Optimal Pole Selection for LPV System Identification with OBFs

A clustering approach

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Control and identification of nonlinear systems - challenges

- Expand the scope beyond the current LTI framework
- Handling systems in different operating regimes
- Attractive structure: LPV for systems with "position"-dependent (linear) dynamics
- Tools for controller synthesis
- LPV model identification

Here: focus on parametrization

Contents of the presentation

- The LPV system class
- LTI system identification via OBFs
- Extension to the LPV framework
- Optimal basis selection
- A Fuzzy clustering approach
- Conclusions

The LPV System class

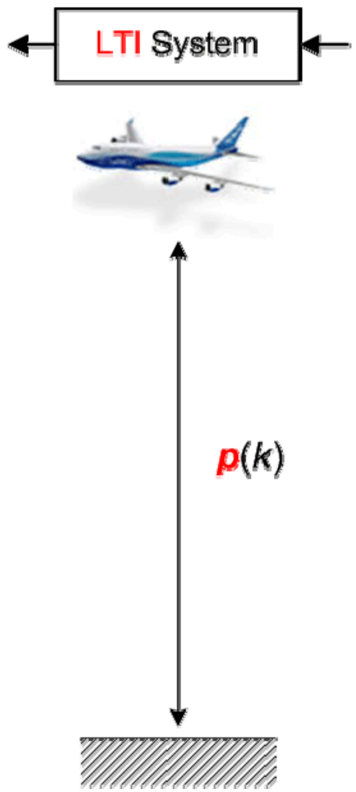
Mathematical formulation

- Parameter dependent models in SSR

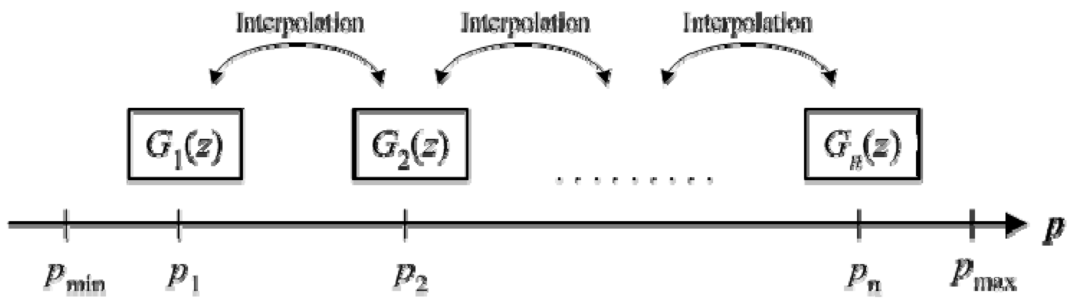
$$x(k+1) = A(p(k))x(k) + B(p(k))u(k)$$
$$y(k) = C(p(k))x(k) + D(p(k))u(k)$$

