Data-driven distributed control: Virtual reference feedback tuning in dynamic networks

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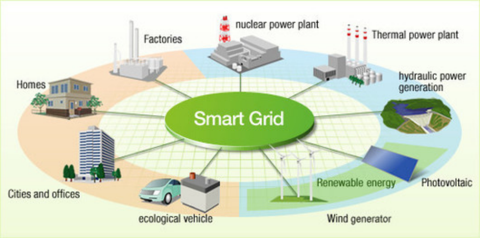
Introduction

Distributed control problems are present in many fields!



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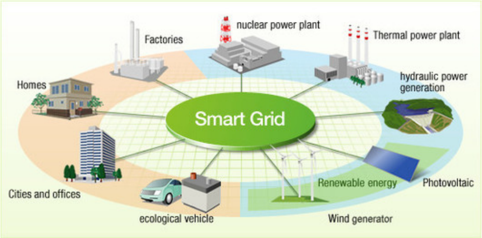

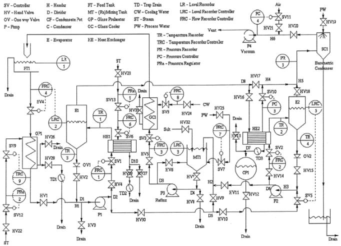

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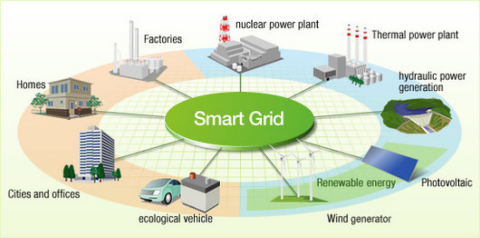

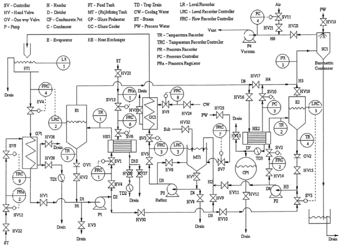

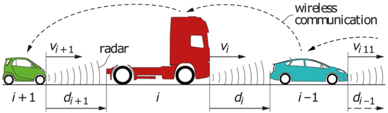

(Ploeg et al., 2013)

Introduction

Applied distributed control for

- Power networks (Jokic et al., 2012), (Riverso et al., 2013), (Bürger et al., 2014), (Schuler et al., 2014), (Tegling, 2018)
- Irrigation networks (Cantoni et al., 2007), (Costa et al., 2014)
- Chemical reactors (Lin et al., 2009), (Christofides et al., 2013), (Chen et al., 2019)
- Multi-agent systems (Rice et al., 2009), (Lunze, 2019)
- Building climate control (Morosan et al., 2010), (Lamoudi, 2013)
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However, models are typically not directly available, but data is!

Introduction

For an (unknown) interconnected system:

How to optimally design a distributed controller from measured data?

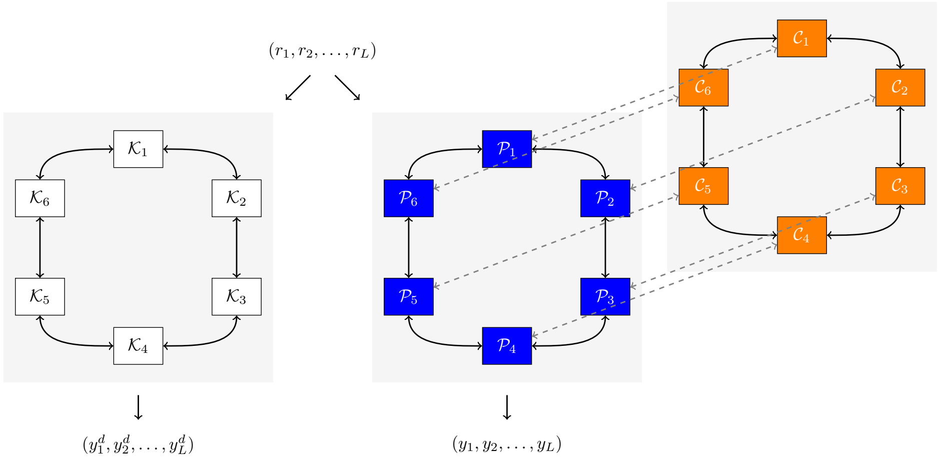
Model-based philosophy:

- Modelling: How to obtain the most relevant model from data?
- Control: What is the optimal distributed controller for a model?

