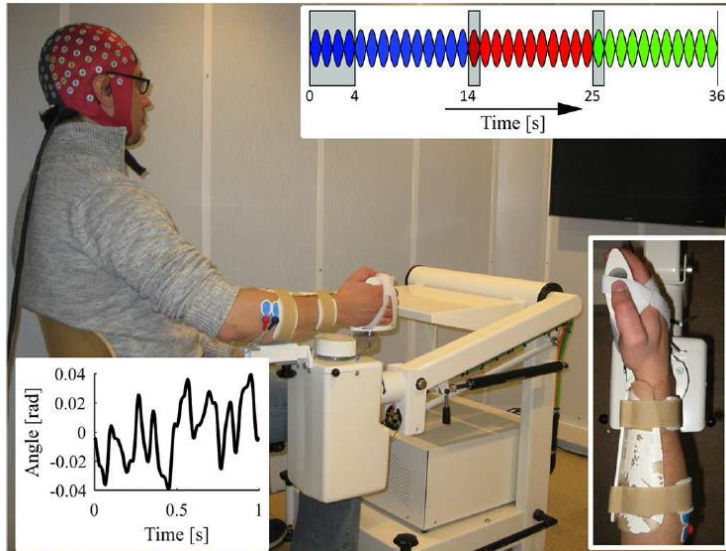


# Detecting Nonlinear Modules in a Dynamic Network: A Step-by-Step Procedure

M. Schoukens and P.M.J. Van den Hof



# What?



# Overview

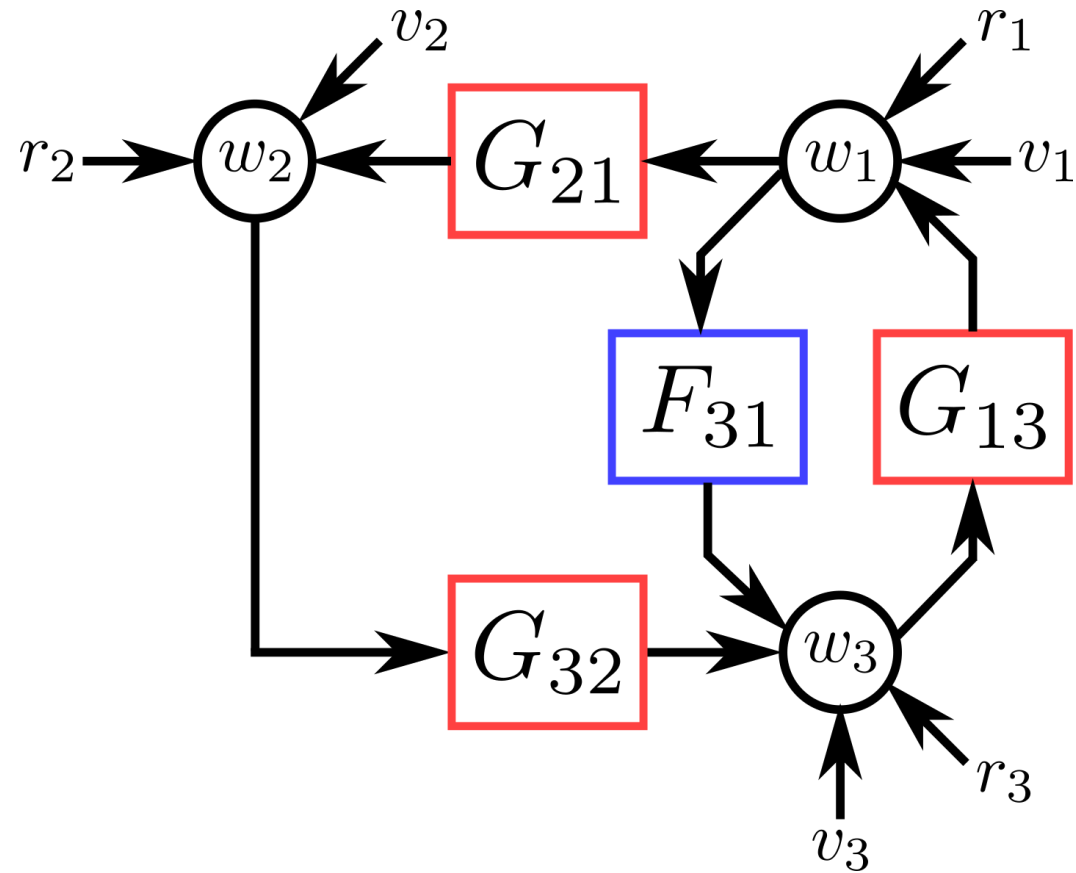
- Dynamic Networks

Best Linear Approximation

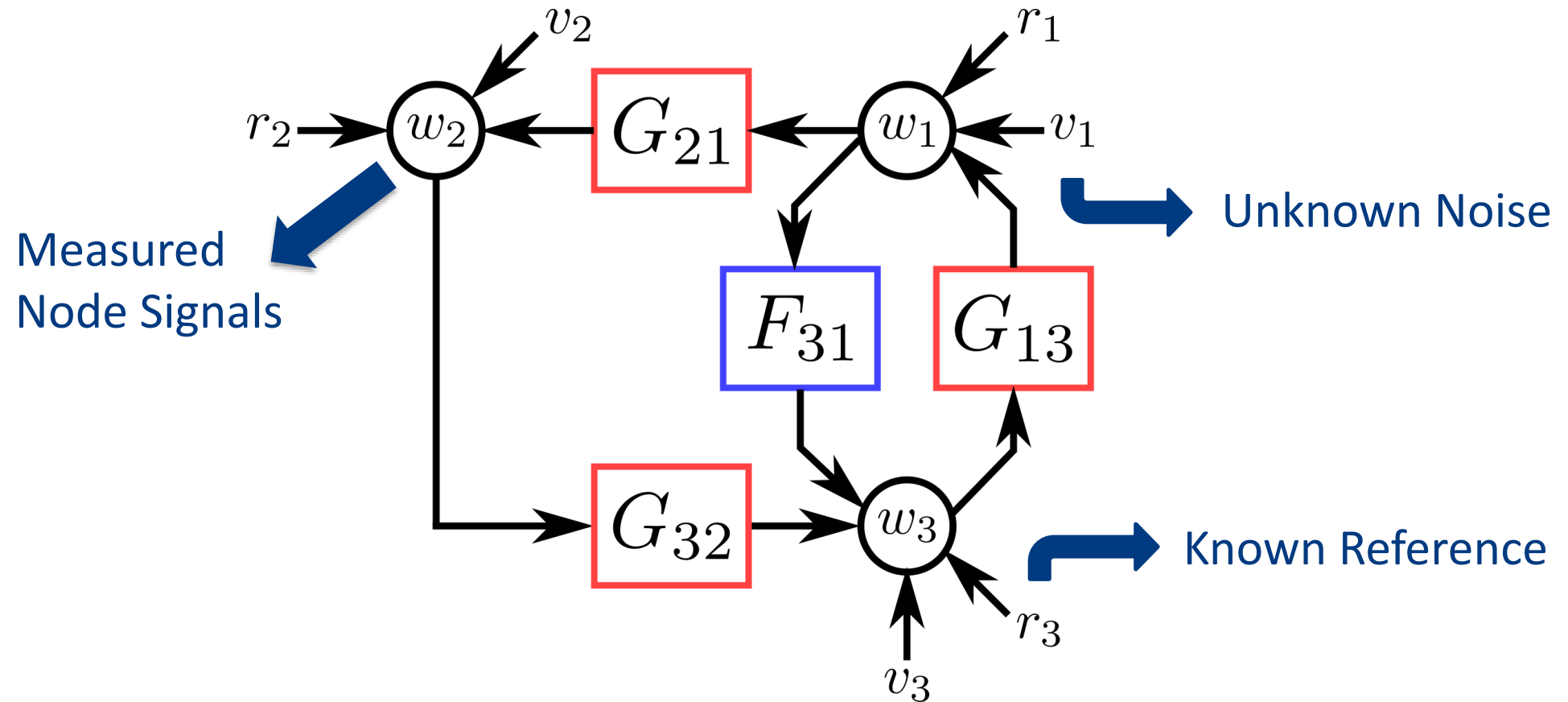
Nonlinearity Detection in Dynamic Networks

