15-th Benelux Meeting on Systems and Control
Minicourse on Closed Loop Identification

by

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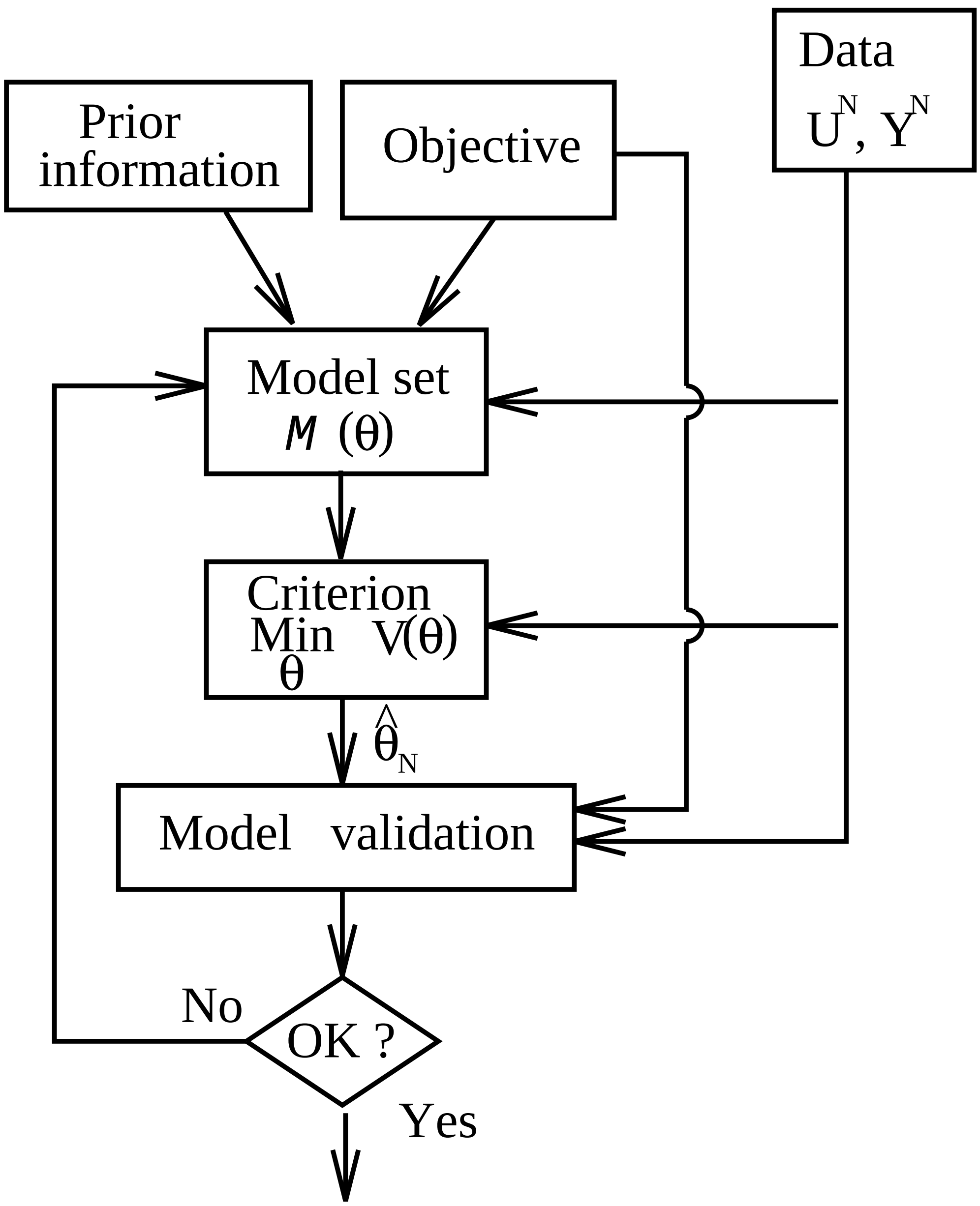
Part I
CLOSED LOOP IDENTIFICATION USING CLASSICAL INPUT-OUTPUT METHODS

Outline

- The prediction error framework
- Identification in open loop
- Bias and variance errors
- Identification in closed loop:
  - Identifiability
  - Bias expressions
  - Variance expressions
  - Controlling the bias distribution of approximate models.

THE GLOBAL SCENARIO



SYSTEM, MODEL, PREDICTOR, ...

True system (conceptually useful) :

$$\mathcal{S} : y_t = P_0(z)u_t \;+\; \underbrace{v_t}_{H_0(z)e_t}$$

e_t : white noise

Model set : $\mathcal{M} = \{M(\theta), \theta \in D_\theta\}$

Model :

$$M(\theta) : y_t(\theta) = P(z,\theta)u_t + H(z,\theta)q_t$$

(q_t : white noise)

Predictor :

$$\begin{aligned} \hat{y}_{t|t-1}(\theta) \triangleq \hat{y}_t(\theta) &= H^{-1}(z,\theta)P(z,\theta)u_t \\ &\quad +[1 - H^{-1}(z,\theta)]y_t \\ &= W_u(z,\theta)u_t + W_y(z,\theta)y_t \end{aligned}$$

$$\boxed{\textbf{Model} \quad \stackrel{1-1}{\longleftrightarrow} \quad \textbf{Predictor}}$$

$$\begin{aligned} W_u(z,\theta) &= H^{-1}(z,\theta)P(z,\theta) \\ W_y(z,\theta) &= 1 - H^{-1}(z,\theta) \\ H(z,\theta) &= 1 - W_y(z,\theta) \\ P(z,\theta) &= [1 - W_y(z,\theta)]W_u(z,\theta) \end{aligned}$$

SOME TYPICAL MODEL SETS

1. ARX models :

$$\begin{aligned} y_t + a_1 y_{t-1} + \ldots + a_n y_{t-n} \\ = b_1 u_{t-1} + \ldots + b_m u_{t-m} + q_t \end{aligned}$$

$$\Leftrightarrow \quad A(z^{-1}) y_t = B(z^{-1}) u_t + q_t$$

$$P(z, \theta) = \frac{B(z^{-1})}{A(z^{-1})}$$

$$H(z, \theta) = \frac{1}{A(z^{-1})}$$

Rewrite as :

$$y_t = [1 - A(z^{-1})] y_t + B(z^{-1}) u_t + q_t$$

Predictor :

$$\hat{y}_{t|t-1}(\theta) = [1 - A(z^{-1})] y_t + B(z^{-1}) u_t$$

2. OE models :

$$
\begin{aligned}
y_t &= \frac{B(z^{-1})}{A(z^{-1})} u_t + q_t \\
P(z, \theta) &= \frac{B(z^{-1})}{A(z^{-1})} \\
H(z, \theta) &= 1
\end{aligned}
$$

Predictor :

$$\hat{y}_{t|t-1}(\theta) = \frac{B(z^{-1})}{A(z^{-1})} u_t$$

THREE IMPORTANT SITUATIONS

$\boxed{\mathcal{S} \in \mathcal{M}}$

$$\begin{aligned} \exists \theta_0 \text{ such that } P(z,\theta_0) &= P_0(z) \\ H(z,\theta_0) &= H_0(z) \end{aligned}$$

$\boxed{\mathcal{S} \notin \mathcal{M}, \text{ but } P_0 \in \mathcal{G}}$

$$\begin{aligned} \exists \theta_0 \text{ such that } P(z,\theta_0) &= P_0(z) \\ \text{but } H(z,\theta_0) &\neq H_0(z) \end{aligned}$$

$\boxed{\mathcal{S} \notin \mathcal{M}}$

This is the practically important and interesting situation of unmodelled dynamics, reduced complexity modeling, system approximation.

In this course we consider almost exclusively $\mathcal{S} \notin \mathcal{M}$. The true system could be nonlinear, time-varying, ...

PREDICTION ERROR IDENTIFICATION

Prediction error

$$\begin{aligned}\varepsilon_t(\theta) &\triangleq y_t - \hat{y}_{t|t-1}(\theta)\\ &= H^{-1}(z,\theta)[(P_0(z) - P(z,\theta))u_t + v_t]\end{aligned}$$

Filtered prediction error

$$\varepsilon_t^f(\theta) = D(z)\varepsilon_t(\theta) \quad D(z): \text{ data filter}$$

Prediction error criterion (Least Squares)

$$\begin{aligned}V_N(\theta) &= \frac{1}{N}\sum_{t=1}^{N}[\varepsilon_t^f(\theta)]^2\\ \Rightarrow \hat{\theta}_N &= \arg\min_{\theta \in D_\theta} V_N(\theta)\\ \Rightarrow \hat{P} &= P(z,\hat{\theta}_N) \qquad \hat{H} = H(z,\hat{\theta}_N)\end{aligned}$$

Often : $\hat{H}$ **= fixed noise model**

CONVERGENCE

Recall :

$$\begin{aligned} V_N(\theta) &\triangleq \frac{1}{N}\sum_{t=1}^{N}[\varepsilon_t^f(\theta)]^2 \\ \hat{\theta}_N &\triangleq \arg\min_{\theta\in D_\theta} V_N(\theta) \end{aligned}$$

Denote :

$$\begin{aligned} \bar{V}(\theta) &\triangleq E[\varepsilon_t^f(\theta)]^2 \\ \theta^* &\triangleq \arg\min_{\theta\in D_\theta} \bar{V}(\theta) \end{aligned}$$

Under reasonable conditions

$$\begin{aligned} V_N(\theta) &\overset{N\to\infty}{\longrightarrow} \bar{V}(\theta) \\ \hat{\theta}_N &\overset{N\to\infty}{\longrightarrow} \theta^* \end{aligned}$$

ACCURACY

Under reasonable conditions :

$$\sqrt{N}(\hat{\theta}_N - \theta^*) \stackrel{N\to\infty}{\longrightarrow} N(0, P_\theta)$$

$\boxed{\text{If } \mathcal{S} \in \mathcal{M}}$

$$\text{i.e. } \exists \theta_0 \;:\; \begin{aligned} P(z,\theta_0) &= P_0(z) \\ H(z,\theta_0) &= H_0(z) \end{aligned}$$

$$\Rightarrow P_\theta = \sigma^2 E\left[\frac{\partial \hat{y}_t}{\partial \theta}\left(\frac{\partial \hat{y}_t}{\partial \theta}\right)^T\right]^{-1}_{\theta_0}$$

$\Rightarrow$ We can get an estimate of P_θ :

$$P_{\hat{\theta}_N} = \left(\frac{1}{2}\sum_1^N \varepsilon_t^2(\hat{\theta}_N)\right)\left[\frac{1}{2}\sum_{t=1}^N \frac{\partial \hat{y}_t}{\partial \theta}\left(\frac{\partial \hat{y}_t}{\partial \theta}\right)^T\right]^{-1}_{\hat{\theta}_N}$$