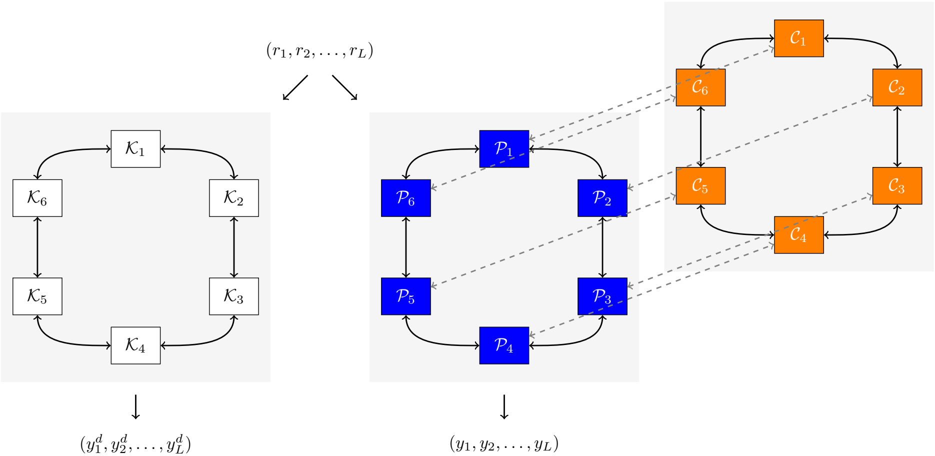
Data-based philosophy:

- How to synthesize an optimal distributed controller directly from data?

Direct data-driven distributed control



Direct data-driven distributed control



Problem

Find controllers $\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_L$ that minimize the global performance criterion $J_{\text{MR}}(\rho) := \bar{E}[y_1^d - y_1]^2 + \cdots + \bar{E}[y_L^d - y_L]^2$ using data.

Single-process case

Single process case (Campi et al., 2002):
Problem: Find a controller $C(\rho)$ that minimizes $J_{\text{MR}}(\rho) = \bar{E}[y_d(t) - y(t)]^2$ using data, for a reference model $y_d = T_d(q)r$.

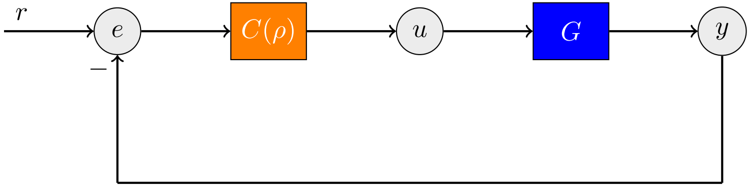


Figure: Standard control loop.

Single process case (Campi et al., 2002):

Solution: Let $C_d(q) := \frac{T_d(q)}{G(q)(1-T_d(q))}$. Then $C(\rho) = C_d$ minimizes $J_{\mathrm{MR}}(\rho) \rightarrow$ identify C_d from data via virtual experiment!

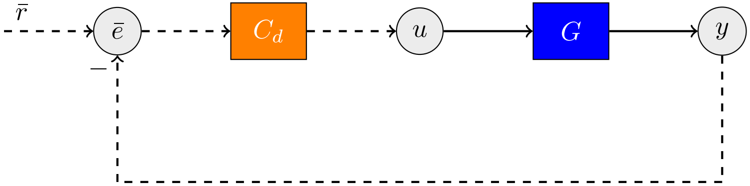
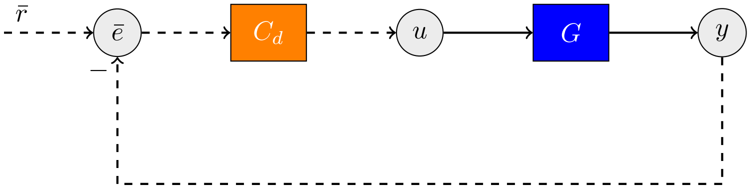


Figure: Virtual experiment setup.

