System Identification in Dynamic Networks

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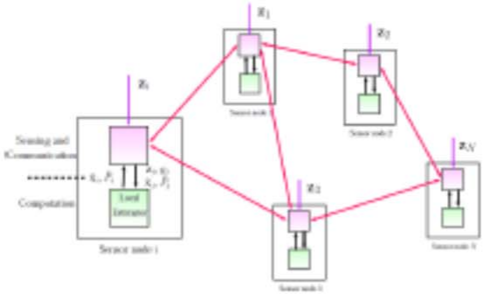
Where innovation starts

Introduction

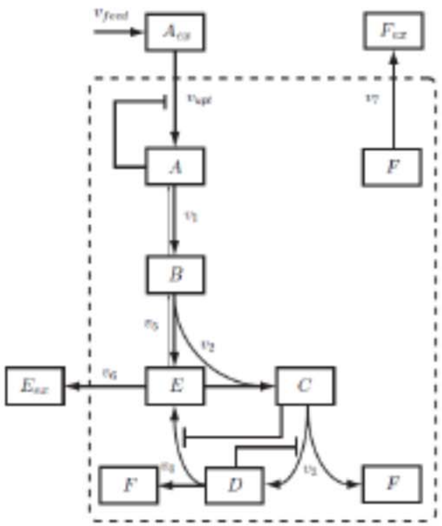
Dynamic networks:

Distributed Control



source: Simonetto 2012

Biological Systems

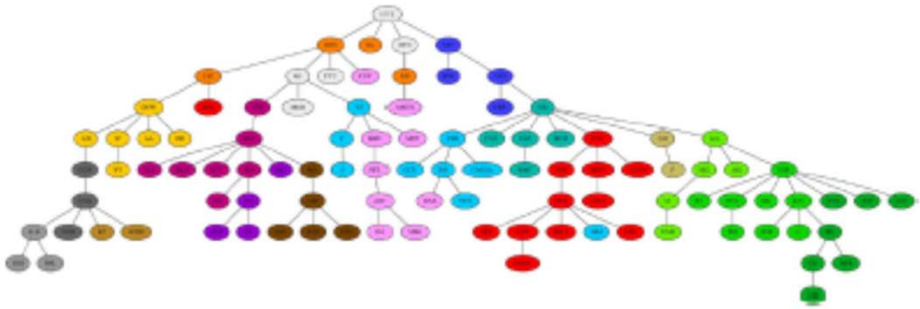


source: Hillen 2012

Power Systems

source: Pierre et al. 2012

Financial Systems



source: Materassi et al. 2010

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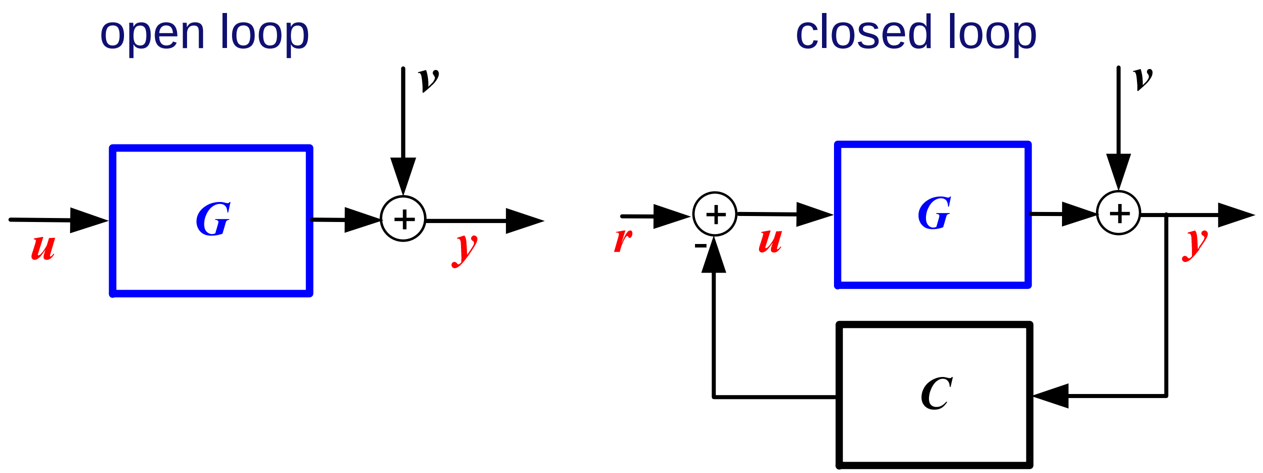
Introduction

The classical identification problems:



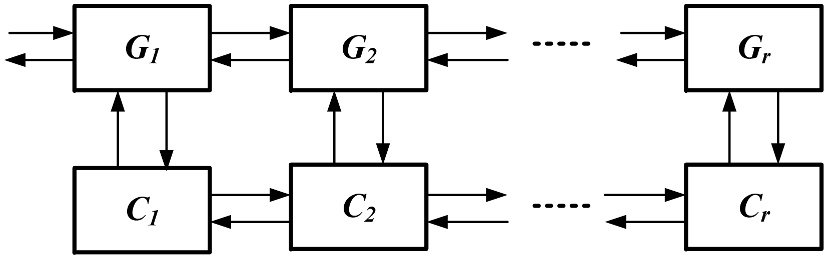
Identify a plant model $\hat{G}$ on the basis of measured signals *u, y* (and possibly *r*)

- **Several classical methods available (PE, subspace, nonparam,..)**
- **Well known results for identification *in known structure* (open loop, closed-loop, possibly known controller)**

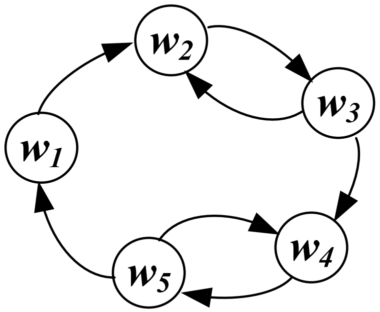
Introduction

Dynamical systems in emerging fields have a more complex structure:

distributed control system



dynamic network

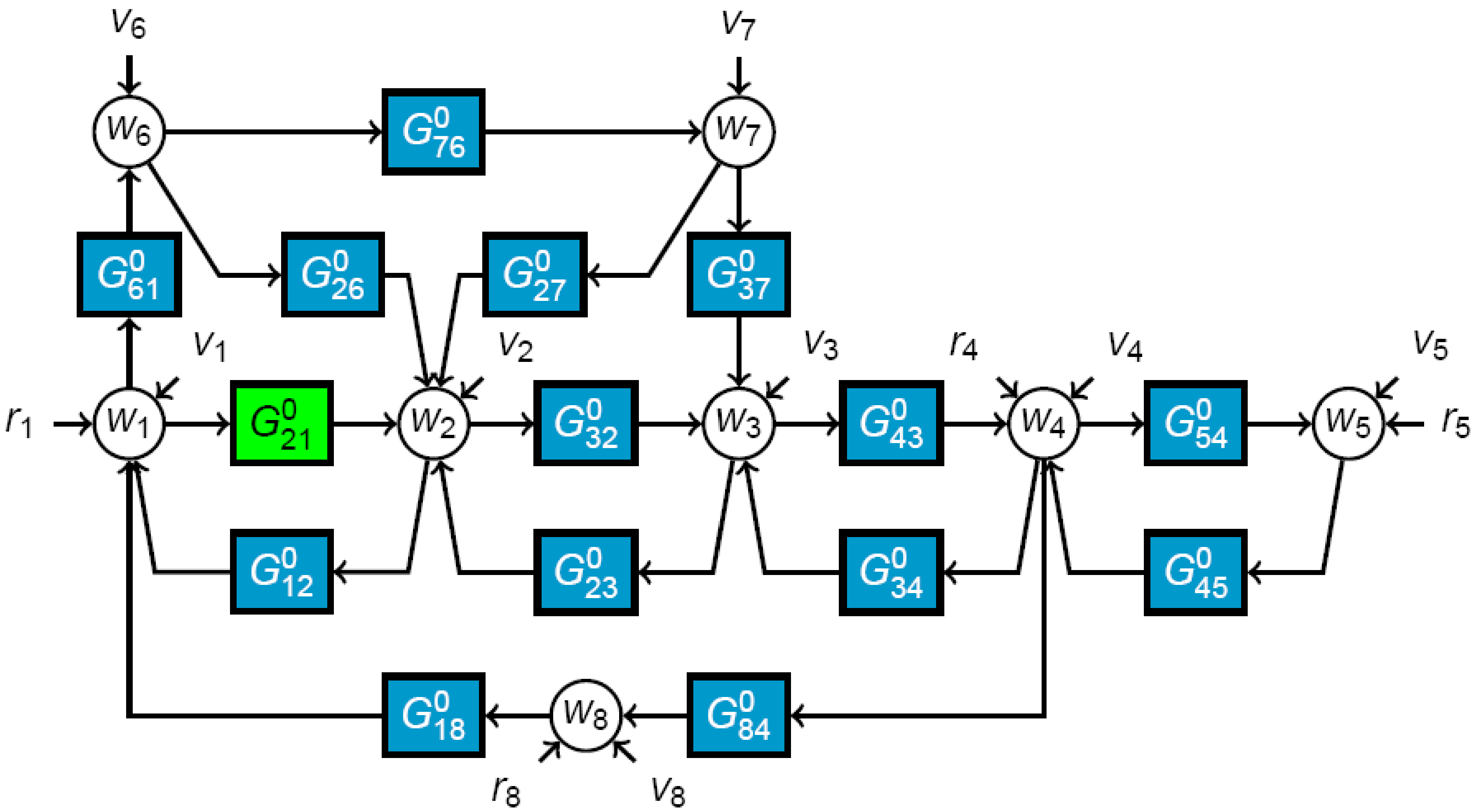


(distributed systems, multi-agent systems, biological networks, smart grids,…..)

