System Identification in Dynamic Networks

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Introduction – dynamic networks
Dynamical systems in emerging fields have a more complex structure:

- distributed control system (1d-cascade)
- dynamic network

(distributed systems, multi-agent systems, biological networks, smart grids,…..)

For on-line monitoring / control / diagnosis it is attractive to be able to **identify**
- (changing) dynamics of particular modules
- (changing) interconnection structure

What are relevant identification questions that appear?
Some modules may be known (e.g. controllers)

- $r_i$: external excitation
- $v_i$: process noise
- $w_i$: node signal
Introduction – relevant identification questions

How to perform “local” identification (i.e. estimating only a single module)?
Where to put sensors and actuators for optimal accuracy?
How to utilize known structure/topology and known modules?
Introduction – relevant identification questions

Can we identify the topology?
Can we deal with sensor noise?
Do we need directions of arrows?
Introduction - identification

The classical identification problems:

open loop

\[ G \]

\[ u \rightarrow y \rightarrow v \]

closed loop

\[ G \]

\[ r \rightarrow u \rightarrow y \rightarrow v \]

Identify a plant model \( \hat{G} \) on the basis of measured signals \( u, y \) (and possibly \( r \))

- We have to move from fixed and known configuration to deal with and exploit structure in the problem.
Network Diagrams

Represented as

Labels of internal variables placed inside summations
Current literature

Numerical fast algorithms for **spatially distributed systems** with identical modules (Fraanje, Verhaegen, Werner), or non-identical ones (Torres, van Wingerden, Verhaegen, Sarwar, Salapaka, Haber)

Contributions to **topology detection**: Chiuso, Materassi, Innocenti, Salapaka, Yuan, Stan, Warnick, Goncalves, Sanandaji, Vincent, Wakin, further exploring and utilizing the concept of Granger causality.

Here: focus on **prediction error methods** and concepts for identification in generally structured (linear) dynamic networks
Towards dynamic network identification

• The basic (prediction error) tools: direct and 2s
• Dynamic network setup
• Single module identification - consistency
  • full MISO models
  • predictor input (sensor) selection
• Sensor noise – the errors-in-variables problem
• Discussion / Wrap-up
Methods for closed-loop identification

1. Direct method

Relying on full-order noise modelling

\[ \epsilon(t, \theta) = H(\theta)^{-1}[y(t) - G(\theta)u(t)] \]

prediction error \( \epsilon(t, \theta) \) to become a white noise signal \( e(t) \) in the optimum.

Using only signals \( u \) and \( y \), discarding \( r \)

\[ \hat{\theta}_N = \arg \min_{\theta} \frac{1}{N} \sum_{t=1}^{N} \epsilon(t, \theta)^2 \]
Methods for closed-loop identification

1. Direct method

Consistency result [Ljung, 1987]

\[ \{G(\hat{\theta}_N), H(\hat{\theta}_N)\} \rightarrow \{G_0, H_0\} \text{ w.p.1, } N \rightarrow \infty \]

if

- full order noise model \( (S \in M) \)
- delay in every loop
- sufficient excitation, i.e.

\[ \Phi_z(\omega) > 0 \quad \forall \omega \quad z = \begin{bmatrix} y \\ u \end{bmatrix} \]

with spectral density

\[ \Phi_z(\omega) = \mathcal{F}\{\bar{E}[z(t)z(t - \tau)]\} \]

Plant representation

\[ y(t) = G_0u(t) + H_0e(t) \]

\( e \) white noise

\( r \) and \( v \) uncorrelated
2. Two-stage/projection/IV method

- Relying on measured external excitation
- Decoupling estimation of $G_0$ and $H_0$

$$\varepsilon(t, \theta) = H(\rho)^{-1}[y(t) - G(\theta)u^r(t)]$$

with $u^r$ the signal $u$ projected onto $r$ such that

$$u = u^r + u^v$$

with $u^r$ and $u^v$ uncorrelated.

Similar least squares criterion.