VRFT

1. Collect data $\{u(t), y(t)\}$ from the plant.
2. Generate a virtual reference signal $\bar{r}(t)$ s.t. $y(t) = T_d(q)\bar{r}(t)$ and a corresponding tracking error $\bar{e}(t) = \bar{r}(t) - y(t)$.
3. Identification problem: Let $\hat{u}(t, \rho) = C(q, \rho)\bar{e}(t)$, with $C(q, \rho) = \rho^\top \bar{C}(q)$. Minimize

$$J_{\mathrm{VR}}(\rho) := \bar{E}[u(t) - \hat{u}(t, \rho)]^2$$



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Why does it work?

VRFT

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$$J_{\mathrm{VR}}(\rho) := \bar{E}[u(t) - \hat{u}(t,\rho)]^2$$

Why does it work?

$$J_{\mathrm{VR}}(\rho) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{|C_d(e^{i\omega})|^2} |(\rho - \rho_d)^\top \bar{C}(e^{i\omega})|^2 \Phi_u(\omega)\, \mathrm{d}\omega$$

with ρ_d s.t. $C(q,\rho_d) = C_d(q)$.

$\Rightarrow$ ρ_d is a global optimum of J_{VR} and it is unique if $\Phi_u(\omega) > 0$ for all ω.

The distributed case

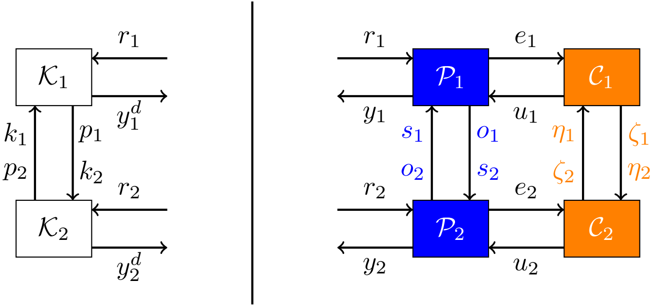
Consider the processes

$$\begin{cases} y_1 = G_1(q)u_1 + W_1(q)s_1 \\ o_1 = F_1(q)y_1 \end{cases} \quad \text{and} \quad \begin{cases} y_2 = G_2(q)u_2 + W_2(q)s_2 \\ o_2 = F_2(q)y_2 \end{cases}$$

describing $\mathcal{P}_1$ and $\mathcal{P}_2$ with $e_i := r_i - y_i$, $i = 1, 2$. Interconnection: $s_1 = o_2$ and $s_2 = o_1$.

Structured analogy to $y_d = T_d(q)r$ is given by the interconnected system

$$\mathcal{K}_1 : \begin{cases} y_1^d = T_1(q)r_1 + Q_1(q)k_1 \\ p_1 = P_1(q)y_1^d \end{cases} \quad \text{and} \quad \mathcal{K}_2 : \begin{cases} y_2^d = T_2(q)r_2 + Q_2(q)k_2 \\ p_2 = P_2(q)y_2^d \end{cases}$$



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Problem

Find controllers $\mathcal{C}_1(\rho_1)$ and $\mathcal{C}_2(\rho_2)$ that minimize $J_{\mathrm{MR}}(\rho_1, \rho_2) = \bar{E}[y_1^d - y_1]^2 + \bar{E}[y_2^d - y_2]^2$ using data.

The distributed case

Key steps to extend data-driven controller design to the distributed case:

1. Determine the ideal distributed controller
2. Relate the global performance J_{MR} to a network identification problem

Ideal controller for the single process is obtained by $T := \frac{GC}{1+GC} = T_d$. Extension to the MIMO case $\Rightarrow$ lose structure!