Total transfer function error

Assume that the true system is linear, time-invariant. At every frequency ω :

$$P_0(e^{j\omega}) - P(e^{j\omega},\hat{\theta}_N) =$$
$$\underbrace{P_0(e^{j\omega}) - P(e^{j\omega},\theta^*)}_{\text{bias error}} + \underbrace{P(e^{j\omega},\theta^*) - P(e^{j\omega},\hat{\theta}_N)}_{\text{variance error}}$$

Bias error influenced by :

- Model set $\{P(z,\theta), H(z,\theta)\}$
- Input spectrum $\phi_u(\omega)$
  and cross-spectrum $\phi_{ue}(\omega)$
- Data filter $D(z)$
  Noise spectrum $\phi_v(\omega)$

Variance error influenced by :

- Model order n
- Number of data N
- Input spectrum $\phi_u(\omega)$
  and cross spectrum $\phi_{ue}(\omega)$

IDENTIFIABILITY IN OPEN LOOP

A. $\mathcal{S} \in \mathcal{M}$

$\exists \theta_0 : P(z, \theta_0) = P_0(z)$ and $H(z, \theta_0) = H_0(z)$

$+$ Richness conditions

$$\Rightarrow \quad \hat{\theta}_N \quad \stackrel{N \to \infty}{\longrightarrow} \theta_0$$

$$\Rightarrow \quad \begin{cases} P(z, \hat{\theta}_N) \stackrel{N \to \infty}{\longrightarrow} P_0(z) \\ \\ H(z, \hat{\theta}_N) \stackrel{N \to \infty}{\longrightarrow} H_0(z) \end{cases}$$

B. $P_0 \in \mathcal{G}$ **but** $\mathcal{S} \notin \mathcal{M}$

$\exists \theta_0 : P(z, \theta_0) = P_0(z)$ **but** $H(z, \theta_0) \neq H_0(z)$.

Then : $P(z, \hat{\theta}_N) \overset{N \to \infty}{\longrightarrow} P(z, \theta_0) = P_0(z)$ **if**

- u_t **is sufficiently rich**

- $P(z, \theta)$ **and** $H(z, \theta)$ **are independently parametrized**

Examples :

OE : $y_t = \frac{B(z)}{F(z)} u_t + q_t$

BJ : $y_t = \frac{B(z)}{F(z)} u_t + \frac{C(z)}{D(z)} q_t$

But not ARX : $A(z) y_t = B(z) u_t + q_t$

BIAS IN OPEN LOOP

Parseval :

$$\begin{aligned}
\mathcal{S} : y_t &= P_0(z)u_t + v_t \quad E\{u_t v_s\} = 0 \quad \forall t, s \\
\mathcal{M} : y_t(\theta) &= P(z,\theta)u_t + H(z,\theta)q_t \\
\Rightarrow \varepsilon_t^f &= \frac{D(z)}{H(z,\theta)}[(P_0(z) - P(z,\theta))u_t + v_t]
\end{aligned}$$

$$E[\varepsilon_t^f]^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \phi_\varepsilon(\omega)d\omega$$

$$\hat{\theta}_N \overset{N\to\infty}{\longrightarrow} \theta^*$$

$$\theta^* = \arg\min_\theta \int_{-\pi}^{\pi} \Big[|P_0(e^{j\omega}) - P(e^{j\omega},\theta)|^2\phi_u(\omega)$$

$$+\phi_v(\omega)\Big] \left|\frac{D(e^{j\omega})}{H(e^{j\omega},\theta)}\right|^2 d\omega$$

Tells us what model we converge to with open loop identification.

BIAS IN OPEN LOOP (cont'd)

$\hat{\theta}_N \longrightarrow \theta^*$

$$\theta^* = \arg\min_{\theta} \int \{|P_0 - P(\theta)|^2 \phi_u + \phi_v\} \frac{|D|^2}{|H(\theta)|^2} d\omega$$

Design variables :

- **model set** $\{P(z,\theta), H(z,\theta)\}$
- **input spectrum**
- **data filter** $D(z)$

EFFECT OF INPUT SPECTRUM ON IDENTIFIED MODEL

Example

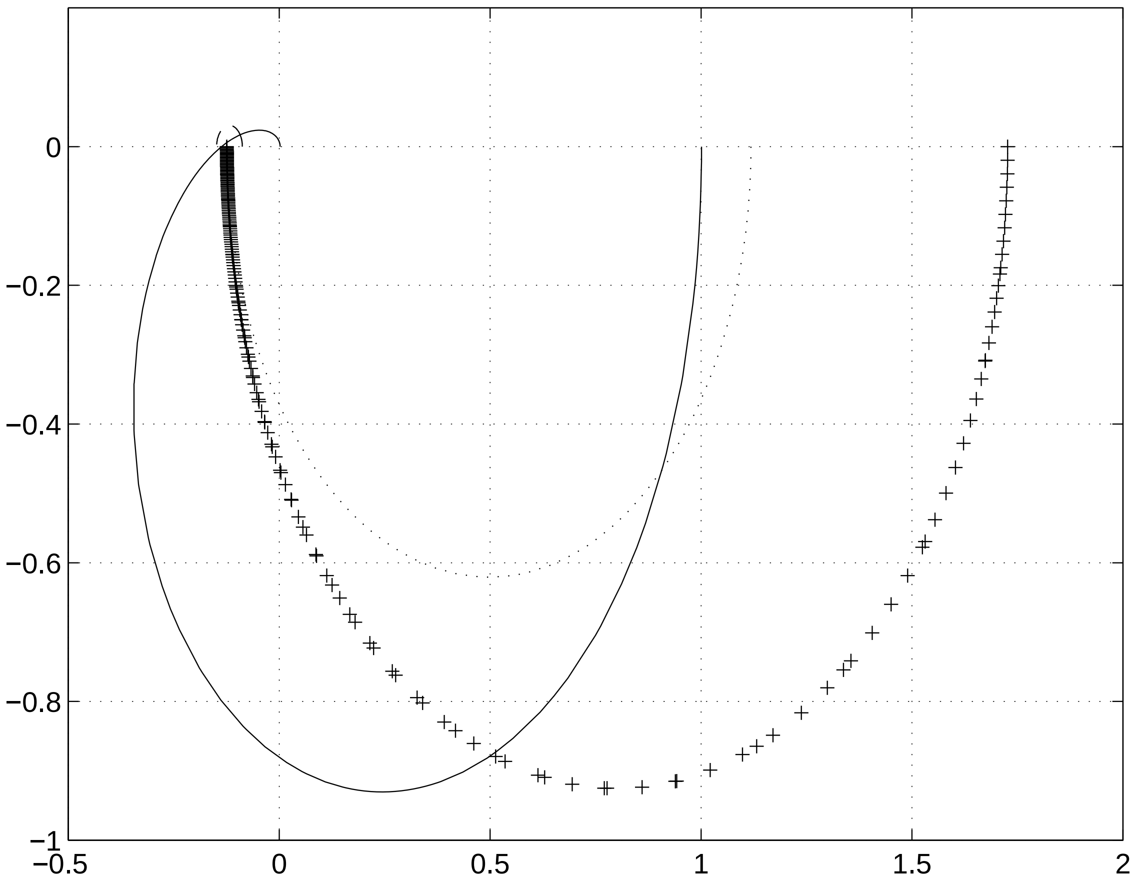
$$\begin{aligned} P_0(z) &= \frac{0.0364z^{-1}(1+1.2z^{-1})}{1-1.6z^{-1}+0.68z^{-2}} \text{ : System} \\ P(z,\theta) &= \frac{bz^{-1}}{1+az^{-1}} \quad \theta = \begin{pmatrix} a \\ b \end{pmatrix} \text{ : Model} \end{aligned}$$

Identification with no noise and with

$$u(t) = \cos(\omega t)$$

$$\begin{aligned} \omega_1 &= 0.1 \text{ rad/sec } \Rightarrow P(z,\hat{\theta}_1) \\ \omega_2 &= 0.2 \text{ rad/sec } \Rightarrow P(z,\hat{\theta}_2) \\ \omega_3 &= 1 \text{ rad/sec } \Rightarrow P(z,\hat{\theta}_3) \end{aligned}$$

Nyquist plot of the true system and 3 estimated models



VARIANCE IN OPEN LOOP

Approximate expression for high order models :

$$E|P(e^{j\omega},\hat{\theta}_N) - P(e^{j\omega},\theta^*)|^2 \simeq \frac{n}{N}\,\frac{\phi_v(\omega)}{\phi_u(\omega)}$$

$$E|H(e^{j\omega},\hat{\theta}_N) - H(e^{j\omega},\theta^*)|^2 \simeq \frac{n}{N}\,\frac{\phi_v(\omega)}{\lambda}$$

Design variables :

- **model order :** n
- **number of data :** N
- **input spectrum :** $\phi_u(\omega)$

IDENTIFICATION IN CLOSED LOOP

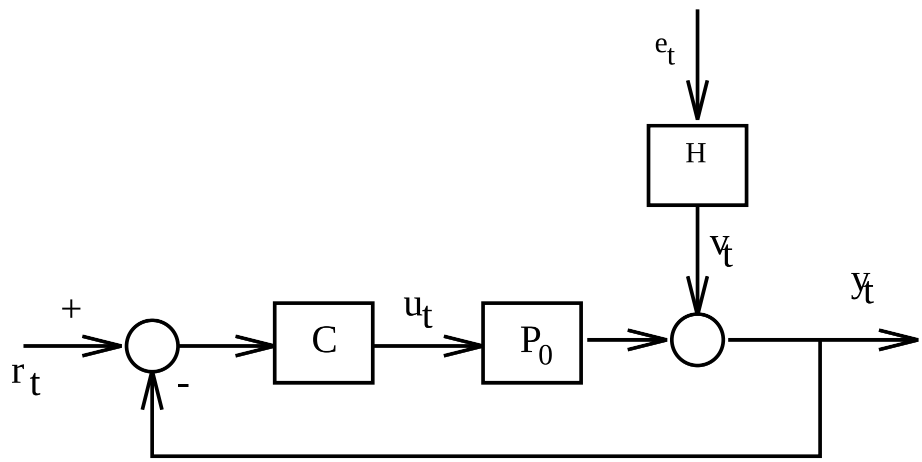
Motivation

- Often the data have been collected on processes that were operating in closed loop

- When the system is unstable, it is often necessary to first stabilize it

- Identification for control typically leads to closed loop identification

IDENTIFICATION IN CLOSED LOOP



Consider different situations :

1. Only $\{(u_t, y_t), t = 1, \ldots, N\}$ is measured

2. $\{(u_t, y_t, r_t), t = 1, \ldots, N\}$ is measured

3. Knowledge of $C(q)$ is used

First we focus on case 1.