Questions to be adressed:

- **How to identify ``single'' transfers in a known (complex) structure?**
- **Can currently available tools from (closed-loop) identification be used for this purpose?**

Introduction

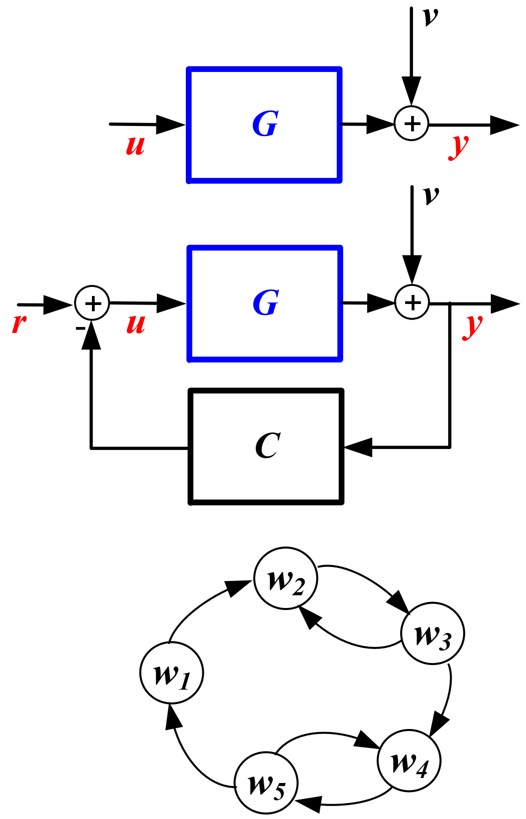


Some modules may be known (e.g. controllers)

Contents

From open-loop and closed-loop identification to dynamic network identification

- Methods for (classical) closed-loop ID
- Dynamic network setup
- Network identification
- Predictor input conditions
- Example
- Discussion



Methods for closed-loop identification

- **Direct method**
  Relying on full-order noise modelling

$$\varepsilon(t,\theta) = H(\theta)^{-1}[y(t) - G(\theta)u(t)]$$

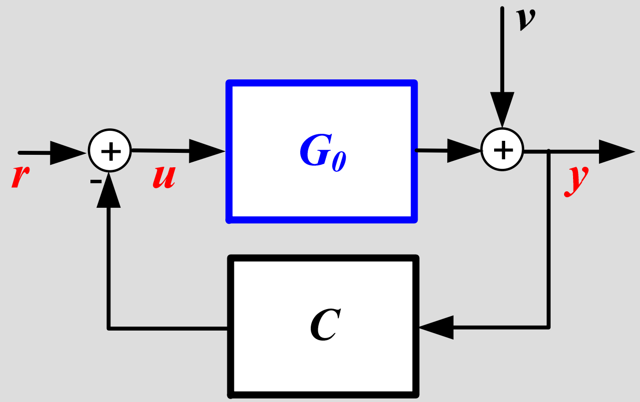
- **Two-stage, indirect, projection, IV**
  Relying on measured external excitation

$$\varepsilon(t,\theta) = H(\rho)^{-1}[y(t) - G(\theta)u^r(t)]$$

with input decomposed:

$$u = u^r + u^v$$

such that u^r and v uncorrelated



Plant representation

$$y(t) = G_0 u(t) + H_0 e(t)$$

e white noise

r and v uncorrelated

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Methods for closed-loop identification

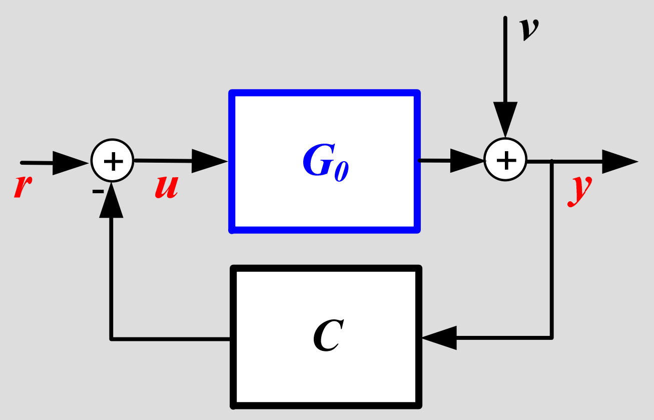
Consistency results for PE identification

- **Direct method** [Ljung, 1987]
  - **full order noise model** $(\mathcal{S} \in \mathcal{M})$
  - **delay in every loop**
  - **sufficient excitation of u, i.e.**

$$\Phi_z(\omega) > 0 \quad \forall\omega \qquad z := vec(y,u)$$

- **Two-stage** [Van den Hof & Schrama, 1993]
  - **no noise model required** $(G_0 \in \mathcal{G})$
  - **no conditions on delays**
  - **sufficient excitation of u^r, i.e**

$$\Phi_{u^r}(\omega) > 0 \quad \forall\omega$$



Plant representation

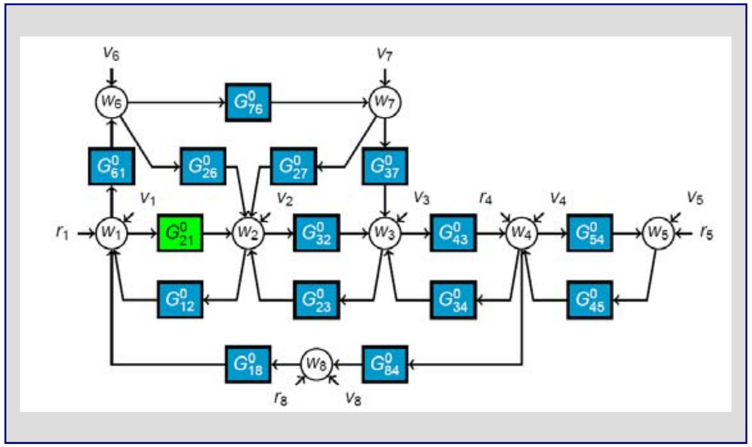
$$y(t) = G_0 u(t) + H_0 e(t)$$

e white noise

r and v uncorrelated

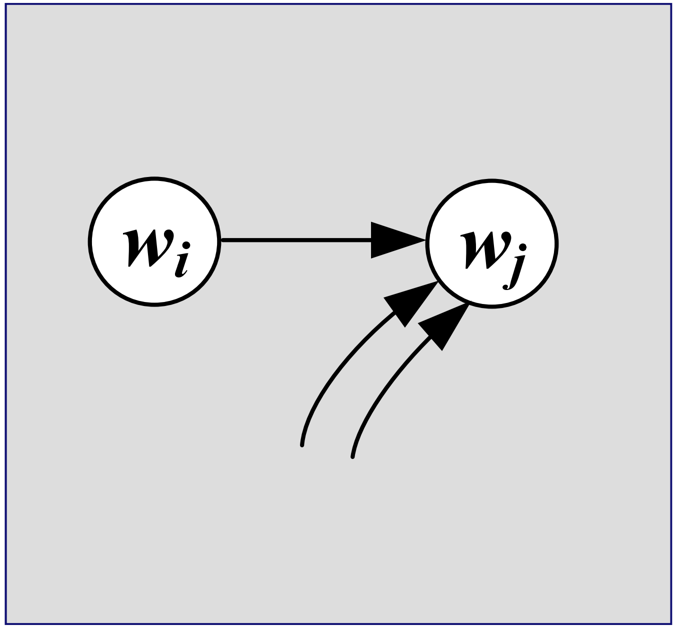
Question

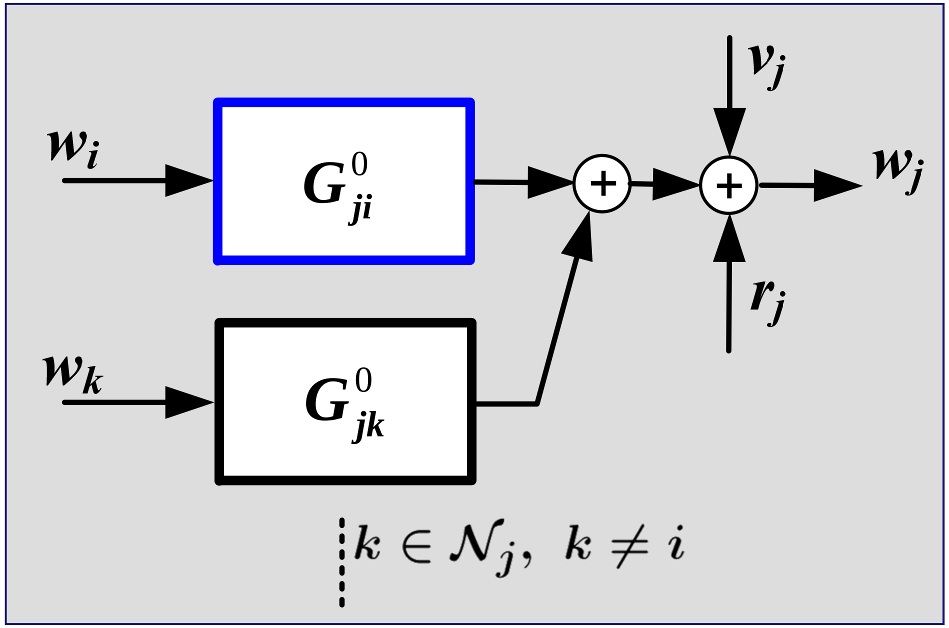
- **Can we utilize these tools for identification of transfer functions in a (complex) dynamic network ?**



Network Setup

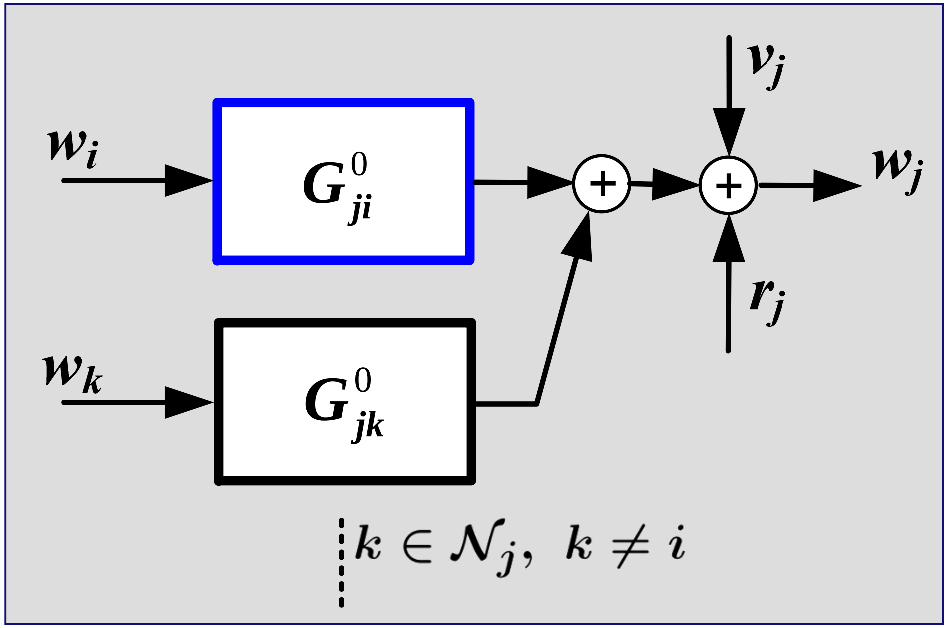
Formalizing one link (transfer between w_i and w_j)





- On each node a disturbance v_j and a reference r_j might be present
- Reference signals are uncorrelated to noise signals
- $\mathcal{N}_j$: set of nodes that has a direct causal link with node j, of which $\mathcal{K}_j$ are known transfers and $\mathcal{U}_j$ unknown.

