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Handling unmeasured disturbances in data-driven distributed control with virtual reference feedback tuning

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Eindhoven University of Technology

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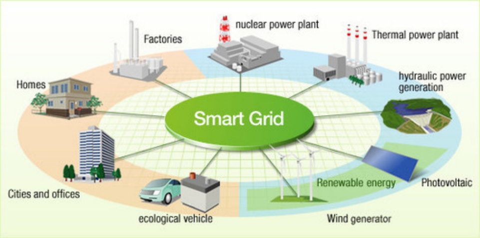
Introduction

Distributed control problems are present in many fields!



(BetterWorldSolutions)

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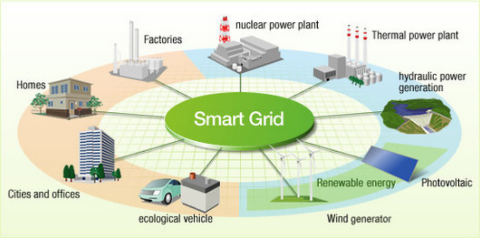

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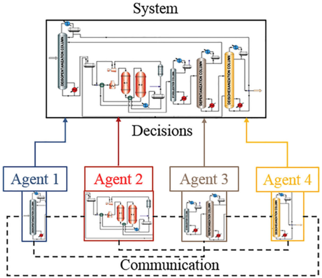

(Tang & Daoutidis, 2019)

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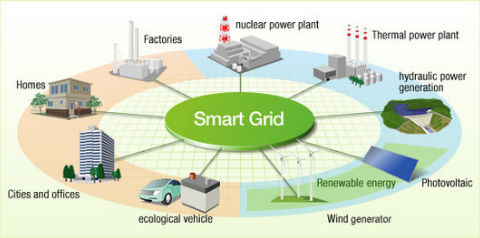

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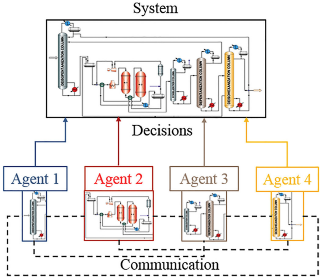

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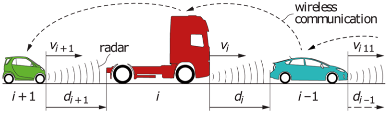

(Ploeg et al., 2013)

Introduction

Applied distributed control for

- Power networks (Jokic et al., 2012), (Riverso et al., 2013), (Bürger et al., 2014), (Schuler et al., 2014), (Tegling, 2018)
- Irrigation networks (Cantoni et al., 2007), (Costa et al., 2014)
- Chemical reactors (Lin et al., 2009), (Christofides et al., 2013), (Chen et al., 2019)
- Multi-agent systems (Rice et al., 2009), (Lunze, 2019)
- Building climate control (Morosan et al., 2010), (Lamoudi, 2013), (Smith et al., 2020)
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However, models are typically not directly available, but data is!

Introduction

For an (unknown) interconnected system:

How to optimally design a distributed controller from measured data?

Model-based philosophy:

- Modelling: How to obtain the most relevant model from data?
- Control: What is the optimal distributed controller for a model?

Data-based philosophy:

- How to synthesize an optimal distributed controller directly from data?

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Virtual reference feedback tuning: a direct method for the design of feedback controllers ☆

