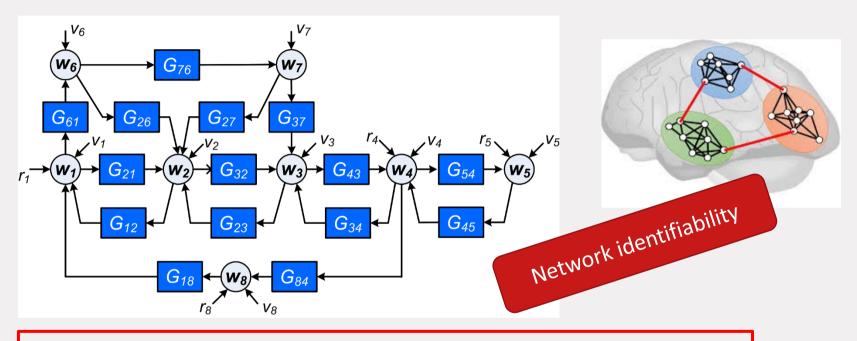


Many challenging data-driven modeling questions can be formulated

Measured time series:

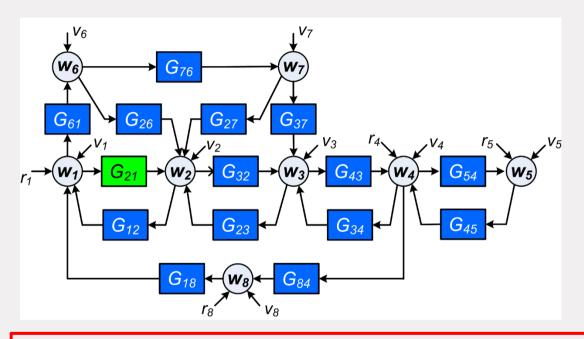
$$\{w_i(t)\}_{i=1,\cdots L}; \ \{r_j(t)\}_{j=1,\cdots K}$$





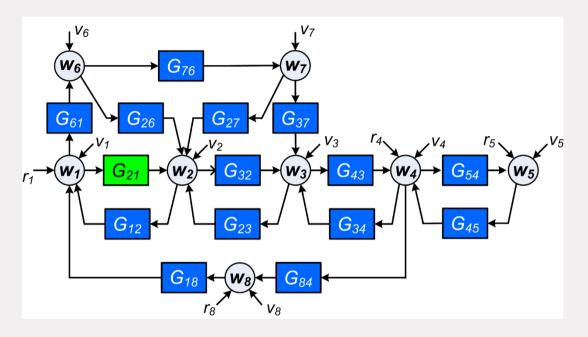
Under which conditions can we estimate the topology and/or dynamics of the full network?





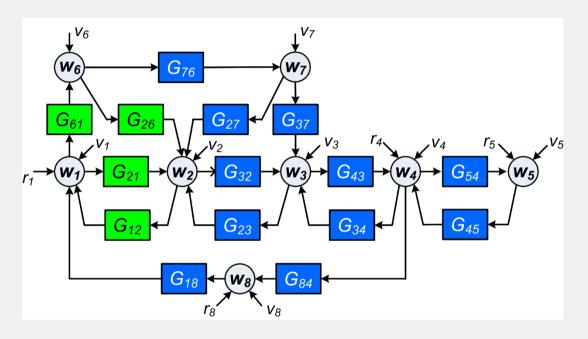
How/when can we learn a local module from data (with known/unkown network topology)? Which signals to measure?





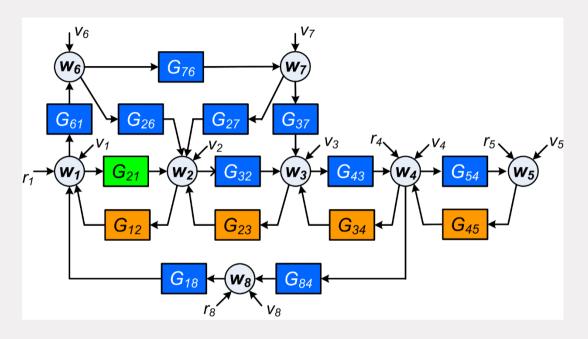
Where to optimally locate sensors and actuators?