What is the ideal distributed controller?

A controller with the same interconnection structure as the process that achieves $J_{\mathrm{MR}}(\rho_1^d, \rho_2^d) = 0$.

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The ideal distributed controller

Application of local canonical controllers (Steentjes, 2018) to our processes leads to

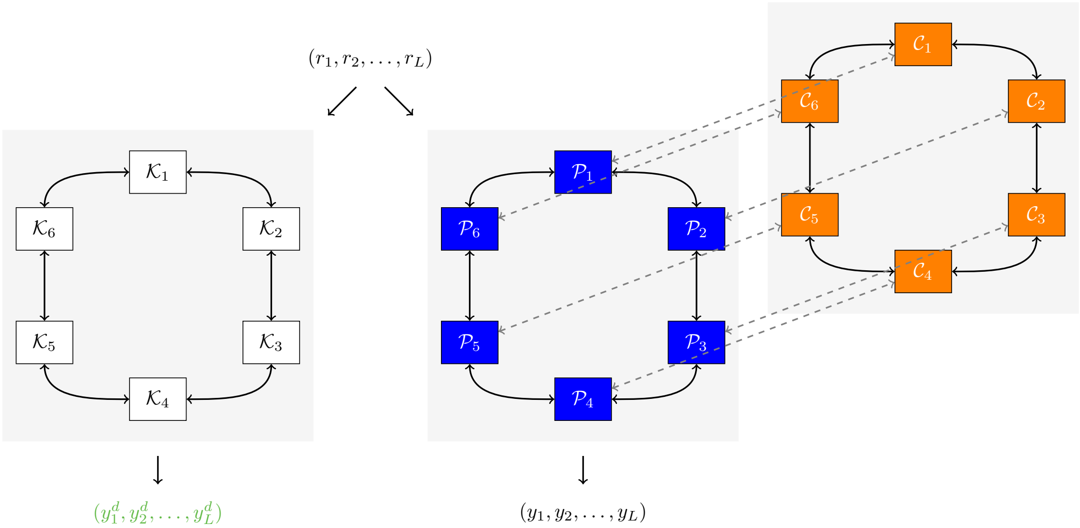
$$\mathcal{C}_i^d : \quad \begin{bmatrix} u_i \\ o_i^c \\ p_i^c \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{T_i}{G_i(1-T_i)} & \frac{-W_i}{G_i} & \frac{Q_i}{G_i(1-T_i)} \\ \frac{F_i T_i}{1-T_i} & 0 & \frac{F_i Q_i}{1-T_i} \\ \frac{P_i T_i}{1-T_i} & 0 & \frac{P_i Q_i}{1-T_i} \end{bmatrix}}_{=: C_i^d(q)} \begin{bmatrix} e_i \\ s_i^c \\ k_i^c \end{bmatrix}$$

Distributed controller is constructed by interconnecting local controllers.

Proposition

$J_{\mathrm{MR}}(\rho_1^d, \rho_2^d) = 0$ for ρ_1^d, ρ_2^d such that $C_1(\rho_1^d) = C_1^d$ and $C_2(\rho_2^d) = C_2^d$.

