Detecting Nonlinear Modules in a Dynamic Network: A Step-by-Step Procedure

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What?
Overview

- Dynamic Networks
  - Best Linear Approximation
  - Nonlinearity Detection in Dynamic Networks
- Conclusions
Dynamic Networks
Dynamic Networks

- Measured Node Signals
- Unknown Noise
- Known Reference

Diagram showing node signals and transformations with labels for nodes and arrows indicating signal flow.
Dynamic Networks

Can we localize and quantify the nonlinear behavior in the network?
Overview

Dynamic Networks

- Best Linear Approximation

Nonlinearity Detection in Dynamic Networks

Conclusions
Best Linear Approximation: Definition

Minimize:

\[ E_u \{ |y(t) - G(q, \theta)u(t)|^2 \} \]
Best Linear Approximation: Experiment

Random Phase Multisine

P  Periods
M  Realizations
Best Linear Approximation: Results

-- FRF
* total distortion
^ noise distortion
Overview

Dynamic Networks
Best Linear Approximation
  • Nonlinearity Detection in Dynamic Networks
Conclusions
Nonlinearity Detection in Dynamic Networks

Step 0: Experiment

Step 1: BLA from References to Nodes

Step 2: BLA in between Nodes

Step 3: Residual Analysis
Step 0: Experiment

(Orthogonal) Random Phase Multisines

- P  Periods
- M  Realizations

All references excited simultaneously
Step 1: BLA from References to Nodes

Standard MIMO BLA:

\[ S_{bla}(q) = \arg \min_{S(q)} \left\{ \sum_{i=1}^{L} \left| \tilde{w}_i(t) - \sum_{j=1}^{L} S_{i,r_j}(q) \tilde{r}_j(t) \right|^2 \right\} \]
Step 1: BLA from References to Nodes
Step 1: BLA from References to Nodes

Node Signal $W_1$

Node Signal $W_2$

Node Signal $W_3$

-- node signals
* NL distortion
^ noise distortion

Nonlinearity Everywhere!
Step 2: BLA in between Nodes

Simulate Noiseless Node Signals:

\[ \tilde{w}_i(t) = \sum_{j=1}^{L} S_{bla,i,r_j}(q) \tilde{r}_j(t) \]

MIMO BLA in Between Nodes:

\[ G_{bla}(q) = \arg \min_{G(q)} \left\{ \sum_{i=1}^{L} \left| \tilde{w}_i(t) - \tilde{r}_i(t) - \sum_{j=1, j \neq i}^{L} G_{i,j}(q) \tilde{w}_j(t) \right|^2 \right\} \]
Step 2: BLA in between Nodes

- FRF

--- true system

* residuals
Step 3: Residual Analysis

Network Simulation:
\[
\tilde{w}_i(t) = \tilde{r}_i(t) + \sum_{j=1, j \neq i}^{L} G_{bla,i,j}(q) \tilde{w}_j(t)
\]

Residual Analysis:
\[
e_i^{[m,p]}(j\omega) = \tilde{w}_i^{[m,p]}(j\omega) - \tilde{w}_i^{[m,p]}(j\omega)
\]

Noise Variance:

Aperiodic Distortion: Variance over the periods \(p\)

Total Variance: Variance over the realizations \(m\)

NL Distortion Variance: Total Variance – Noise Variance

Periodic Nonlinear Distortion
Step 3: Residual Analysis

Only Nonlinearity Detected on Node 3!

-- node signals
* NL distortion
^ noise distortion
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Conclusions

Nonlinearity Detection in Dynamic Networks

MIMO $\leftrightarrow$ Dynamic Network point of view

Ongoing Work:

Combining Single Experiments

Module Level Detection
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