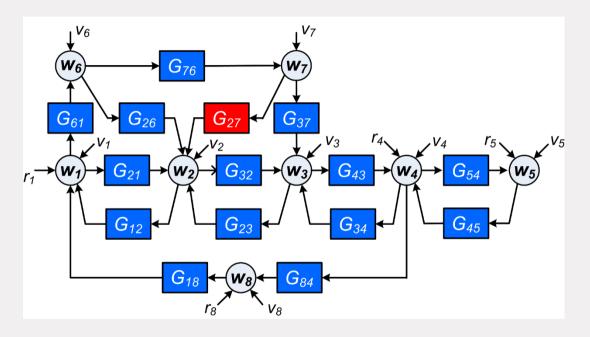
Same questions for a subnetwork





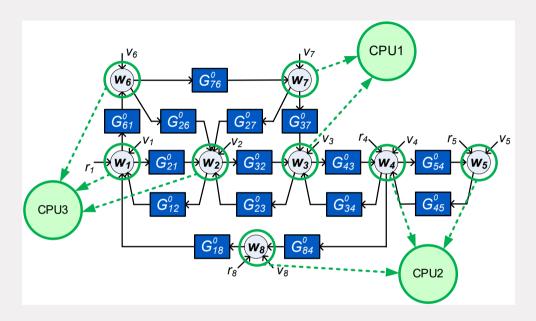
How can we benefit from known modules?





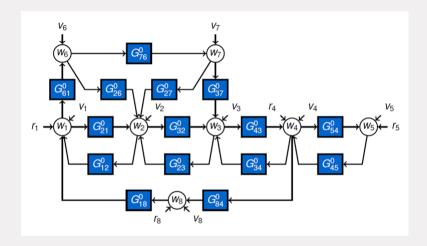
Fault detection and diagnosis; detect/handle nonlinear elements





Can we distribute the computations?





Measured time series:

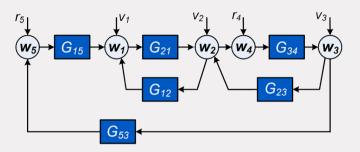
$$\{w_i(t)\}_{i=1,\dots L}; \ \{r_j(t)\}_{j=1,\dots K}$$

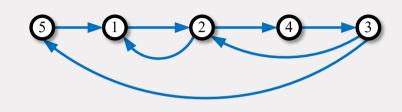
Many challenging data-driven modeling questions can be formulated

- Identification of a local module (known topology)
- Identification of the full network
- Topology estimation
- Identifiability
- Sensor and excitation allocation
- Fault detection
- User prior knowledge of modules
- Distributed identification
- Scalable algorithms



Dynamic network setup – directed graph

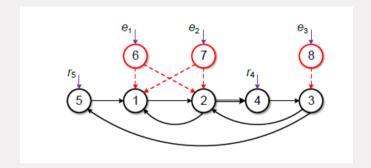




Nodes are vertices; modules/links are edges

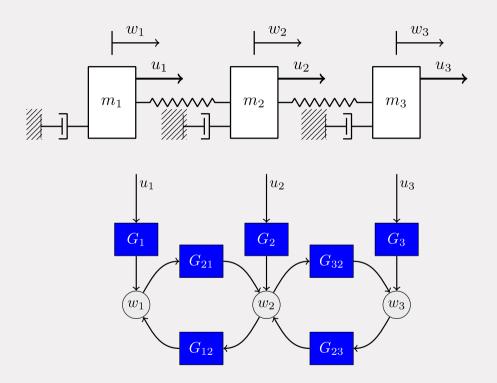
Extended graph:

including the external signals and disturbance correlations





Application: Networks of (damped) oscillators



- Power systems, vehicle platoons, thermal building dynamics, ...
- Spatially distributed
- Bilaterally coupled



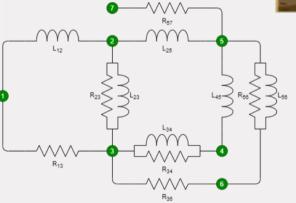
Application: Printed Circuit Board (PCB) Testing

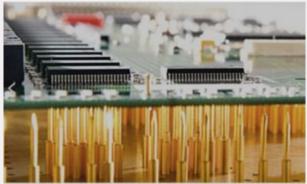


Detection of

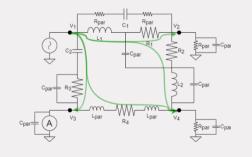
- component failures
- parasitic effects















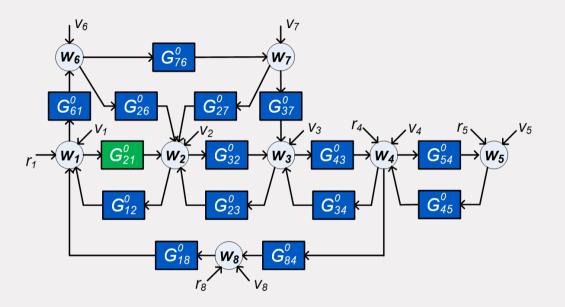
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Single module identification