IDENTIFIABILITY IN CLOSED LOOP

i.e. $\exists\theta_0 : P(z,\theta_0) = P_0(z), H(z,\theta_0) = H_0(z)$

When can feedback cause a problem ?

<u>Example</u> :

$$M(\theta) : y_t + ay_{t-1} = bu_{t-1} + q_t$$

<u>Controller</u> : $u_t = cy_t$

Predictor in closed loop :

$$\hat{y}_t = (bc - a)y_{t-1}$$

$\Rightarrow$ All models $(\hat{a}, \hat{b})$ such that

$$\hat{a} = a + \gamma c \quad \hat{b} = b + \gamma$$

give the same predictor, for any γ

Note : Knowing c does not help !

Classical results on identifiability using (y_t, u_t) data only.

A. $\mathcal{S} \in \mathcal{M}$

Then : $\hat{\theta}_N \to \theta_0$ provided

- r_t is sufficiently rich, or
- controller switches, or
- controller is of sufficiently high order, or
- controller is nonlinear

The case of external excitation : r_t is PE

$$
\begin{aligned}
\mathcal{S} : y_t &= P_0 u_t + H_0 e_t \\
\mathcal{M} : \hat{y}_t &= \hat{H}^{-1}\hat{P}u_t + (1 - \hat{H}^{-1})y_t \\
\mathcal{R} : u_t &= C(y_t - r_t) \\
\varepsilon_t &= y_t - \hat{y}_t
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow \varepsilon_t &= \frac{(P_0 - \hat{P})C}{\hat{H}_0(1 + P_0 C)} r_t + \underbrace{\frac{H_0(1 + \hat{P}C)}{\hat{H}(1 + P_0 C)}}_{\text{monic}} e_t \\
&= T_{\varepsilon r}(\theta) r_t + T_{\varepsilon e}(\theta) e_t \\
&= e_t + \frac{[P_0 - P(\theta)]C}{H(\theta)[1 + P_0 C]} r_t \\
&\quad + \left\{ \frac{H_0[1 + P(\theta)C]}{H(\theta)[1 + P_0 C]} - 1 \right\} e_t
\end{aligned}
$$

Denote

$$\bar{V}(\theta) = E[\varepsilon_t^2]$$

Then

$$\hat{\theta}_N \overset{N\to\infty}{\longrightarrow} \theta^* = \arg\min_{\theta} \bar{V}(\theta)$$

Therefore

$$\bar{V}(\theta) \triangleq E[\epsilon_t^2] \geq E[e_t^2]$$

If r_t is persistently exciting of sufficiently high order, the minimum is achieved for :

- $T_{\varepsilon r}(e^{j\omega}, \theta^*) = 0$
- $T_{\varepsilon e}(e^{j\omega}, \theta^*) = 1$

$\forall \omega$

$$T_{\epsilon r}(e^{j\omega}, \theta^*) = 0 \Rightarrow P(e^{j\omega}, \theta^*) = P_0(e^{j\omega})$$

This, together with

$$T_{\epsilon e}(e^{j\omega}, \theta^*) = 1$$

implies $\Rightarrow H(e^{j\omega}, \theta^*) = H_0(e^{j\omega})$

<u>Conclusion</u>

If $\mathcal{S} \in \mathcal{M}$ and r_t is PE of sufficient order, then

$$P(z, \hat{\theta}_N) \longrightarrow P_0(z)$$
$$H(z, \hat{\theta}_N) \longrightarrow H_0(z)$$

B. $\mathcal{S} \notin \mathcal{M}$ **but** $P_0 \in \mathcal{G}$

$$\begin{aligned} \text{i.e. } \exists\theta_0 &: P(z,\theta_0) = P_0(z), \\ \text{but } \nexists\theta &: P(z,\theta) = P_0(z), H(z,\theta) = H_0(z) \end{aligned}$$

Then prediction error identification using (y_t, u_t) data does not converge to the correct $P_0(z)$. This can be observed by characterizing the minimum of the asymptotic criterion (bias formula).

Conclusion on identifiability using (y_t, u_t) data only.

A. $\mathcal{S} \in \mathcal{M}$

$$\text{i.e. } \exists \theta_0 \text{ s.t. } \begin{aligned} P(z, \theta_0) &= P_0(z) \\ H(z, \theta_0) &= H_0(z) \end{aligned}$$

Then : $\hat{\theta}_N \to \theta_0$ provided

- r_t is sufficiently rich, or
- controller switches, or
- controller is of sufficiently high order, or
- controller is nonlinear

B. $\mathcal{S} \notin \mathcal{M}$ but $P_0 \in \mathcal{G}$

$$\begin{aligned} \text{i.e. } \exists \theta_0 \ &: \ P(z, \theta_0) = P_0(z), \\ \text{but } \nexists \theta \ &: \ P(z, \theta) = P_0(z), H(z, \theta) = H_0(z) \end{aligned}$$

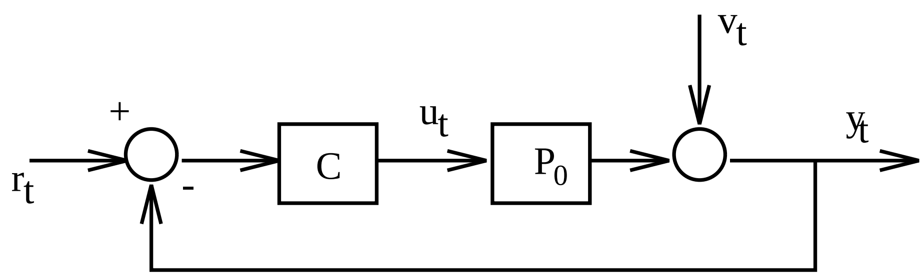
Then : $\hat{\theta}_N \not\to \theta_0$

Bias is governed by the BIAS FORMULA

BIAS IN CLOSED LOOP IDENTIFICATION

$\mathcal{S} : y_t = P_0(z)u_t + v_t$

Controller : $u_t = C(z)(r_t - y_t)$



$$\mathcal{M} : y_t(\theta) = P(z,\theta)u_t + H(z,\theta)q_t$$

$$\Rightarrow \varepsilon_t^f = \frac{D}{\hat{H}(1 + P_0 C)}[(P_0 - \hat{P})Cr_t + (1 + \hat{P}C)v_t]$$

CONVERGENCE POINTS OF $\hat{\theta}_N$

$$\hat{\theta}_N \longrightarrow \theta^* = \arg\min_{\theta} \bar{V}(\theta)$$

$$\bar{V}(\theta) = E[\epsilon_t^2(\theta)]$$

By Parseval :

$$\bar{V}(\theta) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left\{\left|\frac{P_0 - P(\theta)}{1+P_0C}\right|^2 |C|^2\phi_r + \left|\frac{1+P(\theta)C}{1+P_0C}\right|^2 \phi_v\right\}\frac{|D|^2}{|H(\theta)|^2}d\omega$$

This is the BIAS FORMULA for direct closed loop identification using $\{u_t, y_t\}$ data only.

Can show that

$$\arg\min \bar{V}(\theta) = \arg\min \bar{V}^*(\theta)$$

$$\bar{V}^*(\theta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left\{ \left| \frac{P_0 - P(\theta)}{1 + PC} \right|^2 |C|^2 \phi_r \right.$$
$$\left. + \left| \frac{H(1 + P(\theta)C) - H(\theta)(1 + P_0 C)}{1 + P_0 C} \right|^2 \sigma^2 \right\}$$
$$\times \frac{|D|^2}{|H(\theta)|^2} d\omega$$

If $P_0 \in \mathcal{G}$ but $S \notin \mathcal{M}$, the minimum is generically not reached for $\theta = \theta_0$.

$$\begin{aligned}\bar{V}(\theta) &= \int_{-\pi}^{\pi} \Big\{ |P - P(\theta)|^2 |C|^2 \phi_r \\ &\quad + |1 + P(\theta)C|^2 \phi_v \Big\} \\ &\quad\quad \times \frac{|D|^2}{|H(\theta)|^2 |1 + PC|^2} d\omega\end{aligned}$$

Design variables :

- **model set** $\{P(z,\theta), H(z,\theta)\}$
- **reference spectrum** $\phi_r(\omega)$
- **controller** $C(z)$
- **data filter** $D(z)$

Special cases

Assume fixed noise model $\hat{H}(z)$ is used

1. Noise-free identification : $\phi_v = 0$

$$\Rightarrow \theta^* = \arg\min_{\theta} \int_{-\pi}^{\pi} \frac{|P_0 - \hat{P}(\theta)|^2}{|1 + CP|^2} |C^2| \phi_r \frac{|D|^2}{|\hat{H}|^2} d\omega$$

$\hat{P}$ close to P where $\dfrac{|C|^2 \phi_r |D|^2}{|1 + CP_0|^2 |\hat{H}|^2}$ is large.

2. Identification with no excitation : $\phi_r = 0$

$$\Rightarrow \theta^* = \arg\min_{\theta} \int_{-\pi}^{\pi} \left| \frac{1 + C\hat{P}(\theta)}{1 + CP_0} \right|^2 \phi_v \frac{|D|^2}{|\hat{H}|^2} d\omega$$

$\hat{P}$ close to $-\frac{1}{C}$ where $\frac{\phi_v |D|^2}{|1 + CP_0|^2 |\hat{H}|^2}$ is large.

3. Noise and excitation

$$\theta^* = \arg\min \int \{ |P_0 - \hat{P}|^2 |C|^2 \phi_r + |1 + C\hat{P}|^2 \phi_v \} \frac{|D|^2}{|\hat{H}|^2 |1 + CP_0|^2} d\omega$$

Fit occurs where $\frac{|D|^2}{|\hat{H}|^2 |1 + CP_0|^2}$ is large.

Cross-over frequency of closed loop system !

