2018 American Control Conference, Milwaukee, WI, USA

Technische Universiteit
Eindhoven
University of Technology

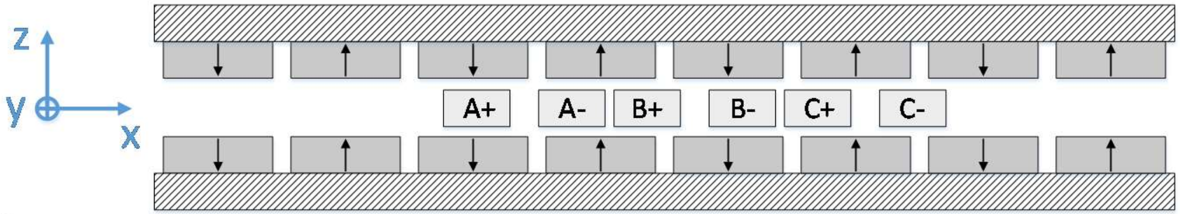
An Instrumental Variable Method for Closed-Loop Identification of Coreless Linear Motors

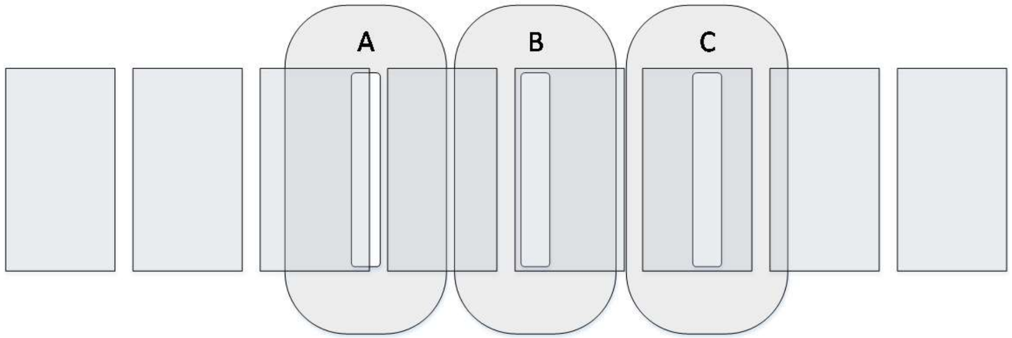
Tuan Nguyen
Mircea Lazar
Hans Butler
Paul M.J. Van den Hof

Where innovation starts

Coreless linear motors

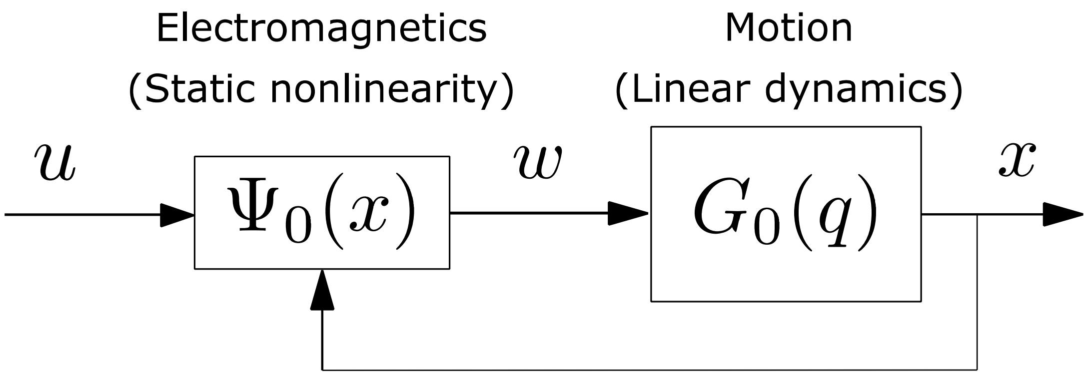
- Generate direct linear motion
- Two main parts
  - Stator: 2 magnet arrays
  - Translator: coil array
- Applications: pick and place machines, scanners, lithography machines…





Research problem

- High accuracy control requires an accurate model
- Identify the model of a coreless linear motor based on measurement data



$u \in \mathbb{R}^{n_u}$ - currents

$w \in \mathbb{R}$ - force

$x \in \mathbb{R}$ - position

- Existing identification methods require current measurement, position measurement and force measurement (known load force)

Research objective:

To identify the model of a coreless linear motor based on current and position measurement data.

