System Identification in Dynamic Networks

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Introduction

Dynamic networks:

- Distributed Control
- Power Systems
- Biological Systems
- Financial Systems

Sources:
- Simonetto 2012
- Pierre et al. 2012
- Hillen 2012
- Materassi et al. 2010
Introduction

The classical identification problems:

Identify a plant model $\hat{G}$ on the basis of measured signals $u, y$ (and possibly $r$)

- Several classical methods available (PE, subspace, nonparam,..)
- Well known results for identification in known structure (open loop, closed-loop, possibly known controller)
Introduction

Dynamical systems in emerging fields have a more complex structure:

- distributed control system
- dynamic network

(distributed systems, multi-agent systems, biological networks, smart grids,.....)

Questions to be addressed:
• How to identify `single” transfers in a known (complex) structure?
• Can currently available tools from (closed-loop) identification be used for this purpose?
Introduction

Some modules may be known (e.g. controllers)
Contents

- Methods for (classical) closed-loop ID
- Dynamic network setup
- Network identification
- Predictor input conditions
- Example
- Discussion

From open-loop and closed-loop identification to dynamic network identification
Methods for closed-loop identification

- **Direct method**
  Relying on full-order noise modelling
  \[ \varepsilon(t, \theta) = H(\theta)^{-1}[y(t) - G(\theta)u(t)] \]

- **Two-stage, indirect, projection, IV**
  Relying on measured external excitation
  \[ \varepsilon(t, \theta) = H(\rho)^{-1}[y(t) - G(\theta)u^r(t)] \]
  with input decomposed:
  \[ u = u^r + u^v \]
  such that \( u^r \) and \( v \) uncorrelated
  
  ![Plant representation diagram]
  
  \[ y(t) = G_0u(t) + H_0e(t) \]
  \( e \) white noise
  \( r \) and \( v \) uncorrelated
Methods for closed-loop identification

Consistency results for PE identification

- **Direct method**  [Ljung, 1987]
  - full order noise model ($S \in \mathcal{M}$)
  - delay in every loop
  - sufficient excitation of $u$, i.e.
  \[ \Phi_z(\omega) > 0 \ \forall \omega \quad z := \text{vec}(y, u) \]

- **Two-stage**  [Van den Hof & Schrama, 1993]
  - no noise model required ($G_0 \in \mathcal{G}$)
  - no conditions on delays
  - sufficient excitation of $u^r$, i.e.
  \[ \Phi_{u^r}(\omega) > 0 \ \forall \omega \]

Plant representation

\[ y(t) = G_0 u(t) + H_0 e(t) \]

- $e$ white noise
- $r$ and $v$ uncorrelated

\[ G_0 \quad C \]
Question

- Can we utilize these tools for identification of transfer functions in a (complex) dynamic network?
Network Setup

Formalizing one link (transfer between $w_i$ and $w_j$)

- On each node a disturbance $v_j$ and a reference $r_j$ might be present.
- Reference signals are uncorrelated to noise signals.
- $N_j$: set of nodes that has a direct causal link with node j, of which $K_j$ are known transfers and $U_j$ unknown.
Network Setup

Assumptions:
• Total of $L$ nodes
• Network is well-posed
  $I - G^0$ invertible
• Stable (all signals bounded)
• All $w_m, m = 1, \cdots L$, measured, as well as all present $r_m$
• Modules may be unstable

\[
\begin{bmatrix}
  w_1 \\
  w_2 \\
  \vdots \\
  w_L
\end{bmatrix}
= 
\begin{bmatrix}
  0 & G^0_{12} & \cdots & G^0_{1L} \\
  G^0_{21} & 0 & \cdots & G^0_{2L} \\
  \vdots & \vdots & \ddots & \vdots \\
  G^0_{L1} & G^0_{L2} & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
  w_1 \\
  w_2 \\
  \vdots \\
  w_L
\end{bmatrix}
+ 
\begin{bmatrix}
  r_1 \\
  r_2 \\
  \vdots \\
  r_L
\end{bmatrix}
+ 
\begin{bmatrix}
  v_1 \\
  v_2 \\
  \vdots \\
  v_L
\end{bmatrix}
\]
Contents

