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A dynamic network approach to identification of physical systems

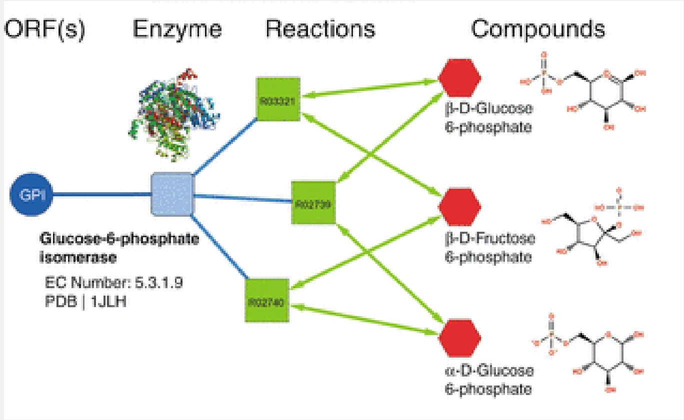
E.M.M. (Lizan) Kivits and Paul M.J. Van den Hof

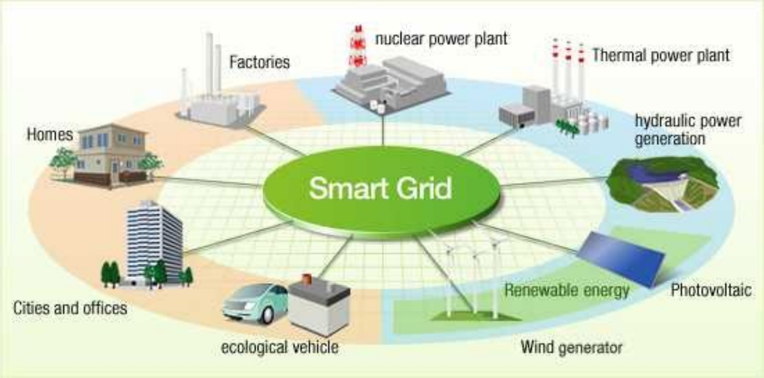
12-12-2019

Department of Electrical Engineering

CDC 2019

Dynamic networks





www.envidia.com
www.betterworldsolutions.eu

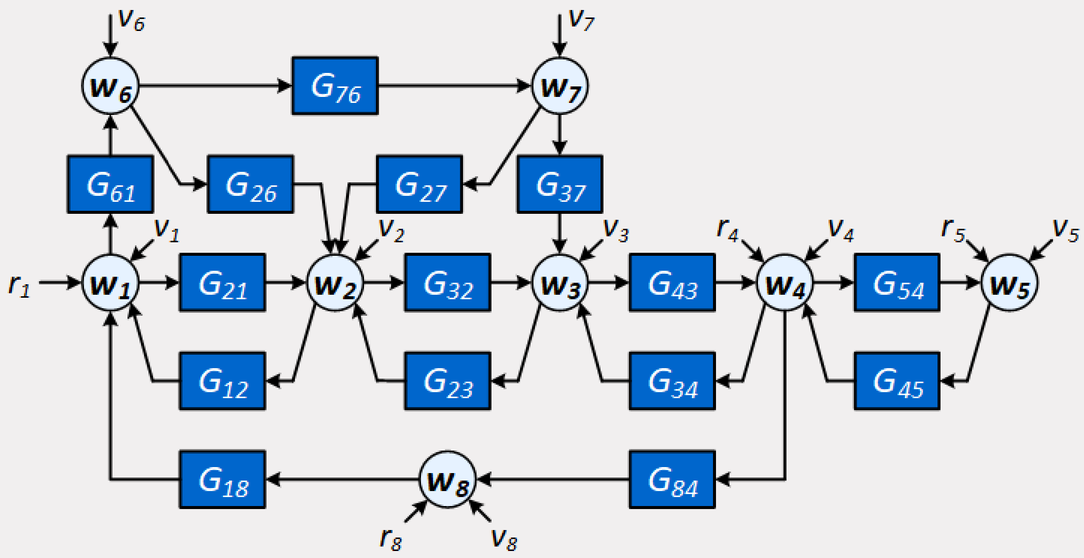
G. Chalancon et al (2013)
www.globallyrecruit.com

Module representation

r_i external excitation
v_i process noise
w_i node signal
G_{ij} module

$$G = \begin{bmatrix} 0 & G_{12} & \cdots & G_{1L} \\ G_{21} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & G_{L-1,L} \\ G_{L1} & \cdots & G_{L,L-1} & 0 \end{bmatrix}$$