The ideal distributed controller



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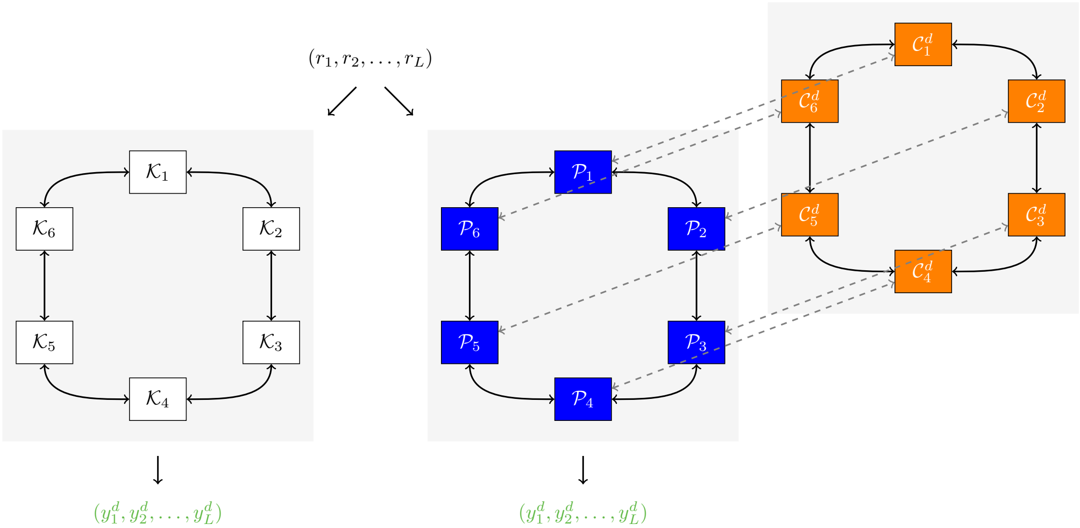
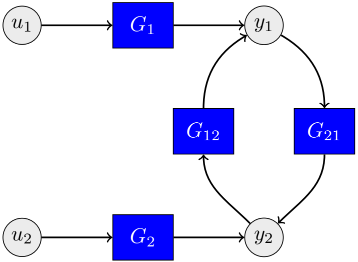


Example

Consider the coupled processes

$$y_1 = G_1(q)u_1 + G_{12}(q)y_1$$
$$y_2 = G_2(q)u_2 + G_{21}(q)y_1$$



with

$$G_1(q) = \frac{c_1}{q - a_1} \qquad G_{12}(q) = \frac{d_1}{q - a_1}$$
$$G_2(q) = \frac{c_2}{q - a_2} \qquad G_{21}(q) = \frac{d_2}{q - a_2}$$

The closed-loop system is desired to behave as two decoupled processes:

$$y_1^d = T_1 r_1$$
$$y_2^d = T_2 r_2$$

$$T_1 = \frac{1 - \gamma_1}{q - \gamma_1}$$
$$T_2 = \frac{1 - \gamma_2}{q - \gamma_2}$$

Example (continued)

The ideal distributed controller is described by

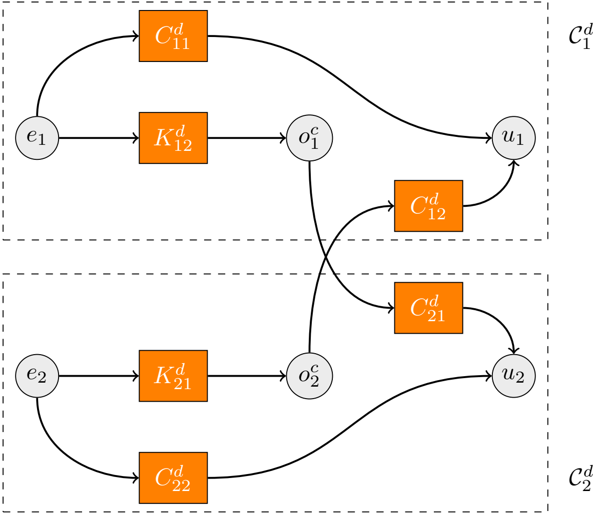
$$\begin{bmatrix} u_1 \\ o_1^c \end{bmatrix} = \begin{bmatrix} C_{11}^d & C_{12}^d \\ K_{12}^d & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ s_1^c \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} u_2 \\ o_2^c \end{bmatrix} = \begin{bmatrix} C_{22}^d & C_{21}^d \\ K_{21}^d & 0 \end{bmatrix} \begin{bmatrix} e_2 \\ s_2^c \end{bmatrix}$$

with $e_i = r_i - y_i$, the interconnections $s_1^c = o_2^c$, $s_2^c = o_1^c$, and

$$C_{11}^d(q) = \frac{1-\gamma_1}{c_1}\frac{q-a_1}{q-1}, \quad C_{12}^d(q) = -\frac{d_1}{c_1}, \quad K_{12}^d(q) = \frac{1-\gamma_1}{q-1}$$

$$C_{22}^d(q) = \frac{1-\gamma_2}{c_2}\frac{q-a_2}{q-1}, \quad C_{21}^d(q) = -\frac{d_2}{c_2}, \quad K_{21}^d(q) = \frac{1-\gamma_2}{q-1}.$$