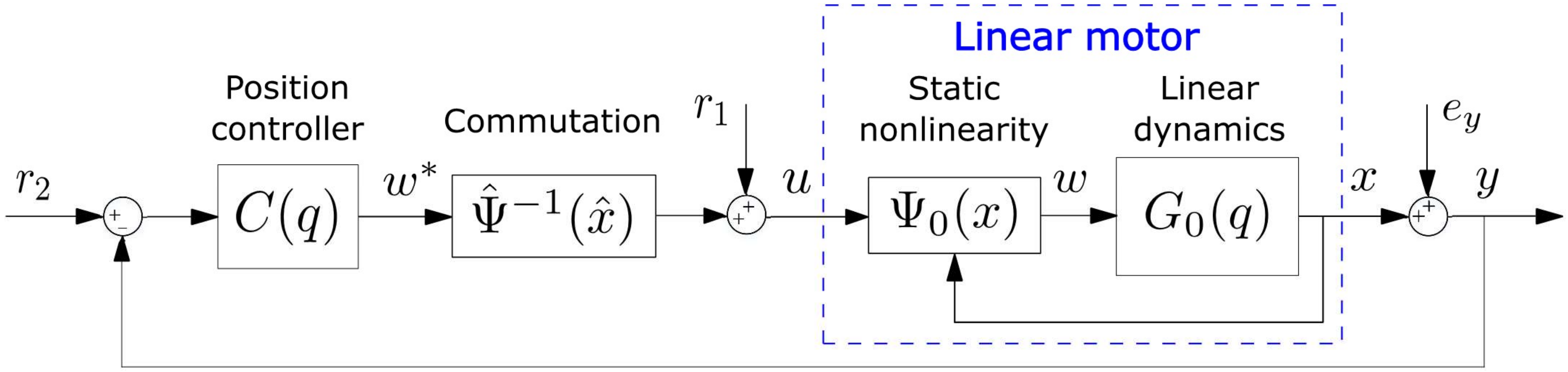
Outline

- Introduction
- Problem formulation
- Instrumental variable framework
- Predictor models
- Numerical example
- Conclusions

Outline

- Introduction
- Problem formulation
- Instrumental variable framework
- Predictor models
- Numerical example
- Conclusions

System description



$u \in \mathbb{R}^{n_u}$ - inputs (currents)

$x \in \mathbb{R}$ - output (position)

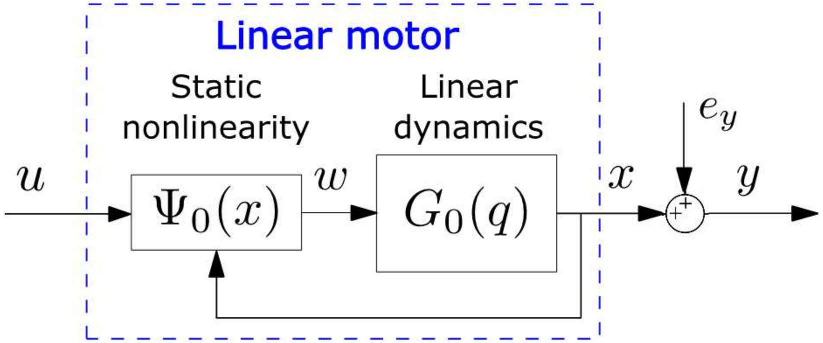
$y \in \mathbb{R}$ - measured output

$w \in \mathbb{R}$ - unavailable internal signal (force)

- Data generating system

$$\begin{cases} y(t) = G_0(q)\Psi_0(x(t))u(t) + e_y(t) \\ u(t) = \hat{\Psi}^{-1}(\hat{x})C(q)(r_2(t) - y(t)) + r_1(t) \end{cases}$$