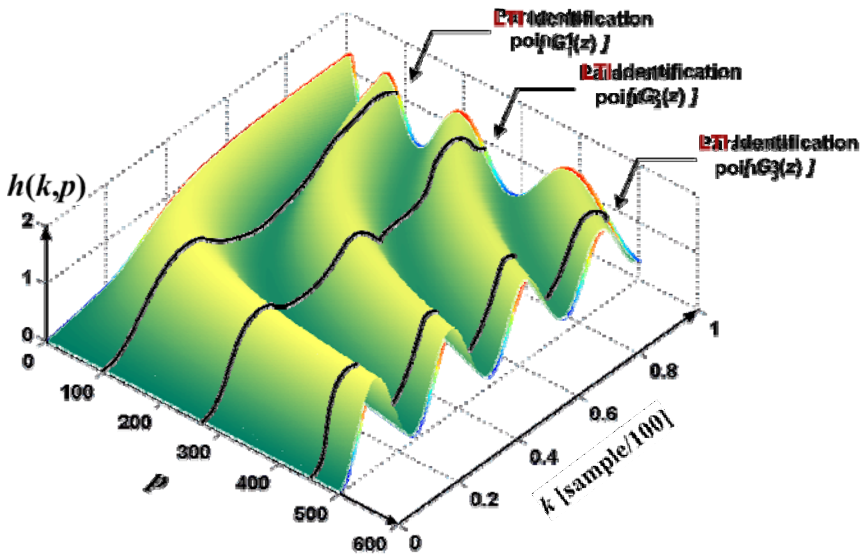
- Transfer function form

$$G(p(k),q) = \frac{B(p(k),q)}{A(p(k),q)}$$



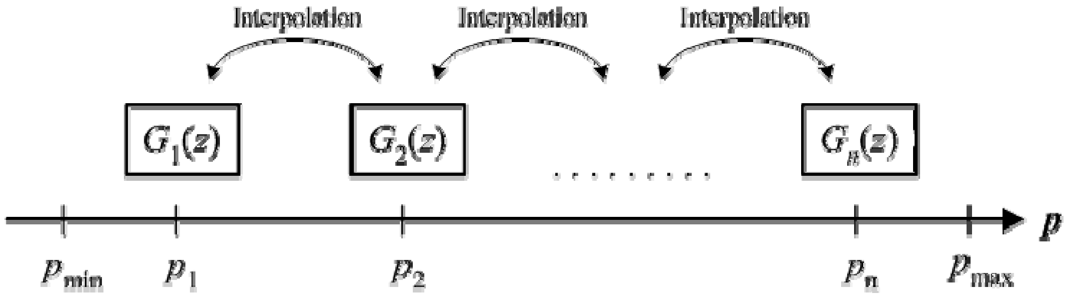
LPV Identification approach

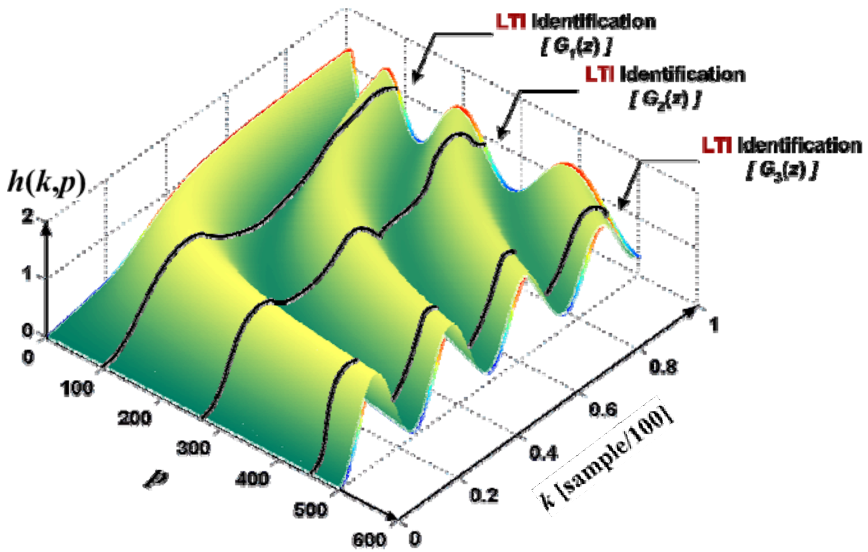




- Procedure (4 step)
  - Local parameter points
  - Model structure selection
  - LTI identification in each parameter point
  - Interpolation of local models ⇨ (LPV)

Extension to the LPV framework

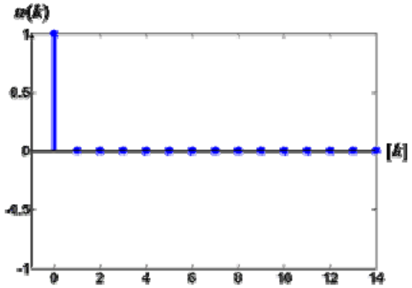




- Questions
  - Selection of local points
  - Model structure
  - Handling models with different McMillan degree
  - Coordinate-free models (OBF's)

LTI system identification via OBFs

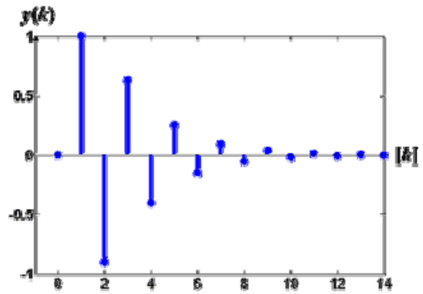
- The OBFs (stable, SISO case)