Example (continued)



Data-driven driven distributed controller

1. Collect input-output data $\{u_i, y_i\}$, $i = 1, 2$ from the processes.
2. Generate virtual reference signals $\bar{r}_1$ and $\bar{r}_2$ satisfying $y_1 = T_1(q)\bar{r}_1$ and $y_2 = T_2(q)\bar{r}_2$ with virtual tracking errors $\bar{e}_1 = \bar{r}_1 - y_1$ and $\bar{e}_2 = \bar{r}_2 - y_2$.
3. Network identification problem: Let

$$\hat{u}_1(\rho_1) = C_{12}(\rho_1)\bar{o}_2^c + C_{11}(\rho_1)\bar{e}_1$$
$$\hat{u}_2(\rho_2) = C_{21}(\rho_2)\bar{o}_1^c + C_{22}(\rho_2)\bar{e}_2$$

with C_{ii}, C_{ij} linear in ρ_i and minimize

$$J_{\mathrm{VR}}(\rho_1, \rho_2) = \bar{E}[u_1 - \hat{u}_1(\rho_1)]^2 + \bar{E}[u_2 - \hat{u}_2(\rho_2)]^2$$

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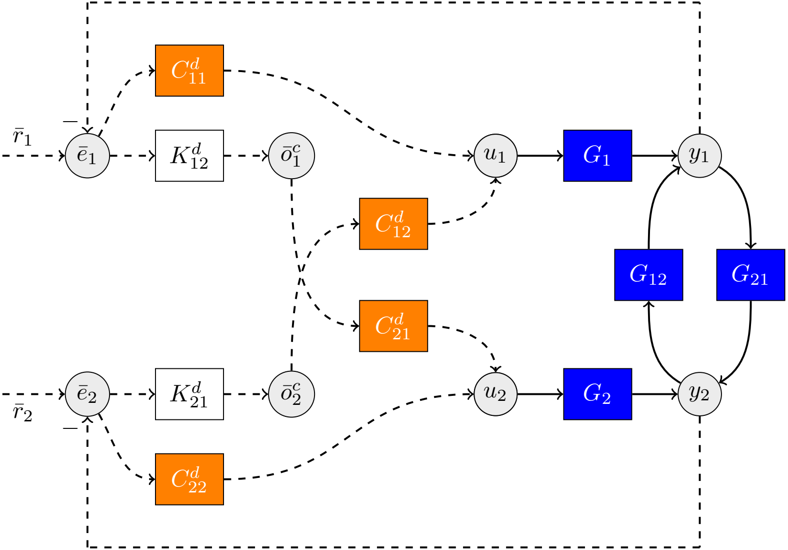
Virtual experiment setup



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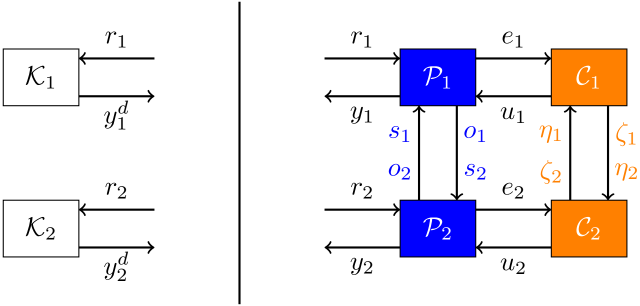
$$J_{\text{VR}}(\rho_1, \rho_2) = \bar{E}[u_1 - \hat{u}_1(\rho_1)]^2 + \bar{E}[u_2 - \hat{u}_2(\rho_2)]^2$$

$\Rightarrow$ (ρ_1^d, ρ_2^d) is a global minimum of J_{VR} and it is unique if $\text{col}(u_1, u_2)$ is persistently exciting of sufficient order.