Model structure



- Linear dynamics

$$G_0(q) = \frac{\sum_{k=1}^{n_b} b_k^0 q^{-k}}{1 + \sum_{j=1}^{n_a} a_j^0 q^{-j}}$$

$$\Rightarrow x(t) = -\sum_{j=1}^{n_a} a_j^0 x(t-j) + \sum_{k=1}^{n_b} b_k^0 w(t-k)$$

- Static nonlinearity

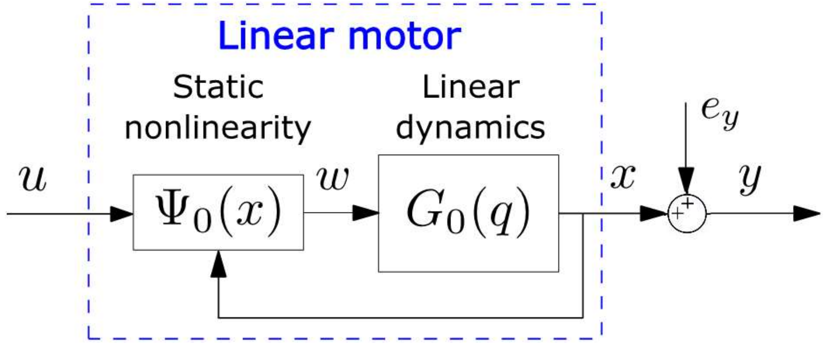
$$\Psi_0(x(t)) = \begin{bmatrix} \sum_{n=1}^{n_F} \left(c_{1,n}^0 \cos(\omega_n x(t)) + d_{1,n}^0 \sin(\omega_n x(t))\right) \\ \vdots \\ \sum_{n=1}^{n_F} \left(c_{n_u,n}^0 \cos(\omega_n x(t)) + d_{n_u,n}^0 \sin(\omega_n x(t))\right) \end{bmatrix}^\top$$

$$\Rightarrow w(t) = \sum_{l=1}^{n_u} \sum_{n=1}^{n_F} \left(c_{l,n}^0 \cos(\omega_n x(t)) + \sum_{n=1}^{n_F} d_{l,n}^0 \sin(\omega_n x(t)) \right) u_l(t)$$

where $\omega_n = \frac{\pi}{\tau_p}$ (τ_p is the magnet pole pitch)

Problem statement

- Model structure



$$y(t) = -\sum_{j=1}^{n_a} a_j^0 x(t-j) + \sum_{k=1}^{n_b} b_k^0 w(t-k) + e_y(t)$$

$$= -\sum_{j=1}^{n_a} a_j^0 x(t-j) + \sum_{k=1}^{n_b}\sum_{l=1}^{n_u}\sum_{n=1}^{n_F} b_k^0 c_{l,n}^0 \cos(\omega_n x(t-k)) u_l(t-k)$$

$$+ \sum_{k=1}^{n_b}\sum_{l=1}^{n_u}\sum_{n=1}^{n_F} b_k^0 d_{l,n}^0 \sin(\omega_n x(t-k)) u_l(t-k) + e_y(t)$$

- Problem statement:

Given N samples of input-output measurements $\{u(t), y(t)\}_{t=1}^{N}$, identify the unknown parameters

$$a = [a_1 \ \ldots \ a_{n_a}]^\top, \quad b = [b_1 \ \ldots \ b_{n_b}]^\top,$$

$$c = [c_{1,1} \ \ldots \ c_{n_u,n_F}]^\top, \quad d = [d_{1,1} \ \ldots \ d_{n_u,n_F}]^\top.$$

The internal signal $w(t)$ is unavailable.

Challenges

- Closed-loop data ⇒ correlation between input and output ⇒ the simple least squares (LS) estimate is biased.

