From closed-loop identification to dynamic networks: generalization of the direct method

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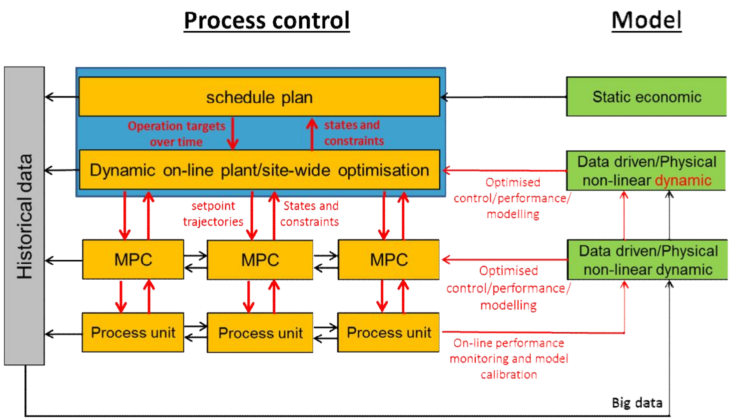
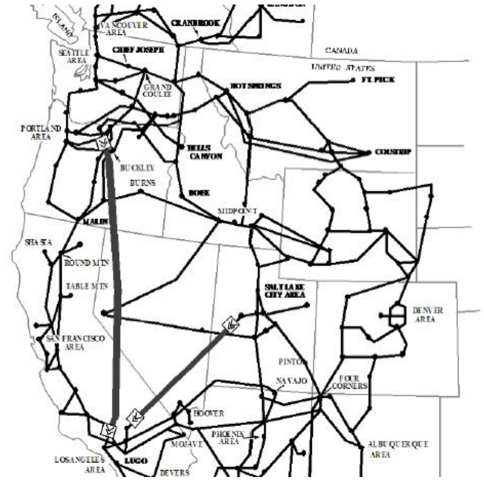
Where innovation starts

Introduction – dynamic networks

Decentralized process control

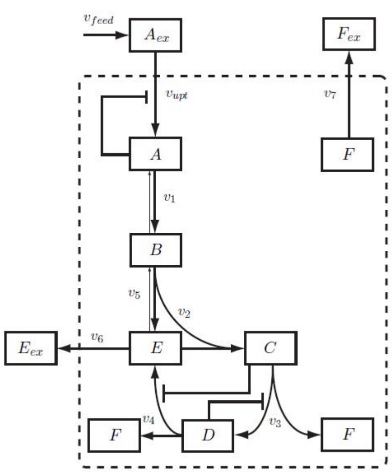


Power grid



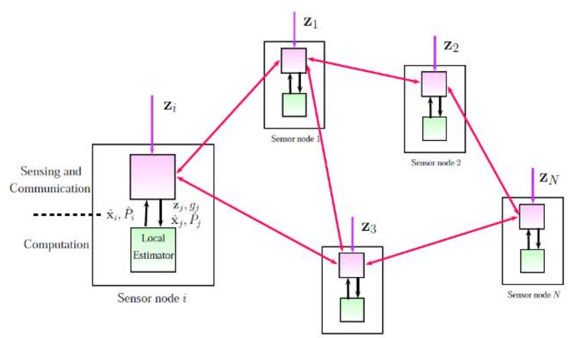
Pierre et al. (2012)

Metabolic network



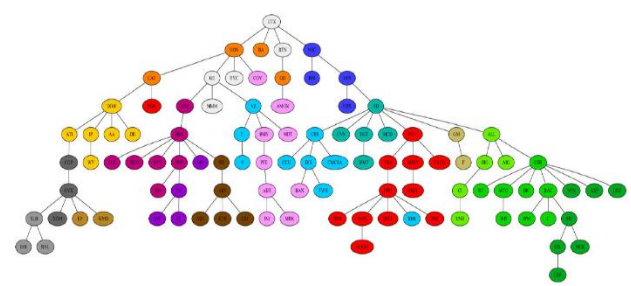
Hillen (2012)

Distributed control (robotic networks)



Simonetto (2012)

Stock market



Materassi et al. (2010)