Single module identification

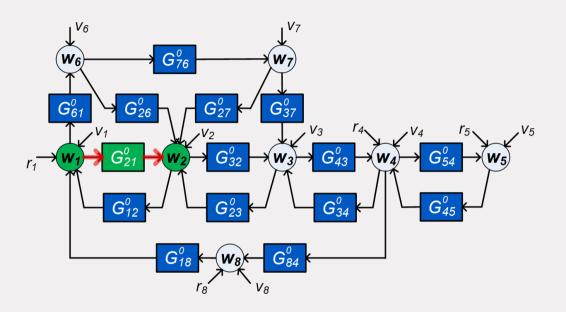


For a network with known topology:

- Identify G_{21}^0 on the basis of measured signals
- Which signals to measure?
 Preference for local measurements
- When is there enough excitation / data informativity?

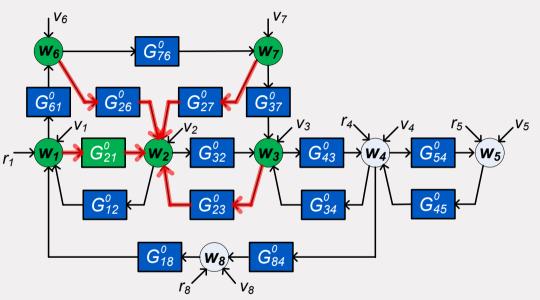


Single module identification



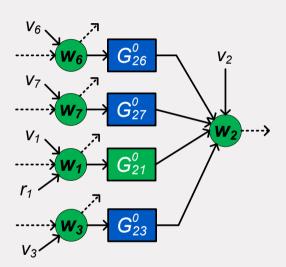
Naïve approach: identify based on w_1 and w_2 : in general does not work.





If noises v_k are correlated it may even be part of a MIMO problem

Identifiying G_{21}^0 is part of a 4-input, 1-output problem





Identifying G_{21}^0 is part of a 4-input, 1-output problem

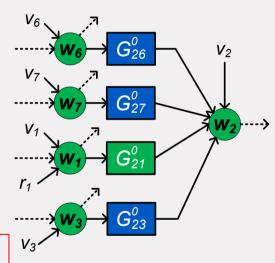
Input signals will be correlated:

similar as in a closed-loop situation

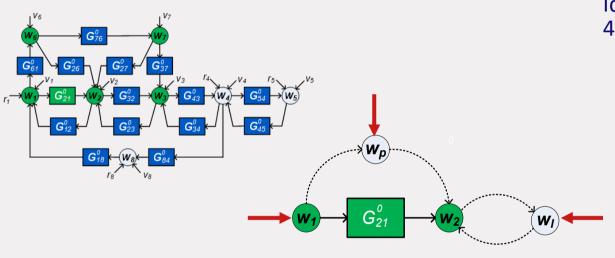
What is required for identifiability / data informativity?

Ability to distinguish between models independent of id-method

Information content of signals dependent on id-method

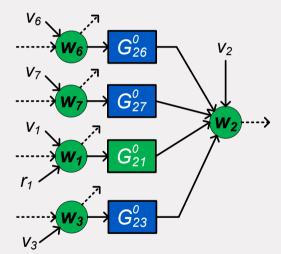






All parallel paths, and loops around the output, plus input w_1 should have an independent external signal r or v and typically need to be blocked by a measured node

Identifying G_{21}^0 is part of a 4-input, 1-output problem

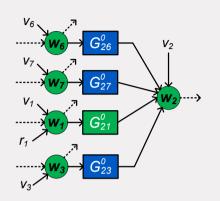




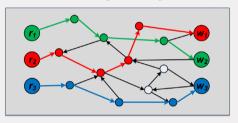
^[1] Weerts et al., Automatica 2018, CDC 2018

^[2] Bazanella et al. CDC2017; Hendrickx et al., IEEE-TAC, 2019.

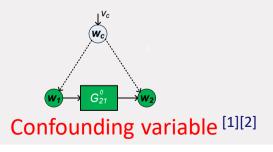
^[3] Dankers et al., TAC 2016



All inputs require an independent excitation (through vertex disjoint paths) from $r,\,e$



If excitation is relying on disturbances and correlated to $oldsymbol{v}_2$

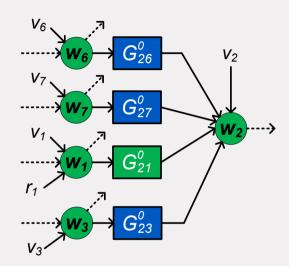


To be handled by:

- Adding more input signals (blocking the cv)
- Including the input as output (MIMO) [3]



Typical solution:



- MISO (sometimes MIMO) estimation problem
- to be solved by any (closed-loop) identification algorithm, e.g. direct/indirect method

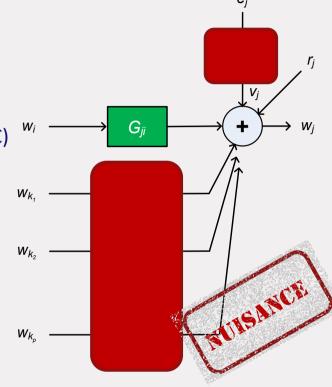


Machine learning in local module identification

- MISO identification with all modules parameterized
- Brings in two major problems :
 - Large number of parameters to estimate
 - Model order selection step for each module (CV, AIC, BIC)
- For 5 modules, combinations = 244,140,625

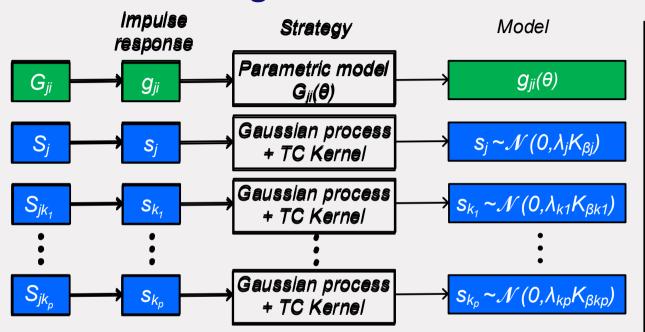


We need only the target module. No NUISANCE!





Machine learning in local module identification



- smaller no. of parameters
- simpler model order selection step
- scalable to large dynamic networks
- simpler optimization problems to estimate parameters

Maximize marginal likelihood of output data: $\hat{\eta} = \underset{n}{\operatorname{argmax}} p(w_j; \eta)$

$$\eta \coloneqq \begin{bmatrix} \theta & \lambda_j & \lambda_{k_1} & ... & \lambda_{k_p} & \beta_j & \beta_{k_1}^{\eta} & ... & \beta_{k_p} & \sigma_j^2 \end{bmatrix}^{\mathsf{T}}$$

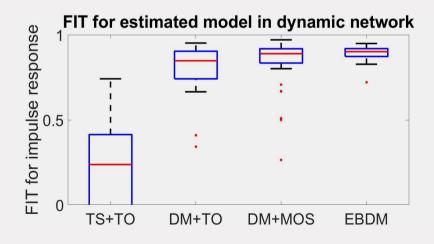


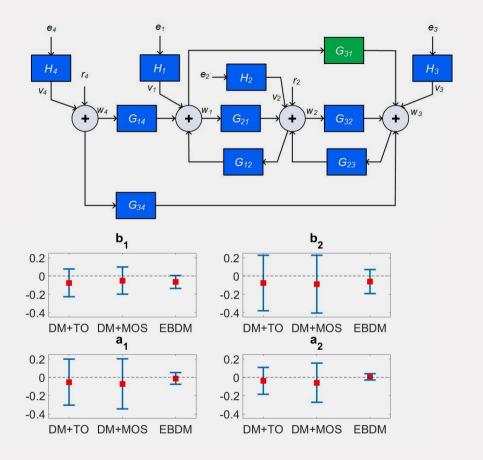
^[1] Everitt et al., Automatica 2017.

^[2] K.R. Ramaswamy et al., Automatica, 2021.

Numerical simulation

- Identify G_{31} given data
- ▶ 50 independent MC simulation
- ▶ Data = 500







Summary single module identification

- Path-based conditions for network identifiability (where to excite?)
- Graph tools for checking conditions
- Degrees of freedom in selection of measured signals sensor selection
- Methods for consistent and minimum variance module estimation, and effective (scalable) algorithms
- A priori known modules can be accounted for

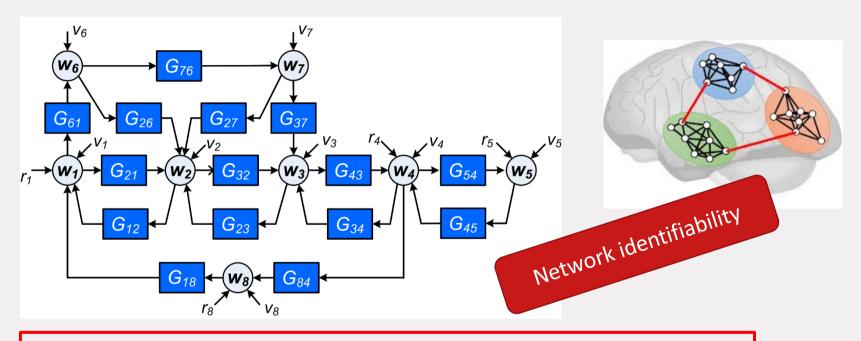




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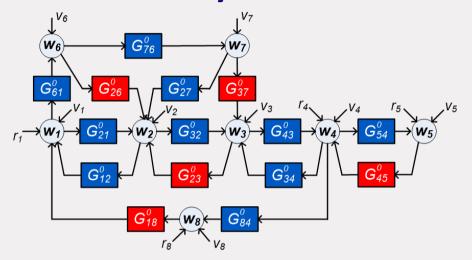
Full network identification