Plant representation

$$y(t) = G_0u(t) + H_0e(t)$$

$e$ white noise

$r$ and $v$ uncorrelated
2. Two-stage/projection/IV method

Consistency result [Van den Hof & Schrama, 1993]

\[ G(\hat{\theta}_N) \rightarrow G_0 \text{ w.p.1, } N \rightarrow \infty \]

if

- full order plant model \((G_0 \in \mathcal{G})\)
- no conditions on loop delays
- sufficient excitation condition:
  \[ \Phi_{u^r}(\omega) > 0 \quad \forall \omega \]

Plant representation

\[ y(t) = G_0 u(t) + H_0 e(t) \]

\(e\) white noise

\(r\) and \(v\) uncorrelated
Question

• Can we utilize these tools for identification of transfer functions in a (complex) dynamic network?
Network Setup

Formalizing one link (transfer between $w_i$ and $w_j$)

- On each node a disturbance $v_j$ and a reference $r_j$ might be present.
- Reference signals are uncorrelated to noise signals.
- $\mathcal{N}_j$: set of nodes that has a direct causal link with node $j$, of which $\mathcal{K}_j$ are known transfers and $\mathcal{U}_j$ unknown.
Network Setup

Assumptions:

- Total of \( L \) nodes
- Network is well-posed
  \( I - G^0 \) causally invertible
- Stable (all signals bounded)
- All \( w_m, m = 1, \ldots, L \), measured, as well as all present \( r_m \)
- Modules may be unstable

\[
\begin{bmatrix}
  w_1 \\
  w_2 \\
  \vdots \\
  w_L \\
\end{bmatrix} =
\begin{bmatrix}
  0 & G^0_{12} & \cdots & G^0_{1L} \\
  G^0_{21} & 0 & \cdots & G^0_{2L} \\
  \vdots & \vdots & \ddots & \vdots \\
  G^0_{L1} & G^0_{L2} & \cdots & 0 \\
\end{bmatrix}
\begin{bmatrix}
  w_1 \\
  w_2 \\
  \vdots \\
  w_L \\
\end{bmatrix} +
\begin{bmatrix}
  r_1 \\
  r_2 \\
  \vdots \\
  r_L \\
\end{bmatrix} +
\begin{bmatrix}
  v_1 \\
  v_2 \\
  \vdots \\
  v_L \\
\end{bmatrix}
\]
Network Setup

Options for identifying a module:

- **Identify the full MIMO system:**
  \[ w = (I - G^0)^{-1}[r + v] \]
  from measured \( r \) and \( w \).

  Global approach with “standard” tools

- **Identify a local (set of) module(s):**
  from a (sub)set of measured \( r_k \) and \( w_\ell \)

  Local approach with “new” tools and structural conditions
How to identify a module:

Suppose we are interested in $G_{21}^0$

Can it be identified from measured input $w_1$ and output $w_2$?

Typically bias will occur due to "neglecting" the rest of the network

- Non-modelled disturbances on $w_2$ can create problems
- The observed transfer between $w_1$ and $w_2$ is not necessarily equal to $G_{21}^0$
Network Setup

How to identify a module:

Two approaches for finding $G_{21}^0$

- **Full MISO approach:**
  Include all node signals that directly map into $w_2$ in an input vector, and identify a MISO model

- **Predictor input selection:**
  Formulate conditions for checking the sufficiency of set of nodes to include as inputs in a MISO model
Towards dynamic network identification

• The basic (prediction error) tools: direct and 2s
• Dynamic network setup
• Single module identification - consistency
  • full MISO models
  • predictor input (sensor) selection
• Sensor noise – the errors-in-variables problem
• Discussion / Wrap-up
Module of interest: $G_{ji}^0$

Separate the remaining modules: $G_{jk}^0$ into known transfers: $G_{jk}, k \in \mathcal{K}_j$
and unknown transfers: $G_{jk}, k \in \mathcal{U}_j^i$

