Exploiting unmeasured disturbance signals in identifiability of linear dynamic networks with partial measurement and partial excitation

Shengling Shi, Xiaodong Cheng, Paul M.J. Van den Hof

Control Systems Group, TU Eindhoven, the Netherlands

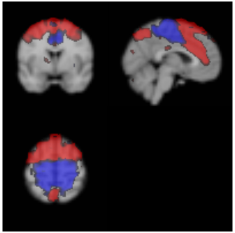
19th IFAC SYSID symposium, 2021

EINDHOVEN UNIVERSITY OF TECHNOLOGY

Outline

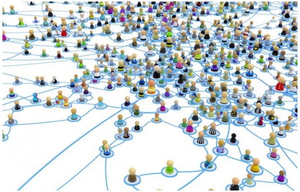
Introduction – dynamic networks

- Appear in a wide range of applications



Biological Network

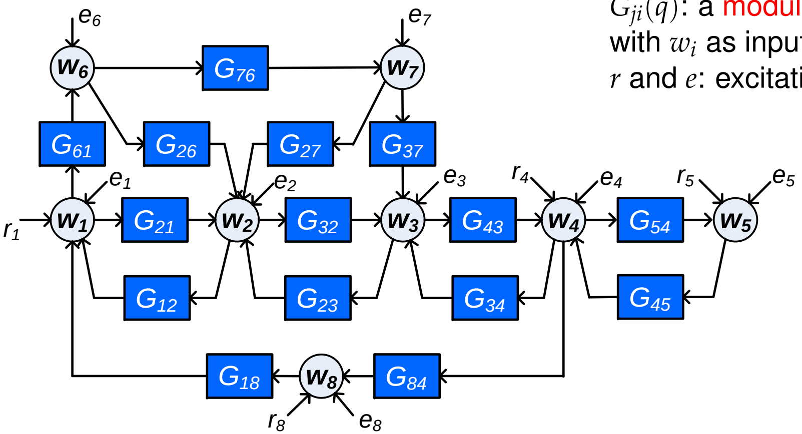
Power Grid



Social Network

- Data-driven modeling of dynamic networks
- A fundamental property: Identifiability represents the ability to distinguish different models based on data

Preliminaries: Dynamic network models



$w_i(t)$: internal signals that are possibly unmeasured;
$G_{ji}(q)$: a module (transfer operator) with w_i as input, w_j as output;
r and e: excitation signals and white noises.

Preliminaries: Dynamic network models

Order and partition the internal signal vector $w(t)$ as

$$w(t) = \begin{bmatrix} w_{\mathcal{C}}(t) \\ w_{\mathcal{Z}}(t) \end{bmatrix}$$

$$\begin{bmatrix} w_{\mathcal{C}} \\ w_{\mathcal{Z}} \end{bmatrix} = G(q,\theta)w(t) + Rr(t) + H(q,\theta)e(t)$$
$$w_{\mathcal{C}}(t) = Cw(t)$$

- Parameter $\theta \in \Theta \subseteq \mathbb{R}^n$
- Matrix C extracts $w_{\mathcal{C}}(t)$ from $w(t)$
- r and $w_{\mathcal{C}}$ are measured

Preliminaries: Dynamic network models

Order and partition the internal signal vector $w(t)$ as

$$w(t) = \begin{bmatrix} w_{\mathcal{C}}(t) \\ w_{\mathcal{Z}}(t) \end{bmatrix}$$

$$\begin{bmatrix} w_{\mathcal{C}} \\ w_{\mathcal{Z}} \end{bmatrix} = G(q,\theta)w(t) + Rr(t) + H(q,\theta)e(t)$$

$$w_{\mathcal{C}}(t) = Cw(t)$$

- Parameter $\theta \in \Theta \subseteq \mathbb{R}^n$
- Matrix C extracts $w_{\mathcal{C}}(t)$ from $w(t)$
- r and $w_{\mathcal{C}}$ are measured

Mapping from external signals to measured internal signals

$$\begin{aligned} w_{\mathcal{C}} &= C(I-G)^{-1}Rr + C(I-G)^{-1}He \\ &= CTRr + CTHe, \text{with } T \triangleq (I-G)^{-1} \end{aligned}$$

- Under mild conditions, the first and the second moments of the measured $w_{\mathcal{C}}$ are reflected by CTR and the power spectrum of $CTHe$, denoted by $C\Phi C^{\top}$
- $(CTR, C\Phi C^{\top})$ as the basis for the identifiability concept

Preliminaries: Network identifiability

Generic identifiability concept [Shi et al., 2020, Shi, 2021]

G_{ji} is generically identifiable if the objects $CT(q,\theta)R$ and $C\Phi(\omega,\theta)C^{\top}$ lead to a unique module $G_{ji}(q,\theta)$ for almost all $\theta \in \Theta$ (generically).

- Extension of the generic identifiability concept in [Bazanella et al., 2017, Hendrickx et al., 2019] by incorporating the power spectrum
- Whether the first and second moments of measured signals lead to a unique network model/module
- Identifiability is a prerequisite for the identification methods that rely on the statistical second-order properties of measured data

Problem formulation

Original definition when $C = I$

G_{ji} is generically identifiable if the objects TR and Φ lead to a unique module G_{ji} generically.

$$w_C = CTRr + CTHe$$

exploiting the spectral factorization of Φ

Problem formulation

Original definition when $C = I$

G_{ji} is generically identifiable if the objects TR and Φ lead to a unique module G_{ji} generically.

$$w_C = CTRr + CTHe$$

exploiting the spectral factorization of Φ

Proposition(Simplified identifiability when $C = I$) [Weerts et al., 2018]

G_{ji} is generically identifiable if and only if TR and TH lead to a unique G_{ji} generically.