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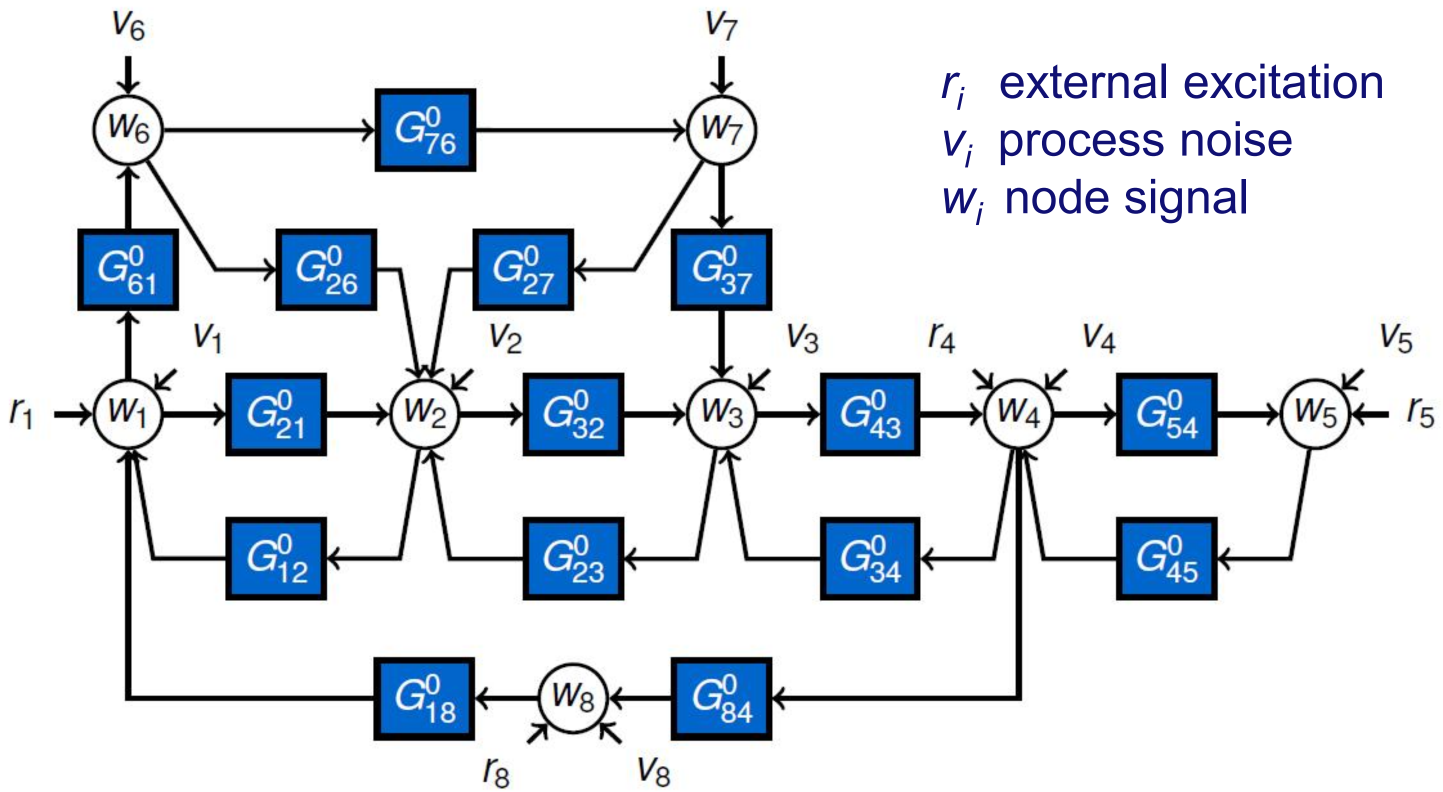
Introduction – dynamic networks



