# Parameter identification in large-scale models for oil and gas production

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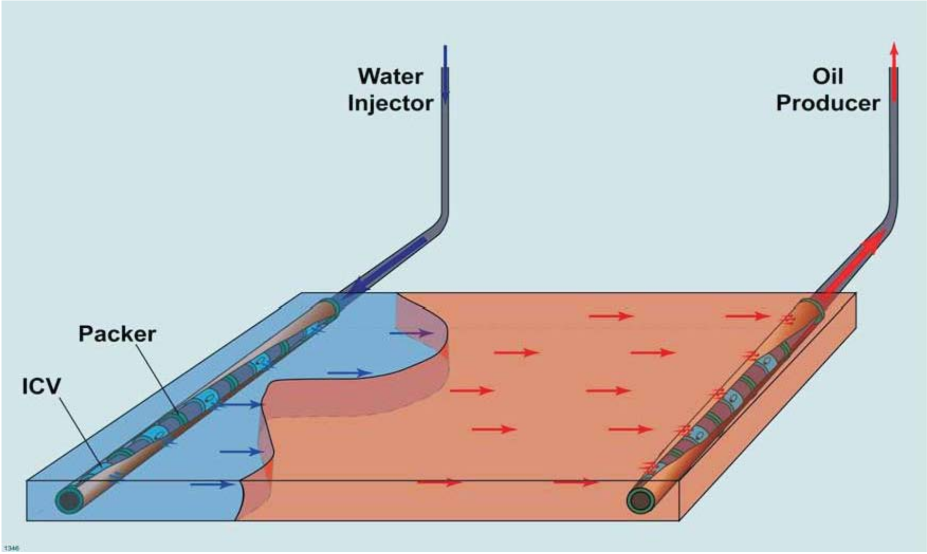
# Motivation:
# an example from oil reservoir engineering

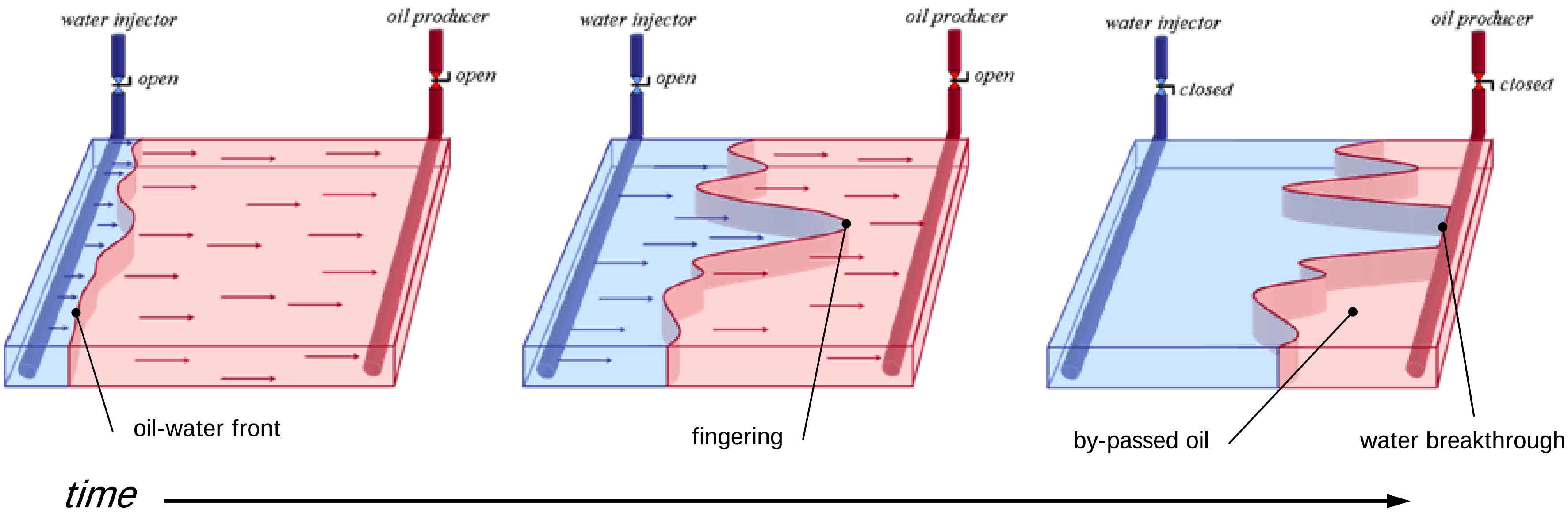
## Water flooding

- Involves the injection of water through the use of injection wells
- Goal is to displace oil by water
- Production is terminated when (too much) water is being produced