Conclusions

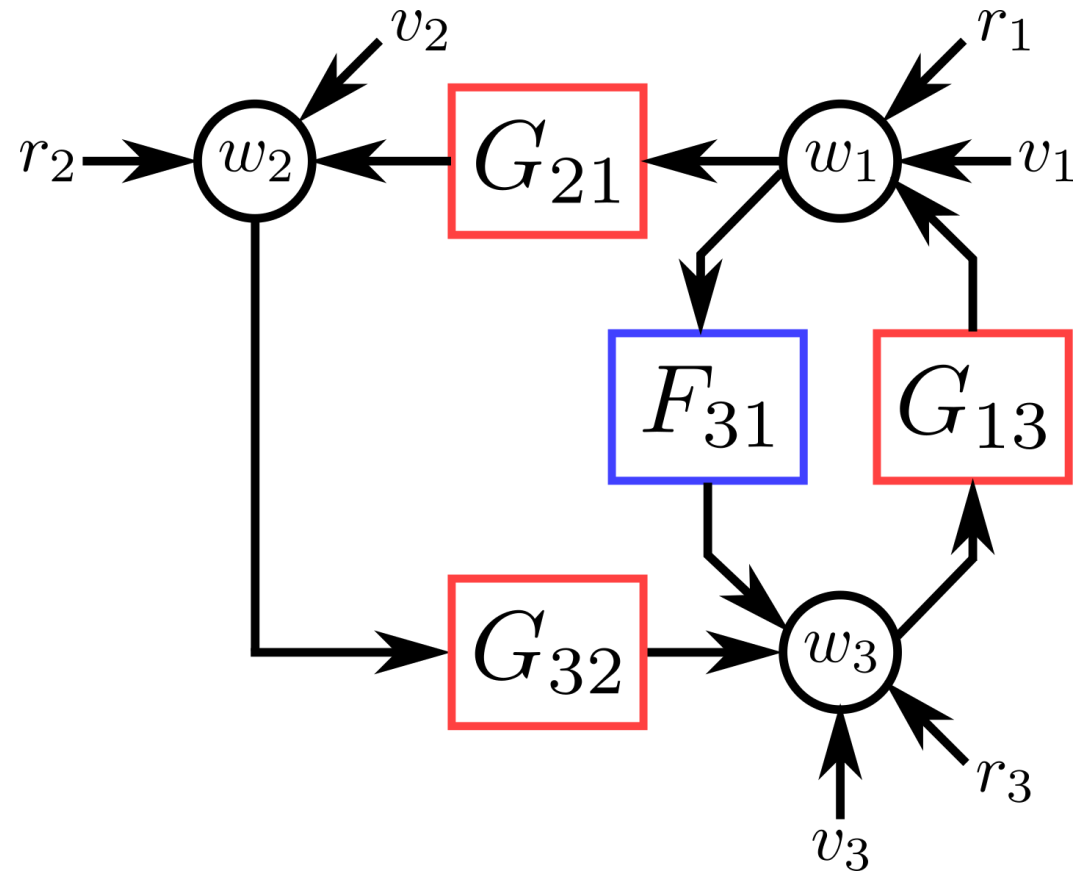
# Dynamic Networks



# Dynamic Networks



# Dynamic Networks



Can we localize and quantify the nonlinear behavior in the network?

