Risk Management in Oil Reservoir Water-Flooding under Economic Uncertainty

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Oil Production

• Production from Oil reservoir
  • Porous rocks with oil in pores
  • \(10^1\) to \(10^4\) \(km^2\) in size
  • Geological structure heterogeneous
    • Very different rock properties within reservoir
  • Life cycle of 10 – 100 years

• Oil production phases
  • Primary production (5-15%)
  • Secondary production (Water-flooding)
  • Tertiary production

Van Essen et al. (2010)
Challenges

- Large-scale, non-convex and non-linear optimization
- Uncertainty:
  - Economic uncertainty
  - Varying oil prices
- Parametric uncertainty

Decision making (model-based economic optimization) under economic uncertainty
Contents

• Introduction

• Model-based economic optimization and Reactive strategy

• Handling risk of uncertainty

• Risk management
  • Worst-case optimization
  • Conditional Value-at-Risk (CVaR ) optimization

• Conclusions
Model-based optimization and Reactive strategy

• Net Present Value (NPV)

\[ J_K(u) = \sum_{k=1}^{K} \frac{\Delta t_k \left[ r_o q_{o,k} - r_{wp} q_{wp,k} - r_{wi} q_{wi,k} \right]}{(1+b_\tau) \tau} \]

• Gradient obtained by solving an adjoint equation

• Reactive strategy:
  • Injection with maximum rates and shut-in production wells when it is no longer profitable.

• Experiment with the ‘standard egg model’
  Jansen et al. (2014)
Model-based optimization and Reactive strategy

Nominal model-based optimization and reactive strategy

It is desirable to include uncertainty in the model-based optimization
Handling risk of uncertainties
Literature Survey

Optimization over an ensemble of possible realizations (geological scenarios)


An ensemble of 100 realizations of standard egg model:

The mean optimization $J_{MO}$ for an ensemble of $N_r = 100$ models

$$J_{MO} = \frac{1}{N_r} \sum_{i=1}^{N_r} J_i (u, \theta_i)$$

Where $J$ is NPV, $u$ is the control input and $\theta_i$ is the uncertainty ensemble
Handling risk of uncertainties

Literature Survey

\[
\max_u J_{MO} = \max_u \frac{1}{N_r} \sum_{i=1}^{N_r} J_i (u, \theta_i)
\]
Handling risk of uncertainties

Literature Survey

- Uncertainty is mapped to the distribution of the objective (NPV)
- Considers uncertainty in optimization framework, does not minimize the negative effect of it

Not a very robust scheme!
Risk management

Risk is unpredicted variability or a potential loss of the expected economic objective.

Risk management is the shaping of gain\loss distribution.
Risk management

Risk measures H. Markowitz (1952), Rockafellar et. al (2000), Capolei et al. (2015b),

- Variance (Portfolio theory):

\[ J_{MVO} = J_{MO} - \gamma J_V \]

\[
J_{MO} = \frac{1}{N_r} \sum_{i=1}^{N_r} J_i(u, \theta_i) \quad \text{(Mean)}
\]

\[
J_V = \frac{1}{N_r-1} \sum_{i=1}^{N_r} (J_i(u, \theta_i) - J_{MO})^2 \quad \text{(Variance)}
\]

Capolei et al. (2015a),
Siraj et al. (2015)
Risk management
Risk measures

- Variance (Portfolio theory):
  - A symmetric measure of risk
  - It penalizes the best cases
  - The decision maker is mainly concerned with the worst cases

Using asymmetric risk measures to improve the worst cases without heavily compromising the best cases
Risk management

Asymmetric Risk measures

Worst-case (Robust optimization):

$$\max_u \min_{\theta_i} J(u, \theta_i)$$

Reformulation:

$$\max_u z \quad s.t. \quad z \leq J(u, \theta_i) \quad \forall i$$
Risk management

Asymmetric Risk measures

Value at Risk (VaR):

$$VaR_\beta(X) = \min \{z \mid F_X(z) \leq \beta\}$$

Conditional Value at Risk (CVaR):

$$CVaR_\beta(X) = E[X \mid X \leq VaR_\beta]$$
Simulation experiments

Experiment details:

- An ensemble of oil price scenarios
- A single realization of reservoir egg model
- For worst-case optimization:

\[
\max_u z \\
\text{s.t. } z \leq J(u, \theta_i) \quad \forall i
\]

Where \( J \) is NPV, \( u \) is the control input and \( \theta_i \) is the uncertainty ensemble.
Worst-case robust optimization

Worst-case increase: 4.41%
Average decrease: 6.18%
Conditional value-at-Risk (CVaR) optimization

Handling economic uncertainty

\[
\max_u J_M - \omega J_{CVaR}
\]

- \( \beta = 0.8 \)
Conditional value-at-Risk (CVaR) optimization

Mean - CVaR optimization

\[ J_M - \omega J_{CVaR} \]

Mean - Variance optimization

\[ J_M - \gamma J_V \]
• Asymmetric risk management using concepts from the theory of risk

• Results highly dependent upon the chosen uncertainty quantification (uncertainty ensemble)

• CVaR and worst-case optimization provide improvement but at the cost of compromising best cases.

Future directions:

• Best cases can be retained by imposing some constraints.

• Semi-variance can be an attractive measure for asymmetric risk shaping (submitted to DYCOPS-CAB 2016)


• A Capolei, B. Foss, and J. B. Jorgensen (2015b), "Profit and risk measures in oil production optimization," in Proc. of 2nd IFAC Workshop on Automatic Control in Offshore Oil and Gas Production in Florianopolis, Brazil
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Thank you
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arigatou gozaimasu
Conditional Value at Risk (CVaR) [1]

Let $X$ be a random variable with the cumulative distribution function $F_X(z) = P\{X \leq z\}$. The $VaR$ of $X$ with confidence level $\beta \in ]0,1[$ is

$$VaR_\beta(X) = \min\{z \mid F_X(z) \geq \beta\}$$

$$CVaR_\beta(X) = E[X \mid X \geq VaR_\beta]$$

CVaR optimization:

$$CVaR_\beta(X) = \min\{\alpha + \frac{1}{1 - \beta}E[X - \alpha]^+]\},$$

where

$$[t]^+ = \max\{t, 0\}$$

And

$$\min_X CVaR_\beta(X) = \min_{(X, \alpha)}\{\alpha + \frac{1}{1 - \beta}E[X - \alpha]^+]\},$$
Worst-case robust optimization

\[ J_{MWCO} = J_{MO} + \lambda J_{WCO} \]
**KNITRO:**

A commercial solver for large-scale non-linear constraint optimization

Both interior-point (barrier) and active-set methods;

Programmatic interfaces: C/C++, Fortran, Java, Python;

Modeling language interfaces: AMPL ©, AIMMS ©, GAMS ©, MATLAB ©, MPL ©, Microsoft Excel Premium Solver ©;