VARIANCE ERROR IN CLOSED LOOP IDENTIFICATION

Assumption : $\mathcal{S} \in \mathcal{M}$

$$E \begin{pmatrix} P(e^{j\omega}, \hat{\theta}_N) - P_0(e^{j\omega}) \\ H(e^{j\omega}, \hat{\theta}_N) - H_0(e^{j\omega}) \end{pmatrix} \begin{pmatrix} \\ \end{pmatrix}^*$$

$$\cong \frac{n}{N} \phi_v(\omega) \begin{bmatrix} \phi_u(\omega) & \phi_{ue}(-\omega) \\ \phi_{eu}(-\omega) & \sigma^2 \end{bmatrix}^{-1}$$

Design variables :

- model order n
- number of data N
- reference spectrum $\phi_r(\omega)$
- controller $C(z)$

VARIANCE OF $P(z,\hat{\theta}_N)$: OPEN AND CLOSED LOOP

Open loop :

$$Var\{\hat{P}_N\} \cong \frac{n}{N}\frac{\phi_v}{\phi_u}$$

Closed loop :

$$\begin{aligned}
u_t &= \frac{C}{1+P_0C}r_t - \frac{CH}{1+P_0C}e_t \\
\phi_u &= \frac{|C|^2}{|1+P_0C|^2}\phi_r + \frac{|C|^2|H|^2}{|1+P_0C|^2}\sigma^2 \\
&\triangleq \phi_u^r + \phi_u^e \\
Var\{\hat{P}_N\} &\cong \frac{n}{N}\phi_v\frac{\sigma^2}{\sigma^2\phi_u - |\phi_{ue}|^2} \\
&= \frac{n}{N}\frac{\phi_v}{\phi_u^r}
\end{aligned}$$

Compare with open loop.

Only the reference contribution in the u signal contributes to knowledge of $\hat{P}$.

VARIANCE OF $H(z, \hat{\theta}_N)$: OPEN AND CLOSED LOOP

Open loop :

$$Var\{\hat{H}_N\} \cong \frac{n}{N}\frac{\phi_v}{\sigma^2}$$

Closed loop :

$$\begin{aligned} Var\{\hat{H}_N\} &\cong \frac{n}{N}\phi_v\frac{\phi_u}{\sigma^2\phi_u - |\phi_{ue}|^2} \\ &= \frac{n}{N}\frac{\phi_v}{\sigma^2}\frac{\phi_u}{\phi_u^r} \\ &= \frac{n}{N}\frac{\phi_v}{\sigma^2}\left(1 + \frac{\phi_u^e}{\phi_u^r}\right) \end{aligned}$$

Compare with open loop.

Conclusions on variance errors

	$O.L.$		$C.L.$
$Var\{\hat{P}_N\}$	$\frac{n}{N}\frac{\phi_v}{\phi_u}$	$<$	$\frac{n}{N}\frac{\phi_v}{\phi_u^r}$
$Var\{\hat{H}_N\}$	$\frac{n}{N}\frac{\phi_v}{\sigma^2}$	$<$	$\frac{n}{N}\frac{\phi_v}{\sigma^2}\left(1+\frac{\phi_u^e}{\phi_u^r}\right)$

Yet, if $\hat{P}_N$ and $\hat{H}_N$ are used <u>for control design</u>, closed loop identification is typically optimal for the minimization of the controller variance, i.e. $Var\{\hat{C}_N\}$ with $\hat{C}_N = C(\hat{P}_N, \hat{H}_N)$.

ALTERNATIVE CLOSED LOOP IDENTIFICATION METHODS

1. The joint input-output identification method

Ref. : Anderson and Gevers (1982)

Assumptions

- $u_t = r_t - Cy_t$
- r_t is PE but not necessarily measurable
- $r_t = W(z)\eta_t, \qquad \eta_t$: white noise

Consider $\begin{pmatrix} u_t \\ y_t \end{pmatrix}$ as a multivariable process driven by white noise. Closed loop system :

$$\begin{pmatrix} u_t \\ y_t \end{pmatrix} = \underbrace{\begin{bmatrix} \frac{W}{1+P_0C} & -\frac{H_0C}{1+P_0C} \\ \\ \frac{P_0CW}{1+P_0C} & \frac{H_0}{1+P_0C} \end{bmatrix}}_{L(z)} \begin{pmatrix} \eta_t \\ e_t \end{pmatrix}$$

Step 1 :

Estimate an innovations models for the joint process $\begin{pmatrix} u_t \\ y_t \end{pmatrix}$ using standard open loop prediction error method
$\Rightarrow \hat{L}(z)$ and $Cov \begin{pmatrix} \eta \\ e \end{pmatrix}$.

Step 2 :

Estimate P, H, C, W and σ^2 from $\hat{L}(z)$ and $Cov \begin{pmatrix} \eta \\ e \end{pmatrix}$.

Comments

1. Uses standard (open loop) prediction error methods in step 1. Step 2 is algebra.

2. Uses $\begin{pmatrix} u_t \\ y_t \end{pmatrix}$ data only. Estimates not only (P, H), but also C and W.

3. Converges under mild conditions to the true P_0, H_0, C, W if $\mathcal{S} \in \mathcal{M}$.

4. Not clear what approximation we get for step 2 from an approximation in step 1.

2. The instrumental variable (IV) method

True system : $y_t = P_0(z)u_t + H_0(z)e_t$

Model :

$$\begin{aligned} A(z^{-1})y_t &= B(z^{-1})u_t + w_t \\ \Longleftrightarrow y_t &= \varphi_t^T\theta_0 + w_t \\ & \quad \text{with } w_t \text{ coloured noise.} \end{aligned}$$

Regressor :

$$\varphi_t = (y_{t-1}, \ldots, y_{t-n_a}, u_{t-1}, \ldots, u_{t-n_b})$$

Construct IV estimator :

$$\begin{aligned} \hat{\theta}_N^{IV} &= \left[\frac{1}{N}\sum_{t=1}^{N}\zeta_t\varphi_t^T\right]^{-1}\left[\frac{1}{N}\sum_{t=1}^{N}\zeta_t y_t\right] \\ &= \theta_0 + \left[\frac{1}{N}\sum_{t=1}^{N}\zeta_t\varphi_t^T\right]^{-1}\left[\frac{1}{N}\sum_{t=1}^{N}\zeta_t w_t\right] \end{aligned}$$

IDENTIFIABILITY WITH IV METHOD

Assumption : $\boxed{\mathcal{S} \notin \mathcal{M} \text{ but } P_0 \in \mathcal{G}}$

$$\hat{\theta}_N \overset{N\to\infty}{\longrightarrow} \theta_0$$

if

- $\frac{1}{N}\sum_{t=1}^{N} \zeta_t \varphi_t \longrightarrow R :$ **nonsingular**

- $\frac{1}{N}\sum_{t=1}^{N} \zeta_t w_t \longrightarrow 0$

Open loop : $\zeta_t \triangleq (u_{t-1}, \ldots, u_{t-n_a-n_b})$
Closed loop : $\zeta_t \triangleq (r_{t-1}, \ldots, r_{t-n_a-n_b})$

These ζ_t satisfy the above conditions.

Summary

- We have studied open loop and closed loop prediction error methods.

- We have given identifiability conditions when $\mathcal{S} \in \mathcal{M}$, and when $\mathcal{S} \notin \mathcal{M}$ but $P \in \mathcal{G}$.

- We have focused on the design variables that affect bias and variance.

- We have seen that, in closed loop identification using $\{u_t, y_t\}$ data, the bias of $\hat{P}$ cannot be controlled when the noise model is incorrect.

- Alternative methods have been given.

Overhead sheets for presentation at
Benelux Meeting 1996, Mierlo, 6-8 March 1996
Minicourse *Closed-loop Identification*
Part - 2

Paul Van den Hof
Michel Gevers

Alternative methods for closed-loop identification using controller or reference signal information

Paul Van den Hof
Michel Gevers

CONTENTS

- What are we aiming for?
- What can be done with "classical" methods (indirect ID)?
- Solutions through reparametrizations:

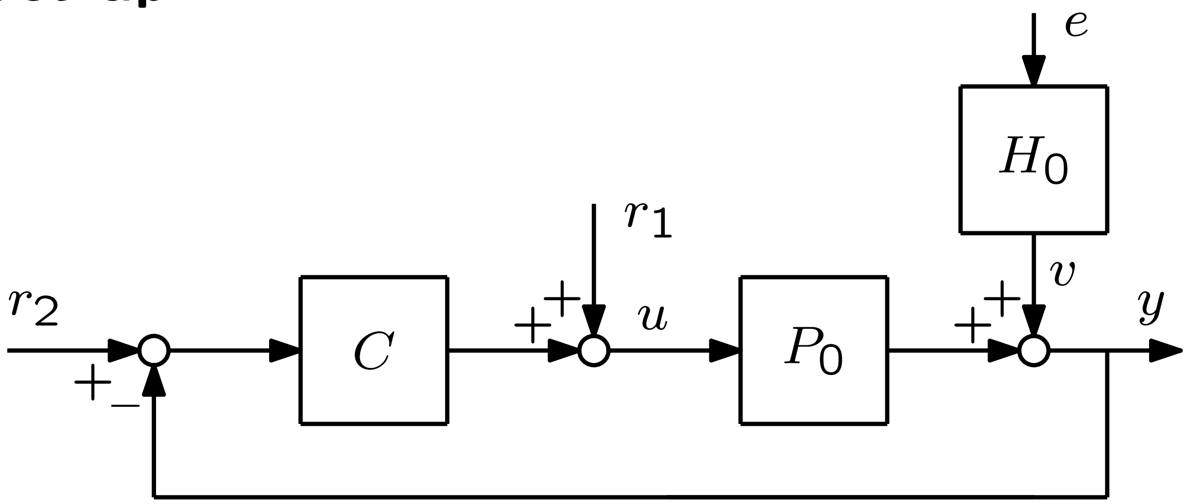
  (1) Two-stage method

  (2) Coprime factor identification

  (3) Identification in dual-Youla parametrization

- A guidebook for choosing your favourite method

Set-up



- Normal operation of many plants.
- Unstable processes have to be stabilized.

Available from measurements / a priori:

- u, y; r_1 and/or r_2
- possibly C

Attention will be limited to prediction error type identification methods

Assumption that e and (r_1, r_2) uncorrelated.

Stability of the closed-loop

Definition

CLosed-loop system is stable if the mapping

$$\begin{bmatrix} r_2 \\ r_1 \end{bmatrix} \to \begin{bmatrix} y \\ u \end{bmatrix}$$

is stable.