# Introduction

- Production to be optimized by manipulating injector and producer valves over life-cycle



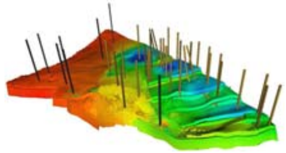

[D.R. Brouwer, 2004]

# The Models

System involves the reservoir, wells and sometimes surface facilities:

| | |
|---|---|
| Inputs: | injector and producer valves |
| Outputs: | flow production rates and bottom hole pressures |

- Partial differential equations – nonlinear
- After discretization in space – large-scale nonlinear ODE’s
- Dynamics is dependent on location of oil-water front
- Process is essentially a (very slow) batch-type process
- Essential parameters are the permeabilities in each grid-block
- How to parametrize / identify these permeabilities?



# Motivation:
# physical structures versus black box

- Black box structures can be used to reduce the number of parameters, but
- Extrapolation capabilities are limited if the (nonlinear) structure is incorrect

With first-principles models:

- *Extrapolate* the model dynamics to (nonlinear) regimes that are not necessarily captured in the data.
- Physical model structures raise the problem of identifiability:
  *can all unknown parameters be estimated uniquely?*

# Contents

# Identifiability

- Consider nonlinear model structure $\hat{\mathbf{y}} = \mathbf{h}(\boldsymbol{\theta}, \mathbf{u}; \mathbf{x}_0)$
  with $\hat{\mathbf{y}}$ being a prediction of the measured outputs $\mathbf{y} := [y_1^T \cdots y_N^T]^T$

The model strucure is

- Locally identifiable in $\boldsymbol{\theta}_m$ for given $\mathbf{u}$ and $\mathbf{x}_0$ if in neighbourhood of $\boldsymbol{\theta}_m$:

$$\{\mathbf{h}(\mathbf{u}, \boldsymbol{\theta}_1; \mathbf{x}_0) = \mathbf{h}(\mathbf{u}, \boldsymbol{\theta}_2; \mathbf{x}_0)\} \Rightarrow \boldsymbol{\theta}_1 = \boldsymbol{\theta}_2$$

[Grewal and Glover 1976]

Global properties are generally very hard to analyze (nonlinear)

- Notion of *identifiability* is instrumental in analyzing model structure properties
- It determines whether it is feasible at all to relate unique values to the physical parameter variables, on the basis of measured data

# Testing local identifiability in model estimation

- Consider quadratic identification criterion based on prediction errors

$$V(\boldsymbol{\theta}) := \frac{1}{2}\boldsymbol{\epsilon}(\boldsymbol{\theta})^T \mathbf{P}_v^{-1} \boldsymbol{\epsilon}(\boldsymbol{\theta}), \quad \boldsymbol{\epsilon}(\boldsymbol{\theta}) = \mathbf{y} - \hat{\mathbf{y}} = \mathbf{y} - \mathbf{h}(\boldsymbol{\theta}, \mathbf{u}; \mathbf{x}_0),$$

- Hessian given by

$$\frac{\partial^2 V(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}^2} = \frac{\partial \hat{\mathbf{y}}^T}{\partial \boldsymbol{\theta}} \mathbf{P}_v^{-1} \left( \frac{\partial \hat{\mathbf{y}}^T}{\partial \boldsymbol{\theta}} \right)^T + \mathbf{S}$$

- Local identifiability test in $\hat{\boldsymbol{\theta}} = \arg\min V(\boldsymbol{\theta})$ : Hessian > 0

- With quadratic approximation of cost function around $\hat{\boldsymbol{\theta}}$:
  Hessian given by $\frac{\partial \hat{\mathbf{y}}^T}{\partial \boldsymbol{\theta}} \mathbf{P}_v^{-1} \left( \frac{\partial \hat{\mathbf{y}}^T}{\partial \boldsymbol{\theta}} \right)^T$

# Testing local identifiability in identification

- Rank test on Hessian through SVD

$$\frac{\partial \hat{\mathbf{y}}^T}{\partial \boldsymbol{\theta}} \mathbf{P}_v^{-\frac{1}{2}} \bigg|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}} = \begin{bmatrix} \mathbf{U}_1 & \mathbf{U}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{\Sigma}_1 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1^T \\ \mathbf{V}_2^T \end{bmatrix}$$