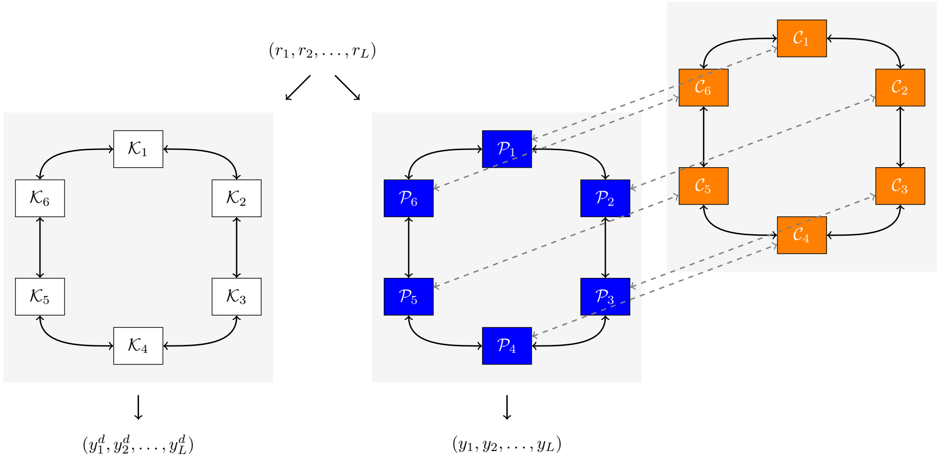
M.C. Campi[a,*], A. Lecchini[b], S.M. Savaresi[c]

Data-driven model reference control design by prediction error identification ☆

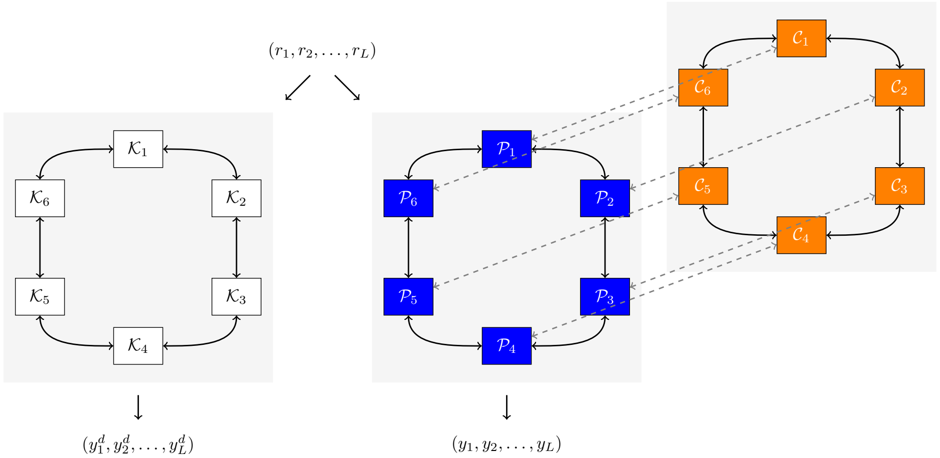
Lucíola Campestrini[a,*], Diego Eckhard[b], Alexandre Sanfelice Bazanella[a], Michel Gevers[c]

data-driven centralized control $\rightarrow$ data-driven distributed control

Direct data-driven distributed control



Direct data-driven distributed control



Problem

Find controllers $\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_L$ that minimize the global performance criterion $J_{\text{MR}}(\rho) := \bar{E}[y_1^d - y_1]^2 + \cdots + \bar{E}[y_L^d - y_L]^2$ using data.

System setup

Reference model:

$$\mathcal{K}_i : \quad y_i^d = T_i(q) r_i$$

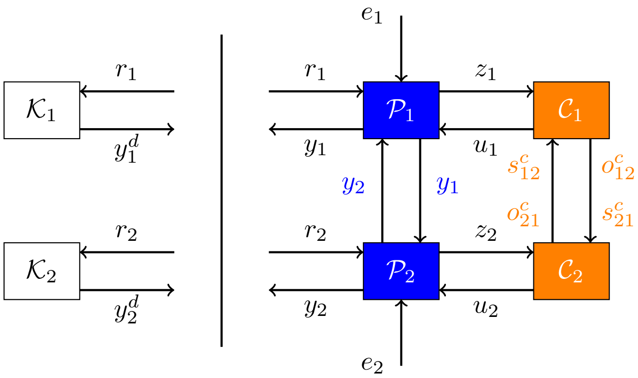
Interconnected system:

$$\mathcal{P}_i : \quad \begin{cases} y_i & = G_i(q) u_i + \sum_{j \in \mathcal{N}_i} G_{ij}(q) y_j + H_i(q) e_i \\ z_i & = r_i - y_i \end{cases}$$