# Overview

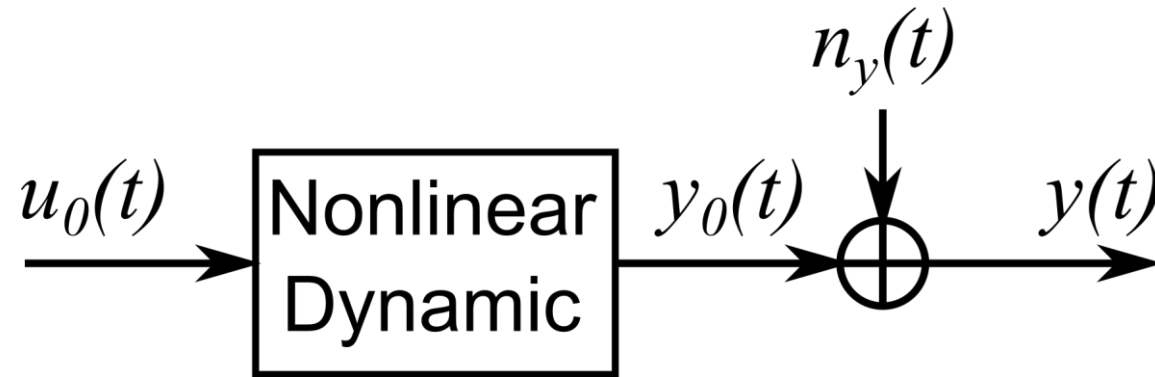
## Dynamic Networks

- Best Linear Approximation

## Nonlinearity Detection in Dynamic Networks

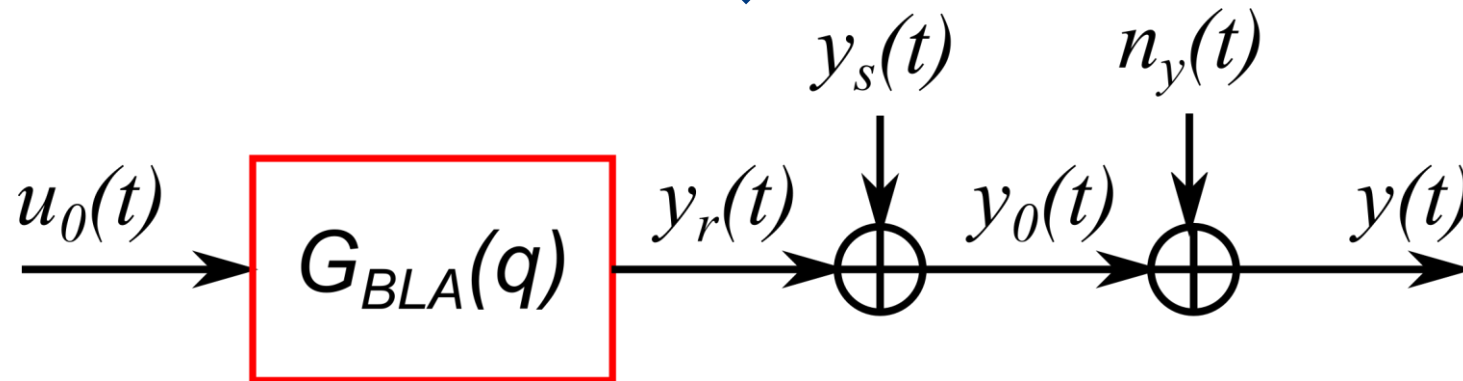
## Conclusions

# Best Linear Approximation: Definition



Minimize:

$$E_u \{|y(t) - G(q, \theta)u(t)|^2\}$$



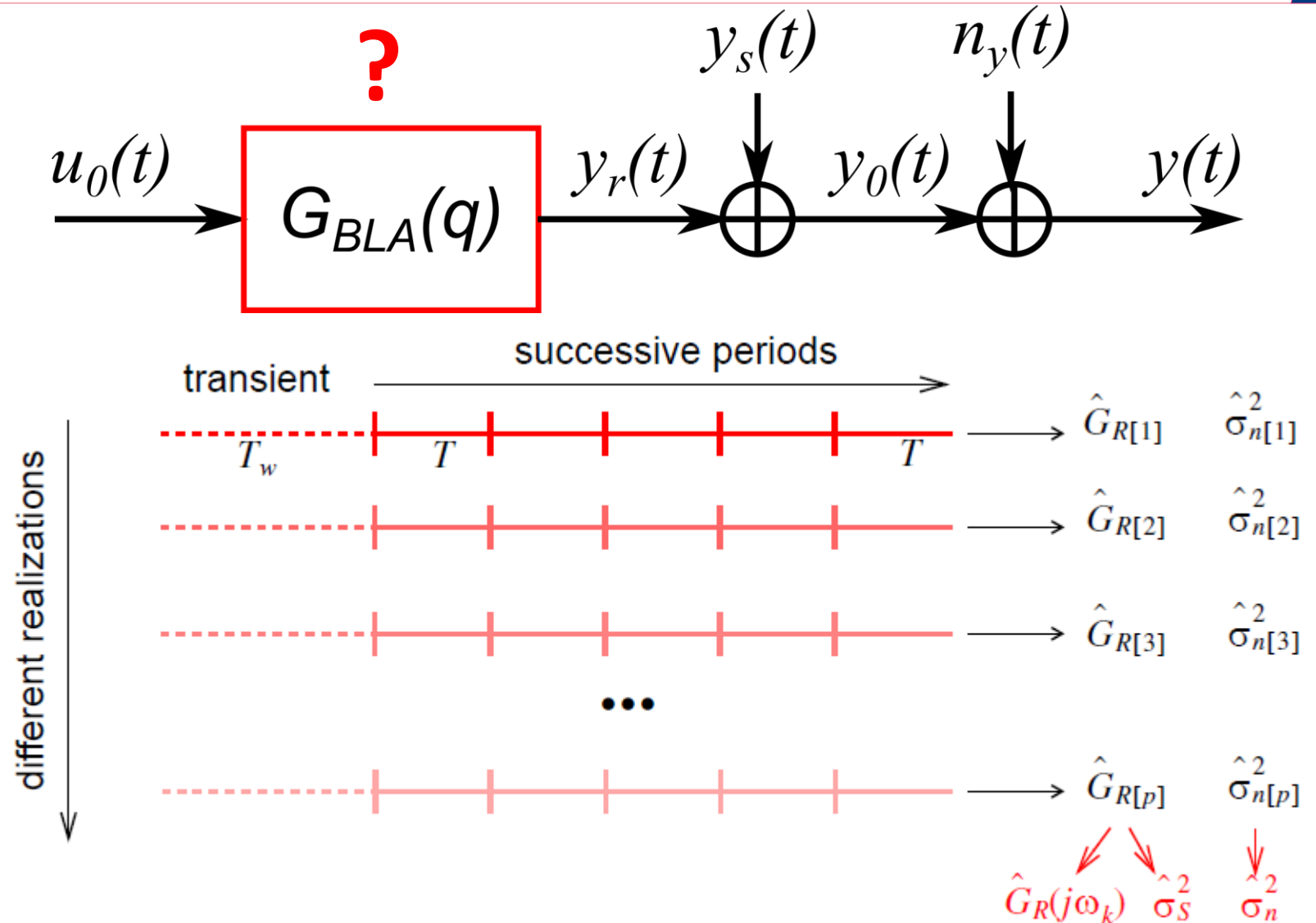


# Best Linear Approximation: Experiment

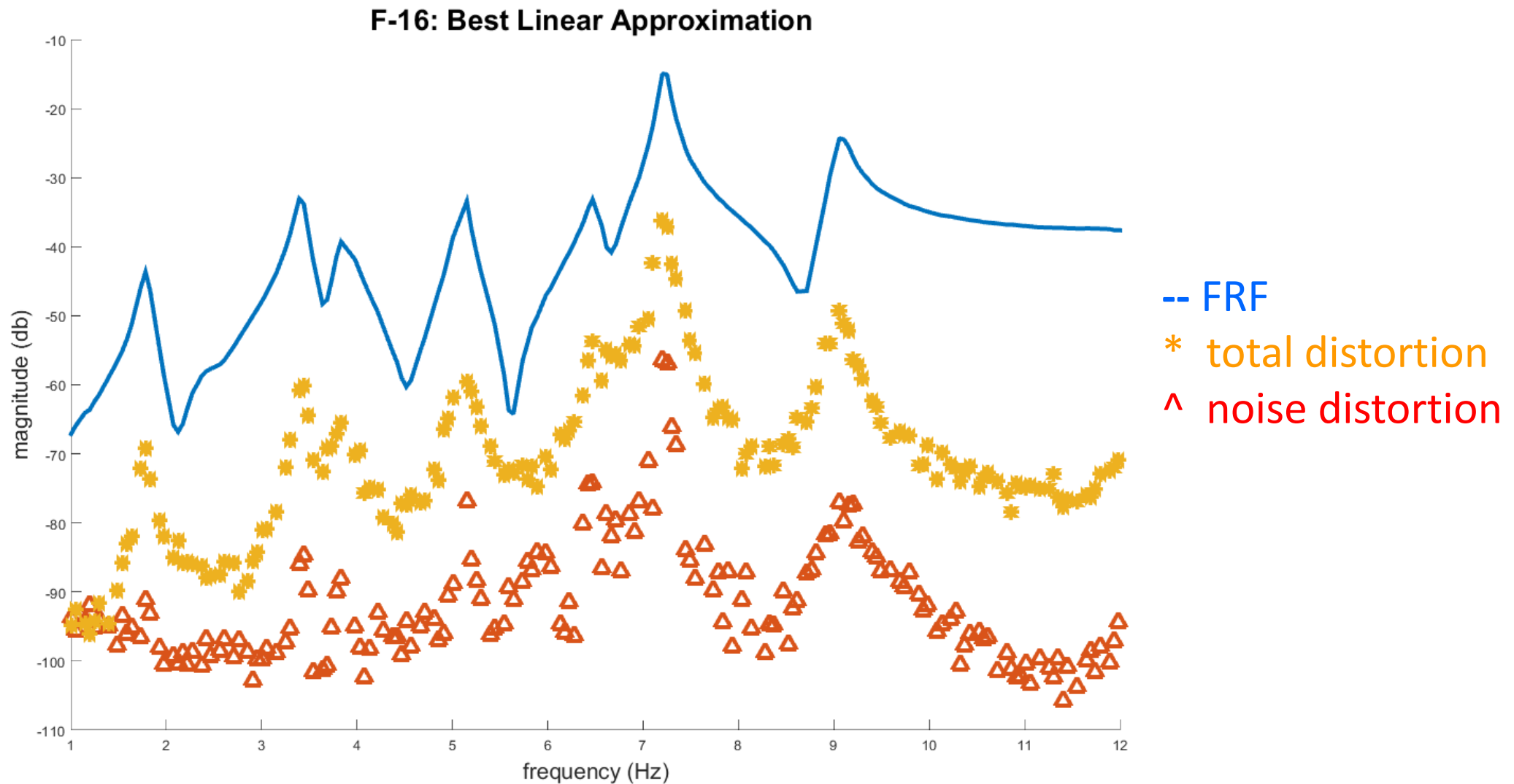
Random Phase Multisine

P Periods

M Realizations



# Best Linear Approximation: Results



# Overview

Dynamic Networks

Best Linear Approximation

- Nonlinearity Detection in Dynamic Networks

Conclusions

# Nonlinearity Detection in Dynamic Networks

Step 0: Experiment

Step 1: BLA from References to Nodes

Step 2: BLA in between Nodes

Step 3: Residual Analysis

