From closed-loop identification to dynamic networks: generalization of the direct method

Paul Van den Hof, Arne Dankers and Harm Weerts

56th IEEE Conference on Decision and Control, Melbourne, 12-15 December 2017
Introduction – dynamic networks

Decentralized process control

Power grid

Metabolic network

Distributed control (robotic networks)

Stock market

Hillen (2012)

Simonetto (2012)

Materassi et al. (2010)

Pierre et al. (2012)
Introduction – dynamic networks

Target: Identify one or more modules on the basis of measured data
Introduction – dynamic networks

So far:

Techniques typically based on (adapted) versions of closed-loop identification methods

- **Direct method** (based on measured node signals)  
  ML properties

- **2-stage/projection/IV method** (including measurements of $r_i^x$)  
  Consistency

- .......

Introduction – dynamic networks

Here:

• Focus on most elementary (2-node) network
• Analyse primal properties of identification methods: targeting for **minimum variance** estimates

To be addressed:

• Direct method
• A generalization: Joint-direct method
• Dealing with sensor noise
The direct method for a network setup

- Typical setup with disturbance on “output” $w_2$
- Prediction error to be minimized:
  \[ \varepsilon_2(t, \theta) = w_2(t) - \hat{w}_2(t|t-1; \theta) \]
  \[ \hat{\theta}_N = \arg \min_{\theta} \frac{1}{N} \sum_{t=0}^{N-1} \varepsilon_2(t, \theta)^2 \]
- Under fairly general conditions, the estimate of $G_{21}^0$ is **consistent** and reaches **minimum variance** (CRLB).
The direct method for a network setup

• Typical setup with disturbance on “output” $w_2$

• Prediction error to be minimized:

$$\varepsilon_2(t, \theta) = w_2(t) - \hat{w}_2(t|t-1; \theta)$$

$$\hat{\theta}_N = \arg\min_\theta \frac{1}{N} \sum_{t=0}^{N-1} \varepsilon_2(t, \theta)^2$$

• Under fairly general conditions, the estimate of $G_{21}^0$ is consistent and reaches minimum variance (CRLB).

• An additional disturbance term $v_1$ is no problem as long as it is uncorrelated with $v_2$. 
The direct method for a network setup

**Quote 1**
Consistency of the direct method is **lost** when the disturbance terms are correlated

**Quote 2**
Adding an external signal $r_2$ does not help

**Quote 3**
Consistency of the direct method is **lost** when the node signals are measured with sensor noise (EIV problem)
The direct method for a network setup

Known in network dynamics / statistics:
Confounding variables:
non-measured variables that affect both input and output

Handling confounding variables = handling correlated disturbances

The joint-direct method

Straightforward step in case of correlated disturbances: model them

Predict both measured node signals, and estimate both modules:

\[ \hat{w}(t|t-1) := \mathbb{E}\{w(t)|w^{t-1}, r^t\} \]

\[ \varepsilon(t, \theta) = w(t) - \hat{w}(t|t-1; \theta) \]

\[ \hat{\theta}_N = \arg \min_{\theta} \frac{1}{N} \sum_{t=0}^{N-1} \varepsilon^T(t, \theta)Q \varepsilon(t, \theta) \]

- Measured excitation signals \( r_1, r_2 \) are included as predictor inputs
- Generalization of classical direct and joint-io method

- L. Ljung, 1999
The joint-direct method

Quote 4
The joint-direct method extends the consistency properties of the direct method to
• the situation of correlated disturbances
• estimates of both modules

provided that \( r_1, r_2 \) are sufficiently exciting.

What if \( r_1 \) and/or \( r_2 \) are missing?
The joint-direct method

**Identifiability analysis** in case $r_1$ and/or $r_2$ are missing:

$$w(t) = G(q, \theta)w(t) + H(q, \theta)e(t) + R(q)r(t)$$

$$G(q, \theta) = \begin{bmatrix} 0 & G_{12}(\theta) \\ G_{21}(\theta) & 0 \end{bmatrix} \quad H(q, \theta) = \begin{bmatrix} H_{11}(\theta) & H_{12}(\theta) \\ H_{21}(\theta) & H_{22}(\theta) \end{bmatrix}$$

$$R(q) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{dependent on presence of } r_1, r_2$$

Denote:

$$\begin{bmatrix} G & H \end{bmatrix} = \begin{bmatrix} 0 & G_{12}(\theta) & H_{11}(\theta) & H_{12}(\theta) \\ G_{21}(\theta) & 0 & H_{21}(\theta) & H_{22}(\theta) \end{bmatrix}$$
The joint-direct method

Composed matrix:

\[
[G \ H] = \begin{bmatrix}
0 & G_{12}(\theta) & H_{11}(\theta) & H_{12}(\theta) \\
G_{21}(\theta) & 0 & H_{21}(\theta) & H_{22}(\theta)
\end{bmatrix}
\]

Theorem (Weerts et al., 2018):
Let the model set satisfy the additional properties that
- All entries are parametrized independently, and sufficiently flexible
- There are \(K\) external excitations and \(p\) white noise processes

Then the elements \([G \ H]_2\) are network identifiable in the model set, if
- \([G \ H]_2\) has maximum \(K+p\) parametrized entries, and
- The transfer function from present excitation signals to \(w_1\) has full row rank

For \(p=2\), 1 external excitation signal is enough to identify \(G^{0}_{21}\)
The joint-direct method

\[
\begin{bmatrix}
G & H
\end{bmatrix} =
\begin{bmatrix}
0 & G_{12}(\theta) & H_{11}(\theta) & H_{12}(\theta) \\
G_{21}(\theta) & 0 & H_{21}(\theta) & H_{22}(\theta)
\end{bmatrix}
\]

**Quote 5.6**
The joint-direct method can consistently identify $G_{21}^0$ in the presence of correlated disturbances, if

- There is at least one excitation signal $r_1, r_2$
- or
- There is no excitation signal, and

$H_{21}(\theta) \equiv 0$

i.e. $v_1$ does not causally affect $v_2$ in agreement with [1].

Dealing with sensor noise

Sensor noise on measured variables is typically more difficult to handle

\[ \tilde{w}(t) = w(t) + s(t) \]

Open-loop situation:

Sensor noise creates correlated disturbances / confounding variables
Dealing with sensor noise

Closed-loop situation: \[ \tilde{w} = G^0 \tilde{w} + v + (I - G^0)s + r \]

- Disturbances \(\tilde{v}_1, \tilde{v}_2\) will be correlated
- Noise dynamics will include plant dynamics
- Theory for handling estimates remains the same
Conclusions

• **Two-node network** as basic building block in dynamic networks

• For correlated disturbances:
  both modules need to be estimated
  → joint-direct method

• Classical “direct method” is a special case

• Confounding variables and sensor noise can be handled as correlated disturbances