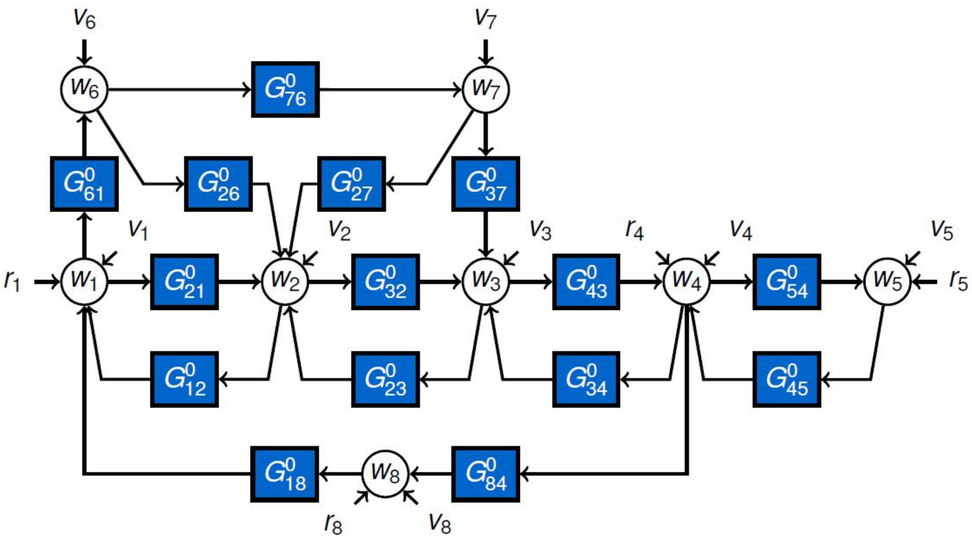
r_i external excitation
v_i process noise
w_i node signal

Target: Identify one or more modules on the basis of measured data

Introduction – dynamic networks

So far:

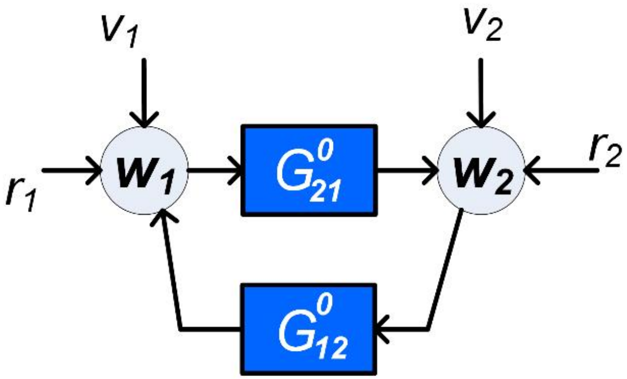
Techniques typically based on (adapted) versions of closed-loop identification methods



- **Direct method** (based on measured node signals)
  ML properties
- **2-stage/projection/IV method** (including measurements of $r_i's$)
  Consistency
- …….

P.M.J. Van den Hof, A.G. Dankers, P.S.H. Heuberger and X. Bombois. *Automatica, 49*, 2994-3006, 2013.

Introduction – dynamic networks



Here:

- Focus on most elementary (2-node) network
- Analyse primal properties of identification methods: targeting for **minimum variance** estimates

To be addressed:

- Direct method
- A generalization: Joint-direct method
- Dealing with sensor noise

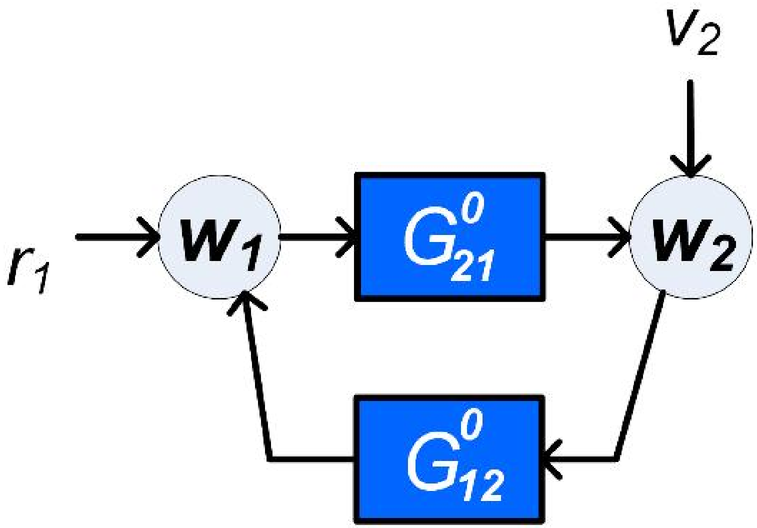
The direct method for a network setup

- Typical setup with disturbance on "output" w_2
- Prediction error to be minimized:

$$\varepsilon_2(t,\theta) = w_2(t) - \hat{w}_2(t|t-1;\theta)$$

$$\hat{\theta}_N = \arg\min_{\theta} \frac{1}{N} \sum_{t=0}^{N-1} \varepsilon_2(t,\theta)^2$$