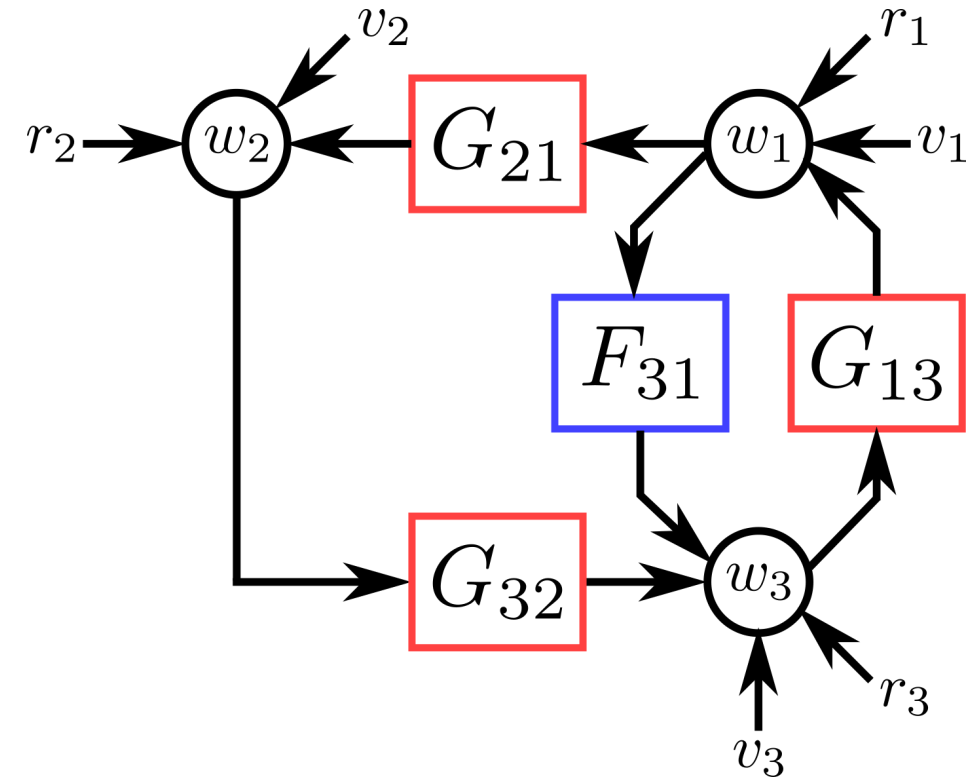
# Step 0: Experiment

(Orthogonal) Random Phase Multisines

P Periods

M Realizations

All references excited simultaneously

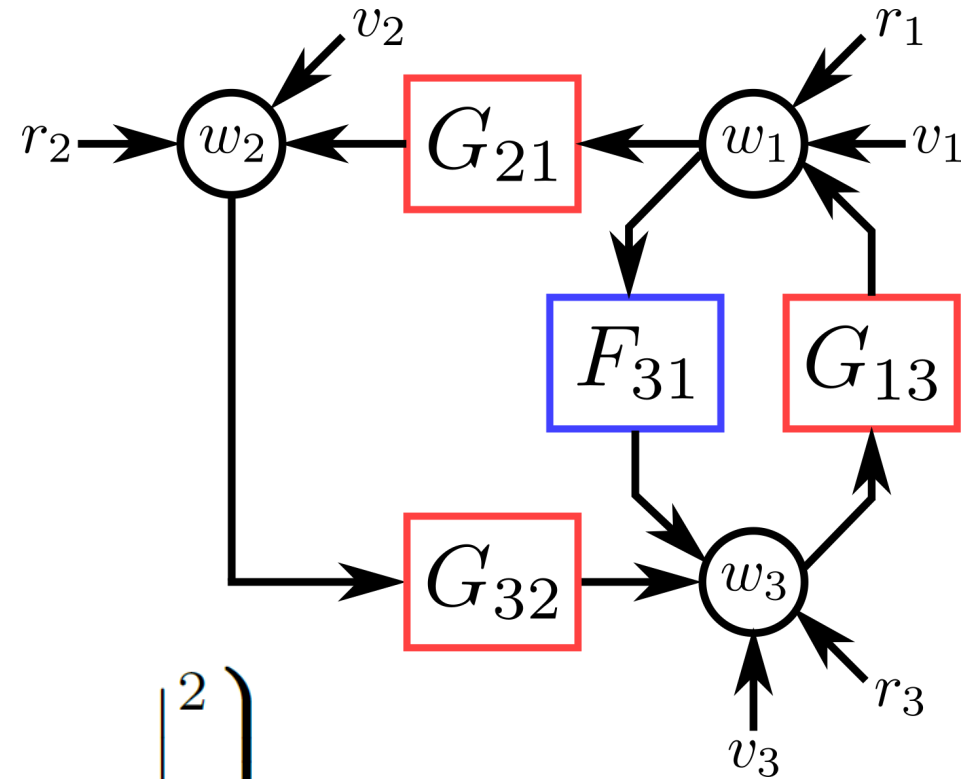


# Step 1: BLA from References to Nodes

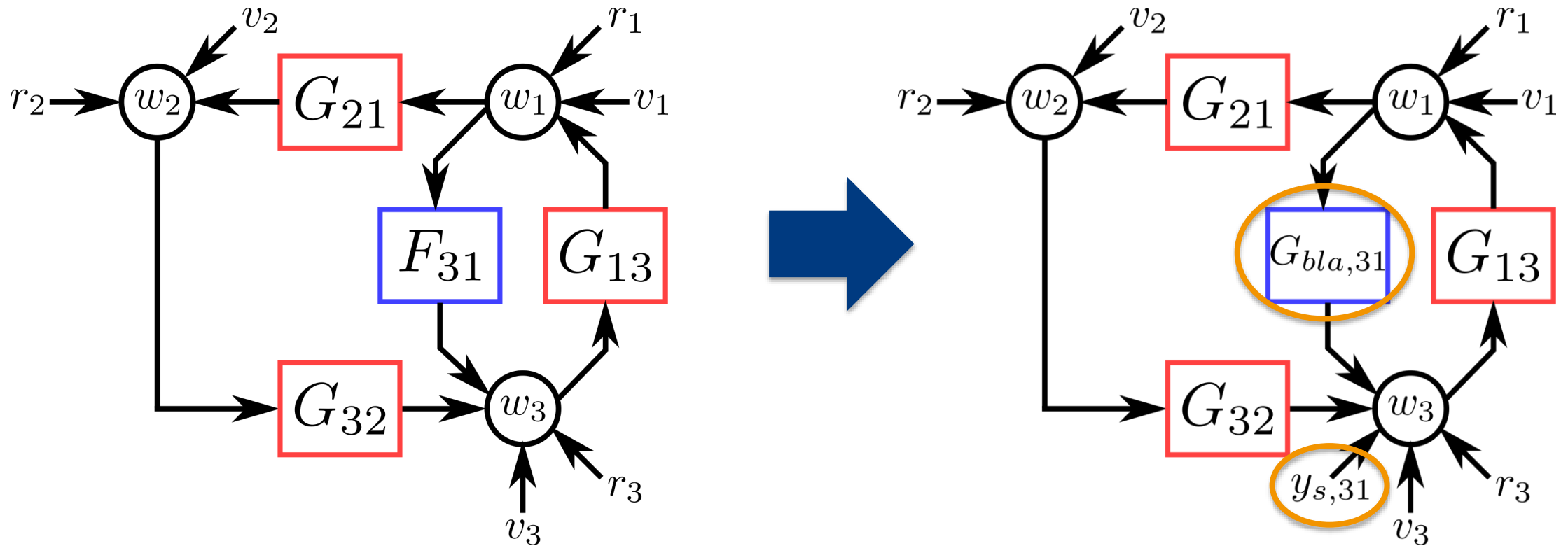
Standard MIMO BLA:

$$\mathbf{S}_{\text{bla}}(q) =$$

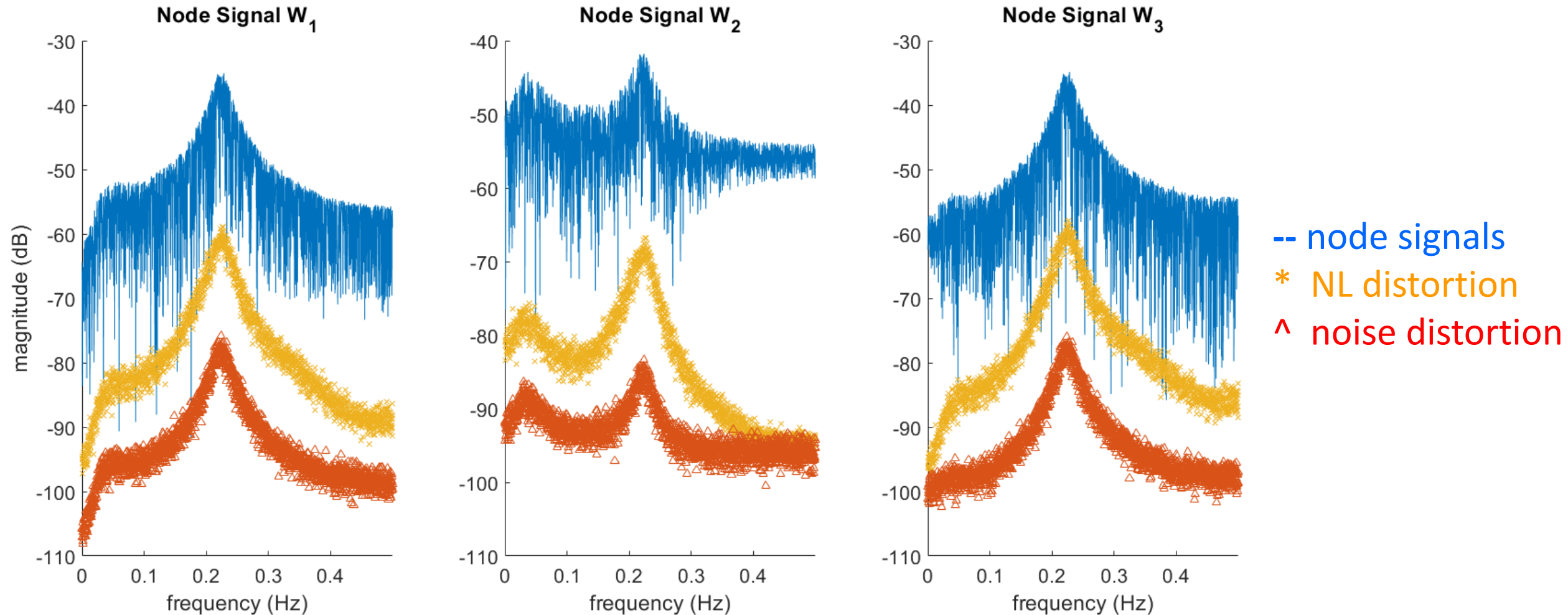
$$\arg \min_{\mathbf{S}(q)} E_{r,v} \left\{ \sum_{i=1}^L \left| \tilde{w}_i(t) - \sum_{j=1}^L S_{i,r_j}(q) \tilde{r}_j(t) \right|^2 \right\}$$