A MISO approach:

$$\varepsilon(t, \theta) = H_j(\theta)^{-1}[w_j - r_j - \sum_{k \in \mathcal{K}_j} G_{jk}^0 w_k - G_{ji}(\theta) w_i - \sum_{k \in \mathcal{U}_j^i} G_{jk}(\theta) w_k]$$

Simultaneous identification of transfers $G_{jk}, k \in \mathcal{U}_j^i$ and a noise model for $v_j$
Network Identification – Direct method
Network Identification – Direct method
Network Identification – Direct method

Result direct method

The plant models $G_{jk}(\theta), \ k \in U_j$ are consistently estimated if:

- All parametrized plant and noise models are correctly parametrized, $G_{jk}(\theta), \ k \in U_j; \ H_j(\theta) \ (S \in \mathcal{M})$
- Every loop in the network that runs through node $j$ has at least one delay (no algebraic loop)
- $\Phi_z(\omega) > 0 \ \forall \omega$, for $z := vec\{w_j, \{w_k\}_{k \in U_j}\}$ (excitation condition)
- Noise source $v_j$ is uncorrelated with all other noise terms in the network

Recall the two-stage/projection/IV approach:

Project $u$ onto an external signal $r$ that is uncorrelated to $v$

$$\varepsilon(t, \theta) = H(\rho)^{-1}[y(t) - G(\theta)u^r(t)]$$

$$u = u^r + u^v$$

with $u^r$ and $u^v$ uncorrelated.

Plant representation

$$y(t) = G_0u(t) + H_0e(t)$$

$e$ white noise

$r$ and $v$ uncorrelated
Network Identification – Two-stage method

Main approach:
• Look for an external reference signal that has a connection with $w_i$
• And that does not act as an unmodelled disturbance on $w_j$
Algorithm:

- Determine whether there exists an $r_m$ such that $w_i^{r_m}$ is sufficiently exciting.
- Construct:

$$\tilde{w}_j = w_j - r_j - \sum_{k \in K_j} G_{jk}^0 w_k$$

known terms

- Identify $G_{ji}^0$ through PE identification with prediction error

$$\varepsilon(t, \theta) = H_j(\rho)^{-1}[\tilde{w}_j - \sum_{k \in U_{is}} G_{jk}(\theta)w_k^{r_m}]$$

where all inputs $k \in U_{is}$ are considered that are correlated to $r_m$.

- This extends to multiple signals $r_m$. 
Network Identification – Two-stage method

Result two-stage method

The plant model $G_{ji}(\theta)$ is consistently estimated if:

- The plant models $G_{jk}(\theta)$ are correctly parametrized $k \in \mathcal{U}_{is}$
- The vector of (projected) input signals is sufficiently exciting
- Excitation signals are uncorrelated to noise disturbances

Network Identification – Two-stage method

Example

- External signal $r_1$
- Input nodes to $w_2$ that are correlated with $r_1$:
  $w_1, w_6, w_7, w_3$
- So 4 input, 1 output problem
- Projected inputs will generally not be sufficiently exciting (we need 4 independent sources)
- Include $r_4, r_5$ and $r_8$ as external signals
- Input nodes remain the same
Network Identification – **Two-stage method**

**Observations:**
- Consistent identification of single transfers is possible, dependent on network topology and reference excitation.
- Full noise models are not necessary.
- No conditions on uncorrelated noise sources, nor on absence of algebraic loops.
- Excitation conditions on (projected) input signals can be limiting.
- Network topology conditions on $r_m$ can simply be checked by tools from graph theory.
Towards dynamic network identification

- The basic (prediction error) tools: direct and 2s
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Predictor input selection

- So far: predictor input choice not very flexible
- What if some signals are hard (expensive) to measure?
- What if we would like to have flexibility in placing sensors?
- Can we formulate (more relaxed) conditions on nodes to be measured, for allowing a consistent module estimate?
There are two basic mechanisms that “deteriorate” the transfer $G_{ji}^0$ when observed through the input/output signals $w_i$ and $w_j$