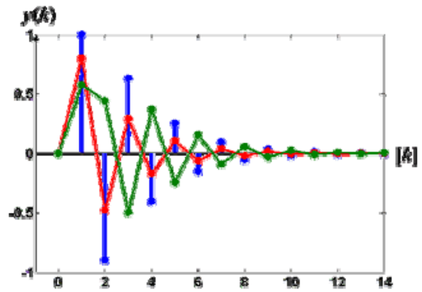
$$G(z) \approx \sum_{k=1}^{n_g} g_k z^{-k}$$

$$G(z) \approx \sum_{k=1}^{n_f} c_k F_k(z)$$

FIR



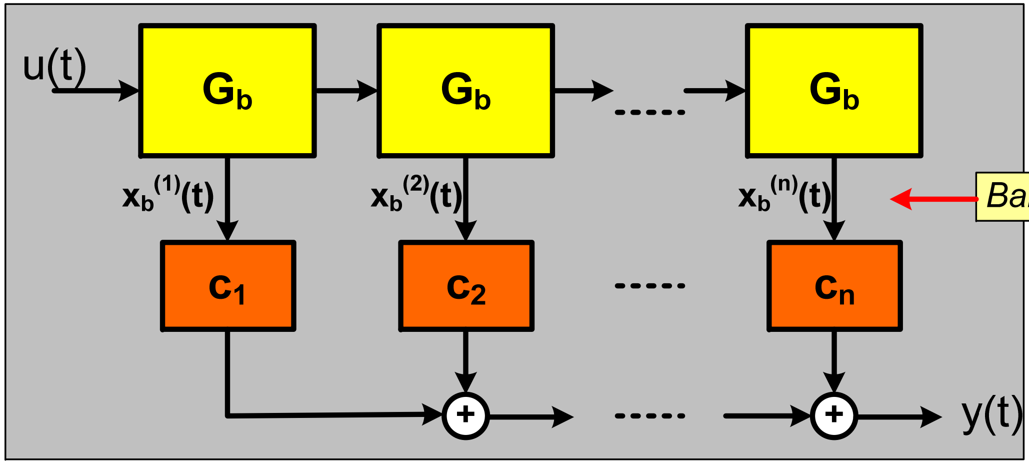
OBF

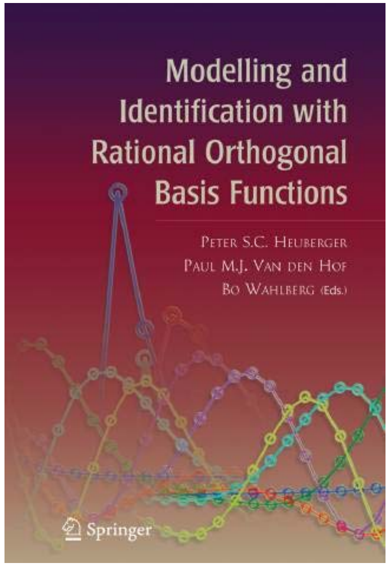


LTI system identification via OBFs

Generation of the OBFs

- By a set of stable poles $\Xi_{n_f} = \{\zeta_1, \ldots, \zeta_{n_f}\} \subset \mathbb{D}$
- By a stable all-pass (inner) function: $G_b(z) = \prod_{k=1}^{n_f} \frac{1 - z\zeta_k^*}{z - \zeta_k}$





(2005)

Choice of poles determines rate of convergence of the series expansion

LTI system identification via OBFs

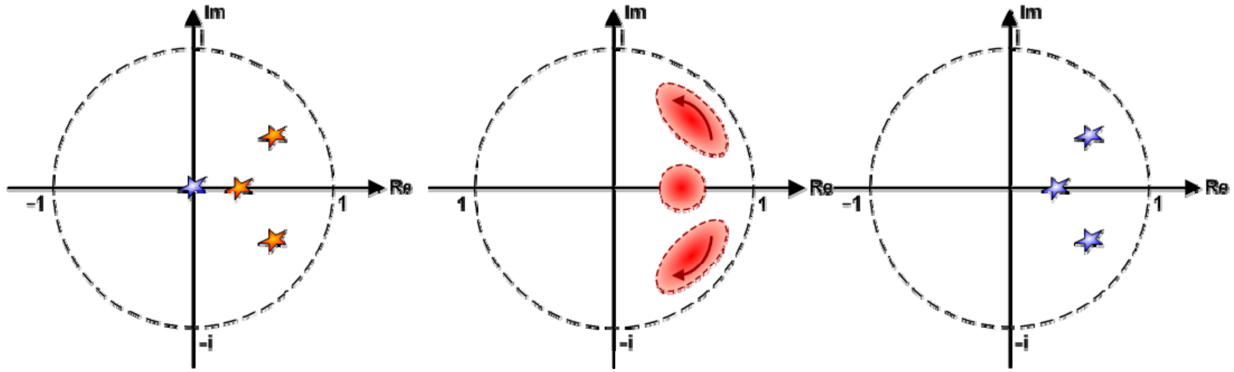
$$G(z) \approx \sum_{k=1}^{n_f} c_k F_k(z)$$