This mapping is characterized by the transfer function

$$T(P_0, C) = \begin{bmatrix} \frac{P_0 C}{1+CP_0} & \frac{P_0}{1+CP_0} \\ \frac{C}{1+CP_0} & \frac{1}{1+CP_0} \end{bmatrix}$$

Problems / Required Properties of Closed-loop Identification Methods

- Consistency of estimate $(\hat{P}, \hat{H})$ provided that $\mathcal{S} \in \mathcal{M}$
- Consistency of estimate $\hat{P}$, provided that $P_0 \in \mathcal{G}$
- Tunable expression for asymptotic bias distribution of $\hat{P}$:

$$P(\theta^*) = \arg\min_{\theta}$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |P_0(e^{i\omega}) - P(e^{i\omega},\theta)|^2 \cdot |W(e^{i\omega})|^2 d\omega$$

  for some known and tunable W

- Ability to handle model class $\mathcal{G}$ of fixed order models
  (control over model order)
- Applicability to situation of unstable plant P_0

A classical method - indirect identification

- Identify a model of the closed-loop:

$$y(t) = G(q,\theta)r_1(t) + K(q,\theta)\varepsilon(t)$$

  where

$$G_0(q) = \frac{P_0}{1+CP_0}; \qquad K_0(q) = \frac{H_0}{1+CP_0}$$

- Reconstruct open loop model $(\hat{P}, \hat{H})$ by solving

$$\hat{G} = \frac{\hat{P}}{1+C\hat{P}}$$

$$\hat{K} = \frac{\hat{H}}{1+C\hat{P}}$$

  Use knowledge of C to construct

$$\hat{P} = \frac{\hat{G}}{1-C\hat{G}}$$

$$\hat{H} = \frac{\hat{K}}{1-C\hat{G}}$$

Properties

- Open-loop type of identification (input and noise uncorrelated)

- Consistency results hold for both $(\hat{P}, \hat{H})$ and $\hat{P}$

- If G and K independently parametrized, then asymptotic model $P(\theta^*)$ is obtained as minimizing argument of

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\frac{P_0}{1+CP_0}-\frac{P(\theta)}{1+CP(\theta)}\right|^2 |L|^2 \Phi_r d\omega$$

- Model order is not under control, because

$$P(\theta) = \frac{G(\theta)}{1 - CG(\theta)}$$

  Generic model order: $n_P = n_G + n_C$

- Unstable plant is no problem

An additional property

Let $\widehat{G}$ be a stable (approximative) model of the closed-loop transfer G_0, and let

$$\hat{P} = \frac{\widehat{G}}{1 - C\widehat{G}}.$$

If additionally C or $\widehat{G}^{-1}$ is stable then

$$\hat{P} \text{ is stabilized by } C$$

Model set is restricted to models stabilized by C.

Similar results for other closed-loop transfer functions

Major shortcoming:
lack of control over model order

Alternative methods based on reparametrization

Remain within "classical" PE framework with LS criteria

Methods

(1) Two-stage method

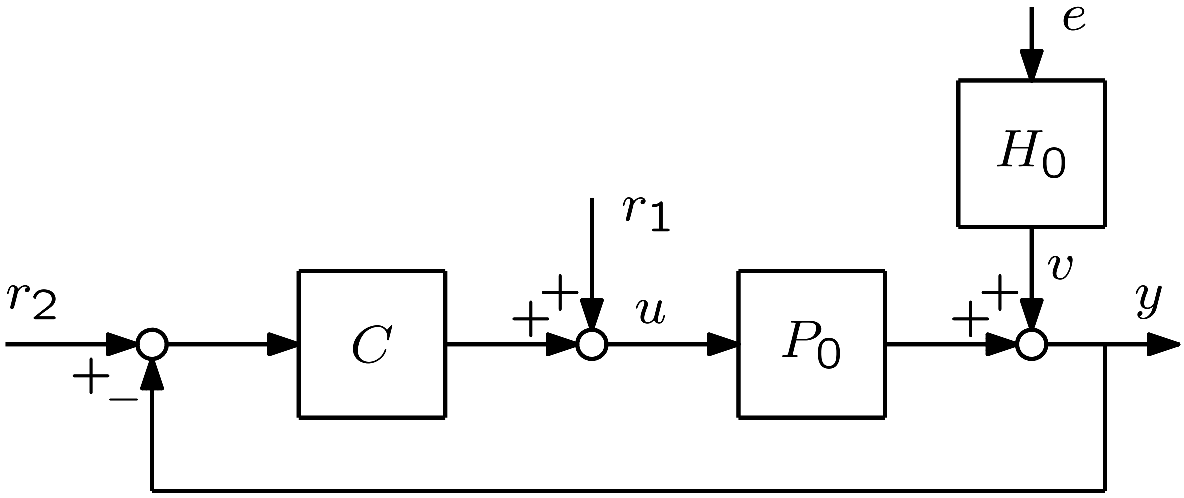
(2) Coprime factor identification

(3) Identification of dual-Youla parameter

Required experimental situation:

- r persistently exciting - (1,2,3)
- Either r measurable or C known - (1,2)
- C known - (3)

1. TWO-STAGE METHOD



Without loss of generality: $r_1 \equiv r$; $\quad r_2 = 0$

Step 1

- Identify transfer $r \to u$

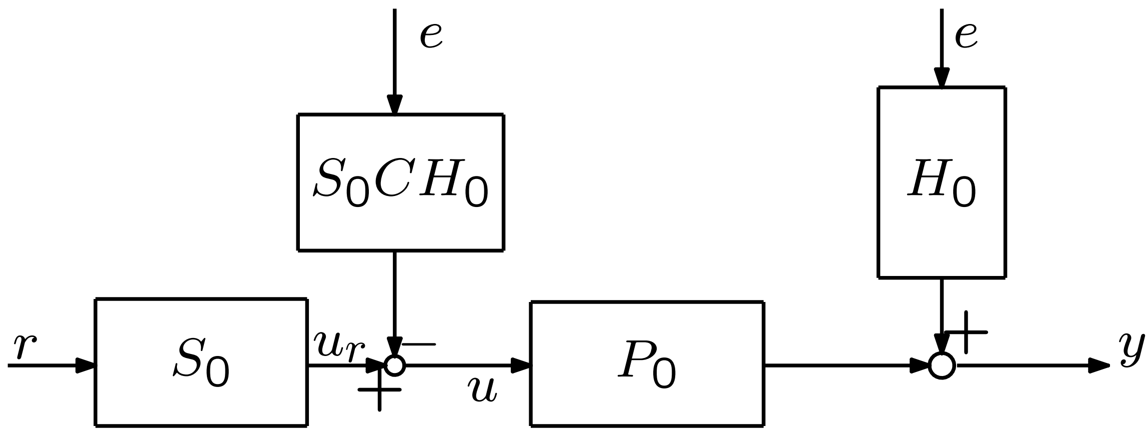
$$u(t) = S_0(q)r(t) - CS_0H_0(q)e(t)$$

  (open loop problem) $\to \widehat{S}$

- Simulate noise-free input $\hat{u}_r(t) = \widehat{S}(q)r(t)$

Step 2

- Identify transfer $\hat{u}_r \to y$
  (open loop problem if $\widehat{S} = S_0$) $\quad \to \widehat{P}$



Step 1 (fixed noise model)

$$\varepsilon_u(t,\beta) = \frac{1}{W(q)}[u(t) - S(q,\beta)r(t)]$$

$$\beta^* = \arg\min \int_{\pi}^{\pi} |S_0 - S(\beta)|^2 \frac{\Phi_r}{|W|^2} d\omega$$

Model set: $S(q,\beta) \in \mathcal{T}$.

Step 2 (fixed noise model)

$$\varepsilon_y(t,\theta) = \frac{1}{K(q)}[y(t) - P(q,\theta)\hat{u}_r(t)]$$

Model set: $P(q,\theta) \in \mathcal{M}$

Consistency / Bias properties

- If $S_0 \in \mathcal{T}$, then $S(\beta^*) = S_0$ and

$$\theta^* = \arg\min \int_{-\pi}^{\pi} |P_0 - P(\theta)|^2 \frac{|S_0|^2 \Phi_r}{|K|^2} d\omega$$

- If $S_0 \in \mathcal{T}$ and $P_0 \in \mathcal{M}$ then

$$P(\theta^*) = P_0$$

(irrespective of modelling noise)

- If $S_0 \notin \mathcal{T}$ then

$$\theta^* = \arg\min \int_{-\pi}^{\pi} |P_0 S_0 - P(\theta) S(\beta^*)|^2 \frac{\Phi_r}{|K|^2} d\omega$$

Note:

$$P_0 S_0 - P(\theta) S(\beta^*) =$$

$$= [P_0 - P(\theta)] S_0 + P(\theta)[S_0 - S(\beta^*)]$$

- **Consistent modelling of P_0 provided first step is accurate enough**
- **Noise-independent bias expression**
- **Model order is not "important" in first step**
- **Control over model order in 2nd step**
- **Method requires** $\{r, u, y\}$ (not C)
- **Easily applicable (standard tools)**
- **Restricted to stable plant models**

Succesfully applied to

- Simulation example
  [Van den Hof and Schrama 1993]

- Compact Disc servo mechanism
  [De Callafon et al. 1993]

- Sugar cane crushing plant
  [Partanen and Bitmead 1993]

- Crystallization plant
  [Eek et al. 1996]

2. COPRIME FACTOR IDENTIFICATION

Data generating system:

$$\begin{aligned} y(t) &= P_0 S_0 r(t) + S_0 H_0 e(t) \\ u(t) &= S_0 r(t) - C S_0 H_0 e(t) \end{aligned}$$

Write: $N_0 = P_0 S_0 \qquad D_0 = S_0$

then $P_0 = \dfrac{N_0}{D_0}$.

Factorization is coprime if factors are stable and there are no cancelling unstable zeros.

N_0, D_0 can be estimated from data $\{r, y, u\}$. (open loop).

$$\begin{aligned} \varepsilon_y(t, \theta) &= y(t) - N(q, \theta) r(t) \\ \varepsilon_u(t, \theta) &= u(t) - D(q, \theta) r(t). \end{aligned}$$

$\widehat{P} = \dfrac{\widehat{N}}{\widehat{D}}$.