Under which conditions can we estimate the topology and/or dynamics of the full network?



Network identifiability



blue = unknown red = known

Question: Can different dynamic networks be *distinguished* from each other from measured signals *w*, *r*?



Network identifiability

The identifiability problem:

The network **model**:

$$w(t) = G(q)w(t) + R(q)r(t) + \underbrace{H(q)e(t)}_{v(t)}$$

can be transformed with any rational P(q):

$$\begin{split} & P(q)w(t) = P(q)\{G(q)w(t) + R(q)r(t) + H(q)e(t)\} \\ & w(t) = (I - P(q))w(t) + P(q)\{G(q)w(t) + R(q)r(t) + H(q)e(t)\} \end{split}$$

to an **equivalent model**:

$$w(t) = ilde{G}(q)w(t) + ilde{R}(q)r(t) + ilde{H}(q)e(t)$$

Nonuniqueness, unless there are structural constraints on G, R, H.



^[1] Weerts, Linder et al., Automatica, 2019.

^[2] Bottegal et al., SYSID 2018

Network identifiability

Consider a **network model set**:

$$\mathcal{M} = \{(G(\theta), R(\theta), H(\theta))\}_{\theta \in \Theta}$$

representing structural constraints on the considered models:

- modules that are fixed and/or zero (topology)
- locations of excitation signals
- disturbance correlation

Generic identifiability of ${\mathcal M}$:

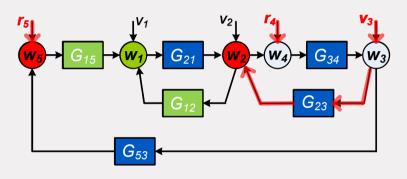
- There do not exist distinct equivalent models (generating the same data)
- for almost all models in the set.



^[1] Weerts et al., SYSID2015; Weerts et al., Automatica, March 2018;

Example 5-node network

Conditions for identifiability rank conditions on transfer function



Full row rank of

$$\begin{bmatrix} v_3 \\ r_4 \\ r_5 \end{bmatrix} {\longrightarrow} \begin{bmatrix} w_2 \\ w_5 \end{bmatrix}$$

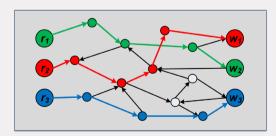
For the **generic case**, the rank can be calculated by a graph-based condition^{[1],[2]}:

Generic rank = number of vertex-disjoint paths

2 vertex-disjoint paths → full row rank 2



The rank condition has to be checked for all nodes.



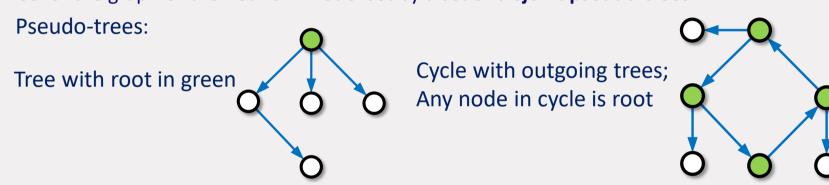
^[1] Van der Woude, 1991

^[2] Hendrickx, Gevers & Bazanella, CDC 2017, TAC 2019

Synthesis solution for network identifiability

Allocating external signals for generic identifiability:

1. Cover the graph of the network model set by a set of disjoint pseudo-trees



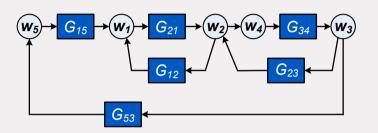
Edges are disjoint and all out-neighbours of a node are in the same pseudo-tree

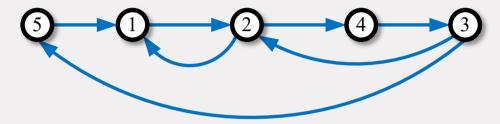
2. Assign an independent external signal ($m{r}$ or $m{e}$) at a root of each pseudo-tree.

This guarantees generic identifiability of the model set.