Distributed controller:

$$\mathcal{C}_i(\rho_i) : \begin{cases} u_i = C_{ii}(q, \rho_i) z_i + \sum_{j \in \mathcal{N}_i} C_{ij}(q, \rho_i) s_{ij}^c \\ o_{ij}^c = K_{ij}(q, \rho_i) z_i + \sum_{h \in \mathcal{N}_i} K_{ijh}(q, \rho_i) s_{ih}^c, \; j \in \mathcal{N}_i \end{cases}$$

System setup



An ideal distributed controller

Application of local canonical controllers (Steentjes, 2018) to our processes leads to

$$\mathcal{C}_i^d : \quad \begin{bmatrix} u_i \\ o_i^c \end{bmatrix} = \underbrace{\begin{bmatrix} \dfrac{T_i}{G_i(1-T_i)} & -\dfrac{1}{G_i} G_{il} \\ \dfrac{T_i}{1-T_i}\mathbf{1} & 0 \end{bmatrix}}_{=: C_i^d(q)} \begin{bmatrix} z_i \\ s_i^c \end{bmatrix}$$

Distributed control architecture is obtained by interconnecting local controllers

Proposition

$J_{\mathrm{MR}}(\rho_1^d, \rho_2^d) = 0$ for ρ_1^d, ρ_2^d such that $C_1(\rho_1^d) = C_1^d$ and $C_2(\rho_2^d) = C_2^d$. $(e = 0)$

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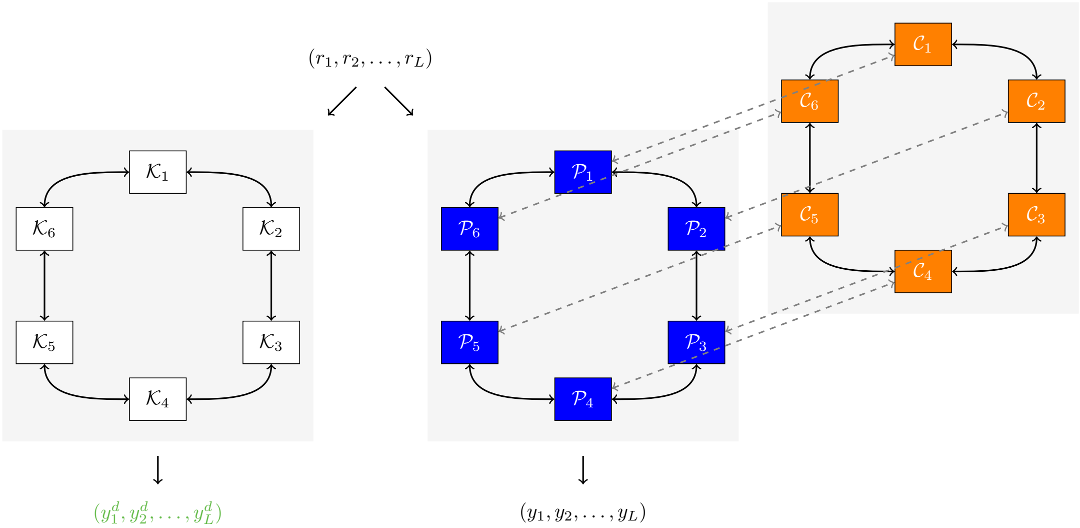
Solution for a coupled reference model also exists! (Steentjes et al., CDC 2020)