Network Setup



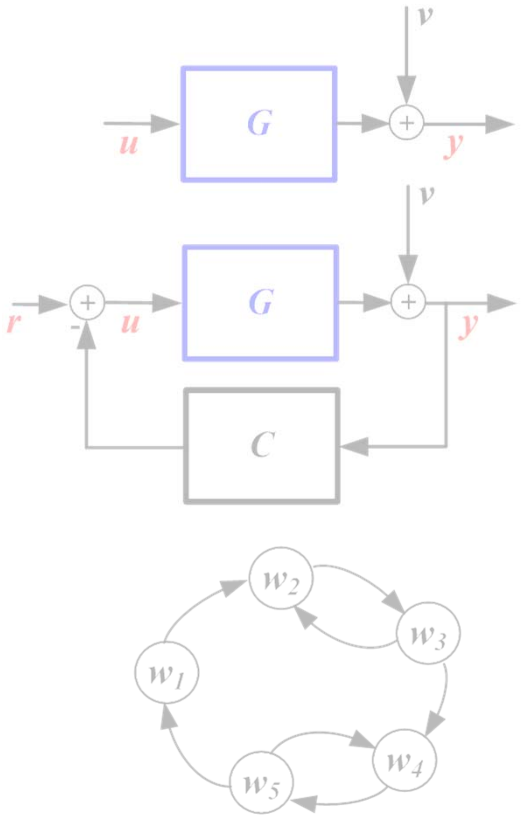
Assumptions:

- Total of L nodes
- Network is well-posed
  $I - G^0$ invertible
- Stable (all signals bounded)
- All $w_m, m = 1, \cdots L,$ measured, as well as all present r_m
- Modules may be unstable

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & G^0_{12} & \cdots & G^0_{1L} \\ G^0_{21} & 0 & \ddots & G^0_{2L} \\ \vdots & \ddots & \ddots & \vdots \\ G^0_{L1} & G^0_{L2} & \cdots & 0 \end{bmatrix}}_{G^0} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_L \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_L \end{bmatrix}$$

Contents

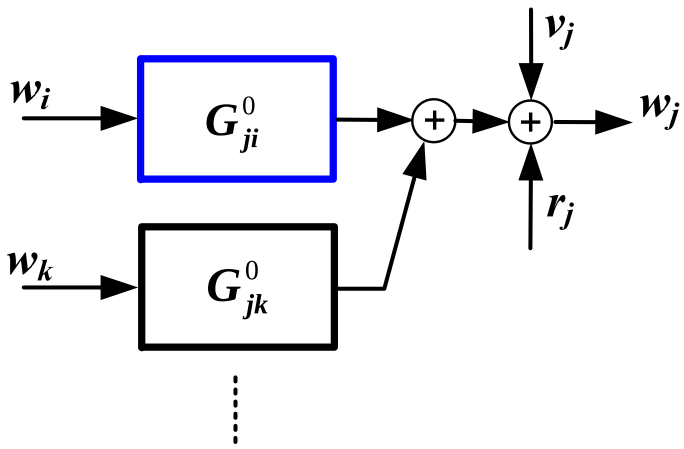
- **Methods for (classical) closed-loop ID**
- **Dynamic network setup**
- **Network identification**
- **Predictor input conditions**
- **Example**
- **Discussion**



Network Identification – Direct method

Applying direct method to input w_i and output w_j will lead to biased results

- if the prediction error can not be whitened, or equivalently
- If there are nodes in $\mathcal{U}_j^i$ that are correlated to w_j

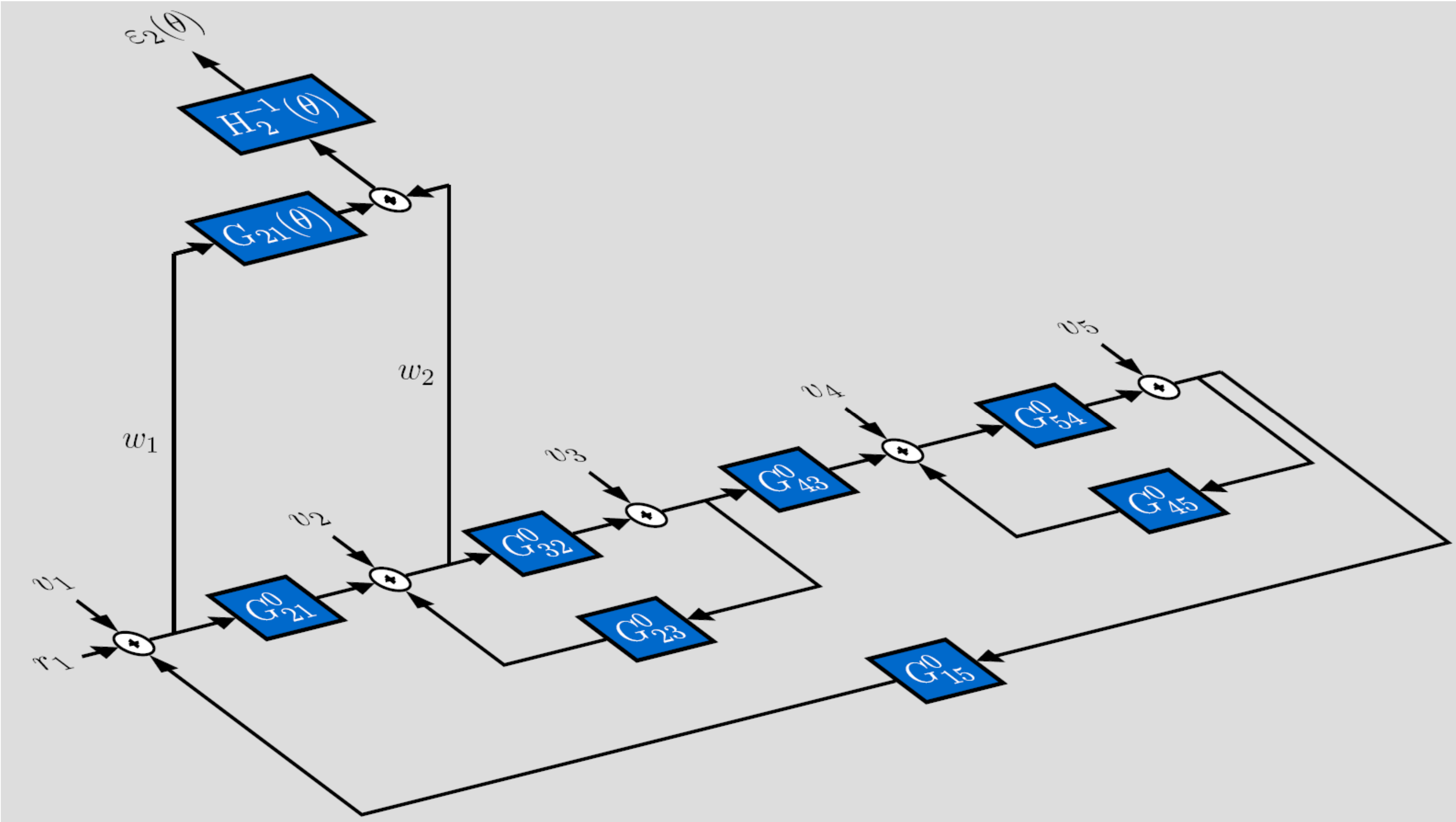


A MISO approach:

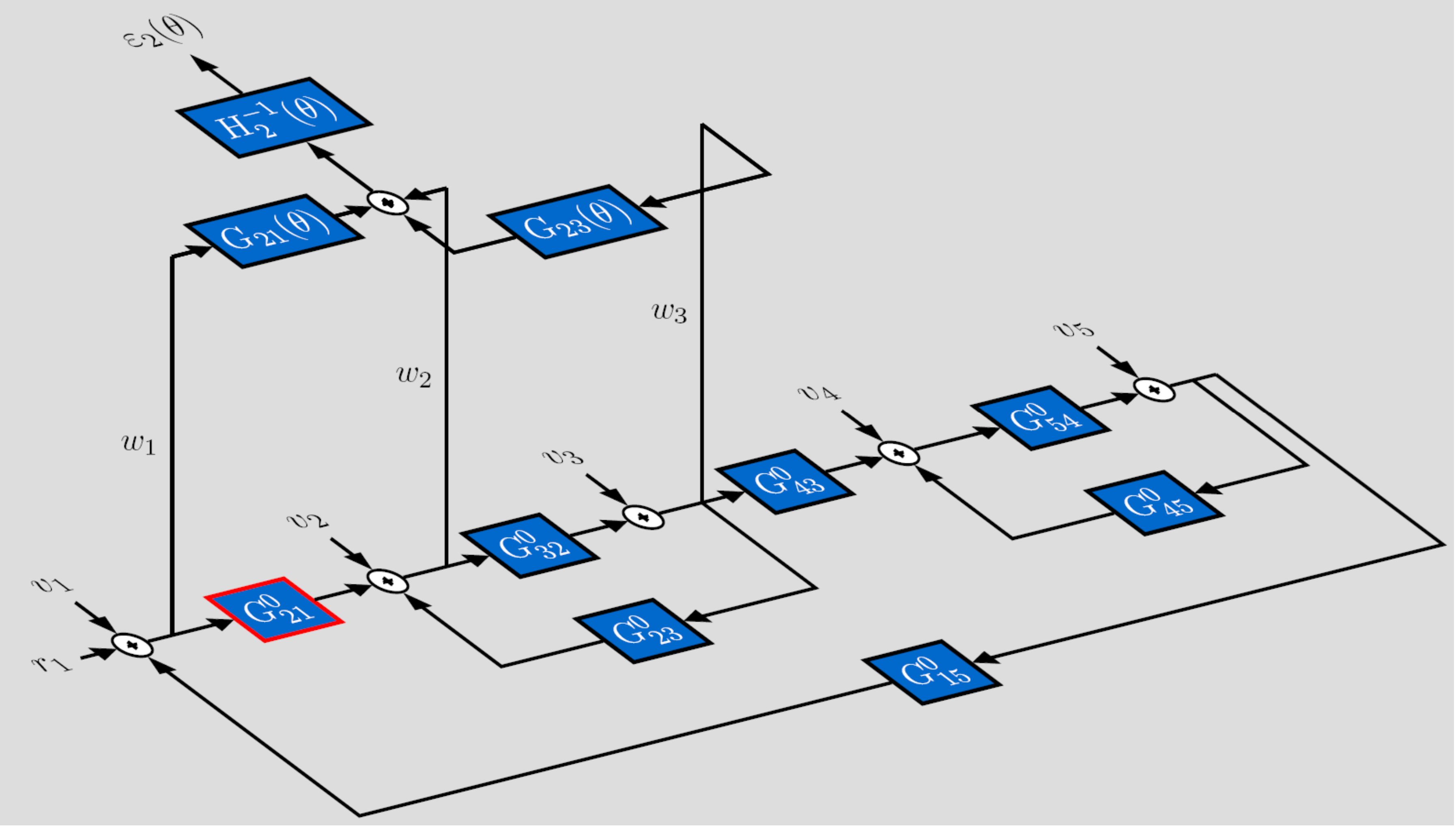
$$\varepsilon(t,\theta) = H_j(\theta)^{-1}[\underbrace{w_j - r_j - \sum_{k\in\mathcal{K}_j} G_{jk}^0 w_k}_{\tilde{w}_j \text{ known}} - G_{ji}(\theta)w_i - \sum_{k\in\mathcal{U}_j^i} G_{jk}(\theta)w_k]$$

⇒ Simultaneous identification of transfers $G_{jk}^0,\ k \in \mathcal{U}_j$ and a noise model for v_j

Network Identification – Direct method



Network Identification – Direct method



Network Identification – Direct method

Result direct method

The plant model $G_{ji}(\theta)$ is consistently estimated if:

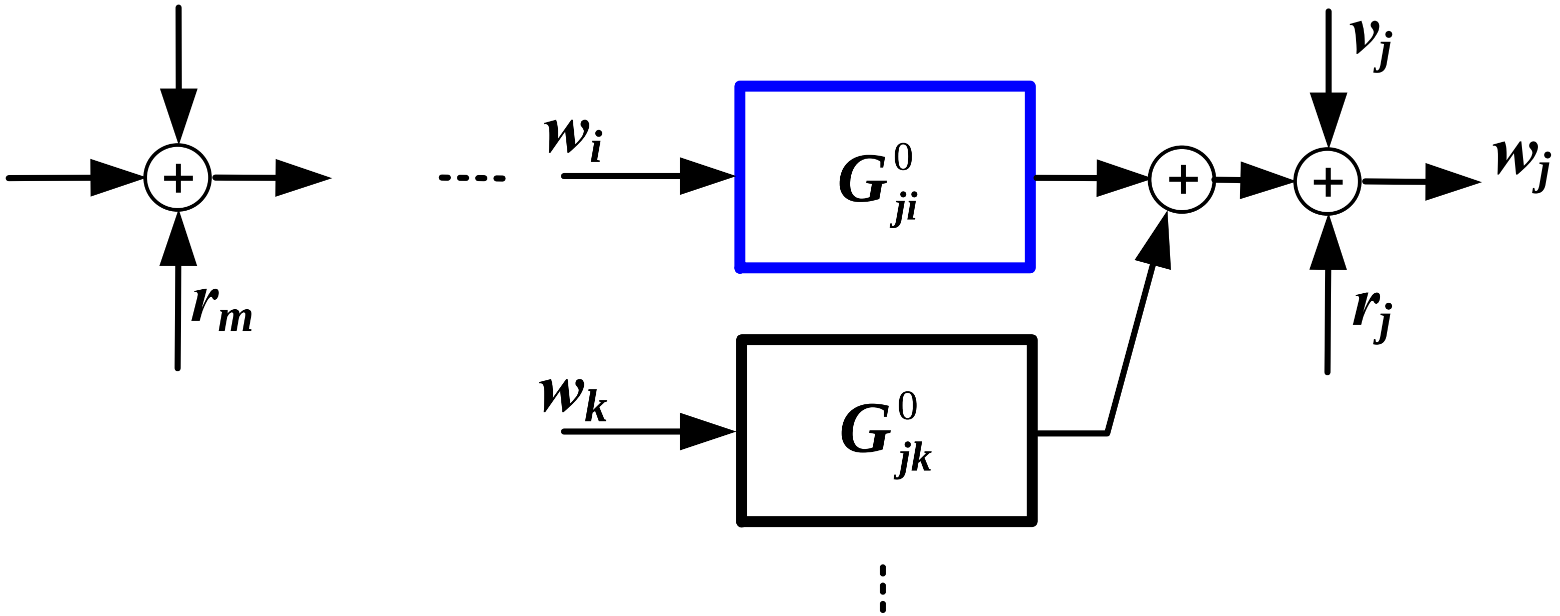
- **All parametrized plant and noise models are correctly parametrized, $G_{jk}(\theta),\ k \in \mathcal{U}_j;\ H_j(\theta)\ \ (\mathcal{S} \in \mathcal{M})$**
- **Every loop in the network that runs through node j has at least one delay (no algebraic loop)**
- **$\Phi_z(\omega) > 0 \ \ \forall\omega$, for $z := vec\{w_j, \{w_k\}_{k\in\mathcal{U}_j}\}$ (excitation condition)**
- **Noise source v_j is uncorrelated with all other noise terms in the network**

[Dankers et al., CDC2012]

Network Identification – Two-stage method

Main approach:

- **Look for an external reference signal that has a connection with w_i**
- **And that does not act as an unmodelled disturbance on w_j**

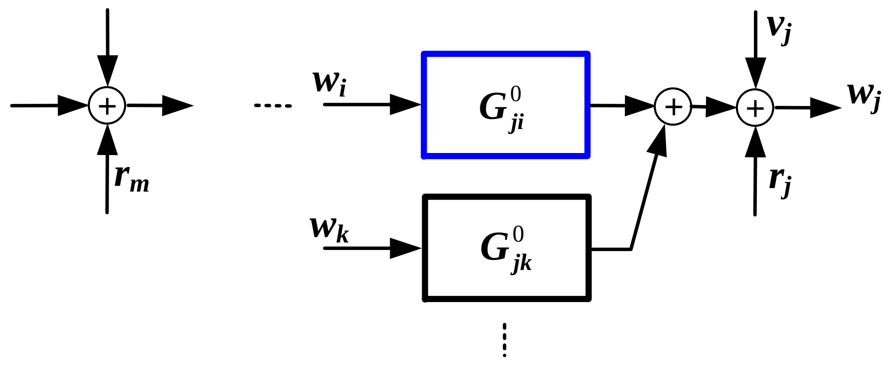


Network Identification – Two-stage method

Algorithm:

- **Determine whether there exists an r_m such that $w_i^{r_m}$ is sufficiently exciting**
- **Construct:**

$$\tilde{w}_j = \underbrace{w_j - r_j - \sum_{k \in \mathcal{K}_j} G^0_{jk} w_k}_{\text{known terms}}$$



- **Identify G^0_{ji} through PE identification with prediction error**

$$\varepsilon(t,\theta) = H_j(\rho)^{-1}[\tilde{w}_j - \sum_{k \in U_{is}} G_{jk}(\theta) w_k^{r_m}]$$

where all inputs $k \in \mathcal{U}_{is}$ are considered that are correlated to r_m

Network Identification – Two-stage method

Result two-stage method

The plant model $G_{ji}(\theta)$ is consistently estimated if:

- **The plant models $G_{jk}(\theta)$ are correctly parametrized $k \in \mathcal{U}_{is}$**
- **The vector of (projected) input signals is sufficiently exciting**
- **Excitation signals are uncorrelated to noise disturbances**

[Van den Hof et al., CDC2012]

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Network Identification – Two-stage method

Observation:

- **Consistent identification of single transfers is possible, dependent on network topology and reference excitation**
- **Full noise models are not necessary**
- **No conditions on uncorrelated noise sources, nor on absence of algebraic loops**
- **Excitation conditions on (projected) input signals**
- **Network topology conditions on r_m can simply be checked by tools from graph theory**

Predictor input selection

What if only a selected number of variables can be measured?

or:

Where to put the sensors?