1. Parallel paths
2. Loops around $w_j$
**First mechanism: parallel paths**

**Objective:** consistently estimate $G_{21}^0$.

**SISO approach.** Try to estimate the dynamics between $w_1$ and $w_2$:

$$w_2 = G_{21}^0 w_1^{(n)} + G_{21}^0 w_1^{(v)} + G_{23}^0 w_3 + v_2$$

unmodeled term

**Problem!** "unmodeled term" (noise term) is correlated to input term, $w_1^{(r1)}$. 

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Objective: obtain an estimate of $G_{ji}^0$

Consistent estimates of $G_{ji}^0$ are possible if:

1. $w_i$ is included as predictor input

2. Each path from $w_i \rightarrow w_j$ passes through a node chosen as predictor input
Second mechanism: loops around the output

**Objective:** consistently estimate \( G_{21}^0 \).

**SISO approach.** Try to estimate the dynamics between \( w_1 \) and \( w_2 \):

\[
 w_2 = G_{21}^0 w_1^{(r_1)} + G_{21}^0 w_1^{(v)} + G_{23}^0 w_3 + v_2
\]

unmodeled term

**Problem!** "unmodeled term" (noise term) is correlated to input term, \( w_1^{(r_1)} \).
Second mechanism: loops around the output

**Objective:** consistently estimate $G_{21}^0$.

**SISO approach.** Try to estimate the dynamics between $w_1$ and $w_2$:

$$w_2 = G_{21}^0 w_1^{(r_1)} + G_{21}^0 w_1^{(v)} + G_{23}^0 w_3 + v_2$$

unmodeled term

**Problem!** ”unmodeled term” (noise term) is correlated to input term, $w_1^{(r_1)}$.

**Solution:** Include $w_3^{(r_1)}$ in the predictor:

$$w_2 = G_{21}^0 w_1^{(r_1)} + G_{23}^0 w_3^{(r_1)} + G_{21}^0 w_1^{(v)} + G_{23}^0 w_3^{(v)} + v_2$$

unmodeled term
**Objective:** obtain an estimate of $G_{ji}^0$

**Consistent** estimates of $G_{ji}^0$ are possible if:

1. $w_i$ is included as predictor input
2. Each path from $w_i \rightarrow w_j$ passes through a node chosen as predictor input
3. Each loop from $w_j \rightarrow w_j$ passes through a node chosen as predictor input
Example with predictor input conditions

**Objective:** Estimate $G_{21}^0$.

**Conditions:** Include variable on every path
- $w_1 \rightarrow w_2$
- $w_2 \rightarrow w_2$

**Conclude:** include $w_1$ and … as predictor inputs
Example with predictor input conditions

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**Conditions:** Include variable on every path
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**Conditions:** Include variable on every path
- $w_1 \rightarrow w_2$
- $w_2 \rightarrow w_2$

**Conclude:** include $w_1$ and ... as predictor inputs
Example with predictor input conditions

**Objective:** Estimate $G_{21}^0$.

**Conditions:** Include variable on every path
- $w_1 \rightarrow w_2 \Rightarrow \text{Include } w_6 \text{ in predictor}$
- $w_2 \rightarrow w_2$

**Conclude:** include $w_1, w_6$ and ... as predictor inputs
Example with predictor input conditions

**Objective:** Estimate $G^0_{21}$.

**Conditions:** Include variable on every path
- $w_1 \rightarrow w_2 \Rightarrow$ Include $w_6$ in predictor
- $w_2 \rightarrow w_2$

**Conclude:** include $w_1, w_6$ and ... as predictor inputs
Example with predictor input conditions

Objective: Estimate $G_{21}^0$.

Conditions: Include variable on every path
- $w_1 \rightarrow w_2 \implies$ Include $w_6$ in predictor
- $w_2 \rightarrow w_2 \implies$ Include $w_3$ in predictor

Conclude: include $w_1$, $w_6$ and $w_3$ as predictor inputs
Result:
The consistency results of both direct and 2s/projection method remain principally valid when the predictor inputs satisfy the formulated conditions on parallel paths and loops around \( w_j \).