Hence, the distributed controller synthesis problem reduces to a network identification problem!

VRFT in dynamic networks



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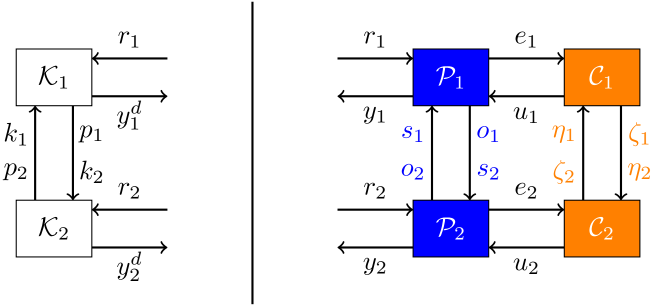

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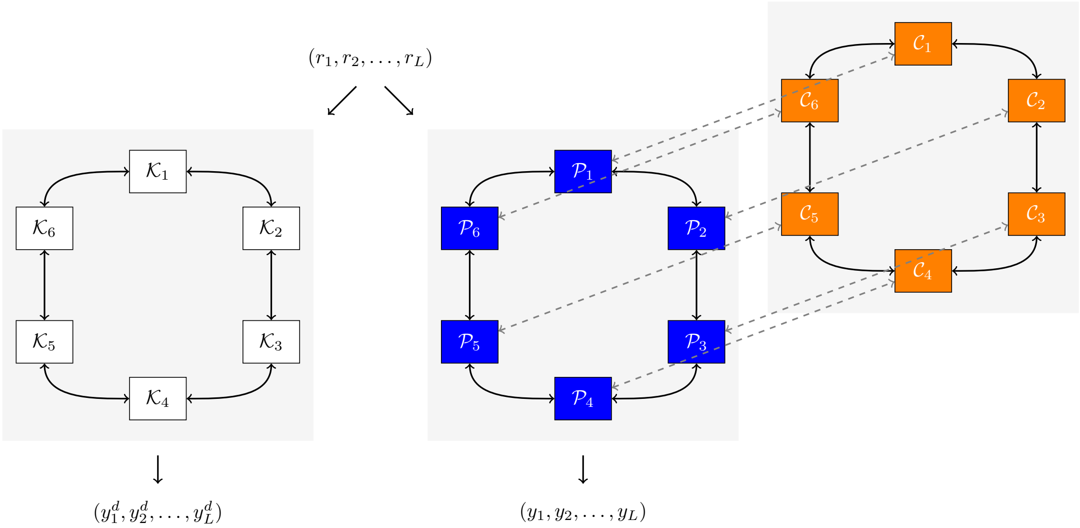

VRFT in dynamic networks

Predictor $\hat{u}_i(\rho_i) := C_{ii}(\rho_i)\bar{e}_i + \sum_{j\in\mathcal{N}_i} C_{ij}^W(\rho_i)\bar{o}_{ji}^c + C_{ij}^Q(\rho_i)\bar{p}_{ji}$.

Theorem

The identification criterion

$$J_i^{\mathrm{VR}}(\rho_i) = \bar{E}[u_i - \hat{u}_i(\rho_i)]^2$$

has a global minimum point at ρ_i^d and is unique if $\Phi_{w_i}(\omega) > 0$ for all $\omega \in [-\pi, \pi]$, $w_i = \mathrm{col}(\bar{e}_i, \mathrm{col}_{j\in\mathcal{N}_i}\, \bar{o}_{ji}^c, \mathrm{col}_{j\in\mathcal{N}_i}\, \bar{p}_{ji})$.