# Step 1: BLA from References to Nodes



# Step 1: BLA from References to Nodes



Nonlinearity Everywhere!



## Step 2: BLA in between Nodes

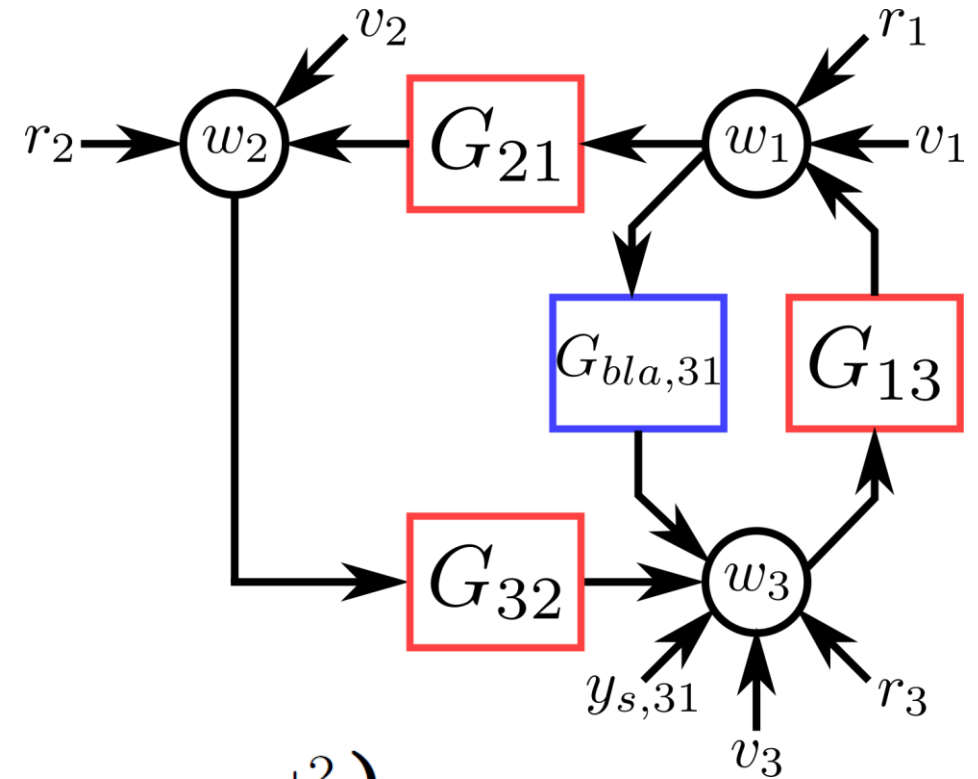
Simulate Noiseless Node Signals:

$$\tilde{w}_i(t) = \sum_{j=1}^L S_{bla,i,r_j}(q) \tilde{r}_j(t)$$

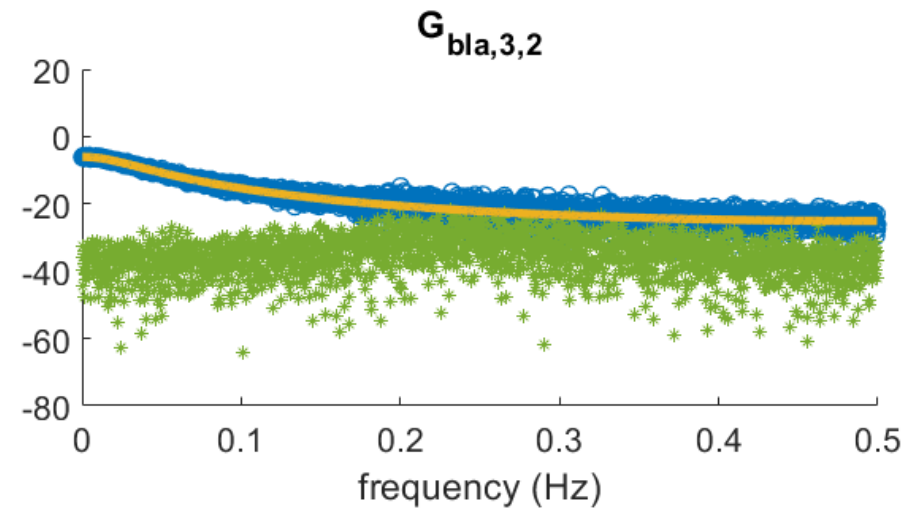
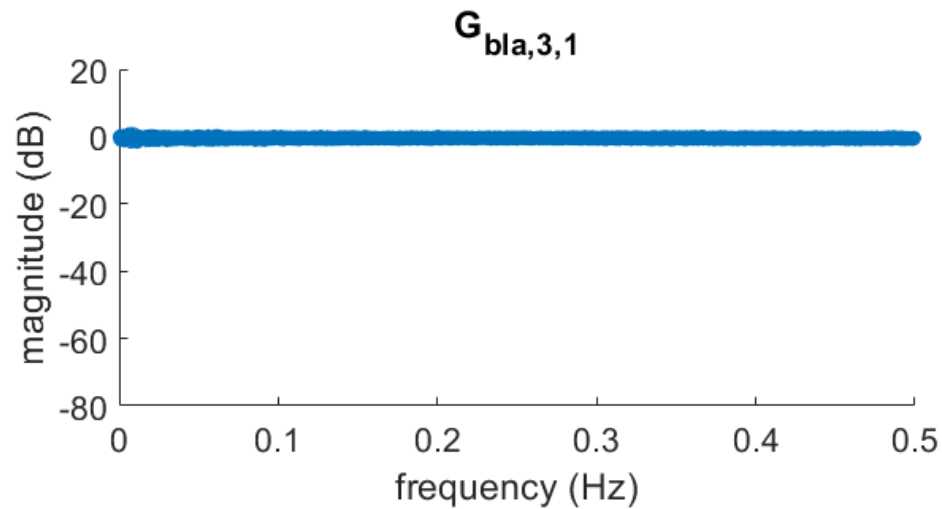
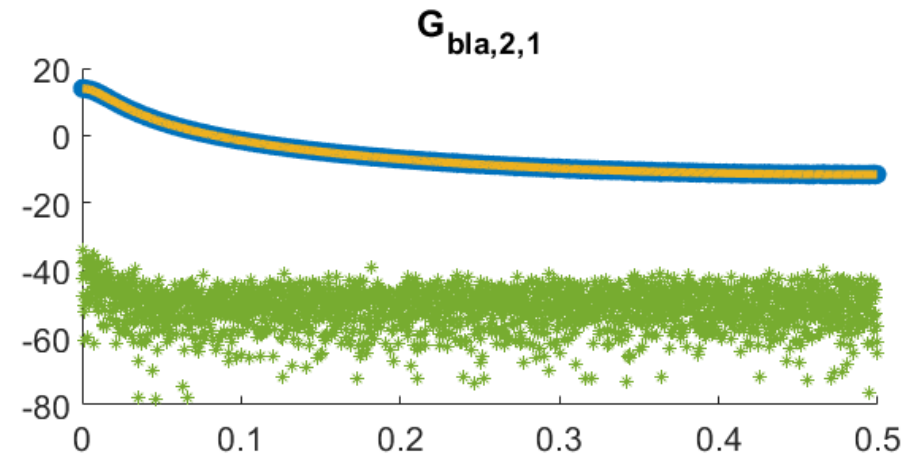
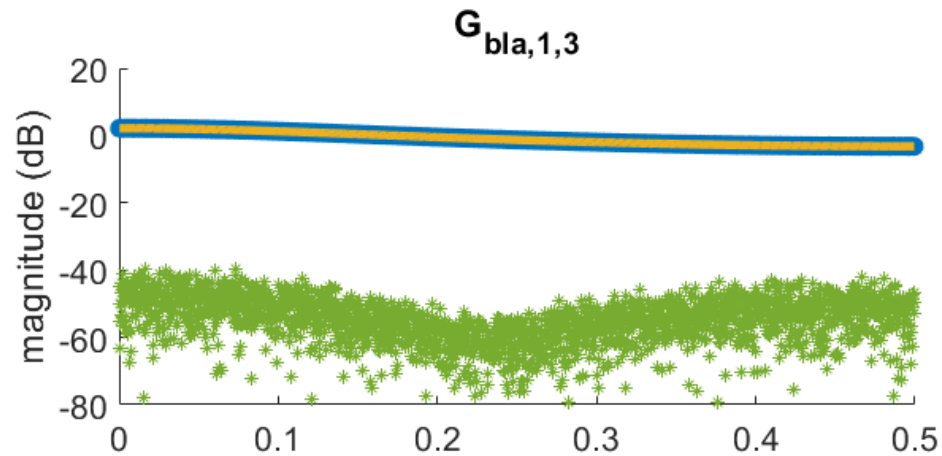
MIMO BLA in Between Nodes:

$$\mathbf{G}_{bla}(q) =$$

$$\arg \min_{\mathbf{G}(q)} E_{r,v} \left\{ \sum_{i=1}^L \left| \tilde{w}_i(t) - \tilde{r}_i(t) - \sum_{j=1, j \neq i}^L G_{i,j}(q) \tilde{w}_j(t) \right|^2 \right\}$$



## Step 2: BLA in between Nodes



- FRF
- true system
- \* residuals

# Step 3: Residual Analysis

Network Simulation:

$$\tilde{\tilde{w}}_i(t) = \tilde{r}_i(t) + \sum_{j=1, j \neq i}^L G_{bla,i,j}(q) \tilde{w}_j(t)$$

Residual Analysis:

$$e_i^{[m,p]}(j\omega) = \tilde{\tilde{w}}_i^{[m,p]}(j\omega) - \tilde{w}_i^{[m,p]}(j\omega)$$

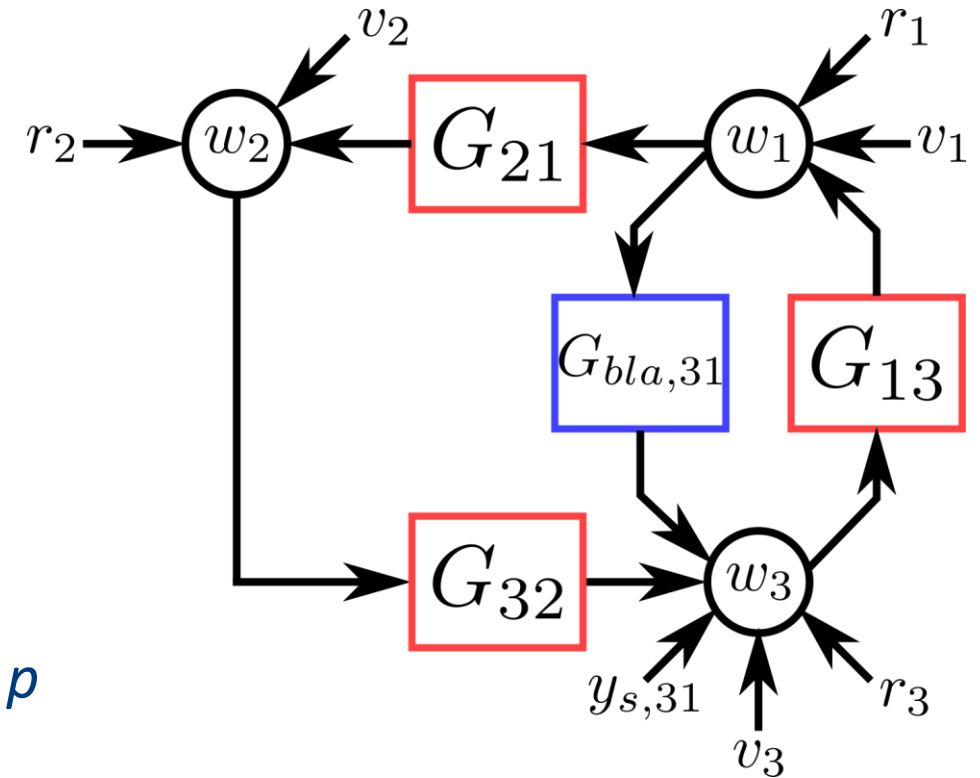
Noise Variance:

➡ Aperiodic Distortion: Variance over the periods  $p$

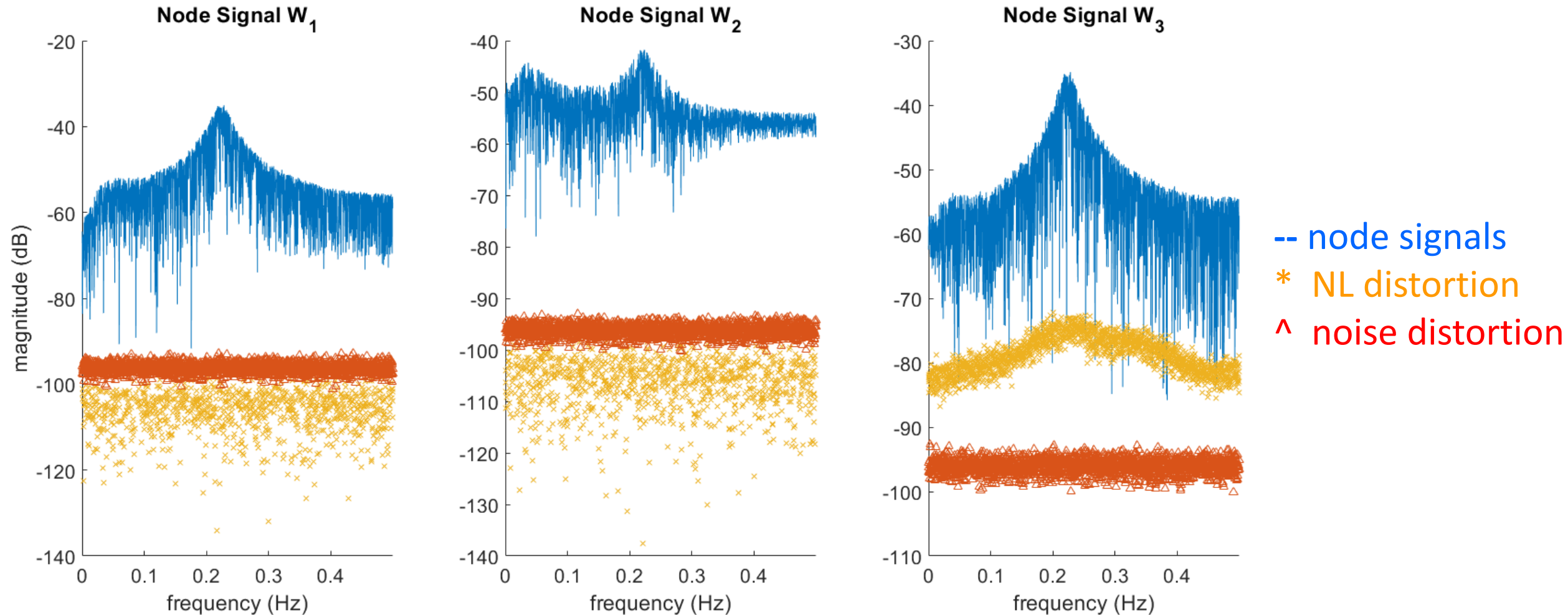
Total Variance: Variance over the realizations  $m$

NL Distortion Variance: Total Variance – Noise Variance

➡ Periodic Nonlinear Distortion



# Step 3: Residual Analysis



Only Nonlinearity Detected on Node 3!

# Overview

Dynamic Networks

Best Linear Approximation

Nonlinearity Detection in Dynamic Networks

- Conclusions

# Conclusions

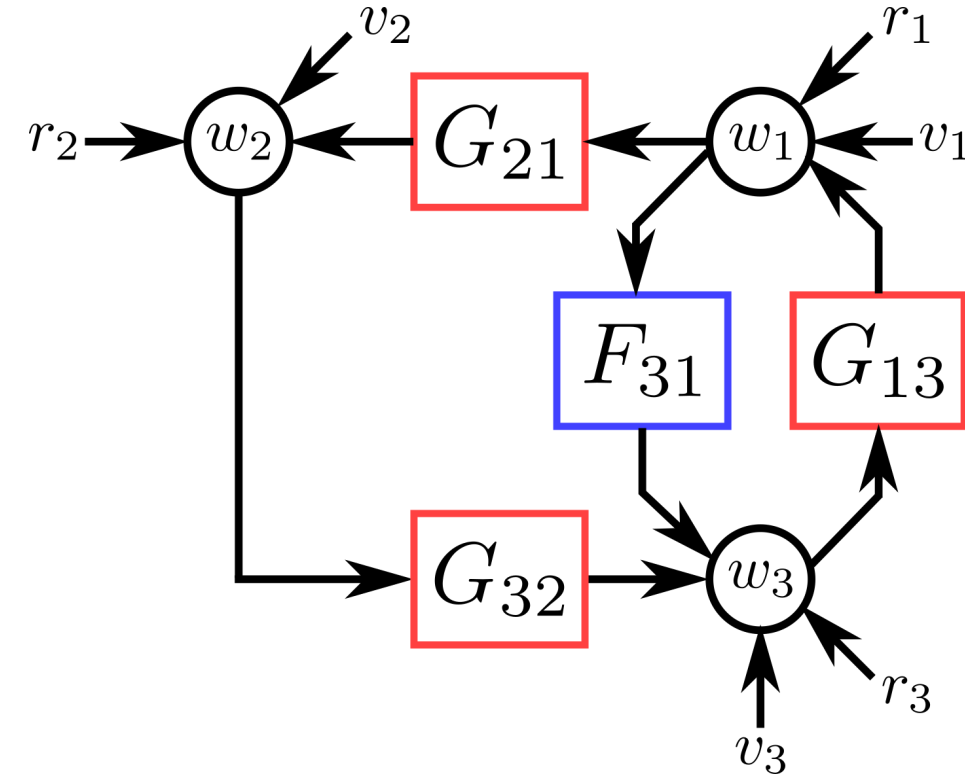
## Nonlinearity Detection in Dynamic Networks

MIMO  $\leftrightarrow$  Dynamic Network point of view

Ongoing Work:

Combining Single Experiments

Module Level Detection



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