  Solution: instrumental variable (IV) framework

- Nonlinear dependency on the unknown noise-free output $x(t)$

$$y(t) = -\sum_{j=1}^{n_a} a_j^0 x(t-j) + \sum_{k=1}^{n_b}\sum_{l=1}^{n_u}\sum_{n=1}^{n_F} b_k^0 c_{l,n}^0 \cos(\omega_n x(t-k))u_l(t-k)$$
$$+ \sum_{k=1}^{n_b}\sum_{l=1}^{n_u}\sum_{n=1}^{n_F} b_k^0 d_{l,n}^0 \sin(\omega_n x(t-k))u_l(t-k) + e_y(t)$$

  ⇒ biased IV estimate

  Solution: bias-correction scheme

Outline

- Introduction
- Problem formulation
- Instrumental variable framework
- Predictor models
- Numerical example
- Conclusions

Instrumental variable framework

- Consider a linear-in-the-parameter predictor

$$\hat{y}(t,\theta) = \varphi^\top(t)\theta$$

$\hat{y}(t,\theta) \in \mathbb{R}$ is the predicted output,

$\theta \in \mathbb{R}^{n_\theta}$ is the parameter vector,

$\varphi(t) \in \mathbb{R}^{n_\theta}$ is the regressor vector.

- Find $\hat{\theta}$ that minimizes the prediction error

$$\min_{\hat{\theta}} \quad \|\varphi^\top(t)\hat{\theta} - y(t)\|^2$$

- Least squares (LS) estimate

$$\hat{\theta}_{\mathrm{LS}} = \left(\frac{1}{N}\sum_{t=1}^{N} \varphi(t)\varphi(t)^\top\right)^{-1} \left(\frac{1}{N}\sum_{t=1}^{N} \varphi(t)y(t)\right)$$

- Instrumental variable (IV) estimate

$$\hat{\theta}_{\mathrm{IV}} = \left(\frac{1}{N}\sum_{t=1}^{N} \zeta(t)\varphi(t)^\top\right)^{-1} \left(\frac{1}{N}\sum_{t=1}^{N} \zeta(t)y(t)\right)$$

where

$\zeta \in \mathbb{R}^{n_\theta}$ is the instrumental vector.

Consistency of IV estimate

- The IV estimate is said to be consistent if

$$\theta_{\mathrm{IV}} \to \theta_0 \text{ with probability 1 as } N \to \infty$$

- The IV estimate is consistent if the following conditions are satisfied
    - C1: $\bar{\mathbb{E}}[\zeta(t)\varphi(t)^\top]$ is nonsingular
    - C2: $\bar{\mathbb{E}}[\zeta(t)(y(t) - \hat{y}(t, \theta_0))] = 0$

- C1 is satisfied if the system is sufficiently excited and $\zeta(t)$ is well correlated with $\varphi(t)$
- C2 is satisfied if $\zeta(t)$ is uncorrelated with the measurement noise and the predictor $\hat{y}(t, \theta_0)$ is selected appropriately

Outline

- Introduction
- Problem formulation
- Instrumental variable framework
- Predictor models
- Numerical example
- Conclusions

NARX predictor

- Model structure

$$y(t) = -\sum_{j=1}^{n_a} a_j^0 x(t-j) + \sum_{k=1}^{n_b}\sum_{l=1}^{n_u}\sum_{n=1}^{n_F} b_k^0 c_{l,n}^0 \cos(\omega_n x(t-k)) u_l(t-k)$$
$$+ \sum_{k=1}^{n_b}\sum_{l=1}^{n_u}\sum_{n=1}^{n_F} b_k^0 d_{l,n}^0 \sin(\omega_n x(t-k)) u_l(t-k) + e_y(t)$$

- Nonlinear autoregressive exogenous (NARX) predictor

$$\hat{y}_{\mathrm{NARX}}(t,\theta) = -\sum_{j=1}^{n_a} a_j y(t-j) + \sum_{k=1}^{n_b}\sum_{l=1}^{n_u}\sum_{n=1}^{n_F} b_k c_{l,n} \cos(\omega_n y(t-k)) u_l(t-k)$$
$$+ \sum_{k=1}^{n_b}\sum_{l=1}^{n_u}\sum_{n=1}^{n_F} b_k d_{l,n} \sin(\omega_n y(t-k)) u_l(t-k) = \varphi_{\mathrm{NARX}}^\top(t)\theta$$

where

$$\theta = [a_1 \quad \dots \quad a_{n_a} \quad b_1 f_1 \quad \dots \quad b_{n_b} f_{n_u}$$
$$b_1 c_{1,1} \quad \dots \quad b_{n_b} c_{n_u,n_F} \quad b_1 d_{1,1} \quad \dots \quad b_{n_b} d_{n_u,n_F}]^\top$$

$$\varphi_{\mathrm{NARX}}(t) = [-y(t-1) \quad \dots \quad -y(t-n_a)$$
$$\cos(\omega_1 y(t-1)) u(t-1) \quad \dots \quad \cos(\omega_{n_F} y(t-k)) u(t-n_b)$$
$$\sin(\omega_1 y(t-1)) u(t-1) \quad \dots \quad \sin(\omega_{n_F} y(t-k)) u(t-n_b)]^\top$$