Direct method for local module identification (Van den Hof et al., 2013)

VRFT in dynamic networks

Corollary

Each global minimum point ρ_i^* of J_i^{VR} satisfies

$$C_{ii}(\rho_i^*) = C_{ii}(\rho_i^d) \text{ and } (C_{ij}^W(\rho_i^*) - C_{ij}^W(\rho_i^d)) + (C_{ij}^Q(\rho_i^*) - C_{ij}^Q(\rho_i^d)P_{ji} = 0,\ j \in \mathcal{N}_i$$

if $\Phi_{\xi_i}(\omega) > 0$ for all $\omega \in [-\pi, \pi]$, $\xi_i = \text{col}(\bar{e}_i, \text{col}_{j\in\mathcal{N}_i}\, \bar{o}_{ji}^c)$.

VRFT in dynamic networks

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$(\rho_1^*, \ldots, \rho_L^*)$ is a global minimum point of J_{MR}!

Illustrative example: 9-systems network

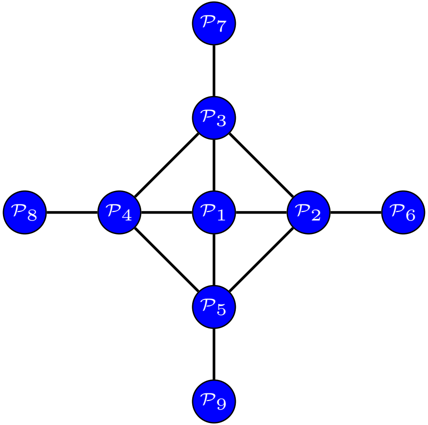
Process:

$$y_i = G_i(q)u_i + \sum_{j \in \mathcal{N}_i} G_{ij}(q)y_j, \quad i = 1, \dots, 9$$

$$G_i = \frac{1}{q - a_i} \quad G_{ij} = \frac{0.1}{q - a_i} \quad a_i \in (0, 1)$$

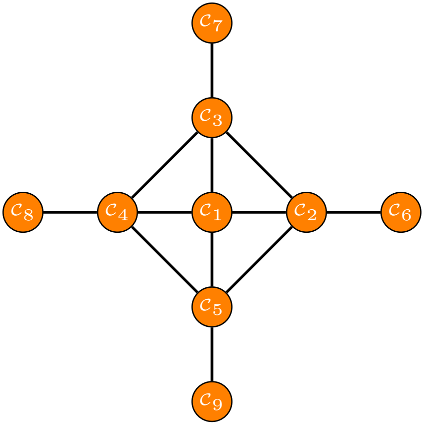
Reference model:

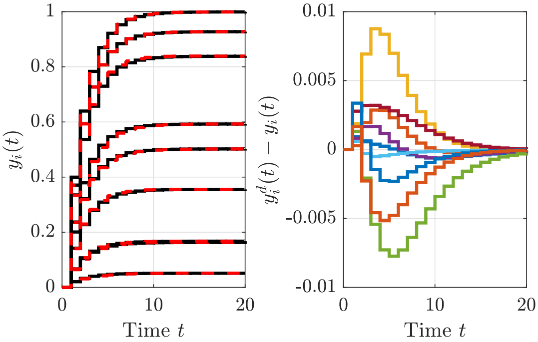
$$y_i^d = T_i(q)r_i \quad T_i = \frac{0.2}{q - 0.8}$$



Data $\{u_i(t), y_i(t), t = 1, \dots, 100\}$ collected in open loop ($\sigma_{u_i} = 1$) with additive noise to y_i ($\sigma = 0.1$)

Illustrative example: 9-systems network (continued)





Concluding remarks

- Extension of direct data-driven control to distributed control possible via
  - extension of the ideal controller to an ideal distributed controller
  - data-driven modelling of the controller via a virtual dynamic network
- Provides a solid link between network identification and distributed control through a global control objective
- Many problems to consider: disturbances, controller classes with reduced-order TFs or absent communication links, updating local controllers, . . .

Thank you for your attention!

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