- If $\boldsymbol{\Sigma}_2 = 0$ then lack of local identifiability

- SVD can be used to reparameterize the model structure through

$$\boldsymbol{\theta} = \mathbf{U}_1 \boldsymbol{\rho}, \quad \dim(\boldsymbol{\rho}) << \dim(\underline{\theta})$$

  in order to achieve local identifiability in $\boldsymbol{\rho}$

- Columns of $\mathbf{U}_1$ are basis functions of the identifiable parameter space

# Testing local identifiability in identification

$$\left.\frac{\partial \hat{\mathbf{y}}^T}{\partial \boldsymbol{\theta}} \mathbf{P}_v^{-\frac{1}{2}}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}} = \left[\begin{array}{cc} \mathbf{U}_1 & \mathbf{U}_2 \end{array}\right] \left[\begin{array}{cc} \boldsymbol{\Sigma}_1 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_2 \end{array}\right] \left[\begin{array}{c} \mathbf{V}_1^T \\ \mathbf{V}_2^T \end{array}\right]$$

- What if $\boldsymbol{\Sigma}_2 \neq 0$ but contains (many) small singular values ?

No lack of identifiability, but possibly very poor variance properties

- The reparametrization $\boldsymbol{\theta} = \mathbf{U}_1 \boldsymbol{\rho}, \quad \dim(\boldsymbol{\rho}) << \dim(\underline{\theta})$ leads to improved variance

$$cov(\hat{\boldsymbol{\theta}}) > cov(\boldsymbol{U_1}\hat{\boldsymbol{\rho}}) \qquad \text{if } \boldsymbol{\Sigma}_2 > 0$$

# Bayesian approach

- Estimate parameters in a sequential-type of (Kalman-like) filter algorithm by extending the system states with parameters
- In sequential (Bayesian) approach to state/parameter estimation lack of identifiability is hardly observed:
- Cost function:

$$V_p(\theta) = V(\theta) + \frac{1}{2}(\theta - \theta_p)^T P_p^{-1}(\theta - \theta_p)$$

$$V(\theta) := \frac{1}{2}\epsilon(\theta)^T P_v^{-1}\epsilon(\theta), \qquad \epsilon(\theta) = \mathbf{y} - \hat{\mathbf{y}}(\theta)$$

- Analysis of $V(\theta)$ can show identifiable directions (locally)

# Two examples of methods to reduce the parameter dimension (and evaluate information content of data)

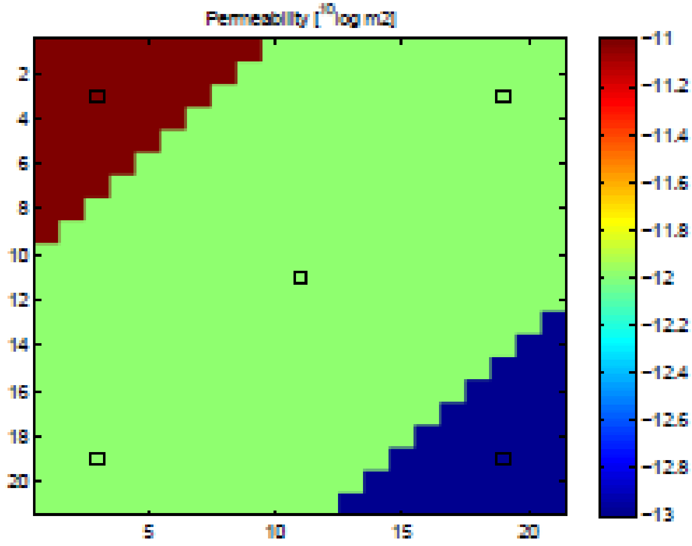
- Iterative algorithm to locally identify parameters in reduced-dimensional space (determined after SVD)
- Apply a physical parametrization in terms of straight permeability channels

# Simple reservoir example

[Van Doren, 2010]

2D two-phase example

(top view)