• Methods for (classical) closed-loop ID
• Dynamic network setup
• Network identification
• Predictor input conditions
• Example
• Discussion
Applying direct method to input $w_i$ and output $w_j$ will lead to biased results

- if the prediction error cannot be whitened, or equivalently
- If there are nodes in $\mathcal{U}_j$ that are correlated to $w_j$

A MISO approach:

$$\varepsilon(t, \theta) = H_j(\theta)^{-1}[w_j - r_j - \sum_{k \in \mathcal{K}_j} G_{jk}^0 w_k - G_{ji}(\theta) w_i - \sum_{k \in \mathcal{U}_j} G_{jk}(\theta) w_k]$$

\[\hat{w}_j \text{ known}\]

Simultaneous identification of transfers $G_{jk}^0$, $k \in \mathcal{U}_j$ and a noise model for $v_j$
Network Identification – Direct method
Network Identification – Direct method
Network Identification – Direct method

Result direct method

The plant model $G_{jj}(\theta)$ is consistently estimated if:

- All parametrized plant and noise models are correctly parametrized, $G_{jk}(\theta)$, $k \in \mathcal{U}_j$; $H_j(\theta)$ ($S \in \mathcal{M}$)
- Every loop in the network that runs through node $j$ has at least one delay (no algebraic loop)
- $\Phi_z(\omega) > 0$ $\forall \omega$, for $z := \text{vec}\{w_j, \{w_k\}_{k \in \mathcal{U}_j}\}$ (excitation condition)
- Noise source $v_j$ is uncorrelated with all other noise terms in the network

[Dankers et al., CDC2012]
Main approach:
- Look for an external reference signal that has a connection with $w_i$
- And that does not act as an unmodelled disturbance on $w_j$
Algorithm:

- Determine whether there exists an $r_m$ such that $w_i r_m$ is sufficiently exciting
- Construct:
  \[
  \tilde{w}_j = w_j - r_j - \sum_{k \in \mathcal{K}_j} G^0_{jk} w_k
  \]
  known terms
- Identify $G^0_{ji}$ through PE identification with prediction error
  \[
  \varepsilon(t, \theta) = H_j(\rho)^{-1} \left[ \tilde{w}_j - \sum_{k \in \mathcal{U}_{is}} G^0_{jk}(\theta) w^K_m \right]
  \]
  where all inputs $k \in \mathcal{U}_{is}$ are considered that are correlated to $r_m$
Network Identification – Two-stage method

Result two-stage method

The plant model $G_{ji}(\theta)$ is consistently estimated if:

- The plant models $G_{jk}(\theta)$ are correctly parametrized $k \in \mathcal{U}_{is}$
- The vector of (projected) input signals is sufficiently exciting
- Excitation signals are uncorrelated to noise disturbances

[Van den Hof et al., CDC2012]
Network Identification – Two-stage method

Observation:
• Consistent identification of single transfers is possible, dependent on network topology and reference excitation
• Full noise models are not necessary
• No conditions on uncorrelated noise sources, nor on absence of algebraic loops
• Excitation conditions on (projected) input signals
• Network topology conditions on $r_m$ can simply be checked by tools from graph theory
Predictor input selection

What if only a selected number of variables can be measured?

or:
Where to put the sensors?
Predictor input selection

- If predictor inputs are not chosen correctly, consistent estimates not possible.
- Considerable flexibility in choosing predictor inputs

Conditions will be derived that the predictor inputs must satisfy.
First mechanism: parallel paths

**Objective:** consistently estimate $G_{21}^0$.

**SISO approach.** Try to estimate the dynamics between $w_1$ and $w_2$:

$$w_2 = G_{21}^0 w_1^{(n)} + G_{21}^0 w_1^{(v)} + G_{23}^0 w_3 + v_2$$

*unmodeled term*

**Problem!** "unmodeled term" (noise term) is correlated to input term, $w_1^{(r)}$. 

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Predictor input selection: condition 1

**Objective:** obtain an estimate of $G_{ji}^0$.

**Consistent** estimates of $G_{ji}^0$ are possible if:

1. $w_i$ and $w_j$ are included as predictor inputs.
2. Each path from $w_i \rightarrow w_j$ passes through a node chosen as a predictor input.
Second mechanism: loops around the output