Observations

- Closed-loop experiments provide access to coprime factors of process ($r \to u$, $r \to y$).
- Unstable plant can be modelled by stable factors
- Identification of these factors is an "open-loop" problem
- Availability of r can be replaced by knowledge of C:

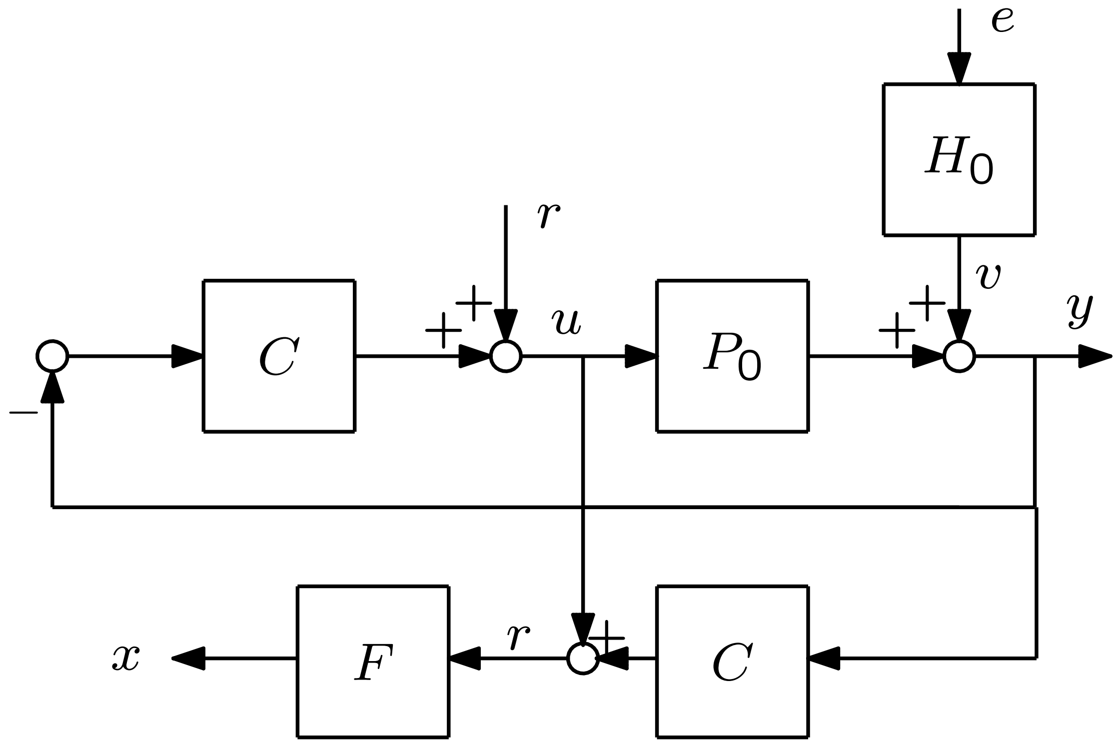
$$r(t) = u(t) + C(q)y(t)$$

- Constructing plant model

$$\hat{P} = \frac{\hat{N}}{\hat{D}}$$

  shows problem of keeping control over model order.

Generalization can provide a solution.



Construct:

$$x(t) := F(q)r(t) = F(q)[u(t) + C(q)y(t)]$$

Then:

$$\begin{aligned} y(t) &= P_0 S_0 F^{-1} x(t) + S_0 H_0 e(t) \\ u(t) &= S_0 F^{-1} x(t) - C S_0 H_0 e(t) \end{aligned}$$

There is access to factorization:

$$\begin{aligned} N_{0,F} &= P_0 S_0 F^{-1} \\ D_{0,F} &= S_0 F^{-1} \end{aligned}$$

Filter F can be used to

- Shape accessible factorizations of P_0

- Obtain minimal order factorizations (McMillan degree of factors equal to McMillan degree of quotient)

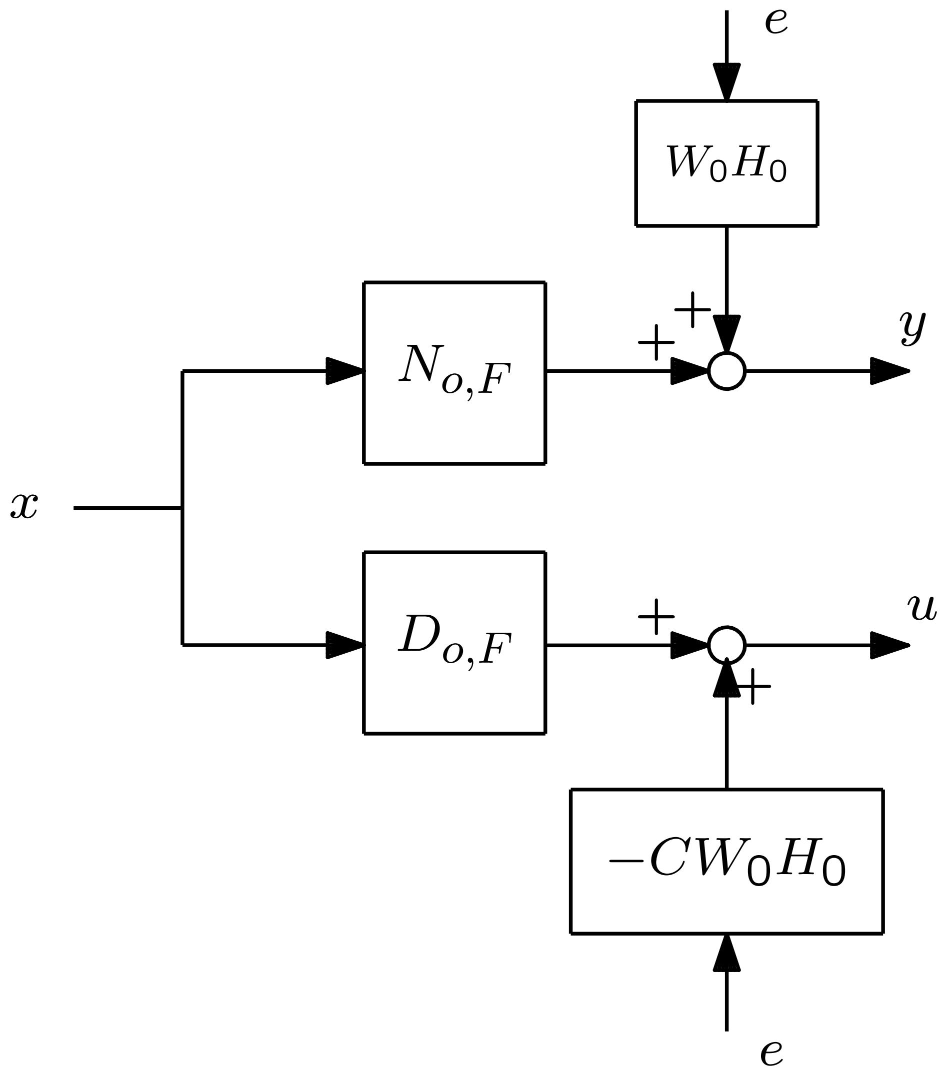
  Using accurate plant model, factorization

$$\begin{aligned} N_{0,F} &= P_0 S_0 F^{-1} \\ D_{0,F} &= S_0 F^{-1} \end{aligned}$$

  can e.g. be made factorization of polynomials

- Shape bias distribution

Block diagram for identification



"Open-loop" identification of coprime factors.

OE model structure

$$\begin{aligned} \varepsilon_y(t,\theta) &= L_y(q)[y(t) - N(q,\theta)]x(t) \\ \varepsilon_u(t,\theta) &= L_u(q)[u(t) - D(q,\theta)]x(t). \end{aligned}$$

$$\theta^* = \arg\min \bar{E}(\varepsilon_y^2(t) + \varepsilon_u^2(t))$$

Residual spectrum

$$\theta^* = \arg\min_\theta \frac{1}{2\pi}\int_{-\pi}^{\pi} \Phi_\varepsilon(\omega,\theta)d\omega$$

with

$\Phi_\varepsilon =$

$$\left\{|P_0 S_0 - N(\theta)F|^2|L_y|^2 + |S_0 - D(\theta)F|^2|L_u|^2\right\}\Phi_r$$

Characterization of F

F yields stable mappings

$$x \rightarrow \begin{pmatrix} y \\ u \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} y \\ u \end{pmatrix} \rightarrow x$$

if and only if F can be written as

$$F = [D_x + CN_x]^{-1}$$

with (N_x, D_x) a coprime factorization of any auxiliary model

$$P_x = \frac{N_x}{D_x}$$

that is stabilized by C.

Now:

$$\begin{aligned} N_{0,F} &= P_0[I + CP_0]^{-1}[I + CP_x]D_x \\ D_{0,F} &= [I + CP_0]^{-1}[I + CP_x]D_x \end{aligned}$$

Choice of F can be replaced by choice of P_x and its factorization $\frac{N_x}{D_x}$.

- **Consistent estimate of** (P_0, H_0) **and** P_0 **for sufficiently large model sets**

- **Tunable bias expression**

- **Method based on** $\{y, u\}$ **and** r **/or** C
(For minimal order factors: C required; see later)

- **No problem with unstable plants/ controllers**

- **Model of** P_0 **obtained by identifying** 2 **transfer functions**

- **There is only** 1 **source of uncertainty (noise)**

In Dual-Youla identification method this redundancy is removed

Successful applications

- Simulation examples
  [Schrama, 1992; Schrama and Van den Hof, 1992]

- X-Y Positioning table
  [Schrama 1992]

- Compact Disc servo mechanism
  [Van den Hof et al. 1995]

3. IDENTIFICATION IN DUAL-YOULA FORM

Youla parametrization

Let $P_0 = \frac{N_0}{D_0}$, and $C_x = \frac{N_c}{D_c}$, be any controller that stabilizes P_0.

Then any LTI controller C stabilizes P_0 if and only if it can be written in a *coprime factorization*

$$C = \frac{N_c + D_0 R_c}{D_c - N_0 R_c}$$

with R_c stable.

Dual-Youla parametrization

Let $C = \dfrac{N_c}{D_c}$, and $P_x = \dfrac{N_x}{D_x}$, be any model that is stabilized by C.
Then any LTI plant P_0 is stabilized by C if and only if it can be written in a *coprime factorization*

$$P_0 = \frac{N_x + D_c R}{D_x - N_c R}$$

with R stable.

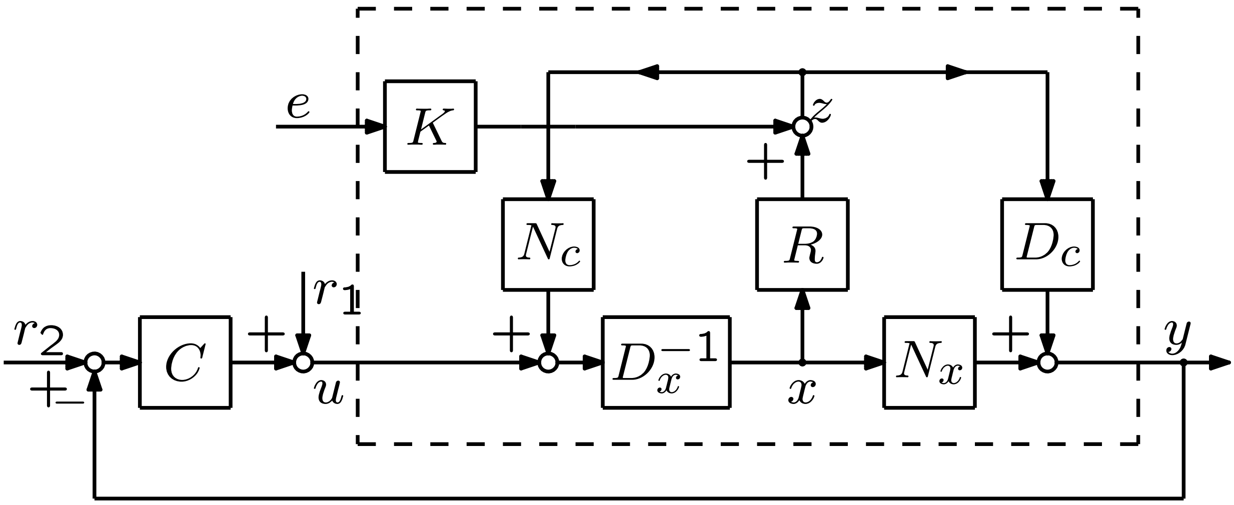
For given P_0,

$$R = R_0 = \frac{(P_0 - P_x)D_x}{D_c(I + CP_0)}$$

and:

$$\begin{aligned} N_x + D_c R_0 &= P_0[I + CP_0]^{-1}[I + CP_x]D_x \\ D_x - N_c R_0 &= [I + CP_0]^{-1}[I + CP_x]D_x \end{aligned}$$

Same factorization as before in coprime factor method



For given (P_0, H_0), $K_0 = D_c^{-1} S_0 H_0$.