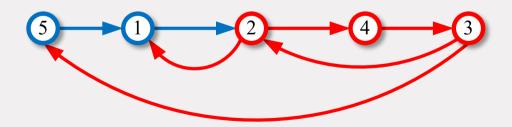
Where to allocate external excitations for network identifiability?





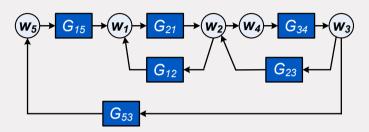
All indicated modules are parametrized

Two disjoint pseudo-trees

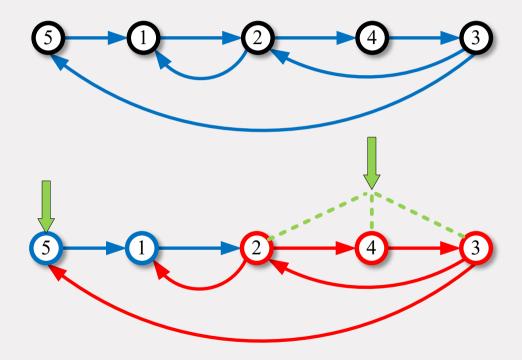




Where to allocate external excitations for network identifiability?

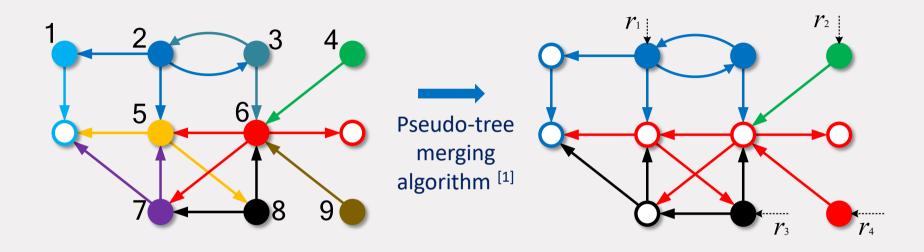


Two independent excitations guarantee generic network identifiability





Where to allocate external excitations for network identifiability?



- Nodes are signals w and external signals (r,e) that are input to parametrized link
- Known (nonparametrized) links do not need to be covered



Summary identifiability of full network

Identifiability of network model sets is determined by

- Presence and location of external signals, and
- Correlation of disturbances
- Topology of parametrized modules
- Graphic-based tool for synthesizing allocation of external signals

Extensions:

Situations where not all node signals are measured [1]



Algorithms for identification of full network

(Prediction error) identification methods will typically lead to large-scale **non-convex** optimization problems

Convex relaxation algorithms are being developed^[1,2] as well as machine learning tools

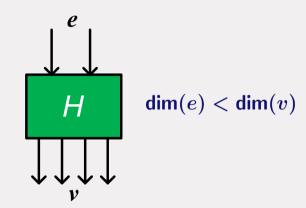


^[1] Weerts, Galrinho et al., SYSID 2018

Algorithms for identification of full network

Particular feature for larger networks:

Modeling disturbances as a **reduced rank process**: (cf dynamic factor analysis^[1])



Consequences for **estimation**^[3,4]:

- Optimization becomes a constrained quadratic problem with ML properties for Gaussian noise
- Reworked Cramer Rao lower bound
- Some parameters can be estimated variance free → regularization effect

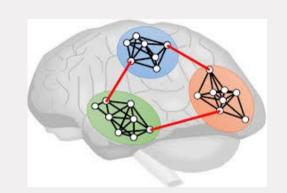


^[1] Deistler et al., EJC, 2010.

^[2] Zorzi and Chiuso, Automatica 2017.

Topology identification

- Topology resulting from full dynamic model
- Alternative: non-parametric models (Wiener filters [1])
 or kernel-based approaches [2][3]



- modeling module dynamics by Gaussian processes,
 kernel with 2 parameters for each dynamic module
- Optimizing likelihood of the data as function of parameters and topology:

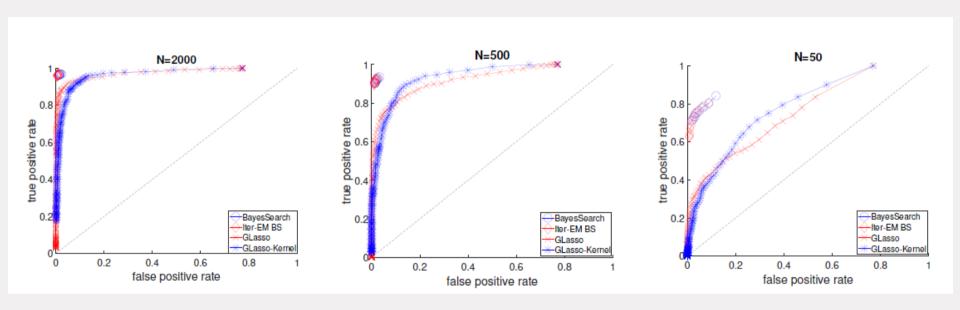
$$p(\{w(t)\}_{t=1}^N | \theta, \mathcal{G})$$