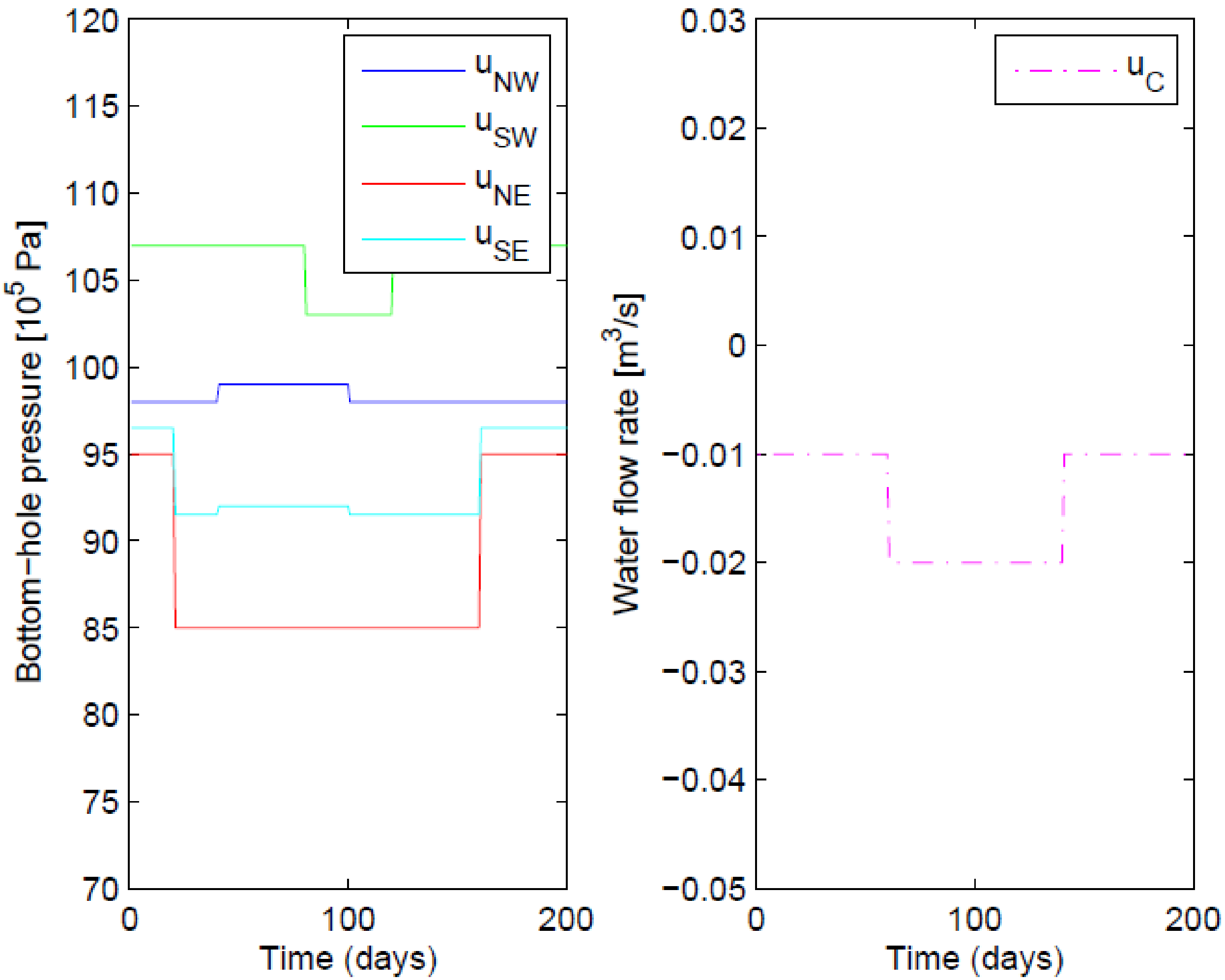



21 x 21 grid block permeabilities
5 wells; 3 permeability strokes

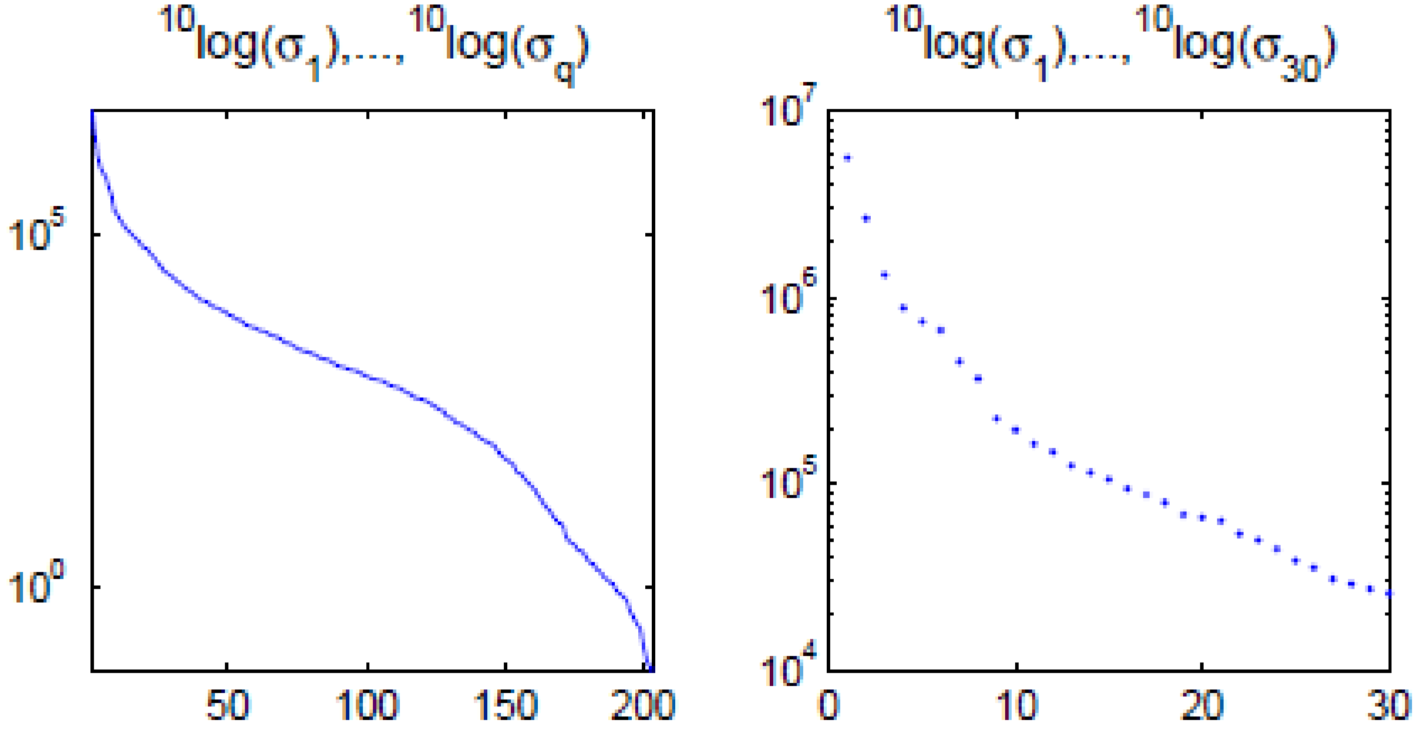
1 injector (centre)
4 producers (corners)

5 inputs: 1 injector flow-rate, and 4 bottom hole pressures