- Identification procedure & properties
  - Orthonormality
  - Linear regression (FIR like structure)
  - Computable variance & bias
  - No bias for noise uncorrelated to the input (OE structure)
  - No undermodelling asymptotically (completeness)
  - Related to fixed-denominator models

Optimal basis selection

In OBF's choice of model structure becomes choice of basis poles

- Selection of basis

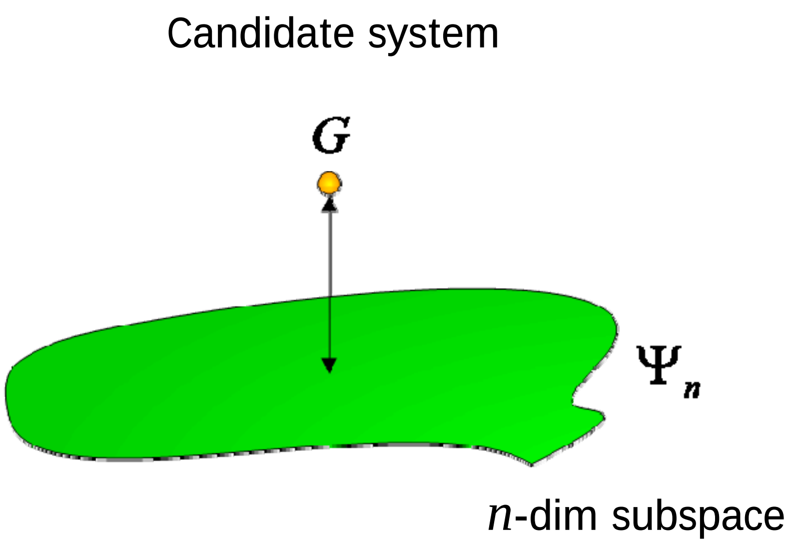


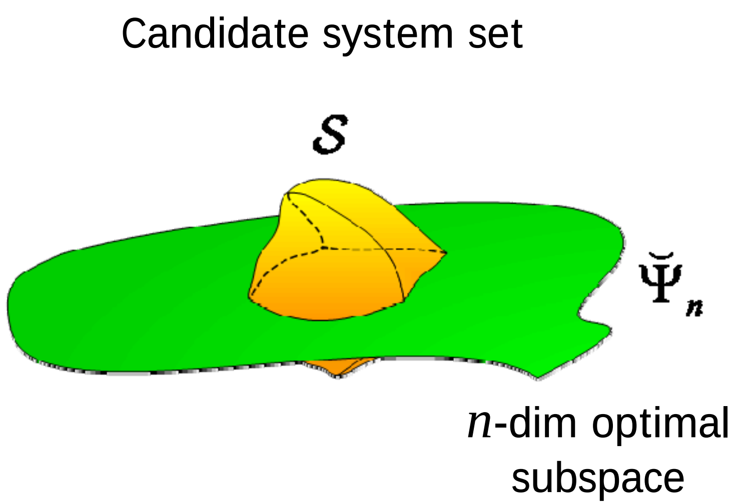
$$G(z) \approx \sum_{k=1}^{n_g} g_k z^{-k} \qquad G(z) = \sum_{k=1}^{3} c_k F_k(z)$$

Choice of the basis should be characterized in terms of a distance between basis poles and the several LTI system poles