- Under fairly general conditions, the estimate of G_{21}^0 is **consistent** and reaches **minimum variance** (CRLB).



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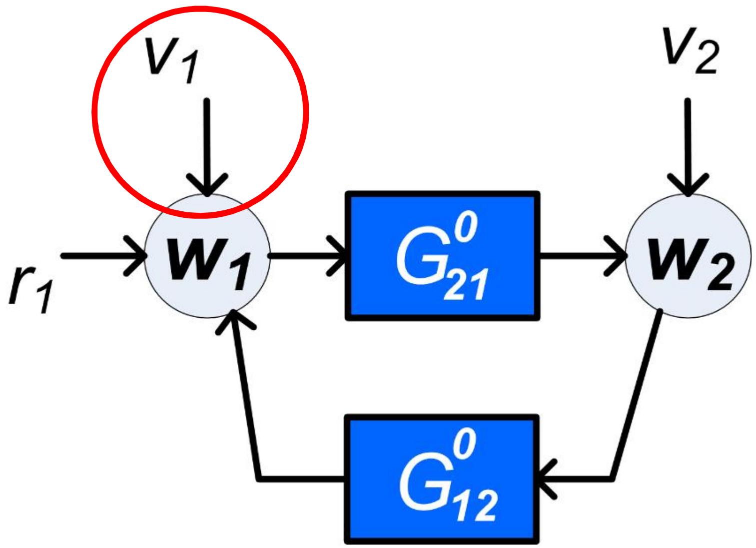
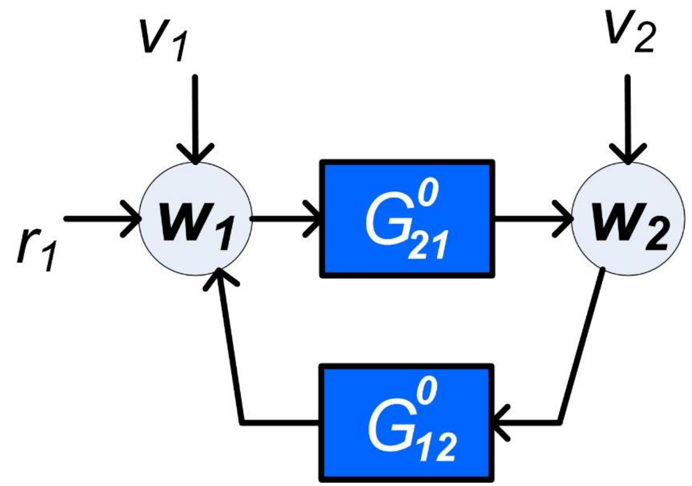


- An additional disturbance term v_1 is no problem as long as it is uncorrelated with v_2 .

The direct method for a network setup

Quote 1

Consistency of the direct method is **lost** when the disturbance terms are correlated



Quote 2

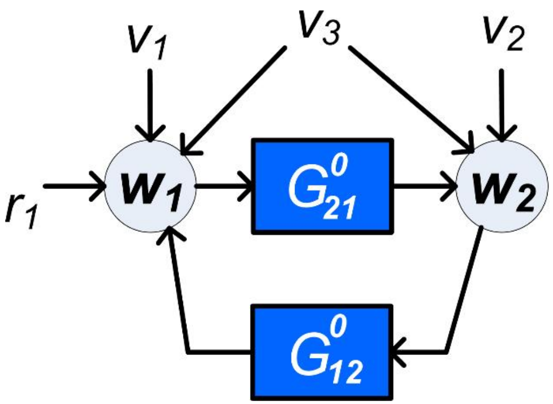
Adding an external signal r_2 does not help