An ideal distributed controller



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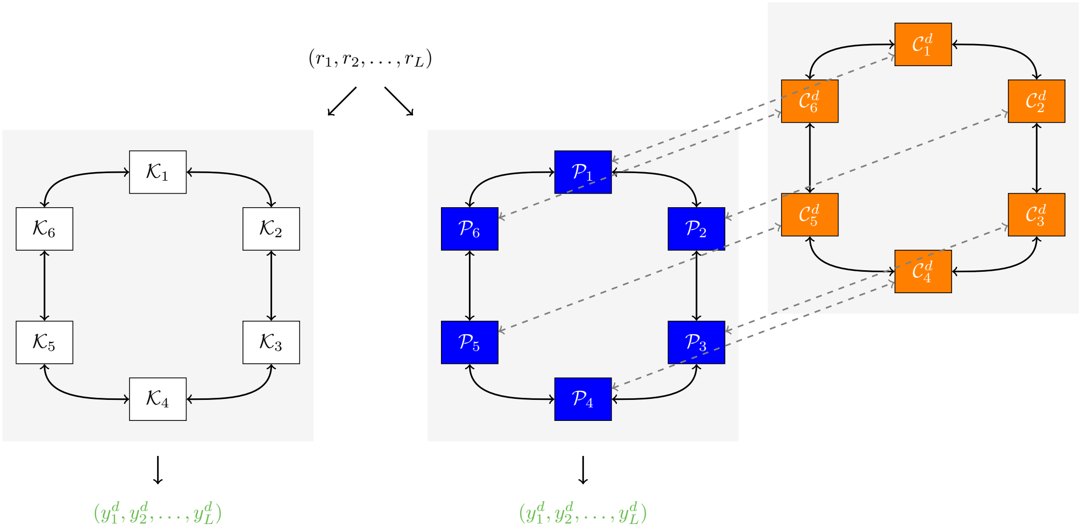
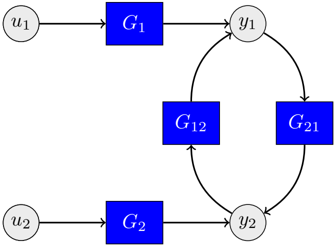


Distributed virtual reference feedback tuning (DVRFT)



DVRFT principle (Steentjes et al., CDC 2020)

- For each $i \in \mathcal{V}$, determine virtual signals: $y_i = T_i \bar{r}_i$, $\bar{z}_i = \bar{r}_i - y_i$
- Identify controller modules C_{ij}^d in virtual reference network

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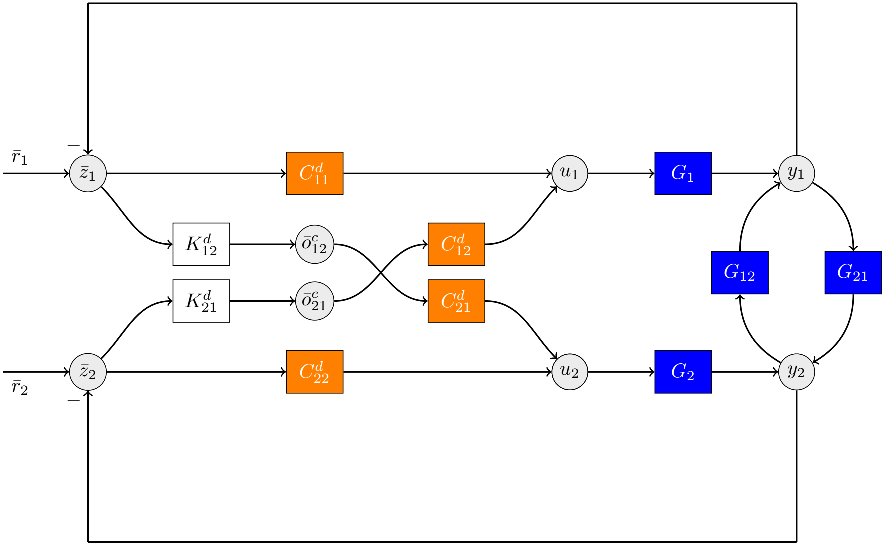