8 outputs: producer flow rates (water and oil)

# Input excitation signals



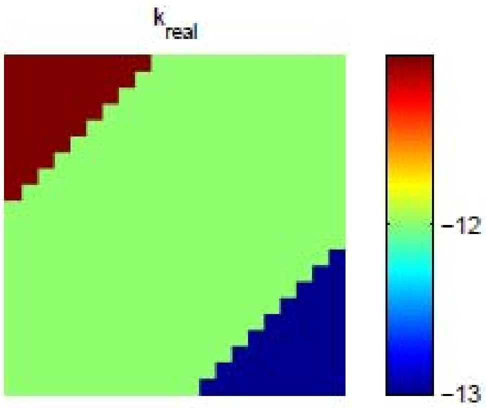
# Simple reservoir example



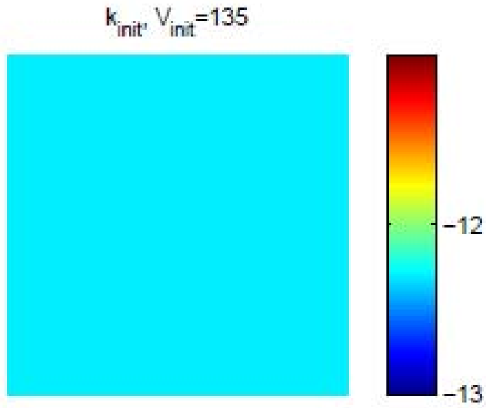

All singular values (left) and first 30 (right)