Simple calculations show:

$$
\begin{aligned}
z(t) &= (D_c + P_x N_c)^{-1}[y(t) - P_x(q)u(t)] \\
x(t) &= (D_x + C N_x)^{-1}[u(t) + C(q)y(t)]
\end{aligned}
$$

leading to

$$z(t) = R_0(q)x(t) + K_0(q)e_0(t)$$

with

$$x(t) = F(q)r(t)$$

- z and x can be reconstructed from measured data

- x is not correlated to the noise disturbance

- R_0 and K_0 can be identified in an "open-loop" manner

Identification (OE structure):

$$\varepsilon(t,\theta) = L(q)[z(t) - R(q,\theta)x(t)]$$

then $\theta^* = \arg\min \bar{E}\varepsilon^2(t,\theta)$.

Resulting model:

$$P(\theta^*) = \frac{N_x + D_c R(\theta^*)}{D_x - N_c R(\theta^*)}$$

Model order of $P(\theta^*)$ is not under control!

Residual spectrum

$$\begin{aligned}\Phi_\varepsilon &= |L|^2 |R_0 - R(\theta)|^2 \Phi_x \\ &= |L|^2 \cdot \left| \frac{P_0 - P(\theta)}{D_c(I + P_0 C)(I + P(\theta)C)} \right|^2 \cdot \Phi_r\end{aligned}$$

Using the expression

$$\frac{1}{1 + P_0 C} - \frac{1}{1 + P(\theta)C} = \frac{[P_0 - P(\theta)]C}{(1 + P_0 C)(1 + P(\theta)C)}$$

this shows that

$$\Phi_\varepsilon = \left|\frac{L}{N_c}\right|^2 \cdot \left| \frac{1}{1 + P_0 C} - \frac{1}{1 + P(\theta)C} \right|^2 \Phi_r$$

weighted difference of plant and model sensitivity

Properties

- **Consistency is no problem**
- **Tunable bias distribution**
- **No control over model order**
- **No problem with unstable plants/ controllers**
- **Identified models are guaranteed to be stabilized by C**

Identification in Dual-Youla Form as generalization of indirect method

If C is stable, then $P_x = \frac{0}{1}$ is stabilized by $C = \frac{C}{1}$.

This factorization leads to

$$R_0 := \frac{(P_0 - P_x)D_x}{D_c(I + CP_0)} = \frac{P_0}{(I + CP_0)}$$

being the closed-loop transfer: $r \to y$.

Literature

- Introduction in [Hansen, 1988]
- Simulation examples
  [Lee, Anderson, Kosut, Mareels, 1992, 1995]
  [Schrama, 1992]

A guidebook for choosing your favourite method

- Most simple to apply:
  direct ID and *2-step method*

- When (considerable) bias is expected from correlation between u and e:
  choose *2-step method*

- When plant unstable:
  Indirect ID, *Cop-Fac* or *Dual-Youla*

- When limited complexity model required:
  2-step method, *Cop-Fac*

- When controller C not accurately known:
  2-step method

	direct	indirect	2-step	CF	Youla
Consistency $(\widehat{P}, \widehat{H})$	+	+	+	+	+
Consistency $\widehat{P}$	-	+	+	+	+
Tunable bias	-	+	+	+	+
Fixed model order	+	-	+	+	-
Unstable plants	□	+	-	+	+
$(\hat{P}, C)$ stable	-	□	-	-	+
C required	no	yes	no	□	yes

15-th Benelux Meeting on Systems and Control
Minicourse on Closed Loop Identification

by

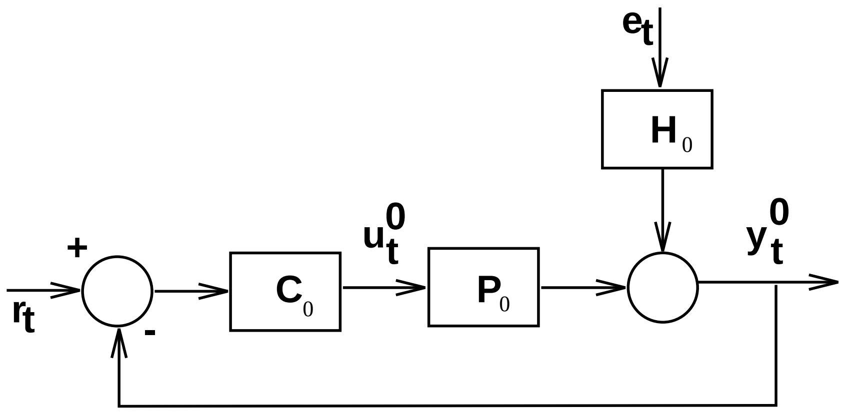
Michel Gevers and Paul Van den Hof

Part III

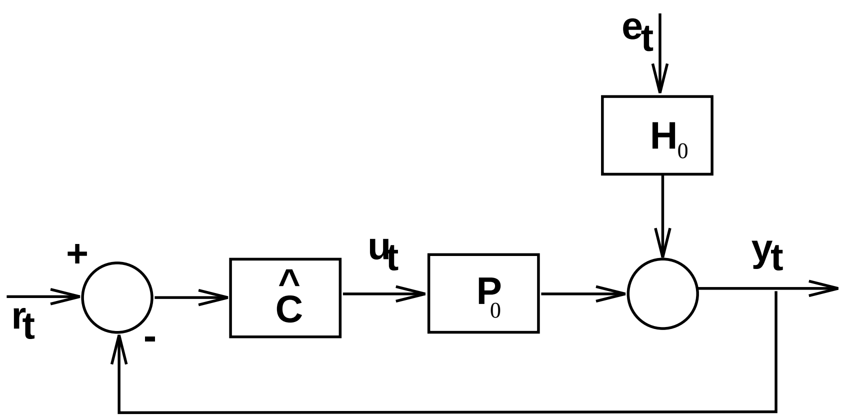
IDENTIFICATION FOR CONTROL : THE OPTIMAL DESIGN APPROACH

The "optimal design" approach

Optimal closed loop system :



Achieved closed loop system :



<u>Goal</u> : Design the identification to minimize $E[y_t^0 - y_t]^2$ given that $\hat{C} = c(P(\hat{\theta}_N), H(\hat{\theta}_N))$

Earlier result (Gevers & Ljung, 1986)

- **Showed that for a number of identification objectives, the optimal identification design was <u>closed loop identification</u>.**

- **This was the first instance in which closed loop identification was shown to be superior to open loop identification.**

- **For Minimum Variance control design, with $\mathcal{S} \in \mathcal{M}$, it was shown that the ideal identification design was to perform closed loop identification with the optimal (but unknown) MV controller in the loop during data collection.**

  $\Rightarrow$ **An apparently useless result.**

New results

(Hjalmarsson, Gevers, Debruyne, 1996)

Assumptions

The system

$$\mathcal{S} : y_t = P_0(z)u_t + H_0(z)e_t$$

The model set

$$\mathcal{M} : y_t = P(z,\theta)u_t + H(z,\theta)e_t$$

Important assumption : $\mathcal{S} \in \mathcal{M}$

$P_0(z) = P(z,\theta_0),\ H_0(z) = H(z,\theta_0)$ **for some** θ_0

If closed loop identification :

$$u_t = C_{id}(z)[r_t - y_t]$$

Prediction error identification - N **data**

$$\Rightarrow \hat{P}_N = P(e^{i\omega},\hat{\theta}_N), \qquad \hat{H}_N = H(e^{i\omega},\hat{\theta}_N)$$

The control design criterion = a mapping from plant to controller:

$$C_0 = c(P_0, H_0), \qquad \hat{C}_N = c(\hat{P}_N, \hat{H}_N)$$

Define identification performance measure

$$\begin{aligned} (P_0, H_0) &\Rightarrow C_0 \Rightarrow \{y_t^0\} \\ (\hat{P}_N, \hat{H}_N) &\Rightarrow \hat{C}_N \Rightarrow \{y_t\} \end{aligned}$$

$$J_V \triangleq E[y_t^0 - y_t]^2$$

Comments:

- Applies to one or two degree of freedom controller

- Can define other measures:

$$\begin{aligned} J_V &= E[y_t^0 - y_t]^2 + \lambda E[u_t^0 - u_t]^2 \\ J_V &= \int_{-\pi}^{\pi} E[|C(e^{i\omega}) - \hat{C}_N(e^{i\omega})|^2] W(e^{i\omega}) d\omega \end{aligned}$$

Optimal solution

Consider one-degree-of-freedom controller.

Denote $\Delta C_N \triangleq \hat{C}_N - C_0$. For small ΔC_N, one can write:

$$y_t^0 - y_t \cong \frac{P_0}{1 + P_0 C_0} \Delta C_N \; y_t^0$$

Therefore, by Parseval:

$$J_V \cong \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{|P_0|^2 \phi_{y^0}}{|1 + P_0 C_0|^2} E |\Delta C_N|^2 d\omega$$

For small ΔC_N, one can write:

$$\Delta C_N = [F_1 \;\; F_2] \begin{bmatrix} \Delta P_N \\ \Delta H_N \end{bmatrix},$$

where:

$$\Delta P_N \triangleq P_0 - \hat{P}_N, \qquad \Delta H_N \triangleq H_0 - \hat{H}_N$$

$$F_1 \triangleq \frac{\partial C}{\partial P}, \qquad F_2 \triangleq \frac{\partial C}{\partial H}$$

Use covariance formula for full order transfer function estimates:

$$E\left[\begin{pmatrix}\Delta P_N\\ \Delta H_N\end{pmatrix}\begin{pmatrix}\Delta P_N\\ \Delta H_N\end{pmatrix}^*\right]$$

$$\cong \frac{n}{N}\frac{|H_0|^2\sigma^2}{\sigma^2\phi_u-|\phi_{ue}|^2}\begin{bmatrix}\sigma^2 & -\phi_{ue}^*\\ -\phi_{eu}^* & \phi_u\end{bmatrix}$$

After some manipulations, we get (for $F_2 \neq 0$):

$$E|\Delta C_N|^2 \cong \frac{n}{N}|H_0|^2|F_2|^2\left(1+\frac{|\sigma^2\frac{F_1}{F_2}-\phi_{ue}|^2}{\sigma^2\phi_u-|\phi_{ue}|^2}\right)$$

Thus, the controller variance is minimized by the design:

$$\phi_{ue}^{opt}(e^{i\omega}) = \frac{F_1(e^{i\omega})}{F_2(e^{i\omega})}\sigma^2$$

$$\phi_{ue}^{opt}(e^{i\omega}) = \frac{F_1(e^{i\omega})}{F_2(e^{i\omega})}\sigma^2$$

What does this mean???