In the “full” MISO case: consistent estimates of all \( G_{jk}^0, \ k \in U_j \)

In the “selected” predictor input case: consistent estimates of \( G_{ji}^0 \)
The two conditions (parallel paths and loops on output) result from an analysis of the so-called immersed network.

The immersed network is constructed on the basis of a reduced number of node variables only, and leaves present node signals invariant.

In the immersed network the module dynamics can change.

Whether dynamics in the immersed network is invariant can be verified with the graph theory/tools of separating sets.

Choosing $w_1$ as the predictor input results in an estimate of

$$\frac{G_{21}^0}{1 - G_{23}^0 G_{32}^0}$$

Removing path through $w_3$ called \textit{lifting a path}.

Network without $w_3$ is called \textit{immersed network}.
Example – Immersed Network

Given measurements of $w_1, w_2, w_4,$ and $w_5$

Immerse this network to contain these nodes only.
Example – Immersed Network

\[
\begin{bmatrix}
  w_1 \\
  w_2 \\
  w_4 \\
  w_5 \\
  w_6 \\
\end{bmatrix} =
\begin{bmatrix}
  0 & 0 & 0 & G_{14}^0 & 0 & 0 \\
  G_{21}^0 & 0 & G_{23}^0 & 0 & 0 & 0 \\
  0 & G_{32}^0 & 0 & 0 & 0 & 0 \\
  0 & 0 & G_{52}^0 & 0 & G_{54}^0 & 0 \\
  0 & 0 & G_{63}^0 & G_{65}^0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
  w_1 \\
  w_2 \\
  w_3 \\
  w_4 \\
  w_5 \\
  w_6 \\
\end{bmatrix}
+ 
\begin{bmatrix}
  v_1 \\
  v_2 \\
  v_3 \\
  v_4 \\
  v_5 \\
  v_6 \\
\end{bmatrix}
\]
Example – Immersed Network

\[
\begin{bmatrix}
w_1 \\
w_2 \\
w_3 \\
w_4 \\
w_5 \\
w_6 \\
\end{bmatrix} = \frac{1}{1 - G_{23}^0 G_{32}^0} \begin{bmatrix}
0 & 0 & G_{14}^0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & G_{52}^0 & G_{54}^0 & 0 \\
0 & G_{63}^0 G_{32}^0 & 0 & G_{65}^0 & 0 \\
\end{bmatrix} \begin{bmatrix}
w_1 \\
w_2 \\
w_4 \\
w_5 \\
w_6 \\
\end{bmatrix} + \frac{1}{1 - G_{23}^0 G_{32}^0} \begin{bmatrix}
V_1 \\
V_2 \\
V_4 \\
V_5 \\
V_6 + G_{63}^0 V_3 \\
\end{bmatrix}
\]
Example – Immersed Network

\[
\begin{bmatrix}
  w_1 \\
  w_2 \\
  w_4 \\
  w_5
\end{bmatrix} =
\begin{bmatrix}
  0 & \frac{G^{0}_{21}}{1-G^{0}_{23}G^{0}_{32}} & 0 & G^{0}_{14} \\
  \frac{G^{0}_{21}}{1-G^{0}_{23}G^{0}_{32}} & 0 & 0 & 0 \\
  0 & \frac{G^{0}_{46}G^{0}_{63}}{1-G^{0}_{56}G^{0}_{65}} & 0 & \frac{G^{0}_{46}G^{0}_{65}}{1-G^{0}_{56}G^{0}_{65}} \\
  0 & \frac{G^{0}_{46}G^{0}_{65}}{1-G^{0}_{56}G^{0}_{65}} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
  w_1 \\
  w_2 \\
  w_4 \\
  w_5
\end{bmatrix} +
\begin{bmatrix}
  v_1 \\
  v_2 + \frac{G^{0}_{23}v_3}{1-G^{0}_{23}G^{0}_{32}} \\
  \frac{v_4 + \frac{G^{0}_{46}G^{0}_{63}v_3}{1-G^{0}_{56}G^{0}_{65}}}{v_5 + \frac{G^{0}_{46}G^{0}_{63}v_3}{1-G^{0}_{56}G^{0}_{65}}}
\end{bmatrix}
\]
Example – Immersed Network

Conclude: only $G_{14}^0$ from the original network is identifiable given this data set
Towards dynamic network identification

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• Discussion / Wrap-up
Sensor noise – the errors-in-variables problem

What if node variables are measured with (sensor) noise?