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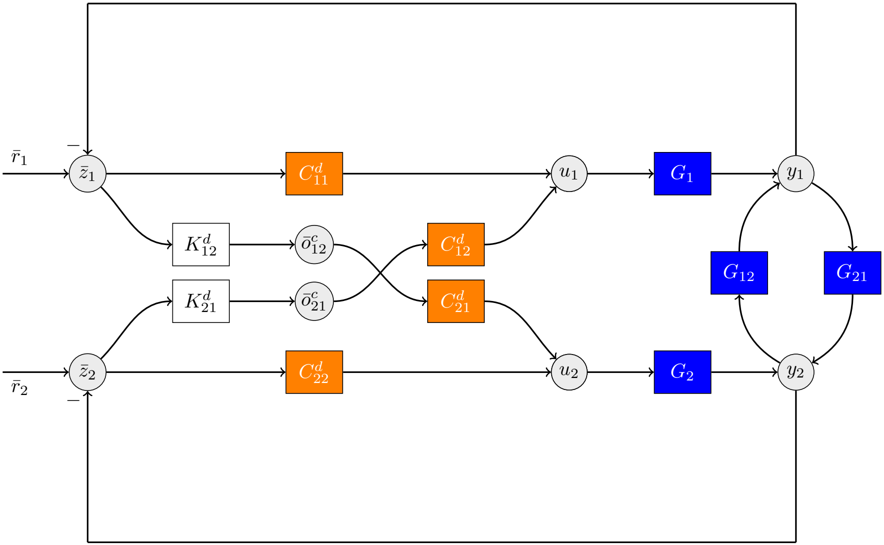
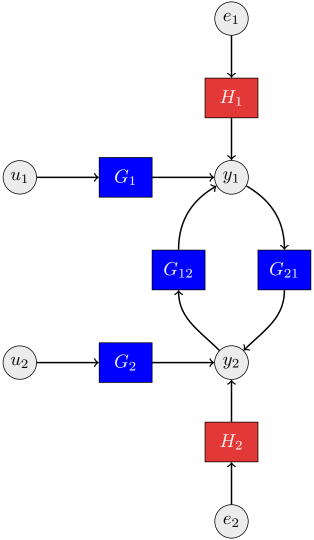
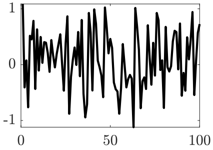
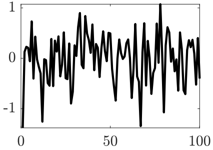


Distributed controller synthesis problem is turned into a network identification problem

Noise in data-driven distributed control: is there a problem?







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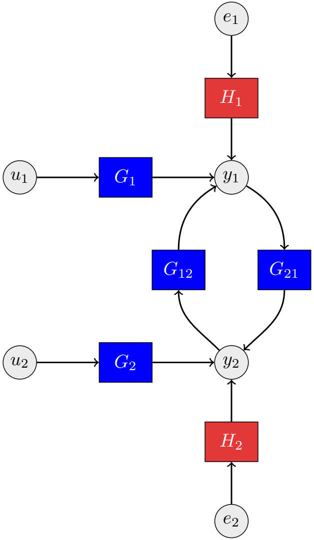
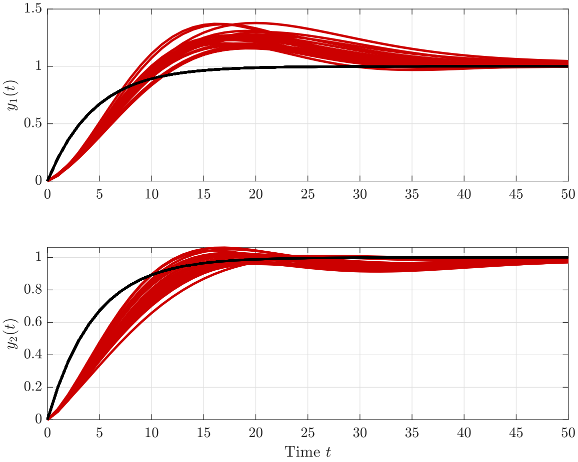
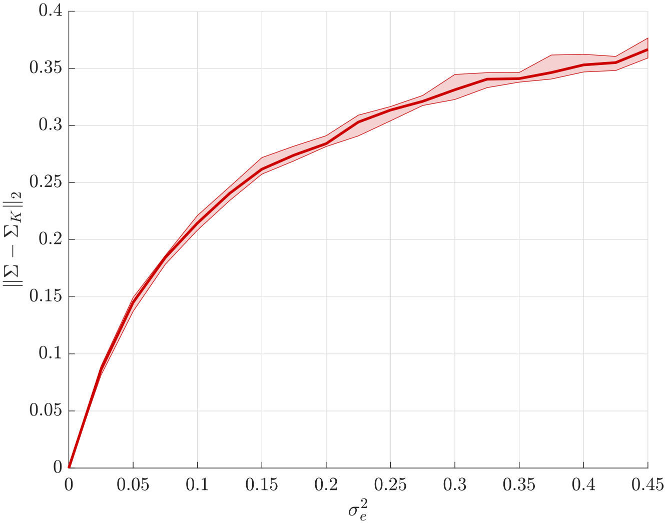




Effect of biased controller estimates

Noise in data-driven distributed control: is there a problem?