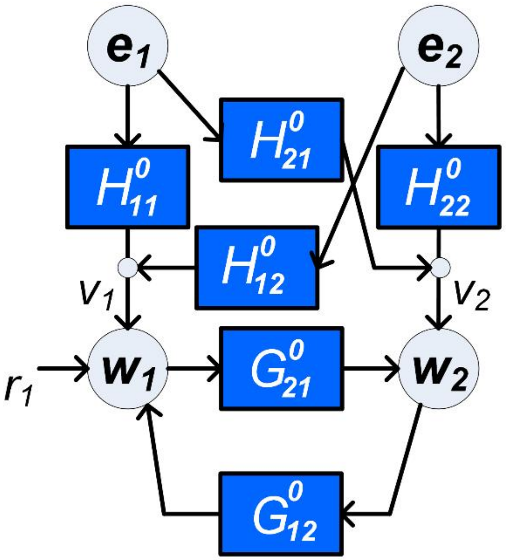
Quote 3

Consistency of the direct method is **lost** when the node signals are measured with sensor noise (EIV problem)

The direct method for a network setup

Known in network dynamics / statistics:
Confounding variables:
non-measured variables that affect both input and output





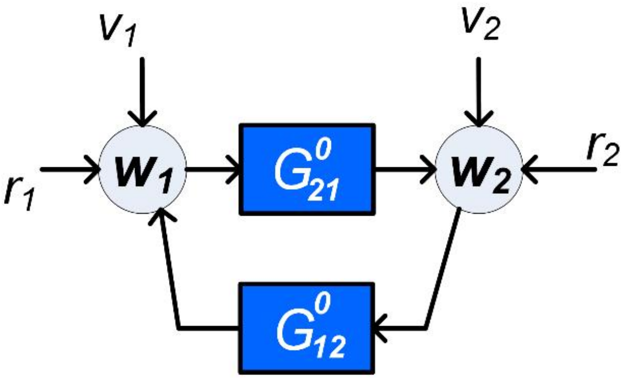
Handling confounding variables = handling correlated disturbances

- J. Pearl, *Stat. Surveys, 3,* 96-146, 2009.
- A.G. Dankers, P.M.J. Van den Hof, A.G. Dankers, X. Bombois and P.S.C. Heuberger, *IEEE Trans. Automatic Control, 61*, 937-952, 2016.

The joint-direct method

Straightforward step in case of correlated disturbances:
model them

Predict both measured node signals,
and estimate both modules:



$$\hat{w}(t|t-1) := \mathbb{E}\{w(t)|w^{t-1}, r^t\}$$

$$\varepsilon(t,\theta) = w(t) - \hat{w}(t|t-1;\theta)$$

$$\hat{\theta}_N = \arg\min_\theta \frac{1}{N}\sum_{t=0}^{N-1} \varepsilon^T(t,\theta)Q\varepsilon(t,\theta)$$

- Measured excitation signals r_1, r_2 are included as predictor inputs
- Generalization of classical direct and joint-io method

- L. Ljung, 1999
- H.H.M. Weerts, P.M.J. Van den Hof and A.G. Dankers, *Proc. 55th IEEE CDC, 2016.*

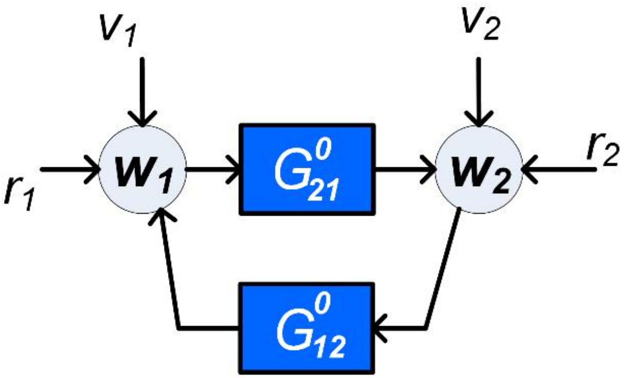
The joint-direct method

Quote 4

The joint-direct method extends the consistency properties of the direct method to

- the situation of correlated disturbances
- estimates of both modules

provided that r_1, r_2 are sufficiently exciting.



What if r_1 and/or r_2 are missing?