‼

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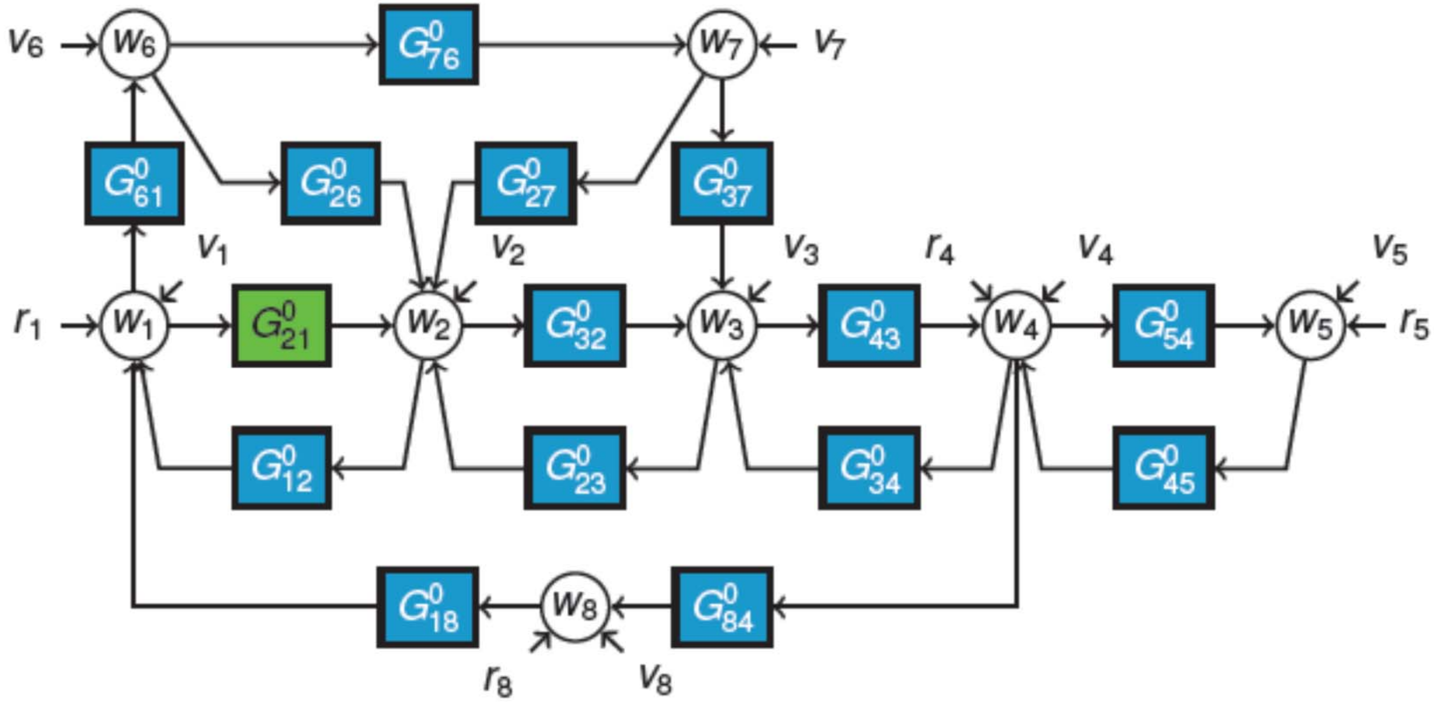
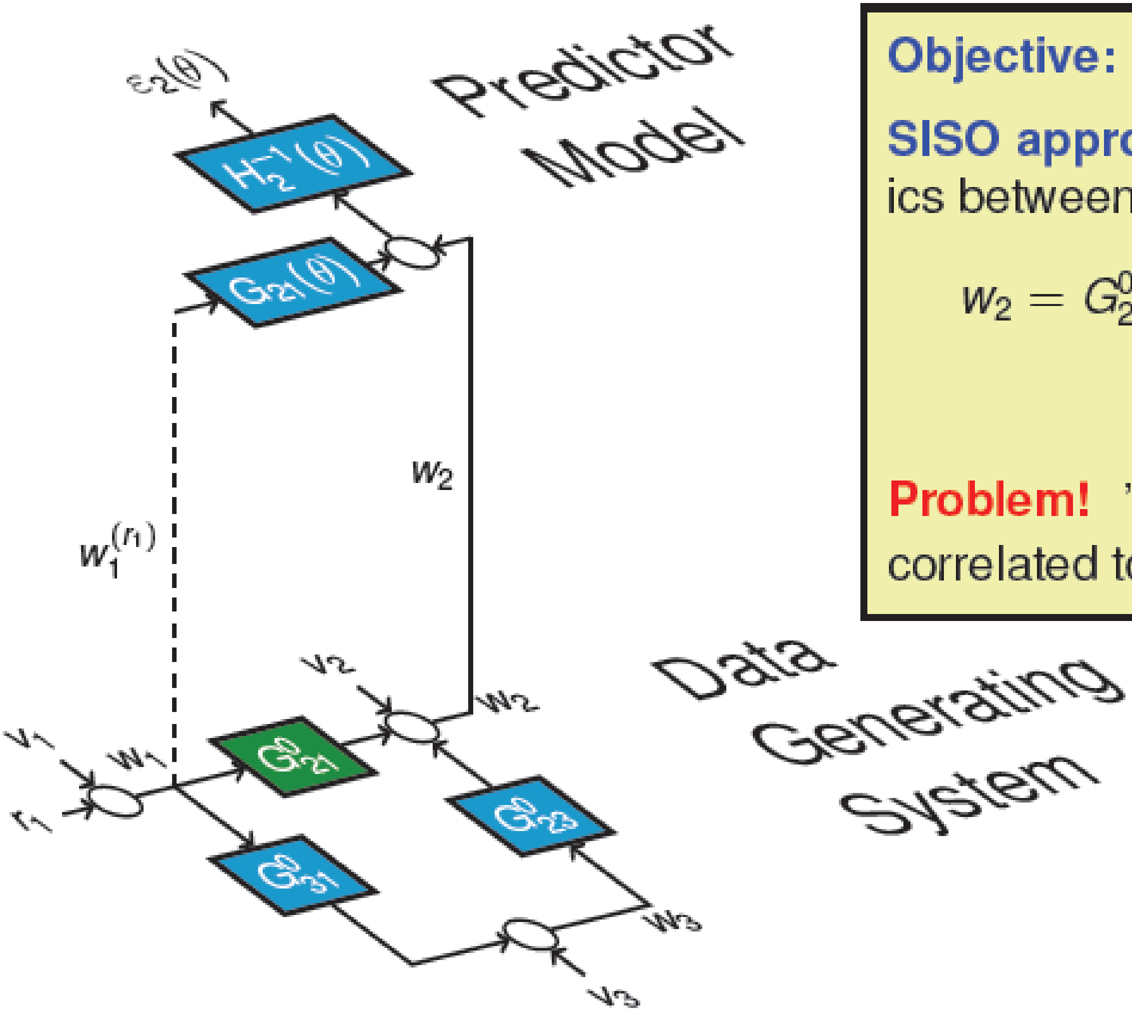
Predictor input selection

- If predictor inputs are not chosen correctly, consistent estimates not possible.
- Considerable flexibility in choosing predictor inputs

Conditions will be derived that the predictor inputs must satisfy.



First mechanism: parallel paths



Objective: consistently estimate G_{21}^0.

SISO approach. Try to estimate the dynamics between w_1 and w_2:

$$w_2 = G_{21}^0 w_1^{(r_1)} + \underbrace{G_{21}^0 w_1^{(v)} + G_{23}^0 w_3 + v_2}_{\text{unmodeled term}}$$

Problem! "unmodeled term" (noise term) is correlated to input term, $w_1^{(r_1)}$.

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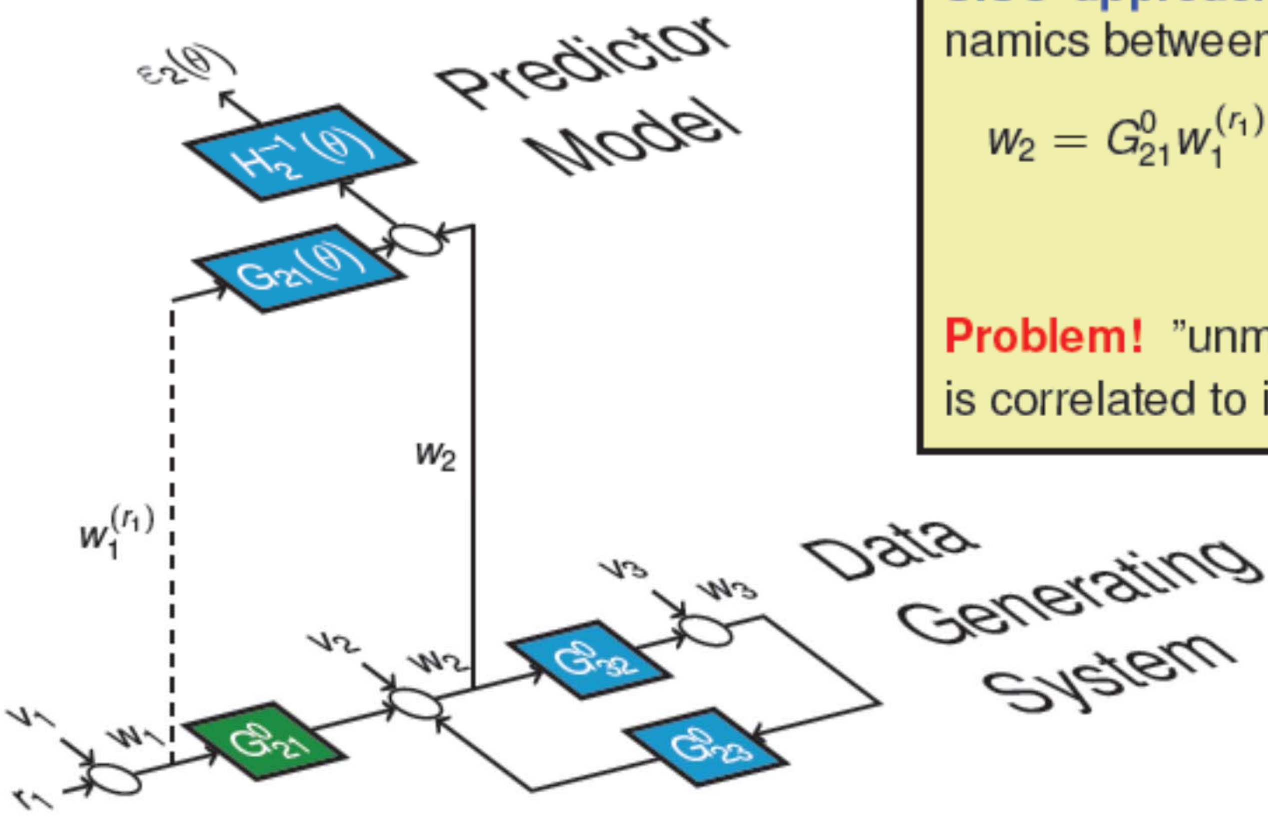
Predictor input selection: condition 1

Objective: obtain an estimate of G^0_{ji}.