[3] Shi, Bottegal, PVdH, ECC 2019

Forward-backward search over topologies + empirical Bayes (EM) for parameters



Topology identification

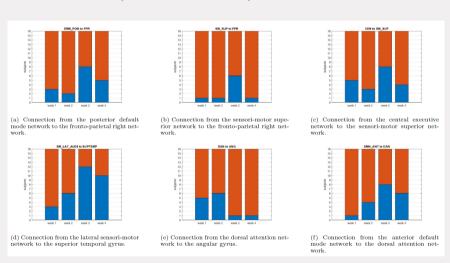


50 MC realizations of network with 6 nodes.



Neurodynamic effect of listening to Mozart music

Identifying changes in network connections in the brain, after intensely listening for one week (Sonate K448)



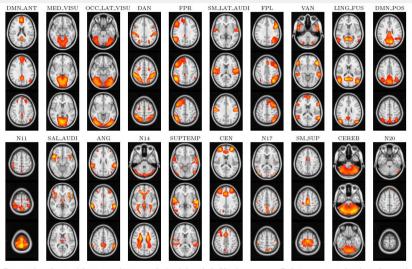


Figure 3: Spatial maps of the 20 active brain networks found through the ICA decomposition. Each image consists of 3 relevant horizontal slices of the brain, where the spatial map is indicated by the red color scale.





Contents

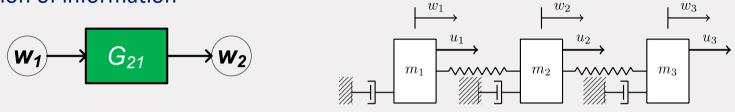
- Introduction and motivation
- How to model a dynamic network?
- Single module identification
- Global network identification
- Diffusively coupled networks
- Extensions Discussion



Diffusively coupled networks

Back to the basics of physical interconnections

In connecting physical systems, there is often no predetermined direction of information [1]



Example: resistor / spring connection in electrical / mechanical system:

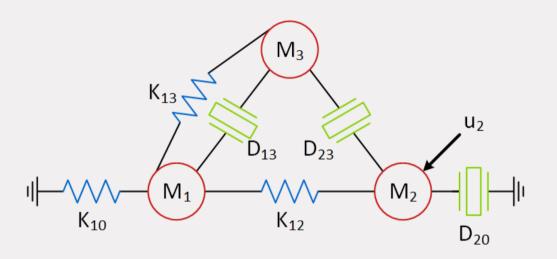
Resistor Spring
$$I = \frac{1}{R}(V_1 - V_2)$$

$$F = K(x_1 - x_2)$$

Difference of node signals drives the interaction: diffusive coupling



Diffusively coupled physical network



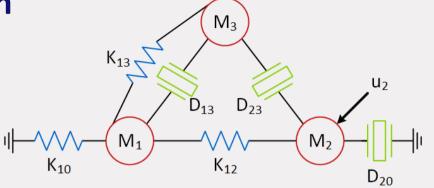
Equation for node *j*:

$$M_j \ddot{w}_j(t) + D_{j0} \dot{w}_j(t) + \sum_{k
eq j} D_{jk} (\dot{m{w}}_j(t) - \dot{m{w}}_k(t)) + K_{j0} w_j(t) + \sum_{k
eq j} K_{jk} (m{w}_j(t) - m{w}_k(t)) = u_j(t),$$



Mass-spring-damper system

- Masses M_j
- Springs K_{ik}
- Dampers D_{jk}
- Input u_j



$$\begin{bmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{bmatrix} \begin{bmatrix} \ddot{w}_1 \\ \ddot{w}_2 \\ \ddot{w}_3 \end{bmatrix} + \begin{bmatrix} 0 & \\ & D_{20} \\ & & \\ & & \end{bmatrix} \begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \end{bmatrix} + \begin{bmatrix} K_{10} & \\ & 0 \\ & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \\ & + \begin{bmatrix} D_{13} & 0 & -D_{13} \\ 0 & D_{23} & -D_{23} \\ -D_{13} & -D_{23} & D_{13} + D_{23} \end{bmatrix} \begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \end{bmatrix} + \begin{bmatrix} K_{12} + K_{13} & -K_{12} & -K_{13} \\ -K_{12} & K_{12} & 0 \\ -K_{13} & 0 & K_{13} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 0 \\ u_2 \\ 0 \end{bmatrix}$$

$$\left[egin{array}{cccc} A(p) &+& B(p) \ diagonal & Laplacian \end{array}
ight] w(t) = u(t) \qquad A(p), B(p) \;\; {
m polynomial} \qquad p = rac{d}{dt}$$