$v = He$, white noise e



$$w(t) = G(q)\,w(t) + R(q)\,r(t) + H(q)\,e(t)$$

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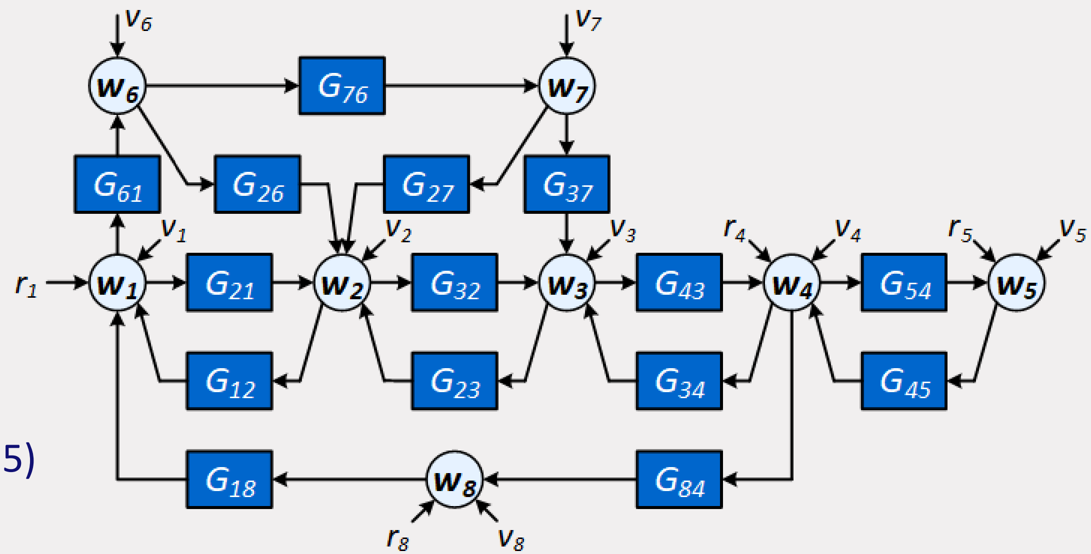
Identification in dynamic networks



Topology estimation

D. Materassi & G. Innocenti (2010)
A. Chiuso & G. Pillonetto (2012)

Full network identification

J. Gonçalves et al (2007)
H. H. M. Weerts et al (2018b)

Local network identification

P. M. J. Van den Hof et al (2013)