Optimal basis selection

- The Kolmogorov n-width theory
  - The distance
  - The optimal subspace





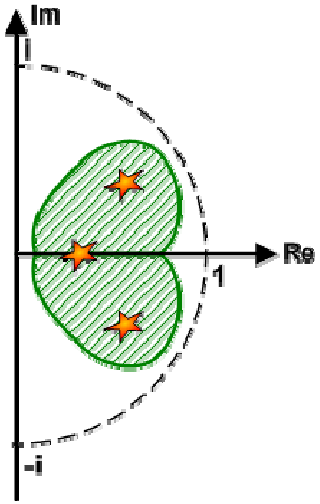
Optimal basis selection

The Kolmogorov n-width theory

- Result (Oliveira e Silva):
  - $G_b(z)$ an inner function
  - Let $\mathcal{S}$ be the set of systems having poles in the region:

$$\left\{ z \in \mathbb{D} \,\middle|\, \left| G_b\left(z^{-1}\right) \right| \leq \rho \right\}$$

  - The OBFs, generated by $G_b(z)$ are optimal in the n -width sense



Optimal basis selection

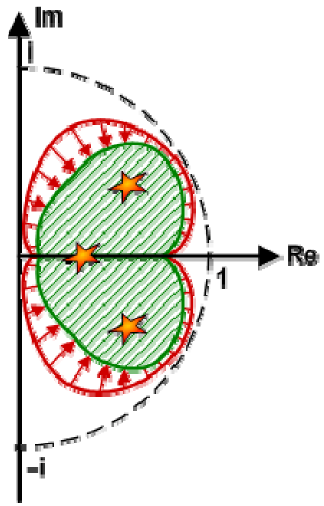
- The inverse Kolmogorov ***n***-width theory
  - Given a region of poles: Ω
  - Try to approximate it as

$$\Omega \approx \Omega(\Xi_n, \rho) = \left\{ z \in \mathbb{D} \,\middle|\, \left| G_b\left(z^{-1}\right) \right| \le \rho \right\}$$

$$\rho = \text{decay rate of error}$$

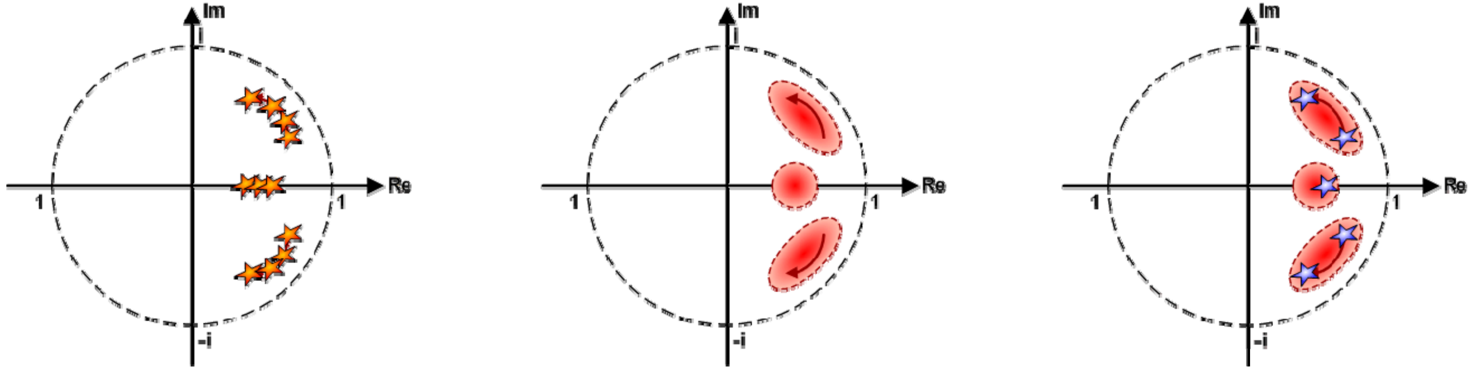
  - The ***n*** optimal OBF poles obtained through (Kolmogorov measure minimization):

$$\min_{\Xi_n \subset \mathbb{D}} \rho = \min_{\Xi_n \subset \mathbb{D}} \max_{z \in \Omega} \left| G_b\left(z^{-1}\right) \right| = \min_{\Xi_n \subset \mathbb{D}} \max_{z \in \Omega} \left| \prod_{k=1}^{n} \frac{z - \zeta_k}{1 - z\zeta_k^*} \right|$$