During closed loop identification:

$$u_t = \frac{1}{1 + P_0 C_{id}} r_t - \frac{H_0 C_{id}}{1 + P_0 C_{id}} e_t$$

Therefore:

$$\phi_{ue} = \frac{-H_0 C_{id}}{1 + P_0 C_{id}} \sigma^2$$

Conclusion

The variance $E|\Delta C_N|^2$ is minimized *at every frequency* if the identification is performed in closed loop with an operating controller:

$$C_{id}^{opt}(z) = -\frac{F_1(z)}{F_1(z)P_0(z) + F_2(z)H_0(z)}$$

This choice of course minimizes J_V.

With ideal closed loop identification:

$$E|\Delta C_N|^2_{idcl} \cong \frac{n}{N}|H_0|^2|F_2|^2$$

$$J_V^{opt} \cong \frac{n}{2\pi N}\int_{-\pi}^{\pi}\frac{|P_0|^2|H_0|^2|F_2|^2}{|1+P_0C_0|^2}\phi_{y^0}d\omega$$

Under open loop identification:

$$E|\Delta C_N|^2_{ol} \cong \frac{n}{N}|H_0|^2|F_2|^2\left(1+\sigma^2\frac{|F_1|^2}{\phi_u|F_2|^2}\right)$$

$$J_V \cong \frac{n}{2\pi N}\int_{-\pi}^{\pi}\frac{|P_0|^2|H_0|^2|F_2|^2}{|1+P_0C_0|^2}\phi_{y^0}\times\left(1+\sigma^2\frac{|F_1|^2}{\phi_u|F_2|^2}\right)d\omega$$

Comments

- Closed loop identification is optimal for control design. $E|\Delta C_N|^2_{idcl} < E|\Delta C_N|^2_{ol}$ whatever the input or reference signal power.

- The optimal design depends on the unknown system $[P_0, H_0]$ (typical) and is therefore not feasible.

- It depends on the control design criterion through the sensitivity functions F_1 and F_2.

- The controller C_{id}^{opt} may not exist or may not be stabilizing. Can define the optimal stabilizing controller for identification (see paper).

- For Model Reference and Minimum Variance control design, $C_{id}^{opt}(z) = C_0(z)$.

Compare two strategies, using N data each.

1. Open loop identification

$$(\hat{P}_N, \hat{H}_N) \Rightarrow J_V^{ol}$$

2. Iterative identification
(Assume $C_{id}^{opt}(z)$ is stabilizing.)

• First open loop identification using $(1-\alpha)N$ data ($0 < \alpha < 1$).
• Using $(\hat{P}_{(1-\alpha)N}, \hat{H}_{(1-\alpha)N})$, compute the corresponding controller $\hat{C}_{id}^{opt}(z)$ and apply to the plant.
• Closed loop identification using αN data.

$$(\hat{P}_N, \hat{H}_N) \Rightarrow J_V^{it}$$

One can show that, if N is large enough, then

$$J_V^{it} < J_V^{ol}$$

Simulation

'True' system

$$(1-1.5q^{-1}+0.7q^{-2})y_t = q^{-1}(1+0.5q^{-1})u_t+e_t$$

Optimal identification design is with the ideal MV regulator:

$$u_t = -\frac{1.5 - 0.7q^{-1}}{1 + 0.5q^{-1}} y_t$$

Experiment 0: Closed loop identification with ideal regulator operating.

Experiment 1: Open loop identification with 1000 data generated with $\phi_u = 1$.

Experiment 2: Open loop with $(1-\alpha)1000$ data, followed by closed loop identification with $\phi_r = 1$ with $\alpha 1000$ data, for all values of α in $[0, 1]$.

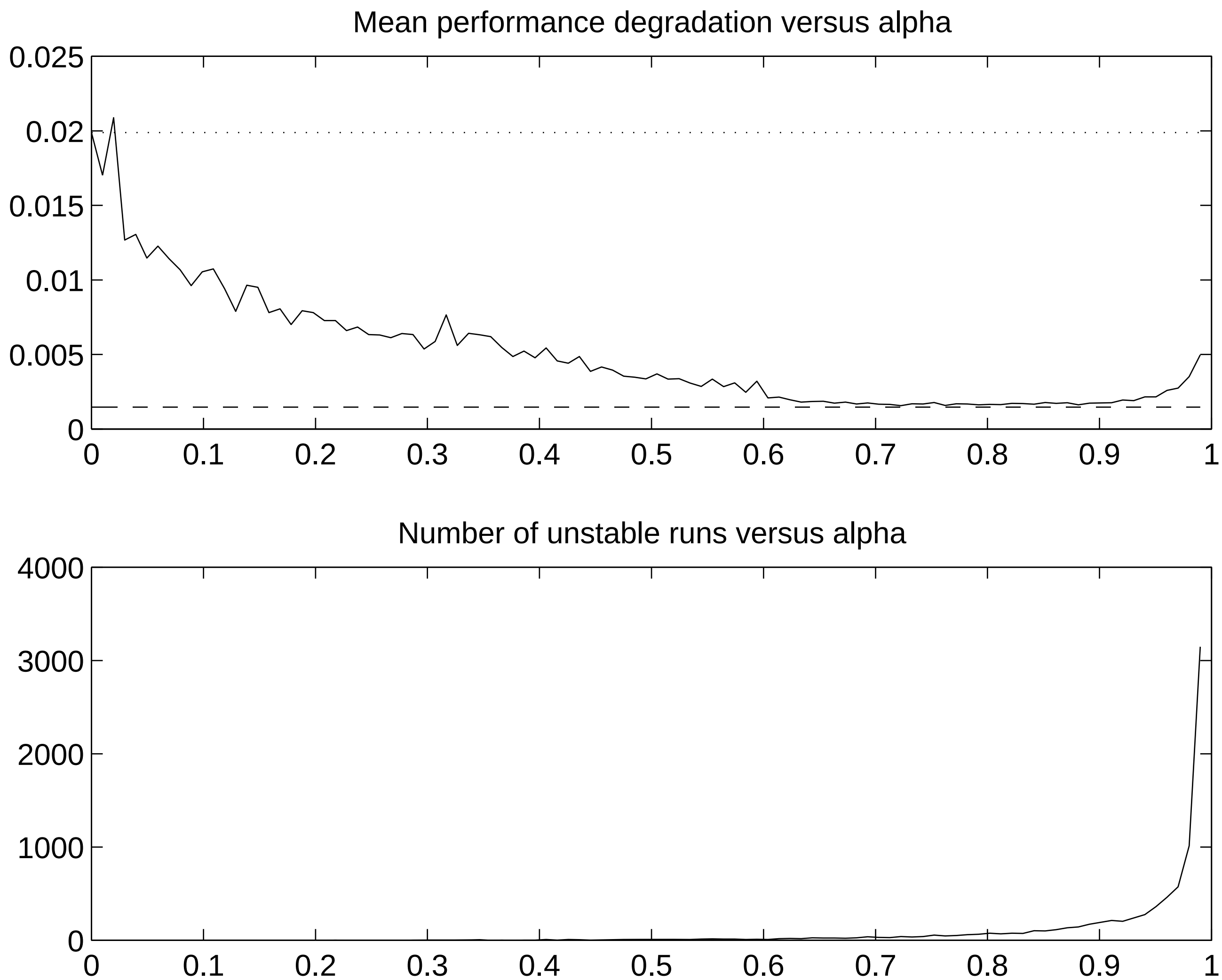
In each case the MV controller $\hat{C}_{1000}$ is computed at the end and the corresponding J_V is computed, using 1000 Monte Carlo simulations.

<u>Results</u>

1. Closed loop identification with ideal MV controller: $J_V^{id} = 0.0014$.

2. Open loop identification: $J_V^{ol} = 0.0199$.

3. Iterative identification: see J_V^{it} on Figure as a function of α.


Mean performance degradation versus alpha
0.025
0.02
0.015
0.01
0.005
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Number of unstable runs versus alpha
4000
3000
2000
1000
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

When is $C_{id}^{opt} = C_0$?

- Recall that

$$C_{id}^{opt}(z) = -\frac{\frac{\partial C}{\partial P}}{\frac{\partial C}{\partial P}P_0 + \frac{\partial C}{\partial H}H_0}$$

- For MV Control, $C_{id}^{opt} = C_0$ was proved in Gevers and Ljung (1986). Note that $C^{MV} = \frac{H_0 - 1}{P_0}$.

$$\Rightarrow \frac{\partial C}{\partial P} = -\frac{H_0 - 1}{P_0^2} \text{ and } \frac{\partial C}{\partial H} = \frac{1}{P_0}$$

Result follows immediately.

The Model Reference case

The mapping $C = c(P, H)$ is defined by

$$\frac{P_0 C_{1,0}}{1 + P_0 C_{2,0}} = \frac{\hat{P}\hat{C}_1}{1 + \hat{P}\hat{C}_2} = T_{yr}$$

$$\frac{H_0}{1 + P_0 C_{2,0}} = \frac{\hat{H}}{1 + \hat{P}\hat{C}_2} = T_{ye}$$

This to leads to

$$[\Delta C_1 \;\; \Delta C_2] = [\Delta P_N \;\; \Delta H_N] \begin{bmatrix} -\frac{C_{1,0}}{P_0} & -\frac{C_{2,0}}{P_0} \\ \frac{C_{1,0}}{H_0} & \frac{1+P_0 C_{2,0}}{P_0 H_0} \end{bmatrix}$$

Sketch of proof :

- substitute in expression of J_V
- use covariance formula for $[\Delta P, \Delta H]$
- optimize w.r.t. $[\phi_u, \phi_{ue}]$.

Very lengthy calculations.

Some conclusions

- If identification is done for the design of a controller which depends on an estimated noise model, and if $\mathcal{S} \in \mathcal{M}$, the ideal identification design is closed loop identification with the ideal controller operating. Apparently useless.

- With an iterative design (open loop followed by closed loop) one can always obtain a more accurate controller provided N is large enough.

- Simulations show that the improvements achieved with iterative design over open loop identification can be substantial.

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