Consistent estimates of G^0_{ji} are possible if:

1. w_i and w_j are included as predictor inputs.
2. Each path from $w_i \rightarrow w_j$ passes through a node chosen as a predictor input.

Second mechanism: loops around the output



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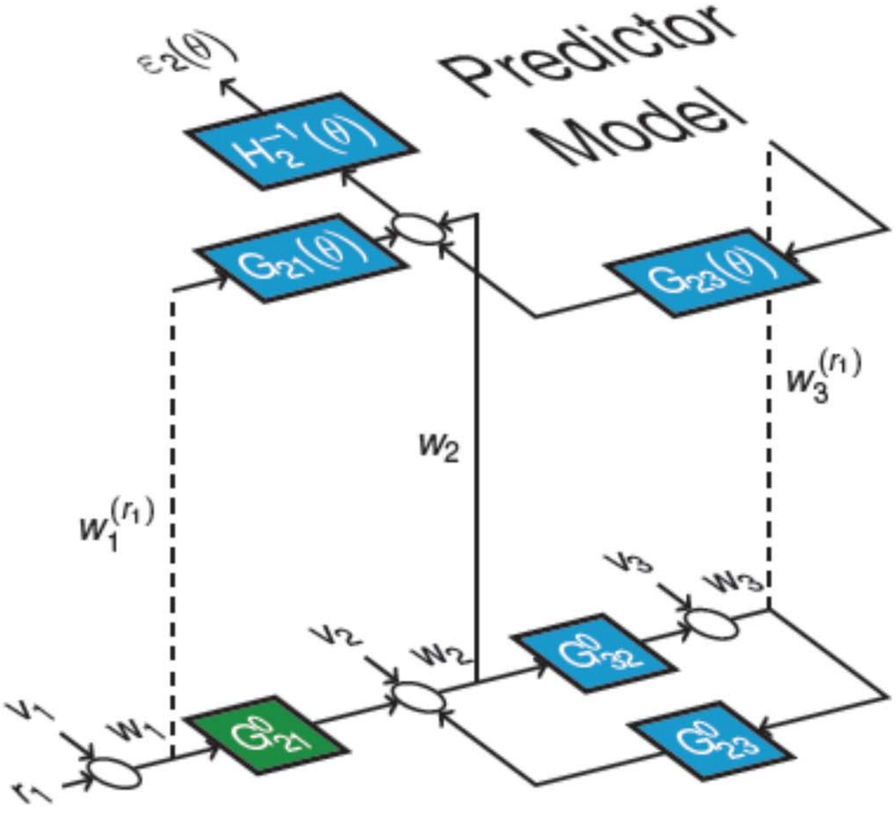

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Problem! "unmodeled term" (noise term) is correlated to input term, $w_1^{(r_1)}$.

Solution: Include $w_3^{(r_1)}$ in the predictor:

$$w_2 = G_{21}^0 w_1^{(r_1)} + G_{23}^0 w_3^{(r_1)} + \underbrace{G_{21}^0 w_1^{(v)} + G_{23}^0 w_3^{(v)} + v_2}_{\text{unmodeled term}}$$

Predictor input selection: conditions

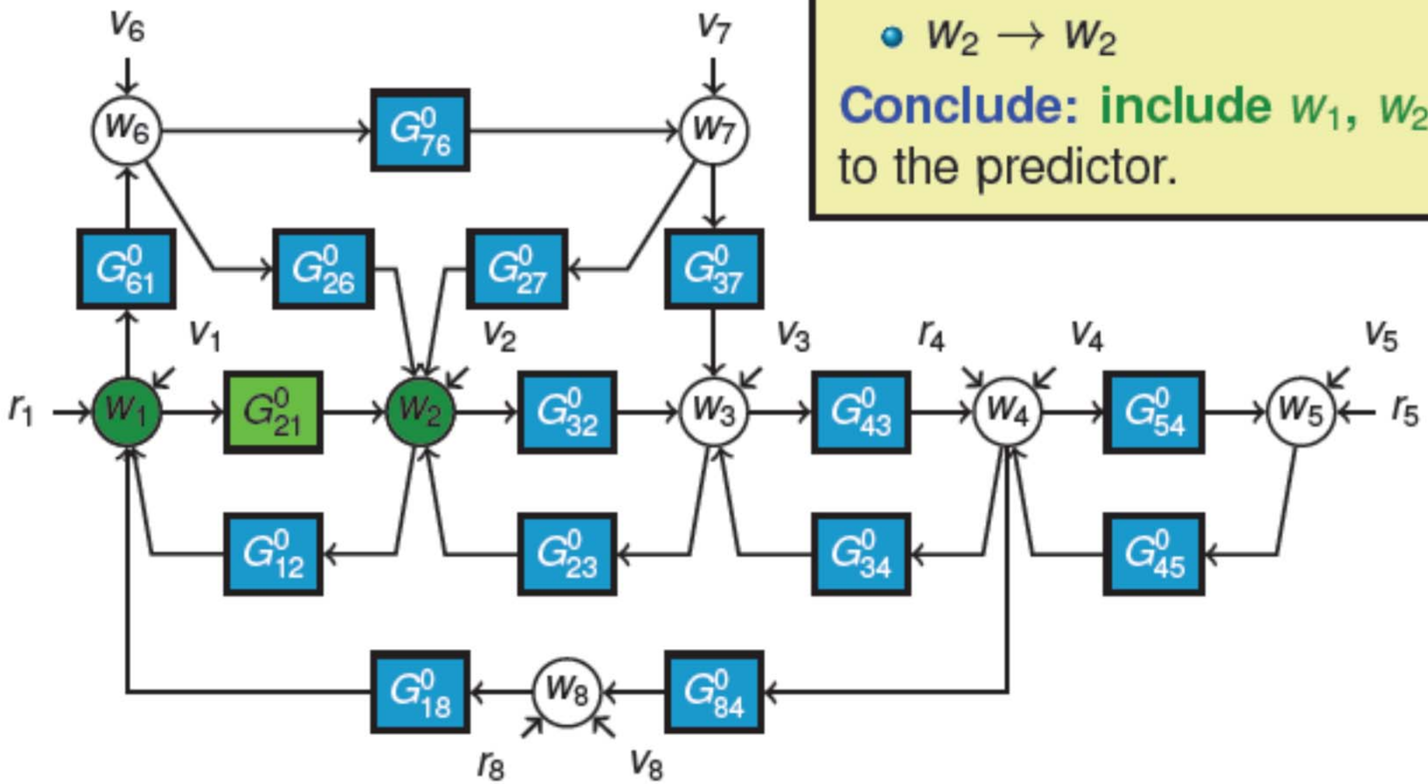
Objective: obtain an estimate of G_{ji}^0.

Consistent estimates of G_{ji}^0 are possible if:

1. w_i and w_j are included as predictor inputs.
2. Each path from $w_i \to w_j$ passes through a node chosen as a predictor input.
3. Each loop from $w_j \to w_j$ passes through a node chosen as a predictor input.

Example with predictor input conditions

Objective: Estimate G_{21}^0.
Project onto r_8, r_4, r_5.



Conditions: Include variable on every path

- $w_1 \rightarrow w_2$
- $w_2 \rightarrow w_2$

Conclude: **include w_1, w_2,** and ... as inputs to the predictor.

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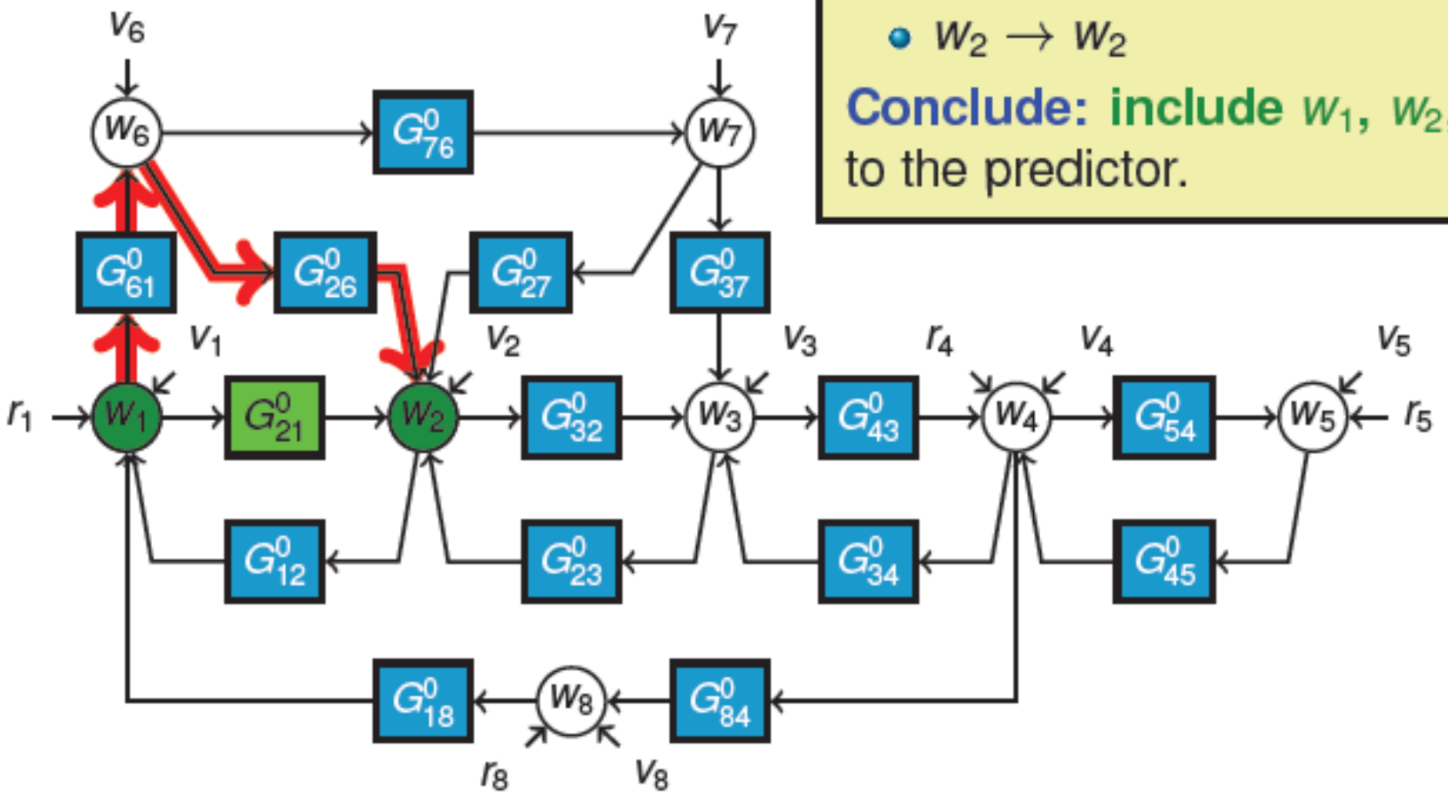