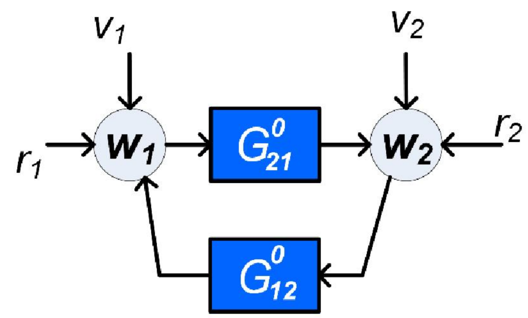
The joint-direct method

Identifiability analysis in case r_1 and/or r_2 are missing:

$$w(t) = G(q,\theta)w(t) + H(q,\theta)e(t) + R(q)r(t)$$

$$G(q,\theta) = \begin{bmatrix} 0 & G_{12}(\theta) \\ G_{21}(\theta) & 0 \end{bmatrix} \qquad H(q,\theta) = \begin{bmatrix} H_{11}(\theta) & H_{12}(\theta) \\ H_{21}(\theta) & H_{22}(\theta) \end{bmatrix}$$

$$R(q) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ dependent on presence of r_1, r_2



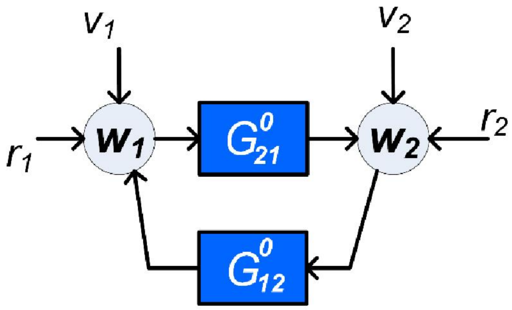
Denote:

$$\begin{bmatrix} G & H \end{bmatrix} = \begin{bmatrix} 0 & G_{12}(\theta) & H_{11}(\theta) & H_{12}(\theta) \\ G_{21}(\theta) & 0 & H_{21}(\theta) & H_{22}(\theta) \end{bmatrix}$$

The joint-direct method

Composed matrix:

$$[G \quad H] = \begin{bmatrix} 0 & G_{12}(\theta) & H_{11}(\theta) & H_{12}(\theta) \\ G_{21}(\theta) & 0 & H_{21}(\theta) & H_{22}(\theta) \end{bmatrix}$$



Theorem (Weerts et al., 2018):

Let the model set satisfy the additional properties that

- All entries are parametrized independently, and sufficiently flexible
- There are ***K*** external excitations and ***p*** white noise processes *e*

Then the elements $[G \quad H]_{2\star}$ are **network identifiable** in the model set, if

- $[G \quad H]_{2\star}$ has maximum K+p parametrized entries, and
- The transfer function from present excitation signals to w_1 has full row rank

For *p*=2, 1 external excitation signal is enough to identify G^0_{21}

H.H.M. Weerts, P.M.J. Van den Hof and A.G. Dankers, *Automatica,* 2018, to appear.

The joint-direct method

$$[G \quad H] = \begin{bmatrix} 0 & G_{12}(\theta) & H_{11}(\theta) & H_{12}(\theta) \\ G_{21}(\theta) & 0 & H_{21}(\theta) & H_{22}(\theta) \end{bmatrix}$$

Quote 5,6

The joint-direct method can consistently identify G_{21}^0 in the presence of correlated disturbances,
if

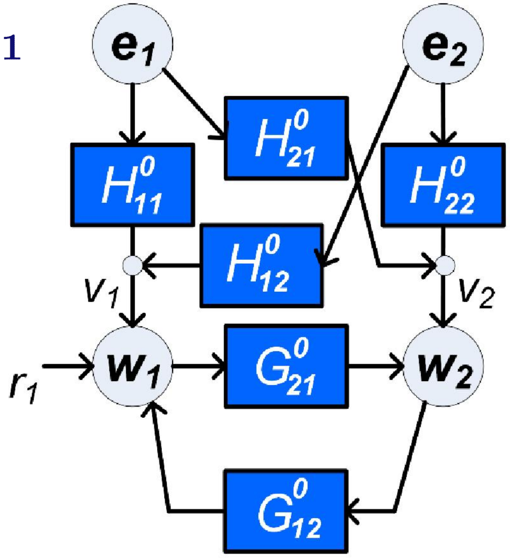
- There is at least one excitation signal r_1, r_2

or

- There is no excitation signal, and

$$H_{21}(\theta) \equiv 0$$

i.e. v_1 does not causally affect v_2
in agreement with [1].