Consistency analysis

- Assumption: $e_y(t)$ is a zero-mean white noise with a symmetric probability density function.
- Condition C2

$$\begin{aligned}\bar{\mathbb{E}}[\zeta(t)(y(t)-\hat{y}_{\mathrm{NARX}}(t,\theta_0))] = & \sum_{k=1}^{n_b}\sum_{l=1}^{n_u}\sum_{n=1}^{n_F} b_k^0 c_{l,n}^0 \left(\bar{\mathbb{E}}\left[\zeta(t)\cos(\omega_n x(t-k))u(t-k)\right]\left(1-\phi_{e_y}(\omega_n)\right)\right) \\ & + \sum_{k=1}^{n_b}\sum_{l=1}^{n_u}\sum_{n=1}^{n_F} b_k^0 d_{l,n}^0 \left(\bar{\mathbb{E}}\left[\zeta(t)\sin(\omega_n x(t-k))u(t-k)\right]\left(1-\phi_{e_y}(\omega_n)\right)\right) \\ \approx\ & 0.\end{aligned}$$

where ϕ_{e_y} is the characteristic function of $e_y(t)$

$$\phi_{e_y}(\alpha) := \mathbb{E}[e^{i\alpha e_y(t)}],\ \text{where } \alpha \in \mathbb{R}.$$

- When $e_y(t)$ is small compared to the magnet pole pitch τ_p, then the bias is negligible because $\phi_{e_y}(\omega_n)$ is very close to 1

Bias-corrected predictor

- Bias-corrected predictor

$$
\begin{aligned}
\hat{y}_{\rm bc}(t,\theta) = & -\sum_{j=1}^{n_a} a_j y(t-j) + \sum_{k=1}^{n_b}\sum_{l=1}^{n_u}\sum_{n=1}^{n_F} b_k c_{l,n}\cos(\omega_n y(t-k))u_l(t-k)\frac{1}{\phi_{e_y}(\omega_n)} \\
& + \sum_{k=1}^{n_b}\sum_{l=1}^{n_u}\sum_{n=1}^{n_F} b_k d_{l,n}\sin(\omega_n y(t-k))u_l(t-k)\frac{1}{\phi_{e_y}(\omega_n)} \\
= & \ \varphi_{\rm bc}^{\top}(t)\theta
\end{aligned}
$$

where

$$
\begin{aligned}
\theta = [a_1 \quad \dots \quad a_{n_a} \quad b_1 f_1 \quad \dots \quad b_{n_b} f_{n_u} \\
b_1 c_{1,1} \quad \dots \quad b_{n_b} c_{n_u,n_F} \quad b_1 d_{1,1} \quad \dots \quad b_{n_b} d_{n_u,n_F}]^{\top}
\end{aligned}
$$

$$
\begin{aligned}
\varphi_{\rm bc}(t) = \Big[& -y(t-1) \quad \dots \quad -y(t-n_a) \\
& \cos(\omega_1 y(t-1))u(t-1)\frac{1}{\phi_{e_y}(\omega_1)} \quad \dots \quad \cos(\omega_{n_F} y(t-k))u(t-n_b)\frac{1}{\phi_{e_y}(\omega_{n_F})} \\
& \sin(\omega_1 y(t-1))u(t-1)\frac{1}{\phi_{e_y}(\omega_1)} \quad \dots \quad \sin(\omega_{n_F} y(t-k))u(t-n_b)\frac{1}{\phi_{e_y}(\omega_{n_F})}\Big]^{\top}
\end{aligned}
$$