Noise entering the network $\Rightarrow$ degraded achieved performance!

Dealing with noise

- Noise in (distributed) virtual reference feedback tuning:
  - Instrumental variables (linear parametrization, high variance)

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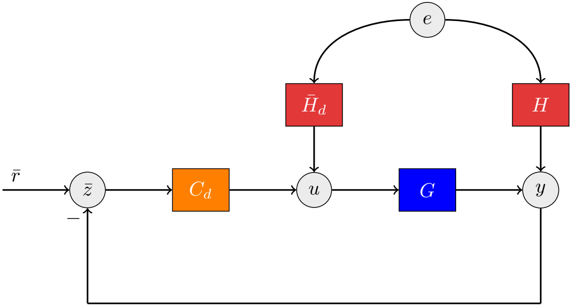
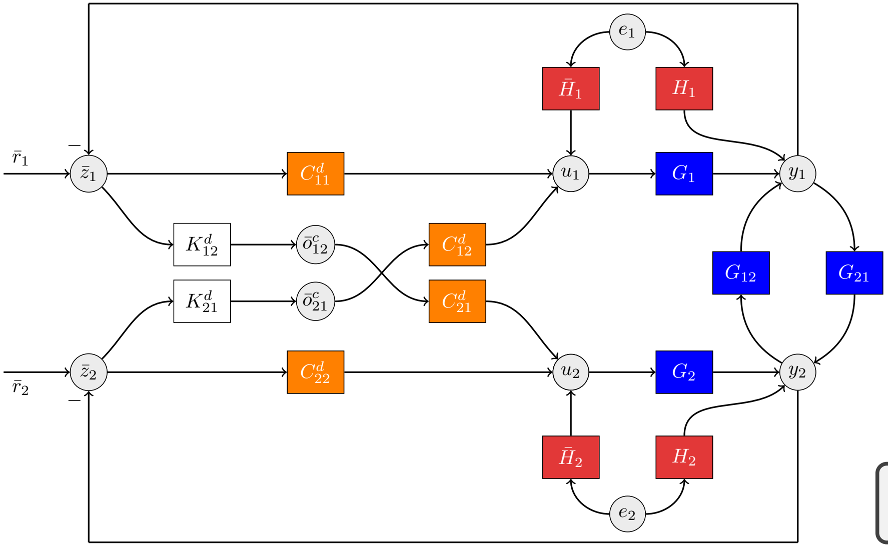
  - Instrumental variables (linear parametrization, high variance)
  - Modelling the noise?



Modelling noise in DVRFT



$$\bar{H}_i = -{G_i}^{-1} H_i$$

Modelling noise in DVRFT

Filter the prediction-error $\varepsilon_i = u_i - \hat{u}_i(\rho_i)$ with G_i?

- Leads to consistent controller estimates if H_i is modelled!
- G_i is unknown $\Rightarrow$ arguably leads to indirect data-driven control

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$$\boxed{\frac{T_i}{1 - T_i} = C_{ii}^d G_i}$$

Predictor $\hat{u}_i(\rho_i) := \bar{H}_i^{-1}(\rho_i)\left(C_{ii}(\rho_i)\bar{z}_i + \sum_{j\in\mathcal{N}_i} C_{ij}(\rho_i)\bar{o}_{ji}^c\right) + (1 - \bar{H}_i(\rho_i)^{-1})u_i$;