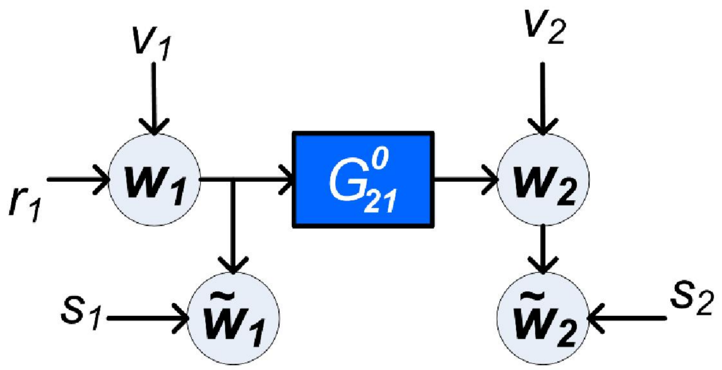
[1] B.D.O. Anderson and M.R. Gevers, *Automatica,* 1982.

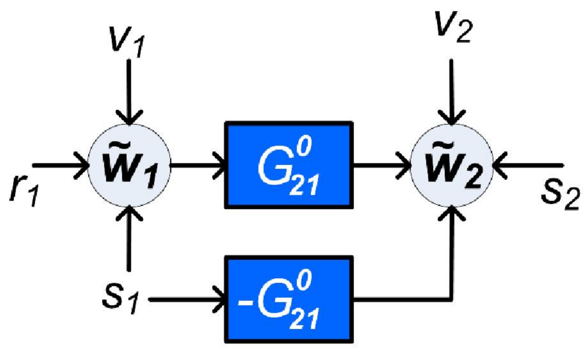
Dealing with sensor noise

Sensor noise on measured variables is typically more difficult to handle

$$\tilde{w}(t) = w(t) + s(t)$$

Open-loop situation:

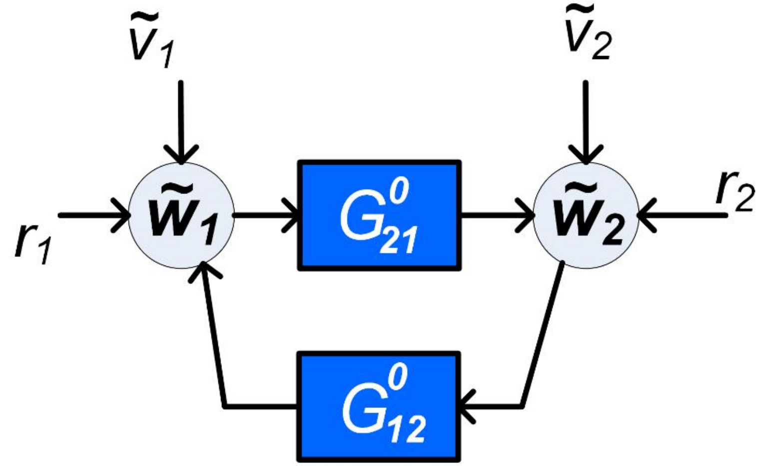




Sensor noise creates correlated disturbances / confounding variables

Dealing with sensor noise

Closed-loop situation: $\tilde{w} = G^0 \tilde{w} + \underbrace{v + (I - G^0)s}_{\tilde{v}} + r$



- Disturbances $\tilde{v}_1, \tilde{v}_2$ will be correlated
- Noise dynamics will include plant dynamics
- Theory for handling estimates remains the same

Conclusions

- **Two-node network** as basic building block in dynamic networks
- For correlated disturbances:
  both modules need to be estimated
  → **joint-direct method**
- Classical “direct method” is a special case
- Confounding variables and sensor noise can be handled as correlated disturbances