Consistency analysis

- Condition C2

$$\bar{\mathbb{E}}[\zeta(t)(y(t)-\hat{y}_{\mathrm{bc}}(t,\theta_0))] = \sum_{k=1}^{n_b}\sum_{l=1}^{n_u}\sum_{n=1}^{n_F} b_k^0 c_{l,n}^0 \left(\bar{\mathbb{E}}\left[\zeta(t)\cos(\omega_n x(t-k))u(t-k)\right]\left(1-\phi_{e_y}(\omega_n)\frac{1}{\phi_{e_y}(\omega_n)}\right)\right)$$
$$+\sum_{k=1}^{n_b}\sum_{l=1}^{n_u}\sum_{n=1}^{n_F} b_k^0 d_{l,n}^0 \left(\bar{\mathbb{E}}\left[\zeta(t)\sin(\omega_n x(t-k))u(t-k)\right]\left(1-\phi_{e_y}(\omega_n)\frac{1}{\phi_{e_y}(\omega_n)}\right)\right)$$
$$=0$$

- The resulting IV estimate is consistent

Outline

Technische Universiteit
Eindhoven
University of Technology

- Introduction
- Problem formulation
- Instrumental variable framework
- Predictor models
- Numerical example
- Conclusions

Example

- Static nonlinearity

$$\Psi_0(x) = \begin{bmatrix} \sum_{n=1}^{2} \left(c_{A,n} \cos(\omega_n x) + d_{A,n} \sin(\omega_n x)\right) \\ \sum_{n=1}^{2} \left(c_{B,n} \cos(\omega_n x) + d_{B,n} \sin(\omega_n x)\right) \end{bmatrix}^{\top} \begin{bmatrix} u_A \\ u_B \end{bmatrix}$$

- Linear dynamics

$$G_0(q) = \frac{b_1 q^{-1} + b_2 q^{-2}}{1 + a_1 q^{-1} + a_2 q^{-2}}$$

- Magnet pole pitch $\tau_p = 40\text{mm}$
- White Gaussian noise, zero mean, $\sigma = 5\mu\text{m}$

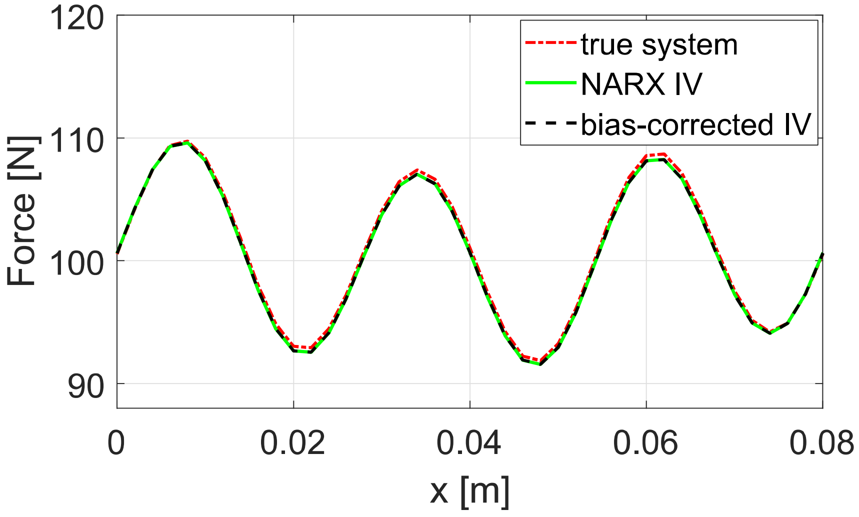
MEAN AND STANDARD DEVIATION OF 150 ESTIMATED MODELS

Parameter	*True value*	*NARX IV*	*Bias-corrected IV*
a_1	-1.9950	-1.9950 ± 0.0001	-1.9950 ± 0.0001
a_2	0.9950	0.9950 ± 0.0001	0.9950 ± 0.0001
$\bar{b}_2$	0.9983	1.0230 ± 0.1881	1.0230 ± 0.1881
$c_{A,1}$	0	-0.0032 ± 0.2716	-0.0032 ± 0.2716
$c_{A,2}$	-0.6988	-0.6965 ± 0.1340	-0.6965 ± 0.1340
$c_{B,1}$	-9.0781	-9.0421 ± 0.8498	-9.0421 ± 0.8498
$c_{B,2}$	-0.2745	-0.2767 ± 0.1126	-0.2767 ± 0.1126
$d_{A,1}$	7.8619	7.8475 ± 0.7323	7.8475 ± 0.7323
$d_{A,2}$	-0.3694	-0.3503 ± 0.1082	-0.3503 ± 0.1082
$d_{B,1}$	-4.5391	-4.5189 ± 0.5322	-4.5189 ± 0.5322
$d_{B,2}$	0.4592	0.4598 ± 0.1288	0.4598 ± 0.1288