# Simple reservoir example

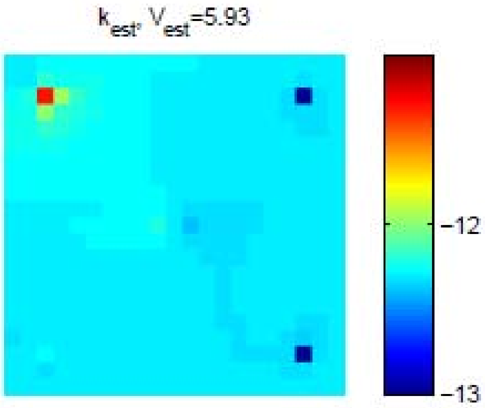
Using the reduced parameter space –iteratively- in estimation:




exact field




initial estimate
(local point)

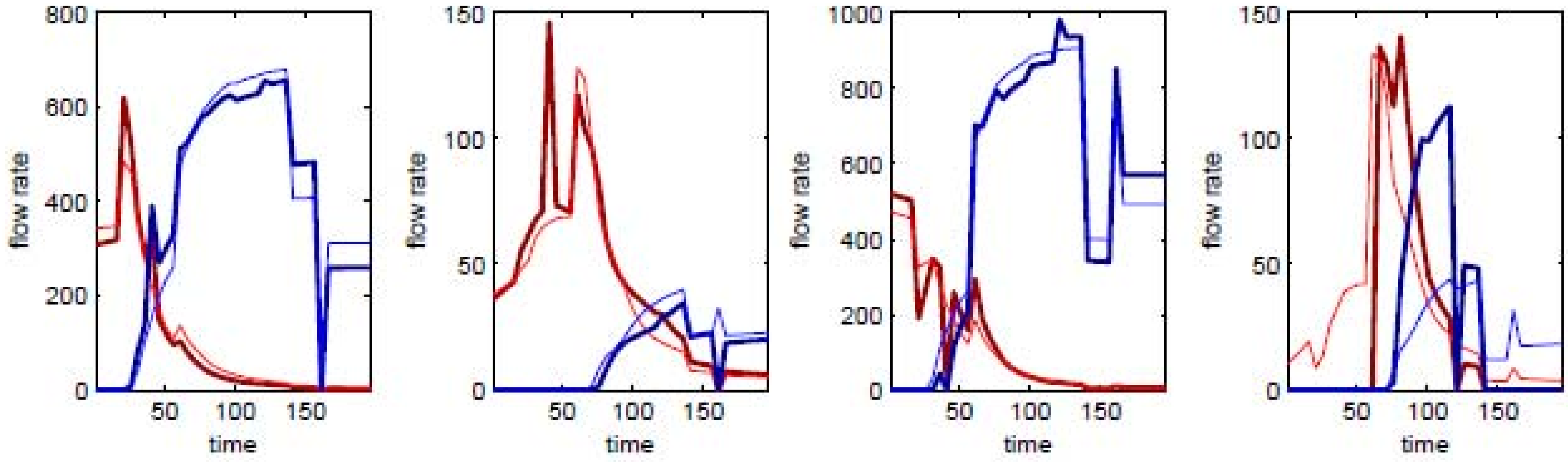



final estimate

**Observation:**

Only grid block permeabilites around well are identifiable.