- Classical (tough) problem in open-loop identification
- In dynamic networks this may become *more simple* due to the presence of multiple (correlated) node signals
Sensor noise – the errors-in-variables problem

Two solution strategies:

1. Use *external signals* in combination with 2s/projection/IV method
2. Apply an *Instrumental Variable (IV)* method with generalized options for selecting IV signals
1. Use *external signals* in combination with 2s/projection/IV method

- If measured predictor input signals \((\tilde{w}_3, \tilde{w}_5)\) are projected onto \(r_1\) and then applied in a 2s-PE criterion, the sensor noise on the inputs is effectively removed.
- When assuming that \(r\)-signals and \(s\)-signals are uncorrelated.
Result:

The consistency result of the 2s/projection method remains valid when sensor noise is present on measured variables, provided that

- Sufficient external excitation is present
- Sensor noise is uncorrelated to excitation signals

Extension of IV-approach to use node signals as IV signals, and including noise models, see:

Discussion / Wrap-up

• So far: focus on (local) consistency results in networks with known structure

• Many additional questions/topics remain:
  Variance of estimates, influenced by
  – Additional (output) measurements
  – Excitation properties

  [See e.g. work of H. Hjalmarsson, B. Wahlberg, N. Everitt, B. Günes, M. Gevers, A. Bazanella]
Identification of the structure/topology addressed in the literature, in particular forms:
- Tree-like structures (no loops)
- Nonparametric methods (Wiener filter)
- Mostly networks without external excitation and uncorrelated process noises on every node

see e.g. Materassi, Innocenti (TAC-2010), Chiuso and Pillonetto (Automatica, 2012)

New identifiability concepts apply to the unique determination of a network topology
see e.g. Goncalves & Warnick (TAC-2008), Weerts et al. (SYSID-2015).

Sparse identification methods can be used in an PE identification setting to identify the topology (non-zero transfers)
Toplogy detection with sparse PE methods

\[
\hat{\theta}_N = \arg \min_{\theta} \frac{1}{N} \sum_{t=1}^{N} \varepsilon^2(t, \theta)
\]
subject to \( \|\theta\|_1 \leq \lambda \)

- Detected: \( \|G_{ij}\|_{\infty} \geq 10^{-5} \)
- 100 simulations

[H. Weerts, 2014]
Network identifiability

Question:

When given measured node signals, can we consistently identify the network and its topology?

This will generally require conditions on
a) Informativity of the data (sufficient excitation), and
b) Ability to distinguish between different network models in the model set

Classical notion of identifiability is addressing a unique relationship between parameters \( \theta \) and predictor filters that map measured signals to predicted values.

\[
\begin{align*}
G(\theta_1) &= G(\theta_2) \\
H(\theta_1) &= H(\theta_2)
\end{align*}
\implies \theta_1 = \theta_2
\]

Instead in dynamic networks we need to incorporate the structural issues in the representation of the network.
Question:

When given measured node signals, can we consistently identify the network and its topology?

This will generally require conditions on
a) Informativity of the data (sufficient excitation), and
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Classical notion of identifiability is addressing a unique relationship between parameters $\theta$ and predictor filters that map measured signals to predicted values.

$$\begin{align*}
G(\theta_1) &= G(\theta_2) \\
H(\theta_1) &= H(\theta_2)
\end{align*} \implies \theta_1 = \theta_2$$

Instead in dynamic networks we need to incorporate the structural issues in the representation of the network.
Many interesting –new- questions pop up!
Bibliography


Papers available at [www.pvandenhof.nl/publications.htm](http://www.pvandenhof.nl/publications.htm)