Mass-spring-damper system

$$\left[egin{array}{cccc} A(p) &+& B(p) \ diagonal & Laplacian \end{array}
ight] w(t) = u(t) \qquad A(p), B(p) ext{ polynomial}$$

$$[\underbrace{Q(p)}_{diagonal} - \underbrace{P(p)}_{hollow\&symmetric}] \ w(t) = u(t)$$

This fully fits in the earlier module representation:

$$w(t) = Gw(t) + \underbrace{Rr(t) + He(t)}_{Q^{-1}(p)u(t)}$$

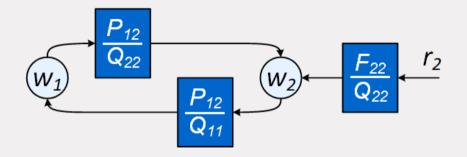
with the additional condition that:

$$G(p) = Q(p)^{-1} P(p)$$
 $Q(p), P(p)$ polynomial $P(p)$ symmetric, $Q(p)$ diagonal



Module representation

Consequences for node interactions:



- Node interactions come in pairs of modules
- Where numerators are the same

Framework for network identification remains the same

Symmetry can simply be incorporated in identification



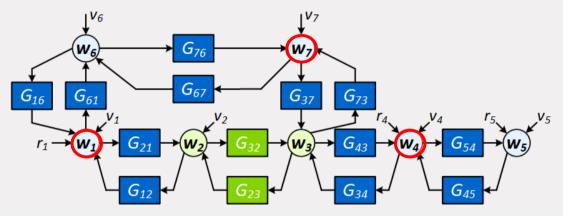
Local network identification

Identification of **one** (physical) interconnection Identification of **two** modules G_{jk} and G_{kj} G₃₇ G₇₃ G_{61}



Immersion conditions

For simultaneously identifying two modules in one interconnection:



The parallel path and loops-around-the-output condition, now simplifies to:

Measuring/exciting all neighbouring nodes of w_2 and w_3 leads to a solution



Summary diffusively coupled networks

- Diffusively coupled networks fit within the module framework (special case)
 - no restriction to second order equations
- Earlier identification framework can be utilized
- Local identification is well-addressed (and stays really local)
- Framework is fit for representing **cyber-physical systems** (combining physical bi-directional links, and cyber uni-directional links).





Extensions - Discussion

Extensions - Discussion

- Including sensor noise [1]
 - Errors-in-variabels problems can be more easily handled in a network setting
- Distributed estimation (MISO models) [2]
 - Communication constraints between different agents
 - Recursive (distributed) estimator converges to global optimizer (more slowly)

- Experiment design [3],[4]
 - design of least costly experiments



[3] Gevers and Bazanella, CDC 2015. [4] Morelli, Bombois et al., ECC 2019;

[2] Steentjes et al., IFAC-NECSYS, 2018.

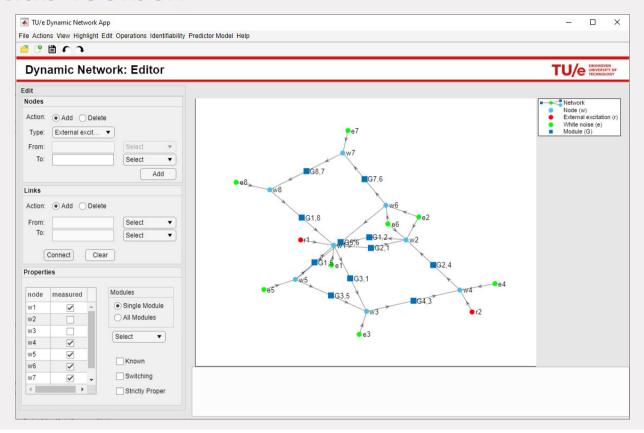


Summary

- Dynamic network modeling:
 - intriguing research topic with many open questions
- The (centralized) LTI framework is only just the beginning
- Further move towards data-aspects related to distributed control
- and large-scale aspects
- and more real-life applications (diagnostics, fault detection)



Matlab Toolbox





ERC SYSDYNET Team: data-driven modeling in dynamic networks

Research team:



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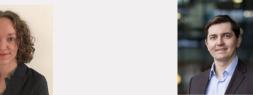
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