Data-driven model learning in interconnected systems

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Introduction – dynamic networks

Decentralized process control

Smart power grid

Complex machines

Brain network

Hydrocarbon reservoirs

P. Hagmann et al. (2008)

Mansoori (2014)

Pierre et al. (2012)
**Introduction**

**Overall trend:**

- (Large-scale) interconnected dynamic systems
- Distributed / multi-agent type monitoring, control and optimization problems, as well as diagnostics
- Data is “everywhere”, big data era, AI/machine learning tools
- Model-based operations require accurate/relevant models
- → **Learning models/actions from data** (including physical insights when available)
Introduction

The classical (multivariable) data-driven modeling problems\textsuperscript{[1]}:

Identify a model of $G$ on the basis of measured signals $u$, $y$ (and possibly $r$), focusing on \textit{continuous LTI dynamics}.

In interconnected systems (networks) the \textbf{structure / topology} becomes important to include.

\textsuperscript{[1]} Ljung (1999), Söderström and Stoica (1989), Pintelon and Schoukens (2012)
Network models

- scalable, describing the physics
- dynamic elements with cause-effect
- handling feedback loops (cycles)
- combining physical and cyber components
- centered around measured signals
- allow disturbances and probing signals
Network models

\[
x(k + 1) = Ax(k) + Bu(k)\\
y(k) = Cx(k) + Du(k)
\]

- States as **nodes** in a (directed) graph
- State transitions (1 step in time) reflected by \(a_{ij}\)
- Transitions are encoded in links
- Ultimate break-down of system structure
- Actuation \((u)\) and sensing \((y)\) reflected by separate links

For data-driven modeling problems:
- Lump unmeasured states in dynamic **modules**
Network models

State space representation [1]

Module representation [2]

Dynamic network models - zooming

Decreasing structural information

Increasing level of detail

$T_{wv}$  $T_{wr}$
Dynamic network setup

$G_{76}$ module

$r_i$ external excitation

$v_i$ process noise

$w_i$ node signal
Dynamic network setup

- $G_{76}^{0}$ module
- $r_i$: external excitation
- $v_i$: process noise
- $w_i$: node signal
Dynamic network setup

- $G_{76}$: module
- $r_i$: external excitation
- $v_i$: process noise
- $w_i$: node signal
Dynamic network setup

$G_{76}$ module

$r_i$ external excitation

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$w_i$ node signal
Dynamic network setup

\[ G_{76} \] module
\[ r_i \] external excitation
\[ v_i \] process noise
\[ w_i \] node signal
Dynamic network setup

Basic building block:

\[ w_j(t) = \sum_{k \in \mathcal{N}_j} G_{jk}^0(q)w_k(t) + r_j(t) + v_j(t) \]

- \( w_j \): node signal
- \( r_j \): external excitation signal
- \( v_j \): (unmeasured) disturbance, stationary stochastic process
- \( G_{jk}^0 \): module, rational proper transfer function, \( \mathcal{N}_j \subset \{ \mathbb{Z} \cap [1, L]\backslash \{j\} \} \)
- \( q \): shift operator, \( q^{-1}w(t) = w(t - 1) \)

Node signals: \( w_1, \cdots, w_L \)
Interconnection structure / topology of the network is encoded in \( \mathcal{N}_j, j = 1, \cdots, L \)
Dynamic network setup

Collecting all equations:

\[
\begin{bmatrix}
  w_1 \\
  w_2 \\
  \vdots \\
  w_L
\end{bmatrix} =
\begin{bmatrix}
  0 & G_{12}^0 & \cdots & G_{1L}^0 \\
  G_{21}^0 & 0 & \cdots & G_{2L}^0 \\
  \vdots & \vdots & \ddots & \vdots \\
  G_{L1}^0 & G_{L2}^0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
  w_1 \\
  w_2 \\
  \vdots \\
  w_L
\end{bmatrix} +
\begin{bmatrix}
  r_1 \\
  r_2 \\
  \vdots \\
  r_K
\end{bmatrix} +
\begin{bmatrix}
  e_1 \\
  e_2 \\
  \vdots \\
  e_p
\end{bmatrix}
\]

Network matrix \( G^0(q) \)

\[
w(t) = G^0(q)w(t) + \underbrace{R^0(q)r(t)}_{u(t)} + v(t); \quad v(t) = H^0(q)e(t); \quad \text{cov}(e) = \Lambda
\]

- Typically \( R^0 \) is just a (static) selection matrix, indicating which nodes have an excitation signal.
- The topology of the network is encoded in the structure (non-zero entries) of \( G^0 \).
- \( r \) and \( e \) are called external signals.
Dynamic network setup