D. Materassi & M. V. Salapaka (2015)
A. Dankers et al (2016)
M. Gevers et al (2018)
K. R. Ramaswamy & P. M. J. Van den Hof (2019)

Identifiability

H. H. M. Weerts et al (2015, 2018a)
H. J. van Waarde et al (2018)
M. Gevers et al (2019)

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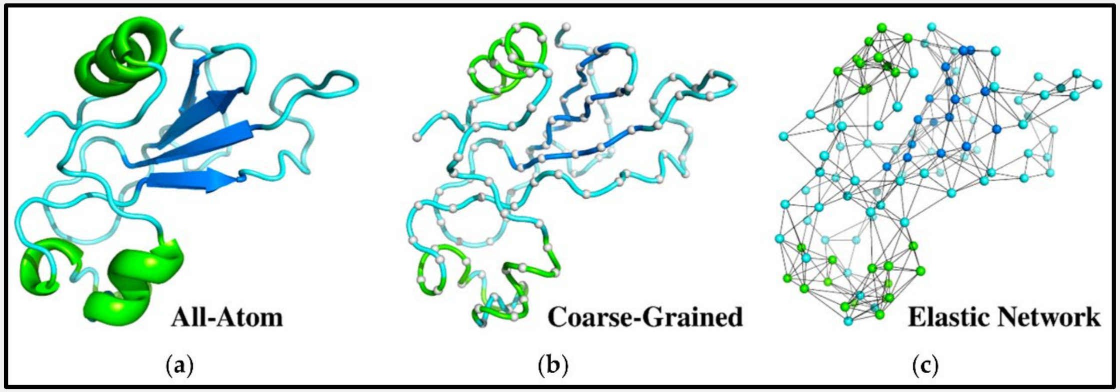
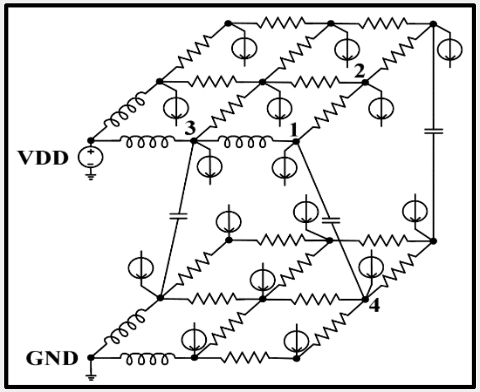
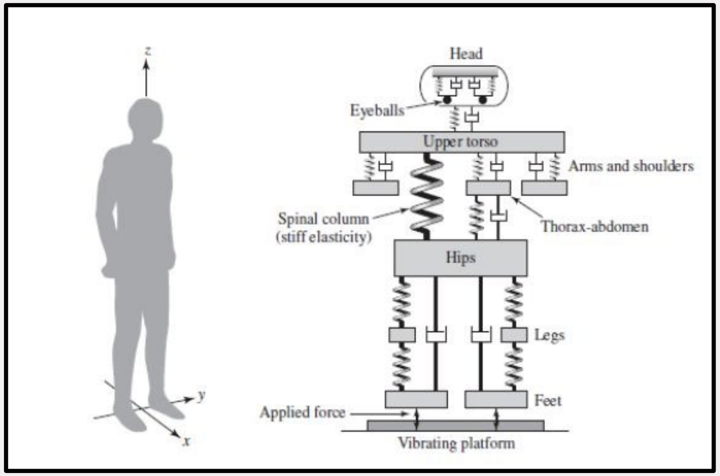
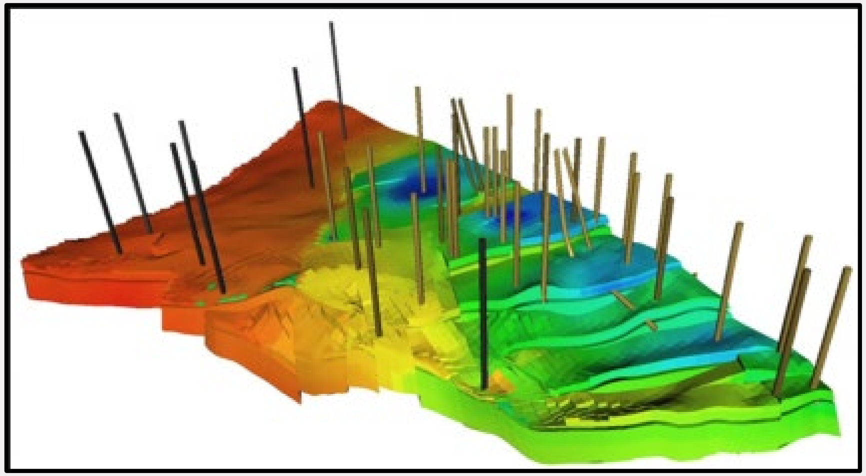
Physical systems