- Important consequence: Both r and e play the same role in the identifiability analysis (serve as excitation sources), even if e is unmeasured
- However, not applicable to the setting with partial measurement, i.e. when $C \neq I$.

Problem formulation

Problem

How to exploit the white noises for network identifiability with unmeasured internal signals, i.e. when $C \neq I$?

- Core issue: Spectral factorization of $C\Phi C^\top$, a submatrix of Φ

Main results: Network equivalence

Through the analysis of the spectral factorization of $C\Phi C^\top$

Definition

Two network models M_1 and M_2 are said to be equivalent if

$$C_1 T_1 R_1 = C_2 T_2 R_2, \text{ and } C_1 \Phi_1 C_1^\top = C_2 \Phi_2 C_2^\top.$$

- Equivalent if the measured signal, generated by the two models, has the same first and second moments
- Two equivalent models can be used to model the same data

Main results: Canonical noise model

Theorem (Canonical noise model)

By the spectral factorization of $C\Phi C^\top$, any network admits an equivalent network as

$$\begin{bmatrix} w_{\mathcal{C}} \\ w_{\mathcal{Z}} \end{bmatrix} = Gw + Rr + \begin{bmatrix} \tilde{H}_{\mathcal{C}} \\ \mathbf{0} \end{bmatrix} \tilde{e}, \quad w_{\mathcal{C}}(t) = Cw(t)$$

Main results: Canonical noise model

Any network can be transformed with a canonical noise model $\Longrightarrow$ Canonical noise model as the standard noise model.

Important consequence:

Proposition (Simplified identifiability concept)

For a network with the canonical noise model, G_{ji} is generically identifiable if and only if CTR and CTH lead to unique G_{ji}.

- Identifiability is simplified: Uniqueness given CTR and CTH, i.e. mappings from r and e to measured internal signals, instead of CTR and $C\Phi C^\top$ as in the original definition
- With the canonical noise model, r and e signals play the same role in identifiability analysis, even when $C \neq I$, i.e. not all internal signals are measured

Main results: Consequence of noise exploitation

- Identifiability conditions require sufficient external excitation sources
- Now both r and e signals can serve as excitation sources, even if e is unmeasured

Main results: Consequence of noise exploitation

- Identifiability conditions require sufficient external excitation sources
- Now both r and e signals can serve as excitation sources, even if e is unmeasured

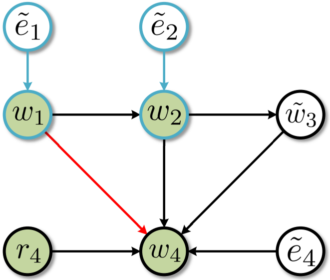
Theorem (Graphical condition for generic identifiability)

G_{ji} is generically identifiable if there exists a disconnecting set $\mathcal{D}$ from $\mathcal{X}_j$ to $\mathcal{N}_j^- \setminus \{w_i\}$ such that

1. $b_{\mathcal{X}_j \rightarrow \{w_i\} \cup \mathcal{D}} = |\mathcal{D}| + 1$;
2. The signals in $\{w_i, w_j\} \cup \mathcal{D}$ are measured

- $\mathcal{X}_j$ contains both r and e signals
- First condition: The maximum number of vertex disjoint paths from $\mathcal{X}_j$ to $\{w_i\} \cup \mathcal{D}$ equals $|\mathcal{D}| + 1$
- Interpretation: the signals $\{w_i\} \cup \mathcal{D}$ are excited by the r and e signals in $\mathcal{X}_j$

Main results: Example



- Each edge denotes a transfer operator, and only green vertices (signals) are measured
- Verify generic identifiability of G_{41}

Applying the previous graphical condition:

- $\mathcal{D} \cup \{w_1\} = \{w_1, w_2\}$ needs to be measured and excited
- G_{41} is generically identifiable due to $\tilde{e}_1$ and $\tilde{e}_2$

Important consequence of the main result:
White noises can act as excitation sources even if they are unmeasured

Conclusions

- Canonical noise model can be used as a standard noise model, as every network admits an equivalent representation with the canonical noise model
- By exploiting the canonical noise model, unmeasured white noises can act as excitation sources in the identifiability analysis with unmeasured internal signals

Conclusions

- Canonical noise model can be used as a standard noise model, as every network admits an equivalent representation with the canonical noise model
- By exploiting the canonical noise model, unmeasured white noises can act as excitation sources in the identifiability analysis with unmeasured internal signals

Thank you for your attention!

References I

Bazanella, A. S., Gevers, M., Hendrickx, J. M., and Parraga, A. (2017).
Identifiability of dynamical networks: which nodes need be measured?
In *Proc. 56th IEEE Conf. Decis Control (CDC)*, pages 5870–5875. IEEE.

Hendrickx, J. M., Gevers, M., and Bazanella, A. S. (2019).
Identifiability of dynamical networks with partial node measurements.
IEEE Trans. Autom. Control, 64(6):2240–2253.

Shi, S. (2021).
Topological aspects of linear dynamic networks: Identifiability and identification.
PhD thesis, Department of Electrical Engineering, Eindhoven University of Technology.

Shi, S., Cheng, X., and Van den Hof, P. M. J. (2020).
Single module identifiability in linear dynamic networks with partial excitation and measurement.
arXiv preprint arXiv:2012.11414.

Weerts, H. H. M., Van den Hof, P. M. J., and Dankers, A. G. (2018).
Identifiability of linear dynamic networks.
Automatica, 89:247–258.