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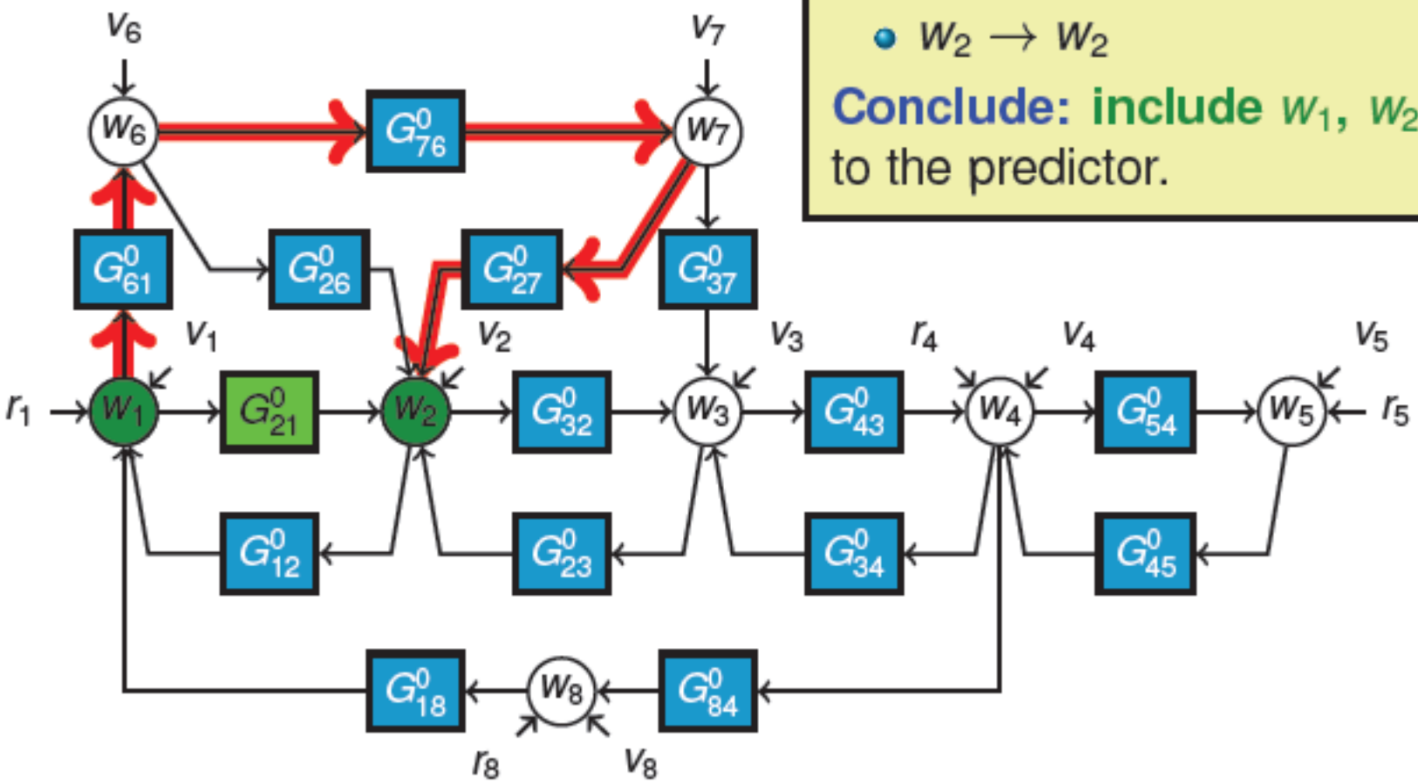

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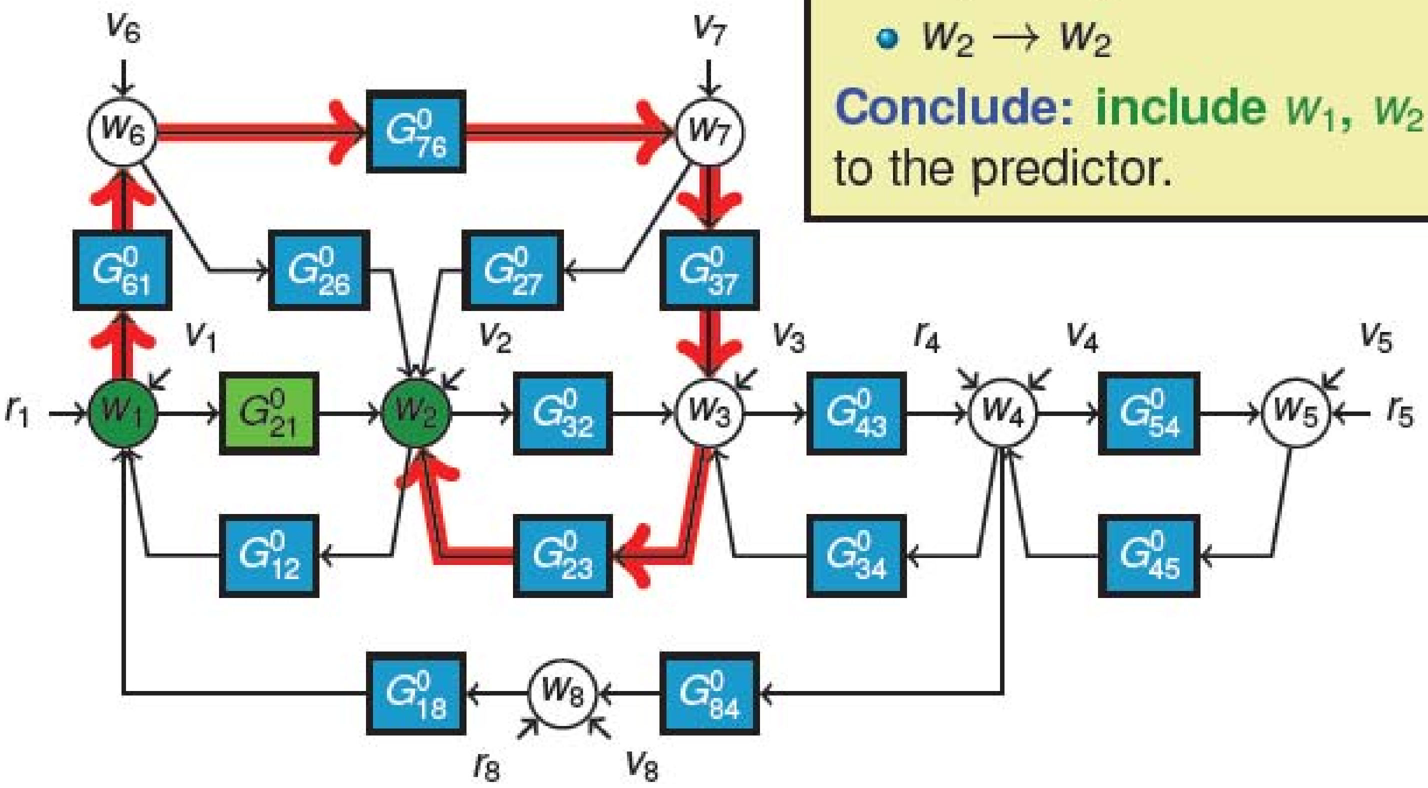

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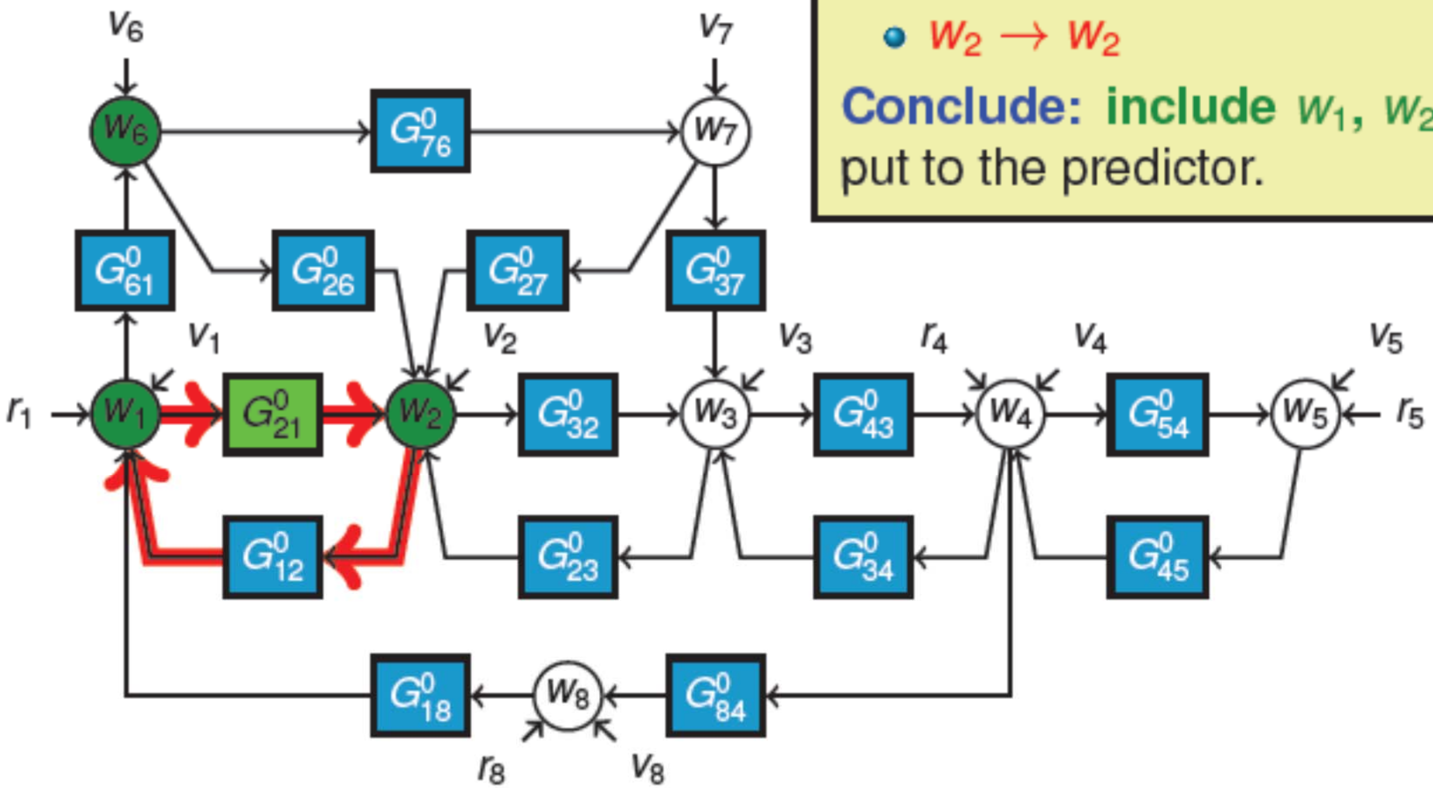