**Objective:** consistently estimate $G_{21}^0$.

**SISO approach.** Try to estimate the dynamics between $w_1$ and $w_2$:

$$w_2 = G_{21}^0 w_1^{(r)} + G_{21}^0 w_1^{(v)} + G_{23}^0 w_3 + v_2$$

unmodeled term

**Problem!** "unmodeled term" (noise term) is correlated to input term, $w_1^{(r)}$. 
Second mechanism: loops around the output

**Objective:** consistently estimate $G_{21}^0$.

**SISO approach.** Try to estimate the dynamics between $w_1$ and $w_2$:

$$w_2 = G_{21}^0 w_1^{(r_1)} + G_{21}^0 w_1^{(v)} + G_{23}^0 w_3 + v_2$$

unmodeled term

**Problem!** "unmodeled term" (noise term) is correlated to input term, $w_1^{(r_1)}$.

**Solution:** Include $w_3^{(r_1)}$ in the predictor:

$$w_2 = G_{21}^0 w_1^{(r_1)} + G_{23}^0 w_3^{(r_1)} + G_{21}^0 w_1^{(v)} + G_{23}^0 w_3^{(v)} + v_2$$

unmodeled term
**Objective:** obtain an estimate of $G^0_{ji}$.

**Consistent** estimates of $G^0_{ji}$ are possible if:

1. $w_i$ and $w_j$ are included as predictor inputs.
2. Each path from $w_i \rightarrow w_j$ passes through a node chosen as a predictor input.
3. Each loop from $w_j \rightarrow w_j$ passes through a node chosen as a predictor input.
Example with predictor input conditions

**Objective:** Estimate $G_{21}^0$. Project onto $r_8, r_4, r_5$.

**Conditions:** Include variable on every path
- $w_1 \rightarrow w_2$
- $w_2 \rightarrow w_2$

**Conclude:** include $w_1, w_2$, and ... as inputs to the predictor.
Example with predictor input conditions

Objective: Estimate $G_{21}^0$. Project onto $r_8$, $r_4$, $r_5$.

Conditions: Include variable on every path
- $W_1 \rightarrow W_2$
- $W_2 \rightarrow W_2$

Conclude: include $w_1$, $w_2$, and ... as inputs to the predictor.
Objective: Estimate $G_{21}^0$.
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- $w_2 \rightarrow w_2$

**Conclude:** include $w_1, w_2$, and ... as inputs to the predictor.
**Objective:** Estimate $G_{21}$.
Project onto $r_8, r_4, r_5$.

**Conditions:** Include variable on every path
- $w_1 \rightarrow w_2 \Rightarrow \text{Include } w_6 \text{ in predictor}$
- $w_2 \rightarrow w_2$

**Conclude:** Include $w_1, w_2, w_6$ and ... as input to the predictor.
Example with predictor input conditions

**Objective:** Estimate $G_{21}^0$.
Project onto $r_8$, $r_4$, $r_5$.

**Conditions:** Include variable on every path
- $w_1 \rightarrow w_2 \Rightarrow$ Include $w_6$ in predictor
- $w_2 \rightarrow w_2$

**Conclude:** include $w_1$, $w_2$, $w_6$ and ... as input to the predictor.
Example with predictor input conditions

**Objective:** Estimate $G_{21}^0$. Project onto $r_8, r_4, r_5$.

**Conditions:** Include variable on every path
- $w_1 \rightarrow w_2 \Rightarrow \text{Include } w_6 \text{ in predictor}$
- $w_2 \rightarrow w_2 \Rightarrow \text{Include } w_3 \text{ in predictor}$

**Conclude:** include $w_1, w_2, w_3,$ and $w_6$ as inputs to the predictor.
Summary

- Current framework for open/closed-loop identification has been extended to dynamic networks
- Methods for closed-loop identification extend to this case with some new properties
- Framework extends (easily) to noise on all variables (EIV)
- They are expected to provide the basic tools for dealing with the structure identification problem also
- So far only consistency considered (no variance)
- Many new questions pop up……

Further reading:
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