# Simple reservoir example



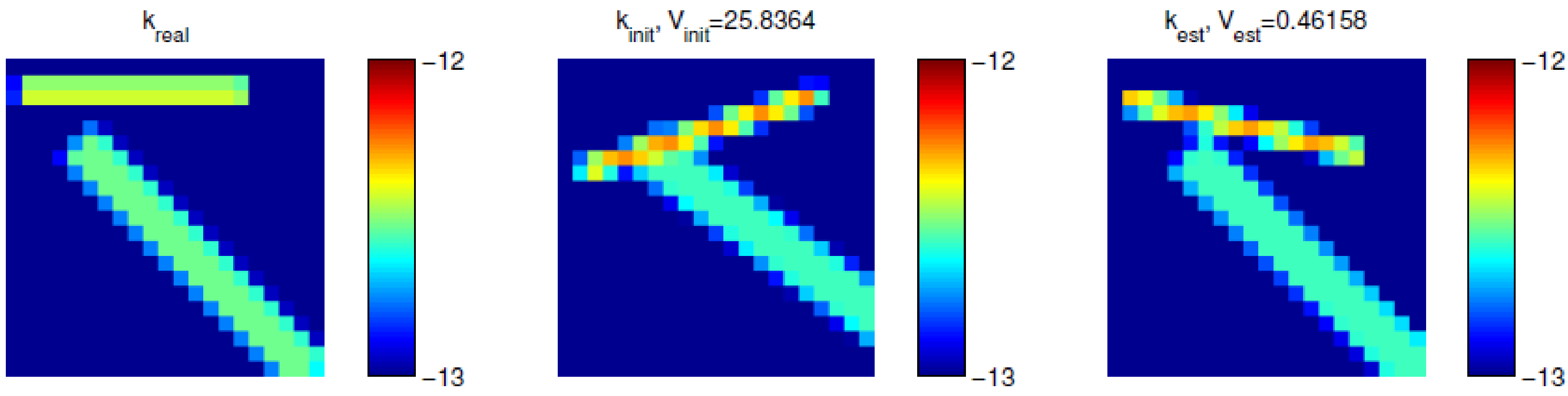

Simulated production of estimated model (thin lines) of water (blue) and oil (red) in the four producer wells

See also Vasco et al. (1997)

# Simple reservoir example

- Model estimation is done in an iterative way:
  - Choosing a local identifiable parametrization
  - Estimating the parameters
  - Repeating the procedure until convergence of the cost function
- During the iterations, the quadratic cost function is reduced from 135 to 5.93.
- “Poor” model seems to be good enough for prediction of production.
- No prior info on permeability structure has been used.

# A parametrization through channel parameters



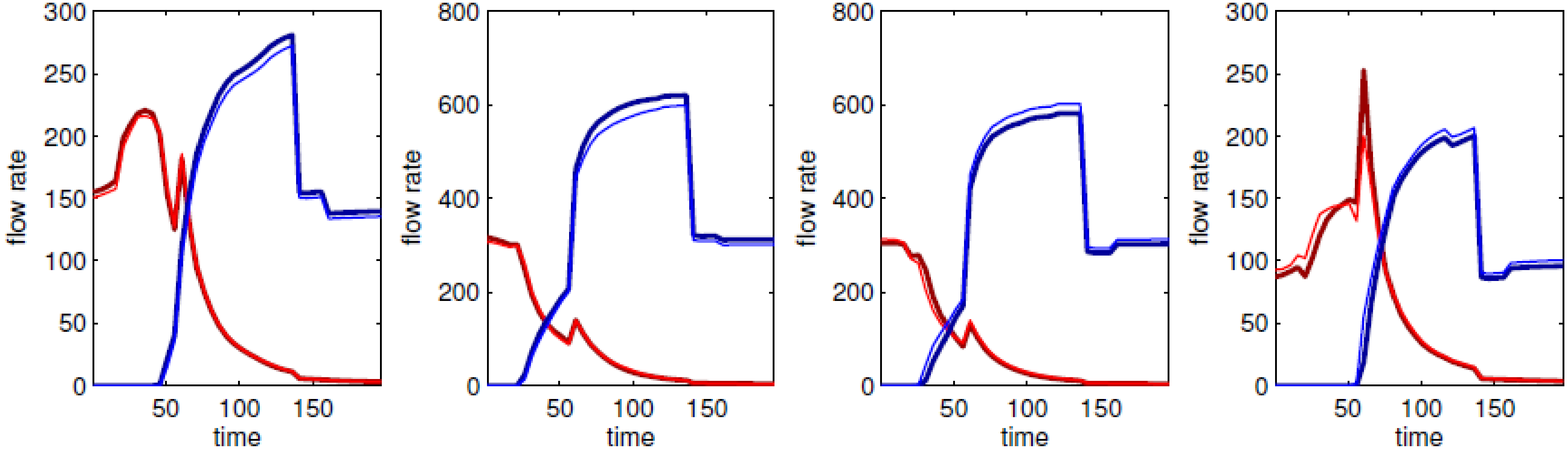

exact field

initial estimate (local point)

final estimate

Parametrization of two straight channels with variable position and orientation, and single permeability value (total 13 param)

# Simulation results




Simulated production of estimated model (thin lines) of water (blue) and oil (red) in the four producer wells

# Summary

- Large-scale reservoir models are not identifiable from production data, nor can they be validated
- When relying on measurement data only, the parameter space needs to be reduced, in order to avoid “arbitrary” results
- A proper balance between data-induced and priors-induced modelling should be achieved when estimating models, focussing at the control-relevant dynamics
- How to structurally determine the control-relevant dynamics is still a challenge

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