Protein structure: Kmiecik et al (2018)
Human body: M.P. Norton (1989)

Power grid: X. Xiong and J. Wang (2011)
Hydrocarbon reservoirs: Mansoori (2014)

Basics of physical interconnections

No pre-determined direction of information[1]

Resistor

$$I = \frac{1}{R}(V_1 - V_2)$$

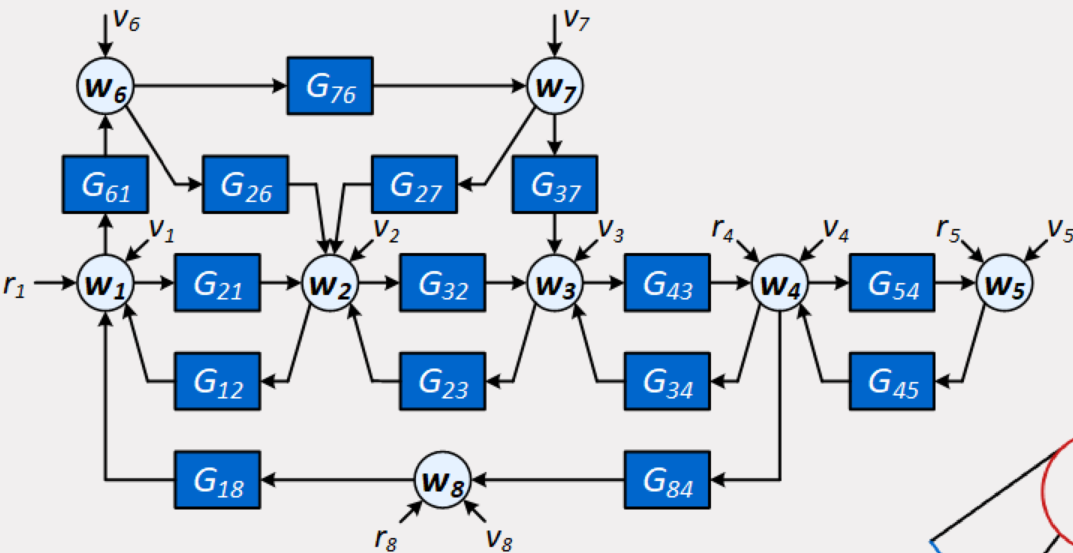
Spring

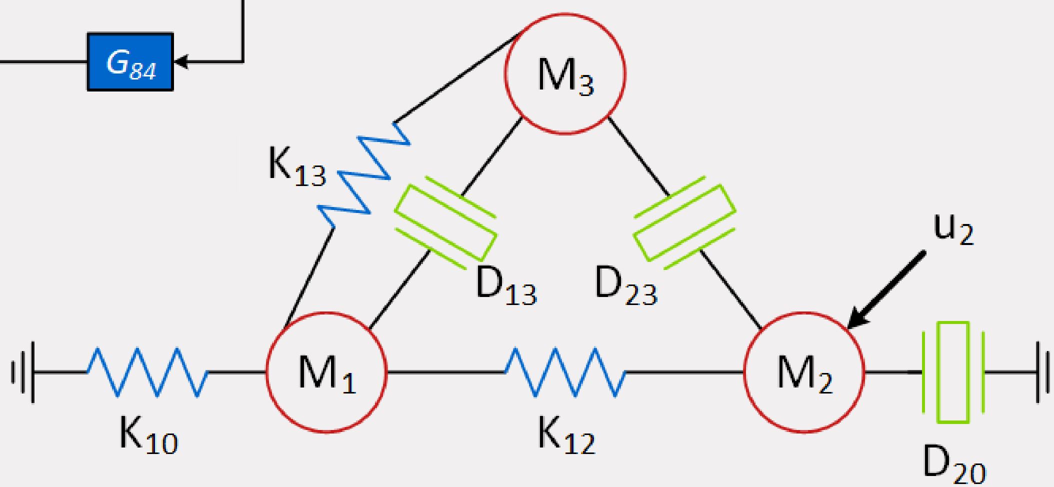
$$F = K(x_1 - x_2)$$

Interaction through difference of node signals: Diffusive coupling

[1] J.C. Willems (2010)

Identification in physical systems?





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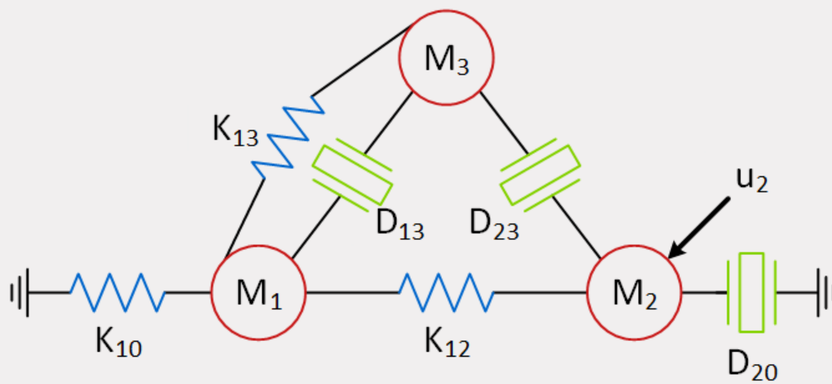
Contents

Contents

Mass-spring-damper system

- Masses M_j
- Springs K_{jk}
- Dampers D_{jk}
- Input u_j
- Positions $w_j(t)$



$$M_j\ddot{w}_j(t) + D_{j0}\dot{w}_j(t) + \sum_{k\neq j} D_{jk}\left(\dot{w}_j(t) - \dot{w}_k(t)\right) + K_{jo}w_j(t) + \sum_{k\neq j} K_{jk}\left(w_j(t) - w_k(t)\right) = u_j(t)$$

$$\begin{bmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{bmatrix}\begin{bmatrix}\ddot{w}_1\\ \ddot{w}_2\\ \ddot{w}_3\end{bmatrix} + \begin{bmatrix} 0 & & \\ & D_{20} & \\ & & 0 \end{bmatrix}\begin{bmatrix}\dot{w}_1\\ \dot{w}_2\\ \dot{w}_3\end{bmatrix} + \begin{bmatrix} K_{10} & & \\ & 0 & \\ & & 0 \end{bmatrix}\begin{bmatrix}w_1\\ w_2\\ w_3\end{bmatrix}$$

$$+\begin{bmatrix} D_{13} & 0 & -D_{13} \\ 0 & D_{23} & -D_{23} \\ -D_{13} & -D_{23} & D_{13}+D_{23} \end{bmatrix}\begin{bmatrix}\dot{w}_1\\ \dot{w}_2\\ \dot{w}_3\end{bmatrix} + \begin{bmatrix} K_{12}+K_{13} & -K_{12} & -K_{13} \\ -K_{12} & K_{12} & 0 \\ -K_{13} & 0 & K_{13} \end{bmatrix}\begin{bmatrix}w_1\\ w_2\\ w_3\end{bmatrix} = \begin{bmatrix}0\\ u_2\\ 0\end{bmatrix}$$