A Fuzzy Clustering approach

- Optimization problem in practice
  - Prior knowledge & local pole observations Ω
  - Clustering of $\mathcal{Z}$ to obtain $\mathcal{Z} = \{z_1, \ldots, z_N\}$
  - Application of Kolmogorov n-width



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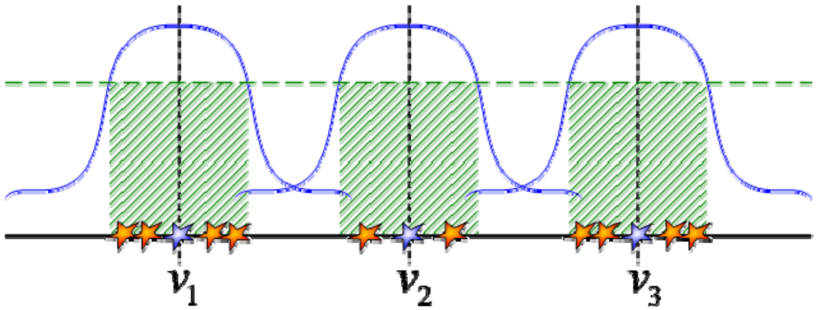
A Fuzzy clustering approach

- The Fuzzy clustering problem
  - Data: N sample pole locations $\mathcal{Z} = \{z_1, \ldots, z_N\}$
  - Clusters: c number of clusters
    - Cluster centers $\mathcal{V} = \{v_1, \ldots, v_c\}$
    - Measure of dissimilarity $d(z_k, v_i)$
    - Membership functions $\mu(z_k, v_i)$



A Fuzzy clustering approach

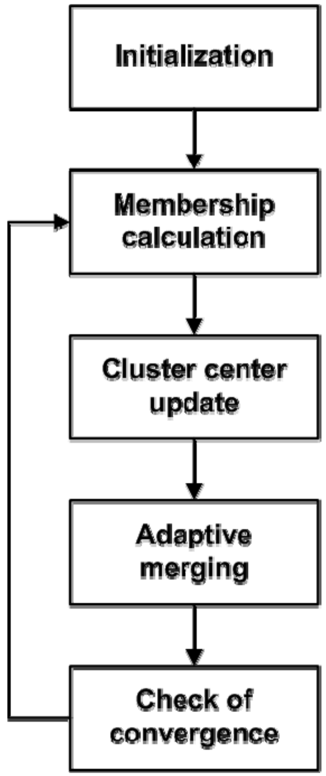
- The Fuzzy clustering problem
  - Optimization problem: Find such a $\mathcal{V}$ and $\mu(z_k, v_i)$ that the following cost function is minimized

$$J_m(\mathcal{V}, \mathcal{Z}) = \sum_{i=1}^{c} \sum_{k=1}^{N} \mu^m(z_k, v_i) d(z_k, v_i)$$

  - Design parameter: $m \in (0, \infty)$
    - Trade off of complexity (fuzziness)

A Fuzzy clustering approach

- The Fuzzy clustering algorithm
  - Alternating minimization of J_m
  - Effect of initial partition
  - Adaptive cluster merging
  - Fast convergence
  - Dissimilarity measures
    - Euclidian
    - Mahalanobis
    - Kolmogorov



A Fuzzy clustering approach

- Dissimilarity measures
  - Euclidian

$$d(z_k, v_i) = |z_k - v_i|^2$$

  - Weighted Euclidian (Mahalanobis)

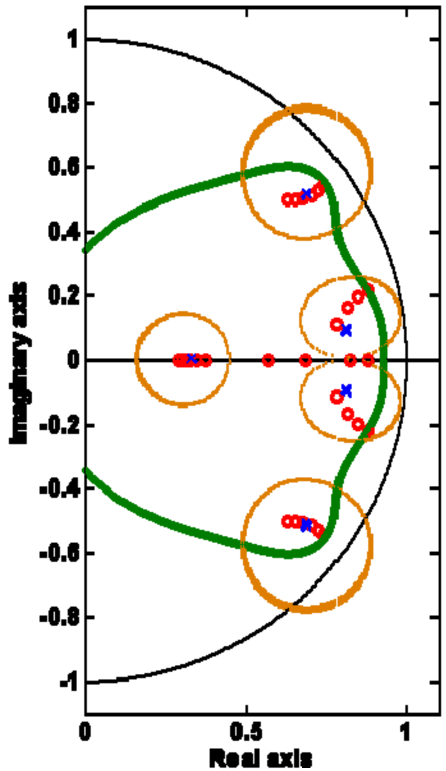
$$d(z_k, v_i) = (z_k - v_i)^T T (z_k - v_i)$$