Conditions: Include variable on every path

- $w_1 \rightarrow w_2 \Rightarrow$ **Include w_6 in predictor**
- $w_2 \rightarrow w_2$

Conclude: **include w_1, w_2, w_6** and ... as input to the predictor.

Example with predictor input conditions

Objective: Estimate G^0_{21}.
Project onto r_8, r_4, r_5.

Conditions: Include variable on every path

- $w_1 \rightarrow w_2 \Rightarrow$ **Include w_6 in predictor**
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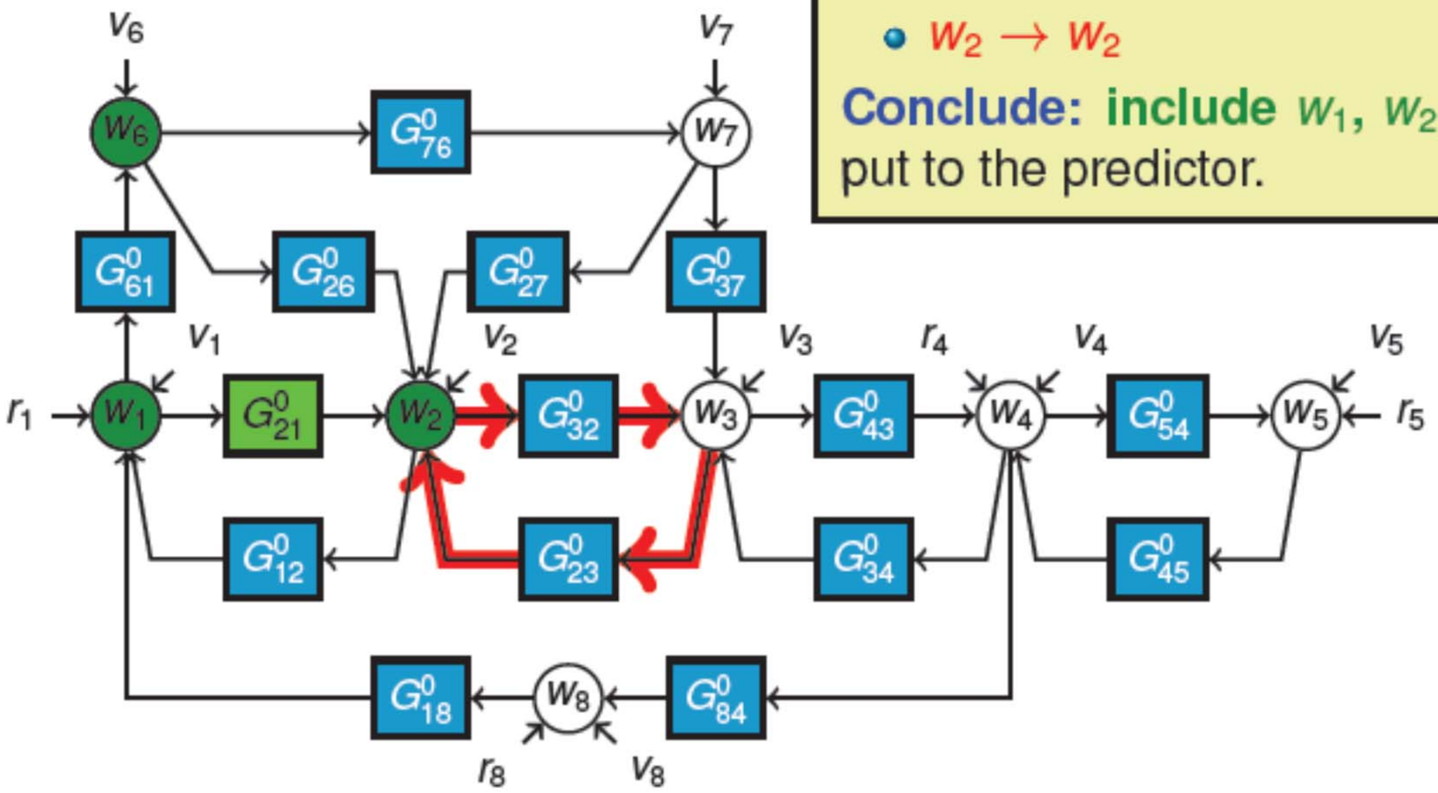
Conclude: **include w_1, w_2, w_6** and ... as input to the predictor.



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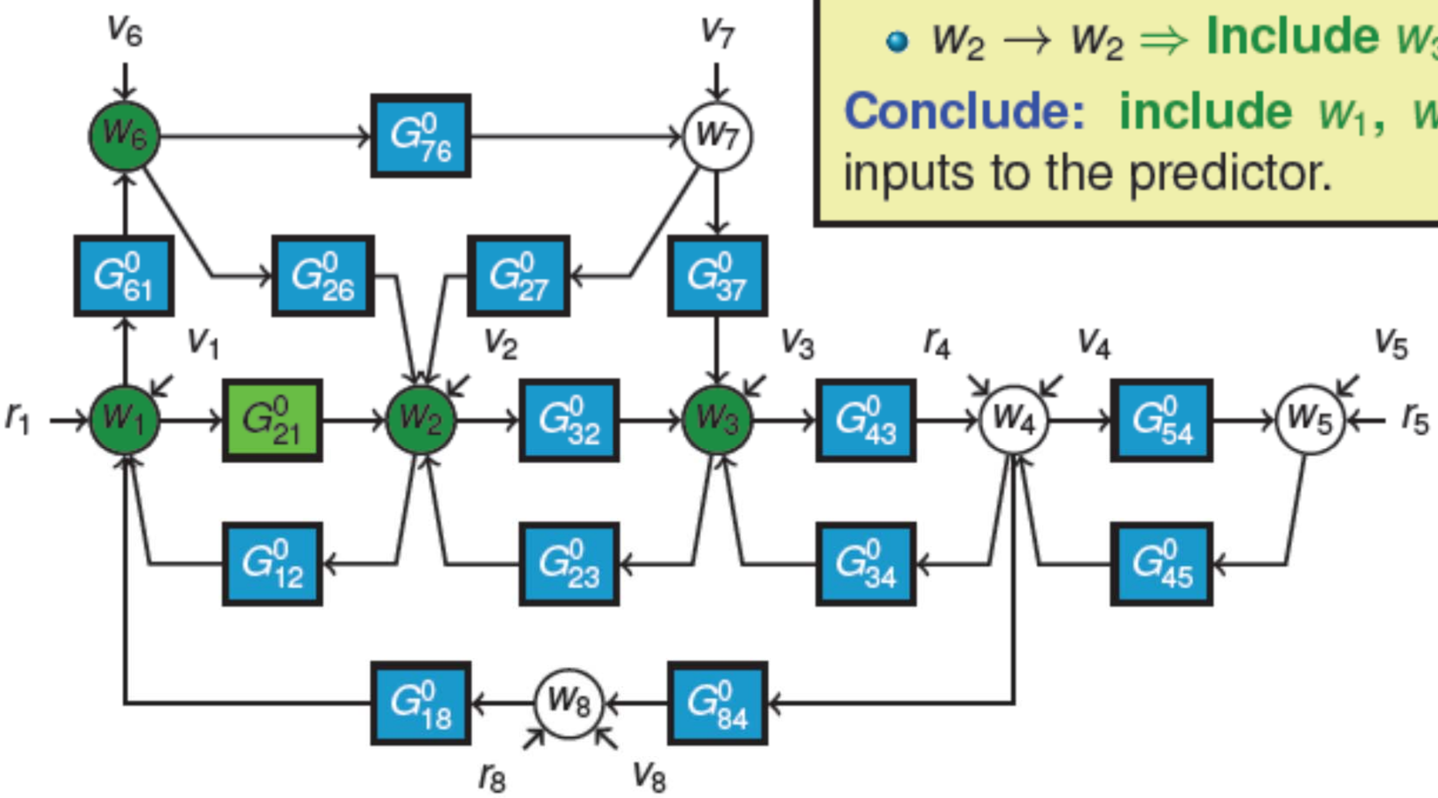

Conditions: Include variable on every path

- $w_1 \rightarrow w_2 \Rightarrow$ **Include w_6 in predictor**
- $w_2 \rightarrow w_2 \Rightarrow$ **Include w_3 in predictor**

Conclude: **include w_1, w_2, w_3, and w_6** as inputs to the predictor.

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Summary

- **Current framework for open/closed-loop identification has been extended to dynamic networks**
- **Methods for closed-loop identification extend to this case with some new properties**
- **Framework extends (easily) to noise on all variables (EIV)**
- **They are expected to provide the basic tools for dealing with the structure identification problem also**
- **So far only consistency considered (no variance)**
- **Many new questions pop up……**

Further reading:

Van den Hof, P.M.J., Dankers, A., Heuberger, P.S.C., and Bombois, X. (2013). Identification of dynamic models in complex networks with prediction error methods – basic methods for consistent module estimates. *Automatica, 49,* 2994-3006.

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