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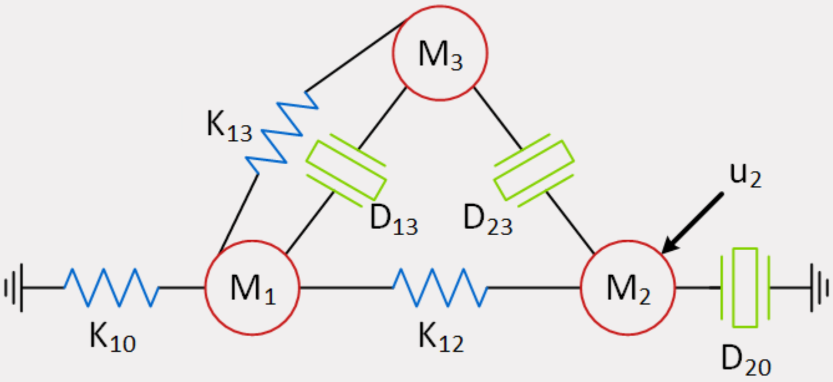
Mass-spring-damper system

- Masses M_j
- Springs K_{jk}
- Dampers D_{jk}
- Input u_j
- Positions $w_j(t)$



$$M\,\ddot{w}(t) + D_0\,\dot{w}(t) + K_0\,w(t) + D\,\dot{w}(t) + K\,w(t) = u(t)$$

$$(M\,p^2 + D_0\,p + K_0)\,w(t) + (D\,p + K)\,w(t) = u(t)$$

$$B(p)\,w(t) + A(p)\,w(t) = u(t)$$

any order

Physical network

$$[B(p) + A(p)]\, w(t) = u(t)$$

$B(p)$ diagonal polynomial

$A(p)$ Laplacian polynomial

$$[Q(p) - P(p)]\, w(t) = u(t)$$

$Q(p)$ diagonal polynomial

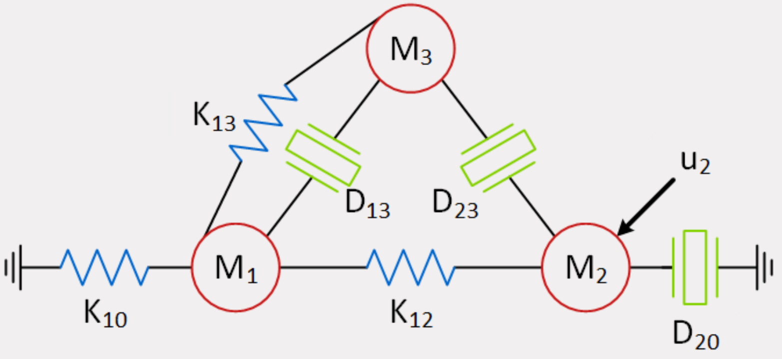
$P(p)$ hollow, symmetric polynomial

$$Q_{11} = M_1\, p^2 + D_{13}\, p + (K_{10} + K_{12} + K_{13})$$
$$Q_{22} = M_2\, p^2 + (D_{20} + D_{23})\, p + K_{12}$$
$$Q_{33} = M_3\, p^2 + (D_{13} + D_{23})\, p + K_{13}$$

Q: elements related to **node**

P: elements in **interconnection**

$$P = \begin{bmatrix} 0 & K_{12} & D_{13}\, p + K_{13} \\ K_{12} & 0 & D_{23}\, p \\ D_{13}\, p + K_{13} & D_{23}\, p & 0 \end{bmatrix}$$



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Discretisation

$[Q(p) - P(p)]\, w(t) = u(t)$

$Q(p)$ diagonal polynomial

$P(p)$ hollow, symmetric polynomial

$$\frac{dw(t)}{dt} = \frac{w(t_d T_s) - w\big((t_d - 1)T_s\big)}{T_s}$$

$[\bar{Q}(q) - \bar{P}(q)]\, w(t) = u(t)$

$\bar{Q}(q)$ diagonal polynomial

$\bar{P}(q)$ hollow, symmetric polynomial

Inputs

$[\bar{Q}(q) - \bar{P}(q)]\, w(t) = u(t)$

$\bar{Q}(q)$ diagonal polynomial

$\bar{P}(q)$ hollow, symmetric polynomial

$$[Q(q) - P(q)]\, w(t) = F\, r(t) + C(q)\, e(t)$$

$Q(q)$ diagonal polynomial

$P(q)$ hollow, symmetric polynomial

F diagonal, binary

$C(q)$ monic, stable, stably invertible

Contents

Physical network

Module representation

Full network identification

Local network identification

Summary

Module representation

$$[Q(q) - P(q)]\, w(t) = F\, r(t) + C(q)\, e(t)$$

$\boldsymbol{P}(q)$ hollow, symmetric
$\boldsymbol{Q}(q), \boldsymbol{F}$ diagonal

$$w(t) = \boldsymbol{Q}^{-1}(q)\boldsymbol{P}(q)\, w(t) + \boldsymbol{Q}^{-1}(q)\boldsymbol{F}\, r(t) + \boldsymbol{Q}^{-1}(q)\boldsymbol{C}(q)e(t)$$