Filter: $$L_i = \frac{T_i}{1 - T_i} = C_{ii}^d G_i$$

Noise model: $$\bar{H}_i(\rho_i) = -C_{ii}(\rho_i)\breve{H}_i(\rho_i)$$

Theorem

Suppose $\Phi_{\zeta_i}(\omega) > 0$ for all $\omega \in [-\pi, \pi]$, $\zeta_i = \mathrm{col}(\bar{z}_i, u_i, \mathrm{col}_{j\in\mathcal{N}_i} \bar{o}_{ji}^c)$, $\exists \rho_i^d$: $C_i^d = C_i(\rho_i^d)$, $H_i = \breve{H}_i(\rho_i^d)$ and G_{ji} contains a delay for $j \in \mathcal{N}_i$.
Then the global minimum point ρ_i^* of

$$V_i^F(\rho_i) = \bar{E}[L_i(u_i - \hat{u}_i(\rho_i))]^2$$

satisfies $C_{ii}(\rho_i^*) = C_{ii}^d$, $C_{ij}(\rho_i^*) = C_{ij}^d$, $j \in \mathcal{N}_i$, and $\breve{H}_i(\rho_i^*) = H_i$.

Example

System dynamics

$$G_1(q) = \frac{c_1}{q - a_1}, \quad G_{12}(q) = \frac{d_1}{q - a_1}, \quad H_1 = \frac{q}{q - a_1}$$

$$G_2(q) = \frac{c_2}{q - a_2}, \quad G_{21}(q) = \frac{d_2}{q - a_2}, \quad H_2 = \frac{q}{q - a_2}$$

Reference model

$$T_i(q) = \frac{0.2}{q - 0.8}, \quad i \in \{1, 2\}$$

- Identify ideal distributed controller from $N = 500$ data samples:
  - Distributed virtual reference feedback tuning (DVRFT)
  - DVRFT with instrumental variables
  - DVRFT with tailor-made noise model

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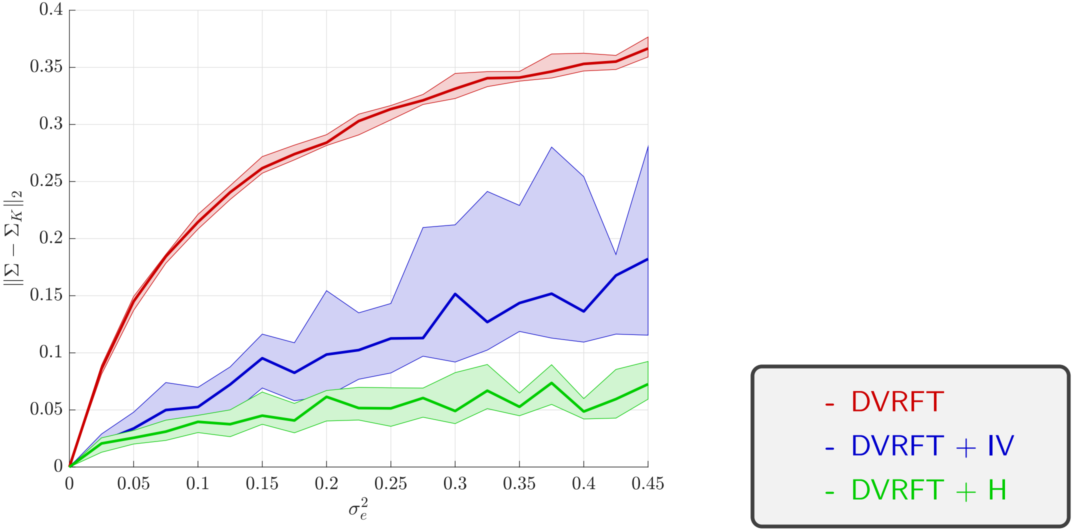

0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
$\|\Sigma - \Sigma_K\|_2$
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
σ_e^2
- DVRFT
- DVRFT + IV
- DVRFT + H

Concluding remarks

Summary:

- Data-driven synthesis of a distributed controller for interconnected systems with disturbances
- Capturing noise with a tailor-made noise model $\Rightarrow$ consistent controller estimates

Remarks & future work:

- Applicable in the SISO case (VRFT)
- Possibly non-linear parametrizations: complexity-performance trade-off
- Multi-step least squares & distributed identification for complexity reduction

Handling disturbances: one step closer to *practical* distributed control from data

Thank you for your attention!

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