  - Kolmogorov

$$d(z_k, v_i) = \left| \frac{z_k - v_i}{1 - z_k v_i^*} \right|$$

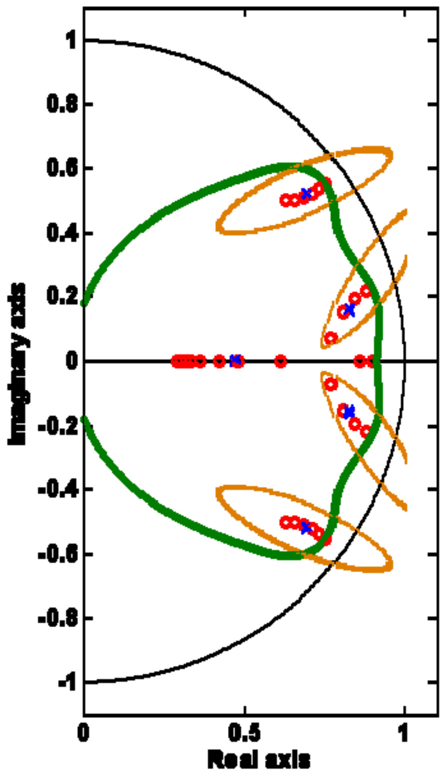
1-width version of the Kolmogorov n-width measure

Example LPV (order 5; 6 local points)



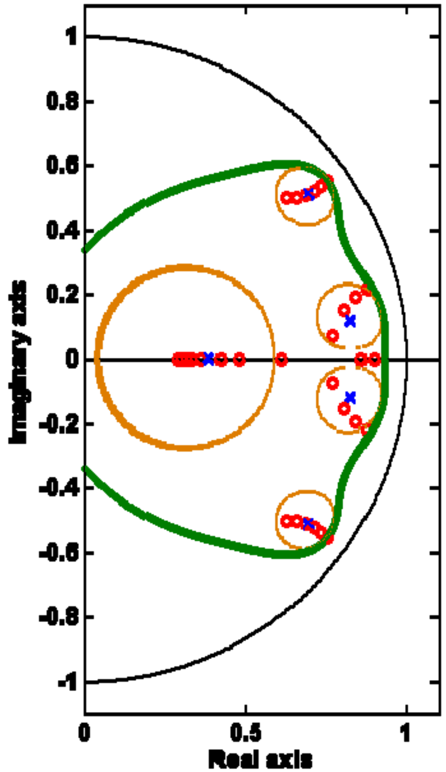
Euclidian

$\rho = -11.65$ dB



Mahalanobis

$\rho = -13.48$ dB



Kolmogorov

$\rho = -12.58$ dB

A Fuzzy clustering approach

	Num. of clus.	Fitting of clus.	Clus. in $\mathbb{D}$	Achieved ρ	Outlying poles
Euclidian	5	Loose	☒	-11.65 dB	☑
Mahalanobis	5	Tight	☒	-13.48 dB	☒
Kolmogorov	5	Tight	☑	-12.58 dB	☑

- Conclusions
  - Mahalanobis: Best quality of cluster shapes & best location of the cluster centers in the Kolmogorov sense
  - Kolmogorov: Direct applicability of the Kolmogorov theory on the resulting clusters & well localized cluster centers

Conclusions

- Selection of optimal OBFs
  - Sample pole locations as "prior knowledge"
  - Clustering pole locations by Fuzzy c-Means clustering
  - Applications different dissimilarity measures to obtain good approximating regions of non-analycity
  - Application of the Kolmogorov n-width result for each pole
- Current research
  - Joint solution of the clustering and the Kolmogorov n-width problem by the Fuzzy-Kolmogorov c-Max method
    [Submittede to CDC 2006]