Module representation

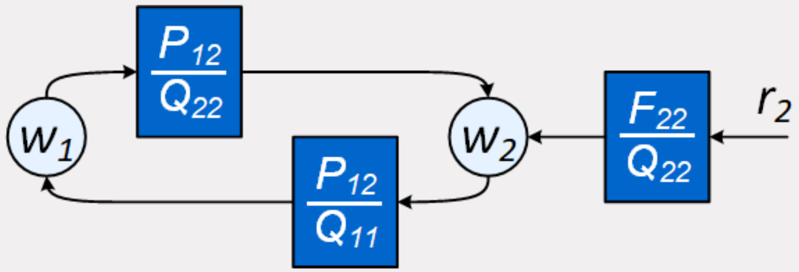
$$w(t) = \boldsymbol{G}(q)\, w(t) + \boldsymbol{R}(q)\, r(t) + \boldsymbol{H}(q)\, e(t)$$

$\boldsymbol{G}(q)$ hollow
$\boldsymbol{R}(q)$ diagonal

Physical networks are module representations with additional structural conditions

Module representation

$$\boldsymbol{G}(q) = \boldsymbol{Q}^{-1}(q)\boldsymbol{P}(q)$$
$$\boldsymbol{R}(q) = \boldsymbol{Q}^{-1}(q)\boldsymbol{F}$$



Q: elements related to **node**

P: elements in **interconnection**

Node interactions in pairs of modules

Framework for network identification remains the same

Incorporated symmetry in identification

Contents

Data generating network

$$\boldsymbol{Q}^0(q)\, w(t) = \boldsymbol{P}^0(q)\, w(t) + \boldsymbol{F}^0\, r(t) + \boldsymbol{C}^0(q)\, e(t)$$

$\boldsymbol{Q}^0(q)$ diagonal polynomial

$\boldsymbol{P}^0(q)$ hollow, symmetric polynomial

$\boldsymbol{F}^0$ diagonal, binary, known

$\boldsymbol{C}^0(q)$ monic, stable, stably invertible

$$w(t) = \boldsymbol{G}^0(q)\, w(t) + \boldsymbol{R}^0(q)\, r(t) + \boldsymbol{H}^0(q)\, e(t)$$

$\boldsymbol{G}^0(q) = (\boldsymbol{Q}^0)^{-1}(q)\boldsymbol{P}^0(q)$

$\boldsymbol{Q}^0(\infty) = \lim_{z\to\infty} Q^0(z)$

$\boldsymbol{R}^0(q) = (\boldsymbol{Q}^0)^{-1}(q)\boldsymbol{F}^0$

$\boldsymbol{H}^0(q) = (\boldsymbol{Q}^0)^{-1}(q)\boldsymbol{Q}^0(\infty)\boldsymbol{C}^0(q)$ monic, stable, stably invertible

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Parameterised model

$$w(t) = \boldsymbol{G}(q,\theta)\, w(t) + \boldsymbol{R}(q,\theta)\, r(t) + \boldsymbol{H}(q,\theta)\, e(t)$$

$$\boldsymbol{G}(q,\theta) = \boldsymbol{Q}^{-1}(q,\theta)\boldsymbol{P}(q,\theta)$$

$$\boldsymbol{R}(q,\theta) = \boldsymbol{Q}^{-1}(q,\theta)\boldsymbol{F}$$

$$\boldsymbol{H}(q,\theta) = \boldsymbol{Q}^{-1}(q,\theta)\boldsymbol{Q}(\infty,\theta)\boldsymbol{C}(q,\theta)$$

$\boldsymbol{Q}(q,\theta)$ diagonal polynomial

$\boldsymbol{P}(q,\theta)$ hollow, symmetric polynomial

$\boldsymbol{F}$ diagonal, binary, known

$\boldsymbol{C}(q,\theta)$ monic, stable, stably invertible

$\boldsymbol{Q}(\infty,\theta) = \lim_{z\to\infty} Q(z,\theta)$ diagonal

[1] H. H. M. Weerts (2018a)

Consistency

Consistent estimates of

$$\boldsymbol{G}(q,\hat{\theta}_N), \qquad \boldsymbol{R}(q,\hat{\theta}_N), \qquad \boldsymbol{H}(q,\hat{\theta}_N)$$

by the joint-direct method[1] with identification criterion

$$\hat{\theta}_N = \arg\min_{\theta} \frac{1}{N} \sum_{t=1}^{N} \varepsilon^\top(t,\theta)\varepsilon(t,\theta)$$

with prediction error

$$\varepsilon(t,\theta) \coloneqq \boldsymbol{Q}^{-1}(\infty,\theta)\boldsymbol{C}^{-1}(q,\theta)\left([\boldsymbol{Q}(q,\theta) - \boldsymbol{P}(q,\theta)]w(t) - \boldsymbol{F}r(t)\right)$$

and under general conditions.