Measured time series:
\[ \{w_i(t)\}_{i=1,...,L}; \ \{r_j(t)\}_{j=1,...,K} \]

Many challenging data-driven modeling and diagnostics challenges appear

- Identification of a local module (known topology)
- Identification of the full network
- Topology estimation
- Identifiability
- Sensor and excitation allocation
- Diagnostics and fault detection
- User prior knowledge of modules
- Distributed identification
- Scalable algorithms
Application: Printed Circuit Board (PCB) Testing

Detection of
• component failures
• parasitic effects
Single module identification
Single module identification

For a network with known topology:

- Identify $G_{21}^0$ on the basis of measured signals
- Which signals to measure? Preference for local measurements
- When is there enough excitation / data informativity?
Single module identification

Local direct method:
(consistency and minimum variance properties)

Select a subnetwork:
• Predicted outputs: \( w_N \)
• Predictor inputs: \( w_D \)
such that prediction error minimization leads to an accurate estimate of \( G_{21}^0 \)

\[
\begin{align*}
\begin{bmatrix} w_D \\
  w_Q \\
\end{bmatrix} & \overset{\bar{G}}{\rightarrow} \\
\begin{bmatrix} w_O \\
  w_Q \\
\end{bmatrix}
\end{align*}
\]

Note: same node signals can appear in input and output
Single module identification

Conditions for arriving at an accurate model:

1. Module invariance: \( \bar{G}_{ji} = G_{ji}^0 \)
2. Handling of confounding variables
3. Data-informativity
4. Technical condition on presence of delays
Single module identification - module invariance

A sufficient condition for module invariance:

**All parallel paths, and loops around the output, should be ”blocked” by a measured node that is present in $w_D$**

All other signals can be removed/immersed from the network

[2] Shi et al., Automatica 2022
Single module identification – confounding variables

Confounding variable [1][2]:
Unmeasured signal that has (unmeasured paths) to both the input and output of an estimation problem.

In networks they can appear in two different ways:

- If $\nu$ disturbances on inputs and outputs are correlated
- If non-measured in-neighbors of $w_\gamma$ affect signals in $w_D$

Can be addressed by adding inputs/outputs to the predictor model[3]

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Single module identification – data-informativity

Predictor model equation:

\[ w_Y(t) = \tilde{G}(q, \theta) w_D(t) + \tilde{H}(q, \theta) \xi_Y(t) + \tilde{J}(q, \theta) u_K(t) + \tilde{S} u_P(t) \]

Typical data-informativity condition:

\[ \kappa \text{ persistently exciting} \quad \Phi_{\kappa}(\omega) > 0 \text{ for almost all } \omega \]

\[ \kappa(t) := \begin{bmatrix} w_D(t) \\ \xi_Y(t) \\ u_K(t) \end{bmatrix} \quad \text{inputs of the predictor model} \]

Rank-based condition can generically be satisfied based on a graph-based condition

Data informativity (path-based condition)

This condition can be verified in a generic sense, by considering the **generic rank** of the mapping from external signals to $\kappa$ \[^1\],[^2]\]

linking to the maximum number of **vertex disjoint paths** between inputs and outputs

\[ b_{\mathcal{R} \rightarrow \mathcal{W}} = 3 \]

Equivalently:  
\[
\dim(w_D) \text{ vertex disjoint paths between } \{u, e\} \setminus \{\xi, u_\kappa\} \text{ and } w_D
\]

[^1]: Van der Woude, 1991  
[^3]: VdH et al., CDC 2020.
Data informativity (path-based condition)

Specific result for networks with full rank disturbances:

Every node signal in $w_Q$ requires an excitation in $u_P$ having a 1-transfer to $w_Y$

$$w_Y(t) = \tilde{G}(q, \theta)w_D(t) + \tilde{H}(q, \theta)\xi_Y(t) + \tilde{J}(q, \theta)u_K(t) + \tilde{S}u_P(t)$$

- For every node in $w_Q$ we need a $u$-excitation
- More expensive experiments with growing # outputs
Single module identification

Conditions for arriving at an accurate model:

1. Module invariance: $\bar{G}_{ji} = G^0_{ji}$
2. Handling of confounding variables
3. Data-informativity
4. Technical conditions on presence of delays

Path-based conditions on the network graph
Algorithms implemented in SYSDYNET Toolbox
Summary single module identification

• Path-based conditions that the predictor model should satisfy

• Different algorithms for synthesizing predictor model

• Degrees of freedom in sensor / actuator placement

• Methods for consistent and minimum variance module estimation, and effective (scalable) algorithms
Related topics...

- Excitation allocation for full network identifiability
- Subnetwork identifiability
- Diffusively coupled networks
- Distributed controller identification
ERC SYSDYNET Team: data-driven modeling in dynamic networks

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