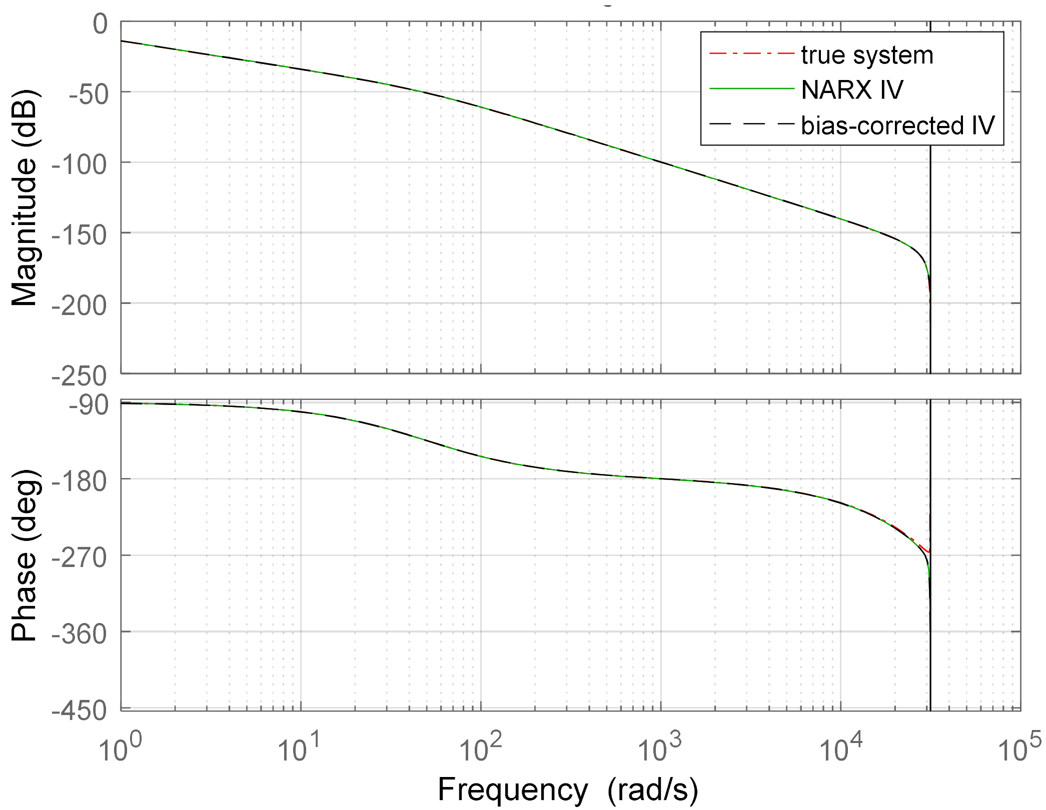
Note: $\bar{b}_2 = b_2 \times 10^7$.

Example

Static nonlinearity



Linear dynamics



Outline

- Introduction
- Problem formulation
- Instrumental variable framework
- Predictor models
- Numerical example
- Conclusions

Conclusions

- Identification of linear motors has two main challenges
  - Closed-loop data
  - Nonlinear dependency on the unknown noise-free output
- The first challenge is addressed by using the IV framework
- The second challenge is addressed by selecting appropriate predictor
  - NARX predictor
    - small bias
    - does not require knowledge of the statistical property of measurement noise
  - Bias-corrected predictor
    - consistent
    - require knowledge of the statistical property of measurement noise
- The method can be applied to other electrical machines
- Future work: improve statistical efficiency of the method

Thank you for your attention!