[1] H. H. M. Weerts (2018a)

Consistency

If in addition

$\boldsymbol{Q}^0(q)$ and $\boldsymbol{P}^0(q)$ left co-prime

$\boldsymbol{F}^0 \neq 0$ (at least 1 $r_j(t)$)

then also consistent estimates of

$$\boldsymbol{Q}(q,\hat{\theta}_N), \qquad \boldsymbol{P}(q,\hat{\theta}_N), \qquad \boldsymbol{C}(q,\hat{\theta}_N).$$

$$\boldsymbol{G}^0(q) = (\boldsymbol{Q}^0)^{-1}(q)\boldsymbol{P}^0(q)$$

$$\boldsymbol{R}^0(q) = (\boldsymbol{Q}^0)^{-1}(q)\boldsymbol{F}^0$$

$$\boldsymbol{H}^0(q) = (\boldsymbol{Q}^0)^{-1}(q)\boldsymbol{Q}^0(\infty)\boldsymbol{C}^0(q)$$

Optimisation

In general non-affine in $\theta \Longrightarrow$ non-convex optimisation

$$\varepsilon(t,\theta) := \boldsymbol{Q}^{-1}(\infty,\theta)\boldsymbol{C}^{-1}(q,\theta)\left([\boldsymbol{Q}(q,\theta)-\boldsymbol{P}(q,\theta)]w(t)-\boldsymbol{F}r(t)\right)$$

Linear regression if $\boldsymbol{C}^0(q)\boldsymbol{Q}^0(\infty) = I$

$$\varepsilon(t,\theta) = \underbrace{[\boldsymbol{Q}(q,\theta)-\boldsymbol{P}(q,\theta)]w(t)}_{\varphi^\top(t)\theta}-\boldsymbol{F}r(t)$$

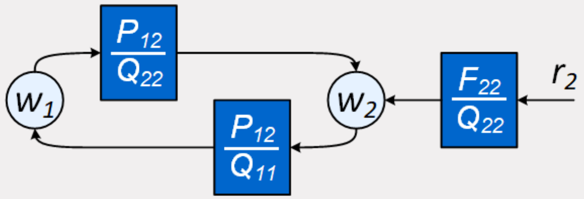
Contents

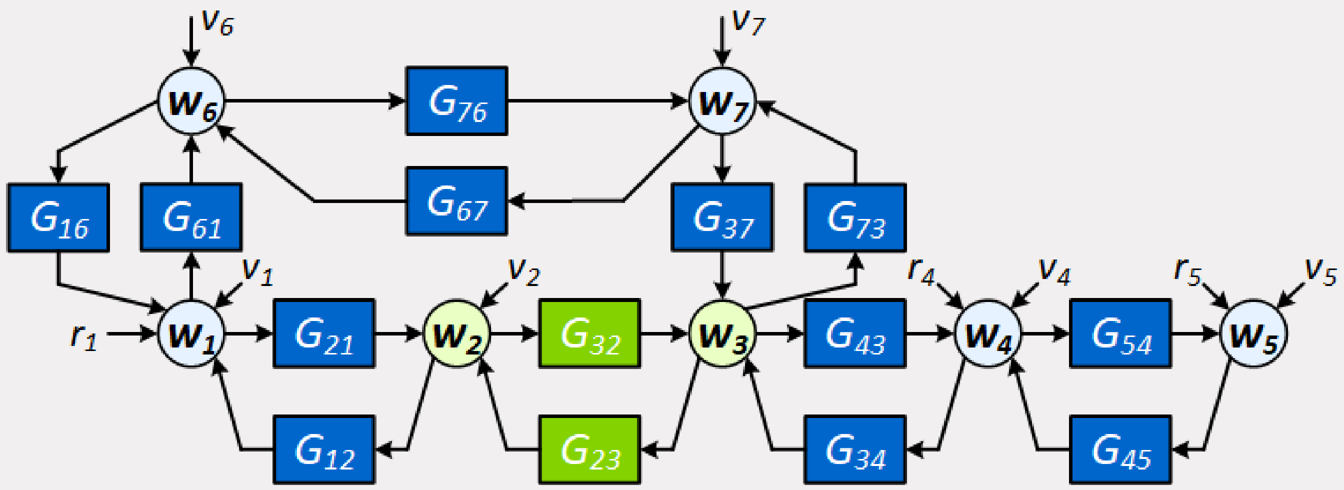
Local network identification

Identification of one physical interconnection

Identification of two modules G_{jk} and G_{kj}

Which node signals to measure?





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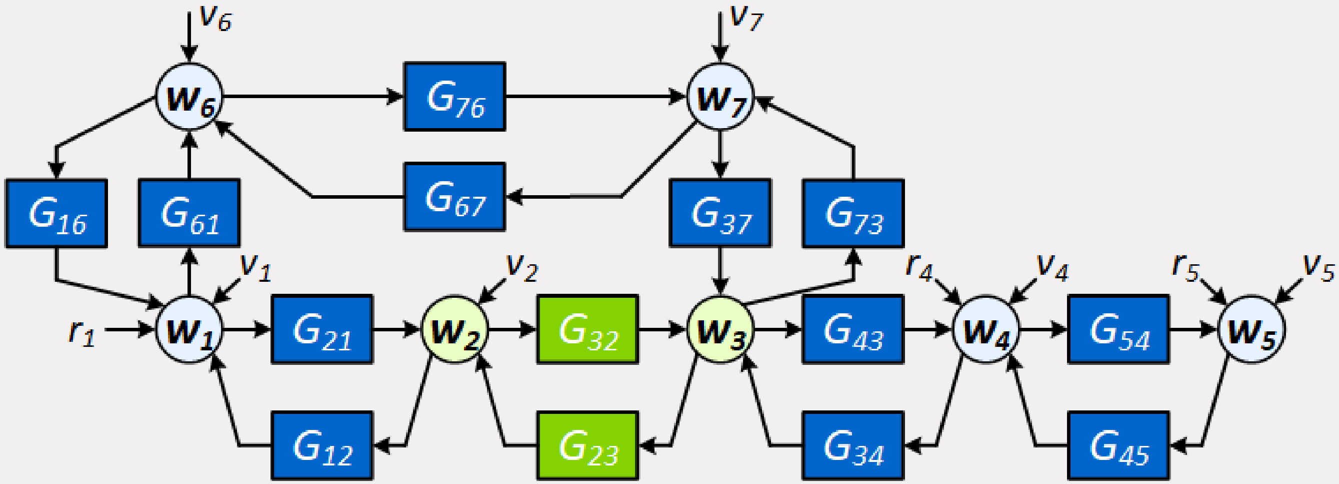
Which signals to measure?

Conditions for invariance of G_{jk}[1]:

1. All parallel paths need to contain a measured node

2. All loops around the output need to contain a measured node



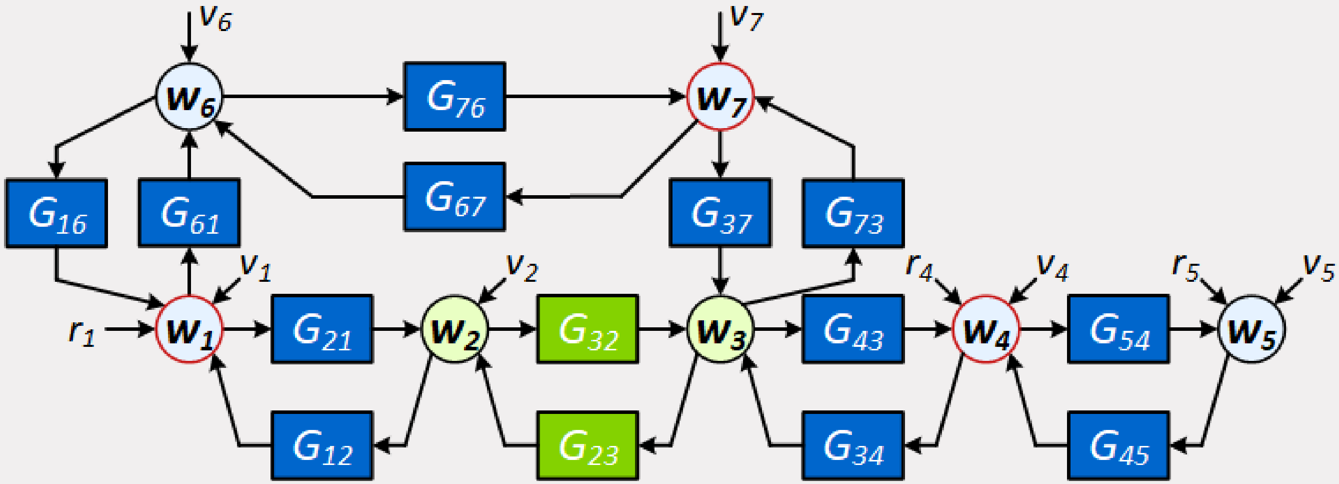
 [1] A. Dankers et al (2016)

Which signals to measure?

Conditions for invariance of G_{jk} and G_{kj}:

All neighbour node signals of $w_j(t)$ and $w_k(t)$ need to be measured

Local identification on the basis of local signals



Contents

Summary

Physical networks = module representations + structural conditions

Full network identification:

- Consistency
- Non-convex optimisation
- Linear regression in special case: $C(q)Q(\infty) = I$

Local network identification:

- Single interconnection
- Neighbour node signals

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